QCD

Problem set #7

Monday, December 2, 10:00, A-1-13

1. Prove that
$$\partial_\mu K^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$
where
$$K^\mu = \varepsilon^{\mu\nu\rho\sigma}\left(A^a_\nu F^a_{\rho\sigma} - \frac{g}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma \right).$$

2. Consider a complex scalar and/or fermion field theory coupled to the nonabelian gauge fields:
$$\begin{aligned}
\mathcal{L}_\phi &= (D_\mu \phi(z))^\dagger (D_\mu \phi(x)) - m^2 \phi^\dagger(x)\phi(x) - V(\phi^\dagger(x)\phi(x)), \\
\mathcal{L}_\psi &= \bar{\psi}(x) \left(i \not{D}_x - m\right) \psi(x),
\end{aligned}$$
with covariant derivative defined as
$$D_\mu = \partial_\mu - ig A^a_\mu(x)\, T^a$$
where T^a are generators of SU(N) group. Lagrangian for the gauge fields reads
$$\mathcal{L}_A = -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\,\mu\nu}(x).$$
Derive by means of the variational approach equations of motion for (analogs of Maxwell equations) the scalar and fermion theory. Prove the identity
$$[D_\mu, F^{\nu\rho}] + [D_\nu, F^{\rho\mu}] + [D_\rho, F^{\mu\nu}] = 0.$$