QCD
problem set 5

1. Check that parton distribution parametrization given in the paper Glück, Reya (linked to the web page) in Eq.(A.1) are properly normalized (quark number, momentum). In Eq. (A.2) the authors include Q^2 dependence. Check normalization for a few values of Q^2 in the range 2 ÷250 GeV^2. Plot these functions for two values of Q^2, one low and one high (all calculations in Mathematica).

2. For the Altarelli-Parisi probabilities defined in lecture notes calculate Mellin moments:
$$\int_0^1 dz\, z^{n-1} P_{ab}(z) = \gamma_{ab}^{(n)}.$$

3. Use Mathematica to calculate traces:
$$\mathrm{Tr}\left[\not{P}\gamma_\lambda\gamma_5\not{P}\gamma_\nu\not{P}\gamma_\mu\right],\ \mathrm{Tr}\left[\not{P}\gamma_5\gamma_\nu\left(\not{P}-\not{k}\right)\gamma_\mu\right]$$

4. Hopf integral.

   Calculate Hopf integral
$$I = \int_{-\infty}^{+\infty} dx\, e^{iax^2},\ a > 0$$
   as a contour integral over the complex plane. Choose the contour in such a way that the integral $\int dt\, e^{-bt^2}$ with positive b and real t appears. When the contribution of the large circle can be neglected? Is the phase of the result unique?

5. In quantum mechanics plane waves are defined as
$$u_p(x) = \langle x|\, p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{i}{\hbar}px}. \tag{1}$$
   Note that the sign is dictated by thenomentum eigenvalue equation
$$-i\hbar\frac{d}{dx}u_p(x) = p\, u_p(x) \tag{2}$$
   and the normalization factor is chosen such that
$$\int dx\, u^*_{p'}(x)u_p(x) = \delta(p-p'). \tag{3}$$
   Calculate the following matrix element
$$\langle x|\exp(\frac{i}{\hbar}\frac{\hat{p}^2}{2m}\epsilon)\exp(V(\hat{x})\epsilon)\,|y\rangle \tag{4}$$
   where ϵ is a short time laps. HINT: Insert identity operator in momentum space expressed as
$$1 = \int \frac{dp}{2\pi}\,|p\rangle\,\langle p|\,.$$