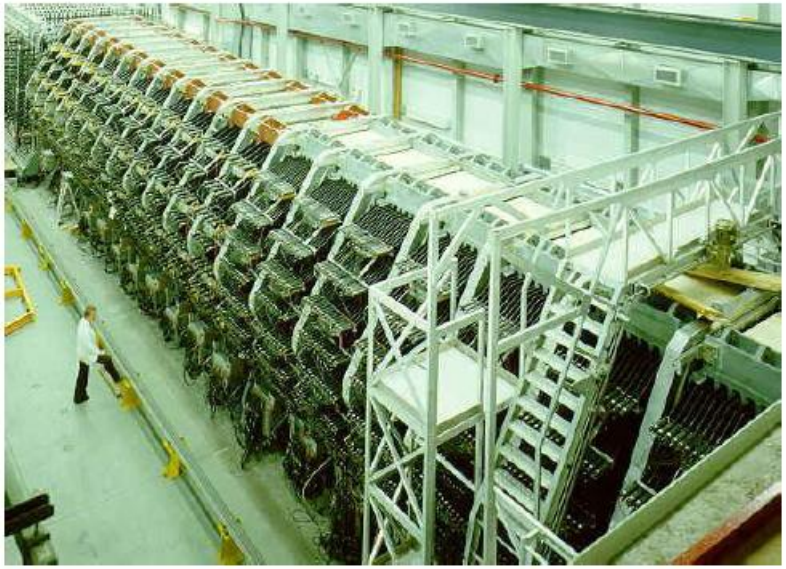
Elementary Particle Physics: theory and experiments

Leptonic Weak Interactions and Neutrino Deep Inelastic Scattering.



Follow the course/slides from M. A. Thomson lectures at Cambridge University

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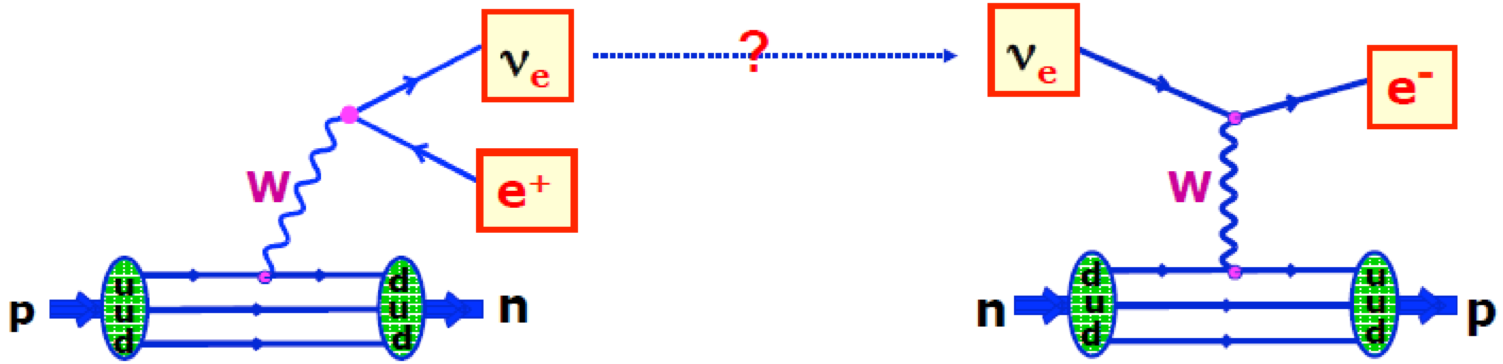
Neutrino Flavours

- Recent experiments (➡ neutrinos have mass (albeit very small)
- The textbook neutrino states, ν_e, ν_μ, ν_τ, are not the fundamental particles; these are ν_1, ν_2, ν_3
- Concepts like "electron number" conservation are now known not to hold.
- So what are ν_e, ν_μ, ν_τ ?
- Never directly observe neutrinos – can only detect them by their weak interactions. Hence by definition ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

