QCD

Problem set #2

Tuesday, October 22, 10:15, B-2-01

1. Show that for free real scalar field theory defined by
$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi \, \partial^\mu \varphi - m^2 \varphi^2\right)$$
the enerdy is given by:
$$T_0^0 = \frac{1}{2}\left(\dot{\varphi}^2 + (\vec{\nabla}\varphi)^2 + m^2\varphi^2\right).$$
Furthermore, for the field decomposition
$$\varphi(t,\vec{r}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left(\frac{a_k}{\sqrt{2\omega_k}} e^{i(\vec{k}\cdot\vec{r}-\omega_k t)} + \frac{a_k^*}{\sqrt{2\omega_k}} e^{-i(\vec{k}\cdot\vec{r}-\omega_k t)} \right)$$
calculate $H = T_0^0$ and momentum $\vec{P} = (T_0^1, T_0^2, T_0^3)$.

2. Langrangian density for classical electrodynamics reads:
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu$$
where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Derive equations of motion by performing variation of the action.

3. Calculate T_0^0. Express Lagrangian density and energy density in terms of electric and magnetic fields $\vec{E}$ and $\vec{B}$.

4. Field Φ is a two dimensional complex scalar field
$$\Phi = \begin{bmatrix} \Phi_A \\ \Phi_B \end{bmatrix} = \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix}$$
that has 4 real components $\phi_{1,2,3,4}$. Lagrangian density takes the following form
$$\mathcal{L}_\Phi = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi).$$
Express $\mathcal{L}_\Phi$ in terms of $\phi_{1,2,3,4}$. $\mathcal{L}_\Phi$ is invariant under the U(1)×SU(2) global transformation
$$\Phi \to \Phi' = e^{-i\theta\tau_0} U \Phi$$
where
$$U = e^{-i\vec{\alpha}\cdot\vec{\tau}}.$$
Here $\vec{\tau}$ are Pauli matrices and τ_0 is a unit matrix. Calculate the change of $\mathcal{L}_\Phi$ if parameters β and $\vec{\alpha}$ are space-time dependent.