Cosmology

Assignment Set B

1. Find the continuum version of the virial theorem. Show that a stationary configuration of the fluid obeing Euler's equations

$$\nabla \cdot (\rho \mathbf{U} \otimes \mathbf{U} + p) = -\rho \nabla \Phi,$$

   satisfies

$$\int d^3x \frac{1}{2}\rho\Phi + 2\int d^3x \frac{1}{2}\rho|\mathbf{U}|^2 + 2\int d^3x \frac{3}{2}p = 0.$$

   Here ρ is the mass density, p is the pressure, Φ is the gravitational potential, and $\mathbf{U}$ denotes the bulk velocity of the fluid.
2. Obtain the above result for a collection of stars (say, a globular cluster).
3. Show that a static (spherically symmetric) configuration of selfgravitating gas (a star) satisfies $U \leq 3GM/(5R)$. Here U is the gravitational energy, M is the total mass, and R the outer radius of the star.