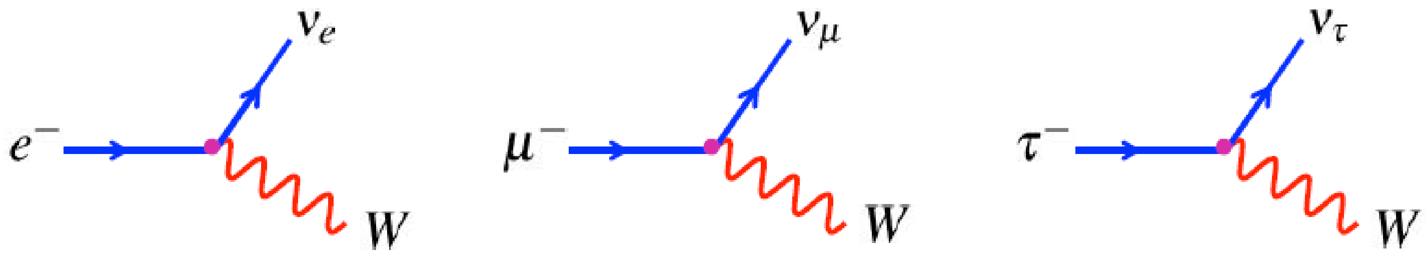
➡ ν_e, ν_μ, ν_τ = weak eigenstates



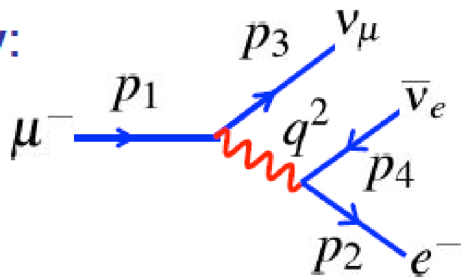
- Unless dealing with very large distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the weak interaction continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

Muon Decay and Lepton Universality

★ The leptonic **charged current** (W±) interaction vertices are:



★ Consider muon decay:



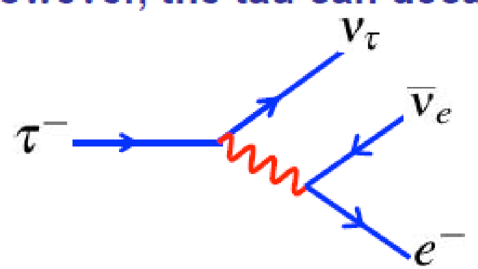
• It is straight-forward to write down the matrix element

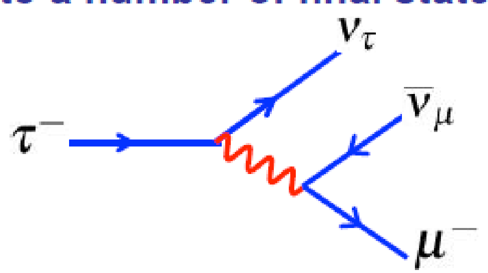
$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2} [\overline{u}(p_3)\gamma^\mu (1-\gamma^5) u(p_1)] g_{\mu\nu} [\overline{u}(p_2)\gamma^\nu (1-\gamma^5) v(p_4)]$$

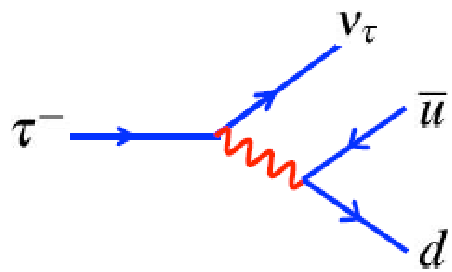
Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$
i.e. in limit of Fermi theory

• Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

- The muon to electron rate $\Gamma(\mu \to e\nu\nu) = \dfrac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \dfrac{1}{\tau_\mu}$ with $G_F = \dfrac{g_W^2}{4\sqrt{2}m_W^2}$
- Similarly for tau to electron $\Gamma(\tau \to e\nu\nu) = \dfrac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$
- However, the tau can decay to a number of final states:







$Br(\tau \to e\nu\nu) = 0.1784(5)$ $Br(\tau \to \mu\nu\nu) = 0.1736(5)$

- Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

- Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

- Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$$

• All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94)\,\mathrm{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\mathrm{s}$$

$$m_\tau = 1.77699(28)\,\mathrm{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\mathrm{s}$$

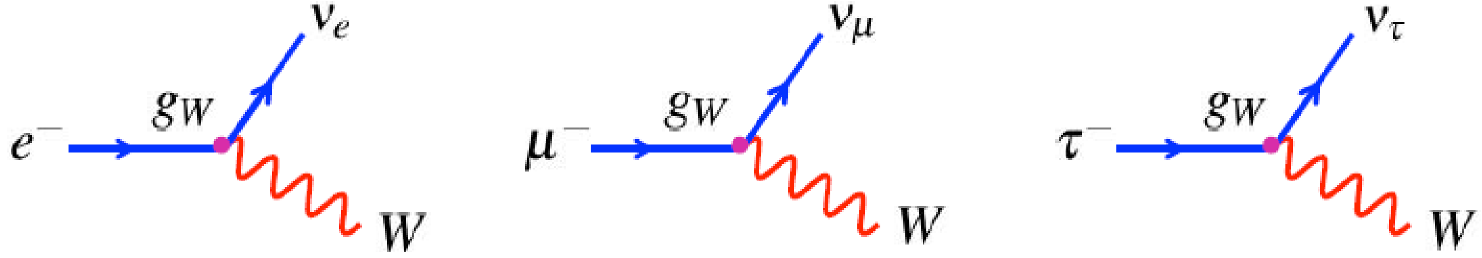
$$Br(\tau \to e\nu\nu) = 0.1784(5)$$

$$\Rightarrow \quad \frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \to e\nu\nu) = 1.0024 \pm 0.0033$$

