QCD

Problem set #8

Monday, December 9, 10:00, A-1-13

1. Winding number of the gauge transformation U is defined as

$$N_{\mathrm{w}} = \frac{1}{24\pi^2}\varepsilon^{ijk}\int d^3r\ \mathrm{Tr}\left[\left(U^\dagger\partial_i U\right)\left(U^\dagger\partial_j U\right)\left(U^\dagger\partial_k U\right)\right] \tag{1}$$

   Calculate (1) for $U = \exp\left(i\,\vec{n}\cdot\vec{\tau}\,\omega(r)\right)$ where $\vec{n} = \vec{r}/r$. What are the boundary conditions for $\omega(r)$ that ensure that N_{w} is an integer.

2. Choose $A_0 = 0$ gauge and calculate the action for the Yang-Mills SU(2) field in terms of electric and magnetic fields $\vec{E}$ and $\vec{B}$ where

$$E_i^a = \dot{A}_i^a,\ B_i^a = \frac{1}{2}\varepsilon_{ijk}\left(\partial_j A_k^a - \partial_k A_j^a + \varepsilon^{abc}A_j^b A_k^c\right). \tag{2}$$

3. Suppose one would like to construct the quantum mechanical hamiltonian where instead of ordinary coordinates one would use A_i^a with the corresponding momenta operators given as

$$-i\frac{\delta}{\delta A_i^a}.$$

   What would be the corresponding hamiltonian and the corresponding potential?

4. Calculate coefficients A and B for the the following Fiertz decomposition of the SU(N) generators

$$T_{ij}^a T_{kl}^a = A\,\delta_{ij}\delta_{kl} + B\,\delta_{il}\delta_{kj}. \tag{3}$$