QCD

Problem set #9

Monday, December 16, 10:00, A-1-13

1. Choose $A_0 = 0$ gauge and calculate the action for the Yang-Mills SU(2) field in terms of electric and magnetic fields $\vec{E}$ and $\vec{B}$ where

$$E_i^a = \dot{A}_i^a,\ B_i^a = \frac{1}{2}\varepsilon_{ijk}\left(\partial_j A_k^a - \partial_k A_j^a + \varepsilon^{abc} A_j^b A_k^c\right). \tag{1}$$

2. Suppose one would like to construct the quantum mechanical hamiltonian where instead of ordinary coordinates one would use A_i^a with the corresponding momenta operators given as

$$-i\frac{\delta}{\delta A_i^a}.$$

What would be the corresponding hamiltonian and the corresponding potential?

3. Calculate coefficients A and B for the the following Fiertz decomposition of the SU(N) generators

$$T_{ij}^a T_{kl}^a = A\,\delta_{ij}\delta_{kl} + B\,\delta_{il}\delta_{kj}. \tag{2}$$

These operators are normalized as

$$\mathrm{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab} \tag{3}$$

Propose graphical illustration of these identities.

4. Calculate so called Casimir operators

$$\sum_{a,j} T_{ij}^a T_{jk}^a = C\delta_{ik} \tag{4}$$

for the fundamental and adjoint representations of the SU(N) group.