• Similarly by comparing $Br(\tau \to e\nu\nu)$ and $Br(\tau \to \mu\nu\nu)$

$$\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004$$

★ Demonstrates the weak charged current is the same for all leptonic vertices



$\Rightarrow$ **Charged Current Lepton Universality**

Neutrino Scattering

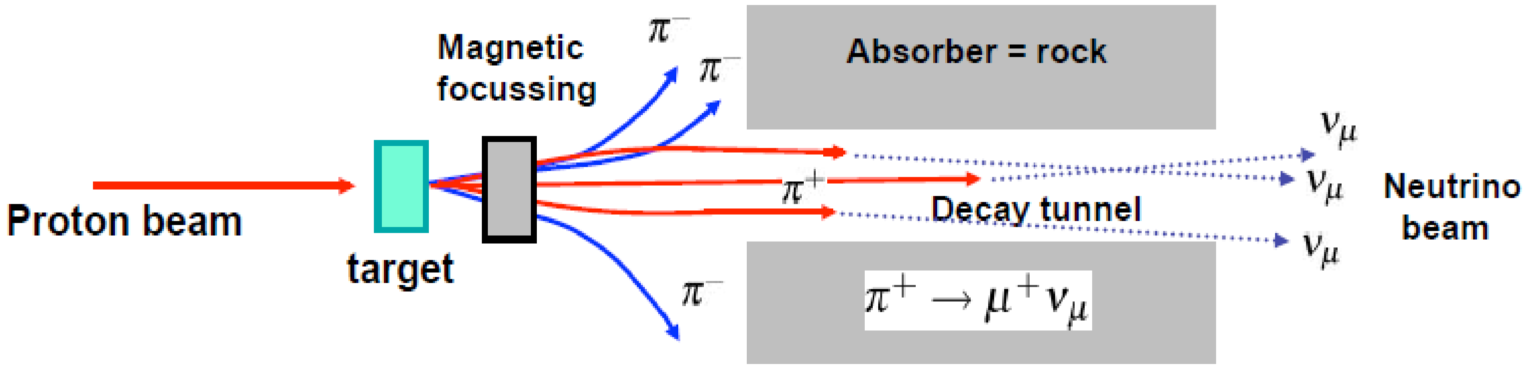
- Can also consider the weak interaction equivalent: **Neutrino Deep Inelastic Scattering** where a virtual W-boson probes the structure of nucleons

  ➡ additional information about parton structure functions

  \+ provides a good example of calculations of weak interaction cross sections.

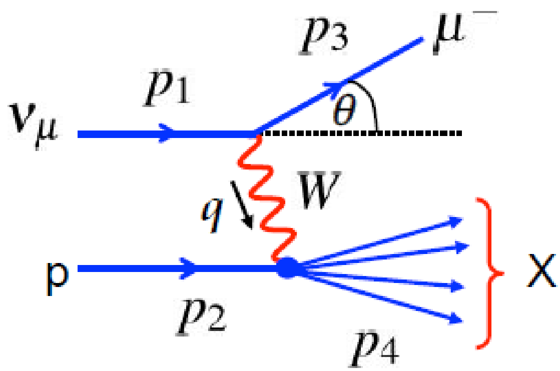
★ **Neutrino Beams:**

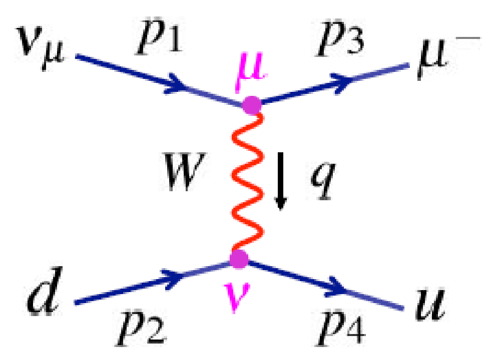
- Smash high energy protons into a fixed target ➡ hadrons
- Focus positive pions/kaons
- Allow them to decay $\pi^+ \to \mu^+ \nu_\mu$ + $K^+ \to \mu^+ \nu_\mu$ $(BR \approx 64\%)$
- Gives a beam of "collimated" ν_μ
- Focus negative pions/kaons to give beam of $\overline{\nu}_\mu$



Neutrino – Quark Scattering

★ For ν_μ-proton Deep Inelastic Scattering the underlying process is $\nu_\mu d \to \mu^- u$





★ In the limit $q^2 \ll m_W^2$ the W-boson propagator is $\approx ig_{\mu\nu}/m_W^2$

