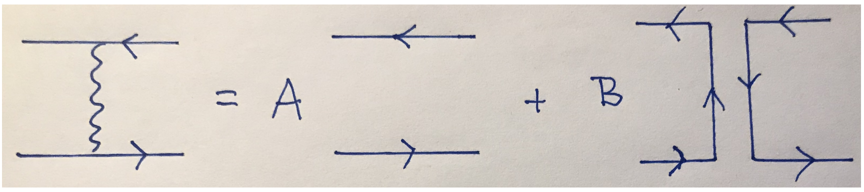
QCD
problem set 3

1. Color factors.

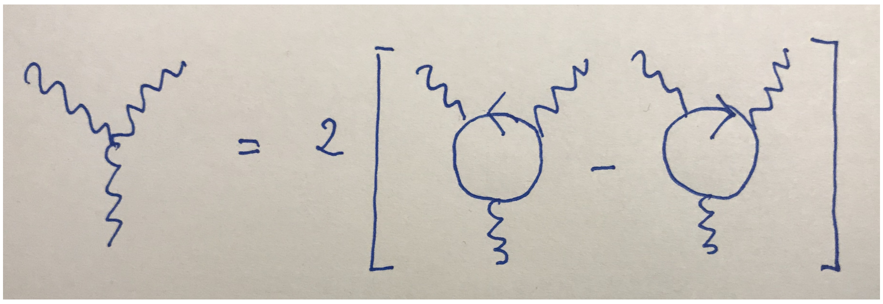
   (a) With the help of the graphical methods for the SU(N) group described at the lecture, find coefficients A and B for the following decomposition of the one gluon exchange:

   and interpret the result for $N \to \infty$.

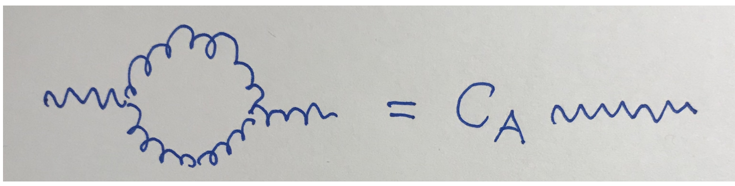
   HINT: contract fermion lines in two possible ways (remember that contractions must preserve the direction of the arrow).

   (b) Prove the following identity:

   HINT: start from the commutation relation for the generators in the fundamental representation and contract it with the generator keeping arrows direction. Then use normalization condition for the generators.

   (c) Using relations from previous problems, first (b) and then (a), calculate C_A – Casimir operator for the adjoint representation:

2. Integrals for the propagator renormalization. Whenever in the calculations you encounter expression

$$\Gamma(1 \mp \varepsilon) = \exp\left(\pm\gamma\varepsilon + \ldots\right)$$

do not expand it, but keep the exponent.

(a) Calculate d-dimensional angular volume

$$\Omega_d = 2\prod_{i=1}^{d-1}\left(\int_0^{\pi} \sin^{i-1}\theta_i d\theta_i\right). \tag{1}$$

Check explicitly the result for $d = 2, 3$ and 4. Write Ω_d explicitly for $d = 4-2\varepsilon$.

(b) Calculate two Euclidean integrals in $d = 4 - 2\varepsilon$ dimensions

$$\{I, I^\mu\} = i\int_0^1 dx \int \frac{d^d k}{(2\pi)^d}\frac{1}{\left(k^2 + M^2\right)^2}\{1, k^\mu - xp^\mu\}$$

where $M^2 = -x(1-x)p^2$. To this end follow the following steps:

- Split $d^d k = k^{d-1}dk d\Omega_d$ and perform angular integration. Note that this integration nullifies k^μ.
- Perform change of variables $k \to r = t/M$ and then $r \to t = r^2$. Remember that M is a function of x.
- Integrations over dx and dt can be now easily expressed in terms of the Euler Beta functions.
- Express Beta functions in terms of the Gamma functions.
- Reduce the resulting expressions to terms involving

$$\left(\frac{4\pi e^{-\gamma}}{-p^2}\right)^{\varepsilon}, \frac{1}{\varepsilon} \text{ and terms without } \varepsilon.$$