List of problems for oral exam on Standard Model: QCD
Fall semester 2020/2021

1. Deep Inelastic Scattering: kinematics, gauge invariant decomposition of the scattering amplitude, Bjorken scaling, parton model and Feynman interpretation of Bjorken scaling, structure functions and Callan-Gross relation, quarks and gluons as partons, evolution equations.
2. Quantum Chromodynamics: formulation of QCD as a non-Abelian gauge theory, transformation of gauge fields – comparison with QED, QCD vertices, transformation properties of covariant derivative and of the field tensor.
3. SU(3) group: commutation relations, Gell-Mann matrices, invariant subgroups, labelling of states, fundamental and adjoint representation, conjugated representations, basis states in the adjoint representation, color factors – graphical methods.
4. Renormalization of QCD: example of quark self-energy, divergences and regularization methods, subtraction of divergences and renormalization schemes, multiplicative renormalization, dimensional transmutation, beta function – comparison with QED, concept of running coupling constant.
5. Functional integrals: Feynman path integral in quantum mechanics, quantal determinant in QM, transition amplitudes and generating functional in QM and in field theory, scalar field propagator from functional integral, functional integral for fermions – Grassmann variables, functional intergal for photons.
6. Gauge fixing: gauge fixing in QED: Landau gauge and general covariant gauge, gauge fixing in QCD: Faddeev–Popov determinant, ghost fields, general formula for an expectation of gauge invariant operators and gluon propagator in covariant gauge and in axial gauge.
7. Chiral symmetry and an anomaly: formulation of chiral symmetry in functional integral approach, non-invariance of fermionic integration measure, chiral anomaly and Fujikawa regularization trick and computation of chiral anomaly, perturbative calculation of axial anomaly.
8. Topological properties of QCD: Euclidean QCD, Attiyah-Singer index theorem, anomaly as full divergence, QCD theta term, topology of gauge fields, homotopy classes and winding number, instantons in QM and in QCD, the problem of zero modes.
9. Consequences of chiral symmetry in QCD: conserved currents for massless quarks, parity of conserved currents, commutation rules for charges, currents and densities, perturbative inclusion of quark masses, QCD spectrum and need for spontaneous chiral symmetry breaking, action of charges on the vacuum state, quark condensate, commutation rule for axial charges and pseudoscalar densities and Goldstone bosons.
10. Effective QCD – theory of Goldstone bosons: group structure of chiral symmetry breaking – symmetry group and vacuum invariance group, concept of cosets, left cosets as Goldstone bosons, Goldstone bosons U field and its transformation properties, invariant effective Lagrangian and the mass term, Gell-Mann–Okubo SU(3) mass relation, PCAC and pion decay, Gell-Mann, Oakes and Renner relation, Goldberger–Treiman relation.