- The Feynman rules give:

$$-iM_{fi} = \left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\frac{ig_{\mu\nu}}{m_W^2}\left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

$$M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

- Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected

• In this limit the helicity states are equivalent to the chiral states and

$$\tfrac{1}{2}(1-\gamma^5)u_\uparrow(p_1) = 0 \qquad \tfrac{1}{2}(1-\gamma^5)u_\downarrow(p_1) = u_\downarrow(p_1)$$

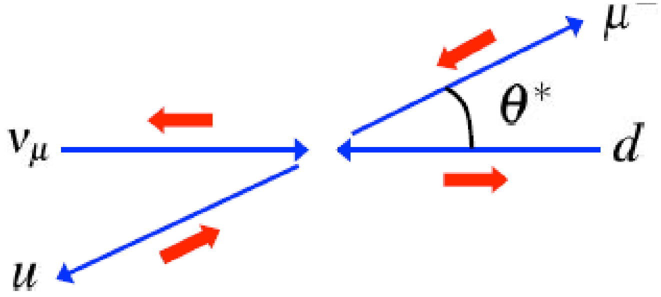
➡ $M_{fi} = 0$ for $u_\uparrow(p_1)$ and $u_\uparrow(p_2)$

• Since the weak interaction "conserves the helicity", the only helicity combination where the matrix element is non-zero is

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

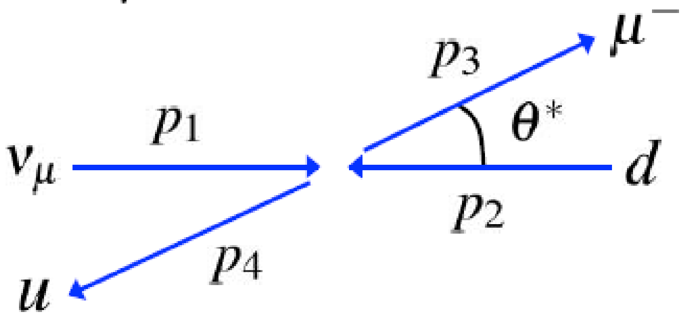
NOTE: we could have written this down straight away as in the ultra-relativistic limit only LH helicity particle states participate in the weak interaction.

★ Consider the scattering in the C.o.M frame



Evaluation of Neutrino-Quark Scattering ME

- Go through the calculation in gory detail (fortunately only one helicity combination)
- In the $\nu_\mu d$ CMS frame, neglecting particle masses:



$$p_1 = (E, 0, 0, E),$$
$$p_2 = (E, 0, 0, -E)$$
$$p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$
$$p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*)$$

- Dealing with LH helicity particle spinors.

for a massless particle travelling in direction (θ, ϕ):

$$u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \qquad c = \cos\frac{\theta}{2}; \quad s = \sin\frac{\theta}{2}$$

- Here $(\theta_1, \phi_1) = (0, 0)$; $(\theta_2, \phi_2) = (\pi, 0)$; $(\theta_3, \phi_3) = (\theta^*, 0)$; $(\theta_4, \phi_4) = (\pi - \theta^*, \pi)$

giving:

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}; \; u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \; u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}; \; u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

- To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

need to evaluate two terms of form

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}$$

- Using

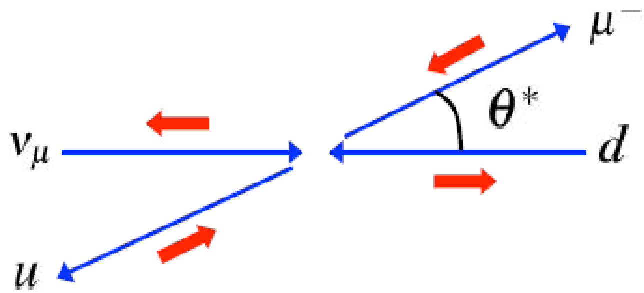
$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix};\ u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}-1\\0\\1\\0\end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix};\ u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}-c\\-s\\c\\s\end{pmatrix}$$

$$\Rightarrow \quad \begin{aligned}
\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2E(c, s, -is, c) \\
\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2) &= 2E(c, -s, -is, -c)
\end{aligned}$$

$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{2m_W^2} 4E^2(c^2 + s^2 + s^2 + c^2) = \frac{g_W^2 \hat{s}}{m_W^2} \qquad \hat{s} = (2E)^2$$

★ Note the Matrix Element is isotropic

$$M_{fi}=\frac{g_W^2}{m_W^2}\hat{s}$$



we could have anticipated this since the helicity combination (spins anti-parallel) has $S_z=0$ ➡ no preferred polar angle

★ As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination (LL→LL) and only 2 possible initial state combinations (the neutrino is always produced in a LH helicity state)

$$\langle|M_{fi}|^2\rangle=\frac{1}{2}\cdot\left|\frac{g_W^2}{m_W^2}\hat{s}\right|^2$$

