QCD

Problem set #4

Tuesday, November 5, 10:15, B-2-01

1. Finish last problem from set 3: Decompose W fields as

$$\boldsymbol{W}_\mu(x) = \begin{bmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{bmatrix} = \begin{bmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{bmatrix},$$

   and calculate the kinetic term

$$-\frac{1}{8}\,\mathrm{Tr}\left(\boldsymbol{W}^{\mu\nu}\boldsymbol{W}_{\mu\nu}\right)$$

   in term of these new fields. Here

$$\boldsymbol{W}_{\mu\nu} = D_\mu(W)\boldsymbol{W}_\nu - D_\nu(W)\boldsymbol{W}_\mu.$$

2. Variables ψ_i and ξ_i $(i = 1, N)$ are two sets of independent Grassmann variables. Prove that

$$J(M) = \int d^N\xi\, d^N\psi\, \exp\left(\psi_i M_{ij} \xi_j\right) = \det(M)$$

   where M is number-valued antisymmetric matrix.

3. Define complex Grassmann variables

$$\chi = \frac{1}{\sqrt{2}}\left(\psi + i\xi\right),\ \bar{\chi} = \frac{1}{\sqrt{2}}\left(\psi - i\xi\right).$$

   Show that

$$d\xi d\psi = i\, d\chi\, d\bar{\chi}.$$

   Calculate

$$\int d\chi\, d\bar{\chi}\, \bar{\chi}\chi.$$

4. Show that for electrodynamics

$$-\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} = -\frac{1}{2}\int d^4x\, \tilde{A}^\mu(k)(g_{\mu\nu}k^2 - k_\mu k_\nu)\tilde{A}^\nu(-k)$$

   where a $\tilde{A}$ is a Fourier transform of A.

5. Try to find an inverse of matrix

$$g_{\mu\nu}k^2 - k_\mu k_\nu$$

solving for α and β the equation

$$\left(g_{\mu\nu}k^2 - k_\mu k_\mu\right)\left(\alpha g^{\nu\rho} + \beta\frac{k^\nu k^\rho}{k^2}\right) = \delta^\rho_\mu$$

and show that there is no solution. Is it possible to invert matrix

$$g_{\mu\nu}k^2 - (1-\xi)k_\mu k_\nu$$

where $\xi \neq 0$ is a real parameter?