QCD
problem set 4

1. Ward identities.

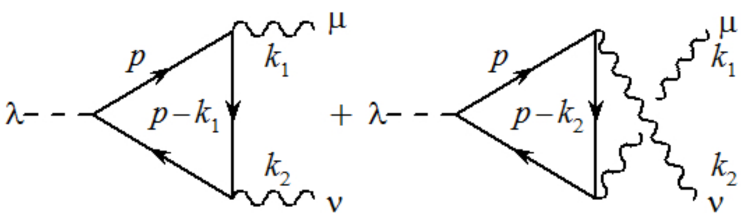
   (a) The vector current in QED (and similarly in QCD) is defined in momentum space as

   $$j^\mu(q) = \bar{u}(p')\gamma^\mu u(p)$$

   where $q = p' - p$ and $u(p)$ is a solution of the Dirac equation $(\not{p} - m)u(p) = 0$. Prove the $q_\mu j^\mu(q) = 0$. What is the result for an axial current

   $$j_5^\mu(q) = \bar{u}(p')\gamma^\mu\gamma_5 u(p).$$

   When the axial current is conserved? Remember that γ_5 anticommutes with all γ^μ's.



Rysunek 1: Loop diagrams contributing to the decay of axial-vector (dashed line) to two photons.

   (b) Consider the following loop contribution to the decay of axial-vector current to two photons (Fig. 1):

   $$\begin{aligned} T^5_{\mu\nu\lambda} &= -i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{(\not{p}-\not{k}_1)-m}\gamma_\mu\right] \\ &\quad -i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\mu\frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\right] \qquad (1)\end{aligned}$$

   where

   $$q = k_1 + k_2. \qquad (2)$$

   Note that the second line (1) and the first line are related by a replacement $\mu \longleftrightarrow \nu$ and $k_1 \longleftrightarrow k_2$. We expect that vector currents are conserved. Check this hypothesis by explicit calculation:

   $$k_1^\mu T^5_{\mu\nu\lambda}, \quad q^\lambda T^5_{\mu\nu\lambda}.$$

   To this end use the identity:

   $$\not{q} = [\not{p} - m] - [(\not{p} - \not{q}) - m]$$

   or a similar one for k_1. Is axial current conserved?

2. In this problem we will discuss parton properties assuming very simple models for quark and gluon distributions. My advice is to use *Mathematica* for these calculations.

   (a) Using properties of parton distributions given at lecture 2 in section *Quarks as Partons* calculate normalization constant A assuming

   $$u_v(x) = \frac{2A}{\sqrt{x}}(1-x)^3, \; d_v(x) = \frac{A}{\sqrt{x}}(1-x)^3$$

   where index v stands for *valence*. Recall that total u or d quark distribution is given as a sum of valence quarks and sea quarks u_s or d_s respectively. We assume that sea quark distributions are equal to antiquark distributions

   $$u_s(x) = \bar{u}(x), \; d_s(x) = \bar{d}(x).$$

   For this problem we assume that there are no strange quarks in the nucleon and we assume isospin symmetry, which says that u and d distributions in neutron, are equal to d and u distributions in proton. Calculate A. Check the value of charge of the proton and neutron. Calculate total momentum carried by the valence quarks. At this point we do not need any information on the sea quarks.

   (b) Gottfried sum rule. Calculate the difference of the structure functions of the proton and neutron:

   $$S_G = \int_0^1 \frac{dx}{x}\left(F_2^p(x) - F_2^n(x)\right).$$

   Experimental value reads $S_G \simeq 0.24$. As you will see S_G will depend on the integral over the distributions of the sea quarks. Assume the sea quark distribution of the following form

   $$\bar{u}(x) = \frac{B}{x}(1-x)^8, \; \bar{d}(x) = \frac{B}{x}(1-x)^\beta.$$

   Note that constant B must be the same in both cases to assure that S_G is finite. From the experimental value of S_G calculate B for several choices of power β taking a few values around 8. Note that the antiquark and sea distributions must be positive. For these choices calculate total momentum carried by quarks. Is it possible to get the value of 100%?

   (c) Choose gluon distribution of the form

   $$g(x) = Cx(1-x)^4.$$

   For given β from the previous problem calculate C from the condition that the total momentum of the proton is 1.