QCD

Problem set #11

Monday, January 20, 10:00

1. Real scalar field lagrangian density reads as follows:

$$\mathcal{L}(x) = \frac{1}{2}\partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m^2 \phi^2(x).$$

   Calculate Hamiltonian. Canonical equal-time quantization rules for real scalar field operators read:

$$\left[\hat{\phi}(t,\vec{x}), \hat{\pi}(t,\vec{x}\,')\right] = i\delta^{(3)}(\vec{x}-\vec{x}\,')$$

   and all other possible commutators are zero. Using decomposition

$$\hat{\phi}(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^2\sqrt{2\omega_k}} \left[e^{-i\,kx}\hat{a}(\vec{k}) + e^{+i\,kx}\hat{a}^\dagger(\vec{k})\right]$$

   show that the canonical quatization rules are satisfied if

$$\left[\hat{a}(\vec{k}), \hat{a}^\dagger(\vec{k}\,')\right] = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}\,')$$

   and the remaining two commutators vanish.

2. Nonlinear realization of the QCD $\mathrm{SU_R}(N)\times\mathrm{SU_L}(N)$ symmetry is implemented by the following transformation:

$$U(x) \to L U(x) R^\dagger$$

   where

$$L, R = e^{-i\theta_a^{L,R} T^a}$$

   where $U(x)$ is expressed in terms of the physical fields

$$U = \exp\left(\frac{2i}{F} T^a \phi^a(x)\right).$$

   and $T^a$are generators of the $\mathrm{SU}(N)$ group, and F is a dimensionfull constant. The simplest possible lagrangian of this effective theory takes the following form:

$$\mathcal{L}_{\text{eff}} = \frac{F^2}{4}\,\mathrm{Tr}\left(\partial_\mu U\,\partial^\mu U^\dagger\right). \tag{1}$$

   For the SU(2) and SU(3) case calculate the chiral effective lagrangian (1) up to 4-field interaction. What is the dimension of F?

3. Use Nother construction to calculate currents associated with L and R transformations. Expand currents up to the lowest possible order in the number of fields $\phi^a(x)$.

4. Add to (1) the symmetry breaking term

$$\mathcal{L}_M = -\frac{F^2 B}{2} \operatorname{Tr}\left(M(U + U^\dagger)\right)$$

where M =diag(m, m, m_s). Calculate $\mathcal{L}_M$ up to the lowest number of fields ϕ.Using

$$\lambda_a \phi^a(x) = \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta^0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta^0 \end{bmatrix}$$

calculate

$$\mathcal{L}_{\text{eff}} + \mathcal{L}_M$$

in terms of the physical fields. Read out the masses of these fields. Since these three masses will be expressed in terms of two "free" parameters Bm and Bm_s, there exits one relation between these masses. Derive this relation and check its numerical accuracy.