The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction

★ From handout 1, in the extreme relativistic limit, the cross section for any 2→2 body scattering process is

$$\frac{d\sigma}{d\Omega^*}=\frac{1}{64\pi^2\hat{s}}\langle|M_{fi}|^2\rangle$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}}\langle|M_{fi}|^2\rangle = \frac{1}{64\pi\hat{s}}\frac{1}{2}\left(\frac{g_W^2 \hat{s}}{m_W^2}\right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2}\right)^2 \hat{s}$$

using $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$ ➡

$$\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$

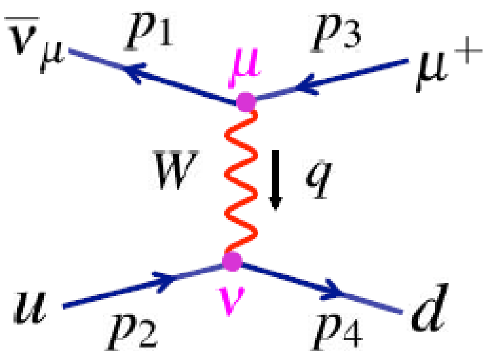
★ Integrating this isotropic distribution over $d\Omega^*$

➡

$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi} \quad (1)$$

• cross section is a Lorentz invariant quantity so this is valid in any frame

Antineutrino-Quark Scattering



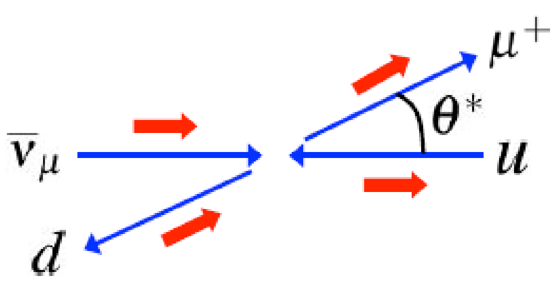
- In the ultra-relativistic limit, the charged-current interaction matrix element is:

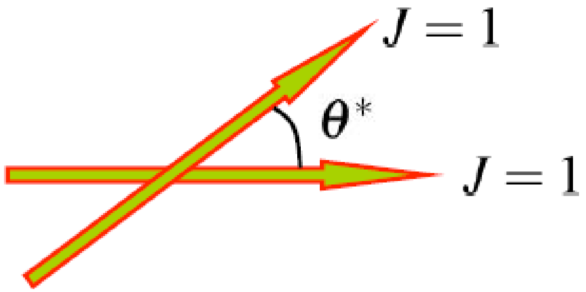
$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}(p_1)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_3)\right] \left[\overline{u}(p_4)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)u(p_2)\right]$$

★ In the extreme relativistic limit only LH Helicity particles and RH Helicity anti-particles participate in the charged current weak interaction:

$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}_\uparrow(p_1)\gamma^\mu v_\uparrow(p_3)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

★ In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum 1 state

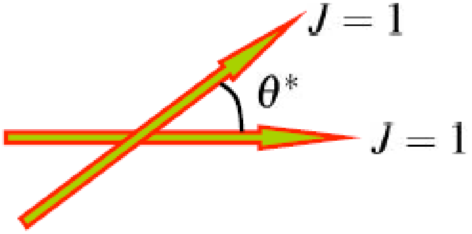




★ In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*}\frac{1}{4}(1+\cos\theta^*)^2$$

- The factor $\frac{1}{4}(1+\cos\theta^*)^2$ can be understood in terms of the overlap of the initial and final angular momentum wave-functions



★ Similarly to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

★ Integrating over solid angle:

$$d\Omega = d\phi\sin\theta d\theta \rightarrow d\phi d(\cos\theta)$$

$$\int(1+\cos\theta^*)^2 d\Omega^* = \int(1+\cos\theta^*)^2 d(\cos\theta^*)d\phi = 2\pi\int_{-1}^{+1}(1+\cos\theta^*)^2 d(\cos\theta^*) = \frac{16\pi}{3}$$

