QCD

Problem set #3

Tuesday, October 29, 10:15, B-2-01

1. Field Φ is a two dimensional complex scalar field
$$\Phi = \begin{bmatrix} \Phi_A \\ \Phi_B \end{bmatrix} = \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix}$$
that has 4 real components $\phi_{1,2,3,4}$. Lagrangian density takes the following form
$$\mathcal{L}_\Phi = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi).$$
Express $\mathcal{L}_\Phi$ in terms of $\phi_{1,2,3,4}$. $\mathcal{L}_\Phi$ is invariant under the U(1)$\times$SU(2) global transformation
$$\Phi \to \Phi' = e^{-i\theta\tau_0} U \Phi \tag{1}$$
where
$$U = e^{-i\vec{\alpha}\cdot\vec{\tau}}.$$
Here $\vec{\tau}$ are Pauli matrices and τ_0 is a unit matrix. Calculate the change of $\mathcal{L}_\Phi$ if parameters β and $\vec{\alpha}$ are space-time dependent. Show that the gauge invariance requires to introduce the gauge fields that modify derivatives:
$$\partial^\mu \to D^\mu = \partial^\mu + i\frac{g_1}{2}B^\mu + i\frac{g_2}{2}\boldsymbol{W}^\mu = \partial^\mu + i\frac{g_1}{2}B^\mu + i\frac{g_2}{2}W_k^\mu \tau_k$$
where $g_{1,2}$ are arbitrary *coupling* constants. How fields B^μ and W_k^μ have to transform to maintain invariance of $\mathcal{L}_\Phi$?

2. After introducing 4 gauge fields B^μ and W_k^μ we have to add to $\mathcal{L}_\Phi$ kinetic terms for these fields. This requires to define field tensors. For B^μ the field tensor is the same as in electrodynamics, however for W_k^μ the field tensor has to be defined differently. Find transformation properties of
$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ \boldsymbol{W}_{\mu\nu} &= D_\mu(W)\boldsymbol{W}_\nu - D_\nu(W)\boldsymbol{W}_\mu \end{aligned}$$
where $D_\mu(W)$ is a covariant derivative for W fields only ($g_1 = 0$) and propose the invariant form of the kinetic terms.

3. Decompose W fields as
$$\begin{aligned} \boldsymbol{W}_\mu(x) &= \begin{bmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{bmatrix} \\ &= \begin{bmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{bmatrix}, \end{aligned}$$
and calculate the proposed kinetic terms in term of these new fields.