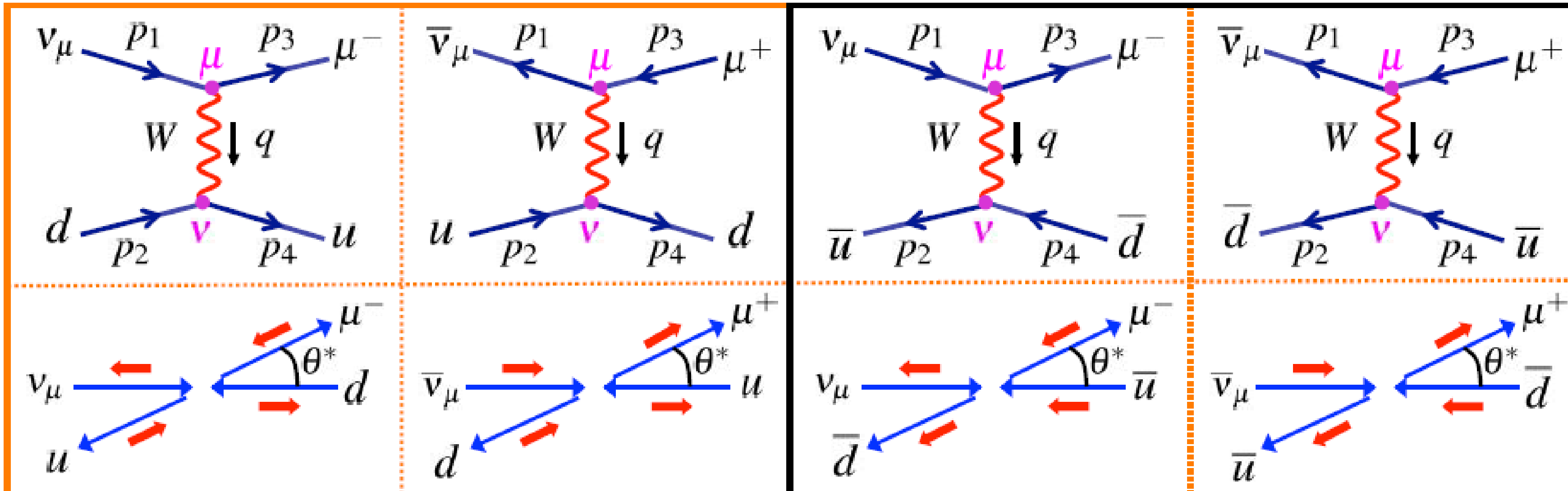
$$\Rightarrow \quad \sigma_{\overline{\nu}q} = \frac{G_F^2\hat{s}}{3\pi}$$

★ This is a factor three smaller than the neutrino quark cross-section

$$\frac{\sigma_{\overline{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}$$

(Anti)neutrino-(Anti)quark Scattering

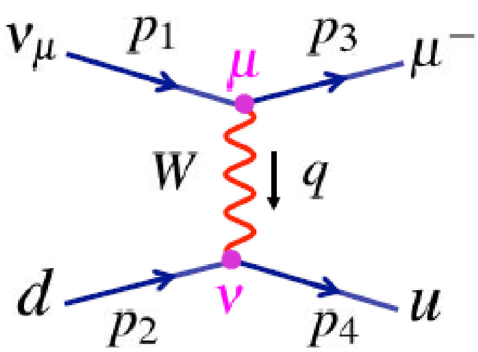
- Non-zero anti-quark component to the nucleon $\Rightarrow$ also consider scattering from $\overline{q}$
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include **LH particles** and **RH anti-particles**



$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\overline{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\overline{\nu}\overline{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\overline{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu \overline{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\overline{\nu}\overline{q}} = \frac{G_F^2\hat{s}}{\pi}$

Differential Cross Section dσ/dy

★ Derived differential neutrino scattering cross sections in C.o.M frame, can convert to Lorentz invariant form



- As for DIS use Lorentz invariant $y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$
- In relativistic limit y can be expressed in terms of the C.o.M. scattering angle

$$y = \frac{1}{2}(1 - \cos\theta^*)$$

- In lab. frame $y = 1 - \frac{E_3}{E_1}$

★ Convert from $\frac{d\sigma}{d\Omega^*} \rightarrow \frac{d\sigma}{dy}$ using

$$\frac{d\sigma}{dy} = \left|\frac{d\cos\theta^*}{dy}\right|\frac{d\sigma}{d\cos\theta^*} = \left|\frac{d\cos\theta^*}{dy}\right| 2\pi\frac{d\sigma}{d\Omega^*} = 4\pi\frac{d\sigma}{d\Omega^*}$$

- Already calculated (1) $\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$

- Hence:

$$\boxed{\frac{d\sigma_{\nu q}}{dy} = \frac{d\sigma_{\overline{\nu}\overline{q}}}{dy} = \frac{G_F^2}{\pi}\hat{s}}$$

and
$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

becomes
$$\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{4\pi}(1+\cos\theta^*)^2\hat{s}$$

from $y = \frac{1}{2}(1-\cos\theta^*) \rightarrow 1-y = \frac{1}{2}(1+\cos\theta^*)$

and hence
$$\boxed{\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}}$$

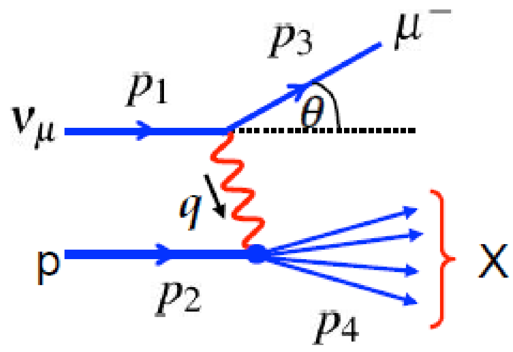
★ For comparison, the electro-magnetic $e^{\pm}q \rightarrow e^{\pm}q$ cross section is:

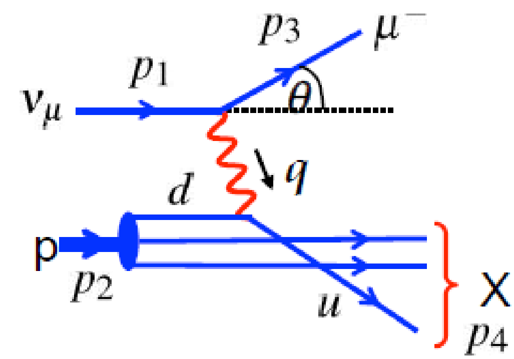
QED
$$\frac{d\sigma_{e^{\pm}q}}{dy} = \frac{2\pi\alpha^2}{q^4}e_q^2\left[1+(1-y)^2\right]\hat{s}$$

DIFFERENCES: Interaction +propagator ($\frac{2\pi\alpha^2}{q^4}e_q^2$ vs. $\frac{G_F^2}{\pi}$) — Helicity Structure ($\left[1+(1-y)^2\right]$ vs. $(1-y)^2$)

WEAK
$$\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}$$

Parton Model For Neutrino Deep Inelastic Scattering





Scattering from a proton with structure functions

Scattering from a point-like quark within the proton

★ Neutrino-proton scattering can occur via scattering from a down-quark or from an anti-up quark

• In the parton model, number of down quarks within the proton in the momentum fraction range $x \rightarrow x + \mathrm{d}x$ is $d^p(x)\mathrm{d}x$. Their contribution to the neutrino scattering cross-section is obtained by multiplying by the $\nu_\mu d \rightarrow \mu^- u$ cross-section derived previously

$$\frac{\mathrm{d}\sigma^{\nu p}}{\mathrm{d}y} = \frac{G_F^2}{\pi} \hat{s} d^p(x)\mathrm{d}x$$

where $\hat{s}$ is the centre-of-mass energy of the $\nu_\mu d$

- Similarly for the $\overline{u}$ contribution

$$\frac{\mathrm{d}\sigma^{\nu p}}{\mathrm{d}y} = \frac{G_\mathrm{F}^2}{\pi}\hat{s}(1-y)^2\overline{u}^p(x)\mathrm{d}x$$

★ Summing the two contributions and using $\hat{s} = xs$

$$\Rightarrow \quad \frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{\pi}sx\left[d^p(x) + (1-y)^2\overline{u}^p(x)\right]$$

★ The anti-neutrino proton differential cross section can be obtained in the same manner:

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{\pi}sx\left[(1-y)^2u^p(x) + \overline{d}^p(x)\right]$$

★ For (anti)neutrino – neutron scattering:

$$\frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{\pi}sx\left[d^n(x) + (1-y)^2\overline{u}^n(x)\right]$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{\pi}sx\left[(1-y)^2u^n(x) + \overline{d}^n(x)\right]$$

- As before, define neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x); \qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$$

$$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x); \qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$$

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_{\mathrm{F}}^2}{\pi} sx\left[d(x) + (1-y)^2\overline{u}(x)\right] \qquad (2)$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_{\mathrm{F}}^2}{\pi} sx\left[(1-y)^2 u(x) + \overline{d}(x)\right] \qquad (3)$$

$$\frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_{\mathrm{F}}^2}{\pi} sx\left[u(x) + (1-y)^2\overline{d}(x)\right] \qquad (4)$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_{\mathrm{F}}^2}{\pi} sx\left[(1-y)^2 d(x) + \overline{u}(x)\right] \qquad (5)$$

★ Because neutrino cross sections are very small, need massive detectors. These are usually made of Iron, hence, experimentally measure a combination of proton/neutron scattering cross sections

★ For an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon:

$$\frac{\mathrm{d}^2\sigma^{\nu N}}{\mathrm{d}x\mathrm{d}y} = \frac{1}{2}\left(\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} + \frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y}\right)$$

$$\Rightarrow \quad \boxed{\frac{\mathrm{d}^2\sigma^{\nu N}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{2\pi} sx\left[u(x) + d(x) + (1-y)^2(\overline{u}(x) + \overline{d}(x))\right]}$$

• Integrate over momentum fraction x

$$\boxed{\frac{\mathrm{d}\sigma^{\nu N}}{\mathrm{d}y} = \frac{G_\mathrm{F}^2}{2\pi} s\left[f_q + (1-y)^2 f_{\overline{q}})\right]} \qquad (6)$$

where f_q and $f_{\overline{q}}$ are the total momentum fractions carried by the quarks and by the anti-quarks within a nucleon

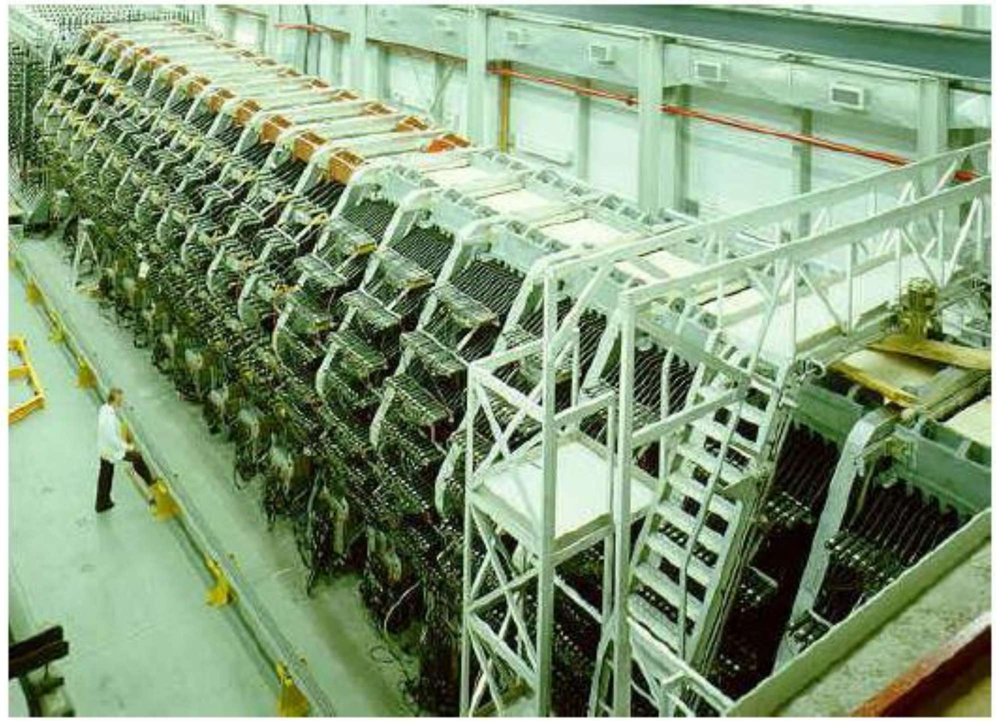
$$f_q \equiv f_d + f_u = \int_0^1 x\left[u(x) + d(x)\right]\mathrm{d}x; \quad f_{\overline{q}} \equiv f_{\overline{d}} + f_{\overline{u}} = \int_0^1 x\left[\overline{u}(x) + \overline{d}(x)\right]\mathrm{d}x$$

• Similarly

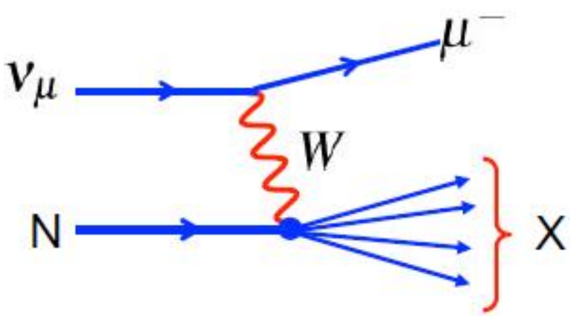
$$\boxed{\frac{\mathrm{d}\sigma^{\overline{\nu} N}}{\mathrm{d}y} = \frac{G_\mathrm{F}^2}{2\pi} s\left[(1-y)^2 f_q + f_{\overline{q}})\right]} \qquad (7)$$

e.g. CDHS Experiment (CERN 1976-1984)

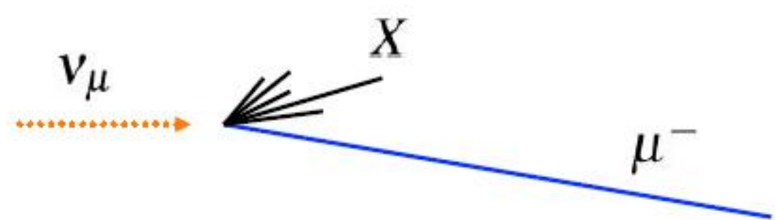
- 1250 tons
- Magnetized iron modules
- Separated by drift chambers



Study Neutrino Deep Inelastic Scattering

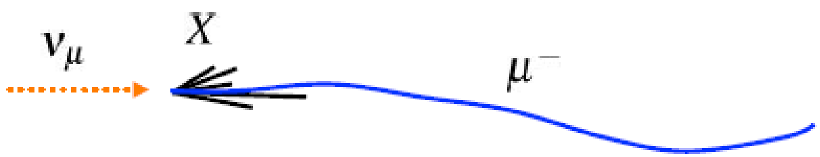


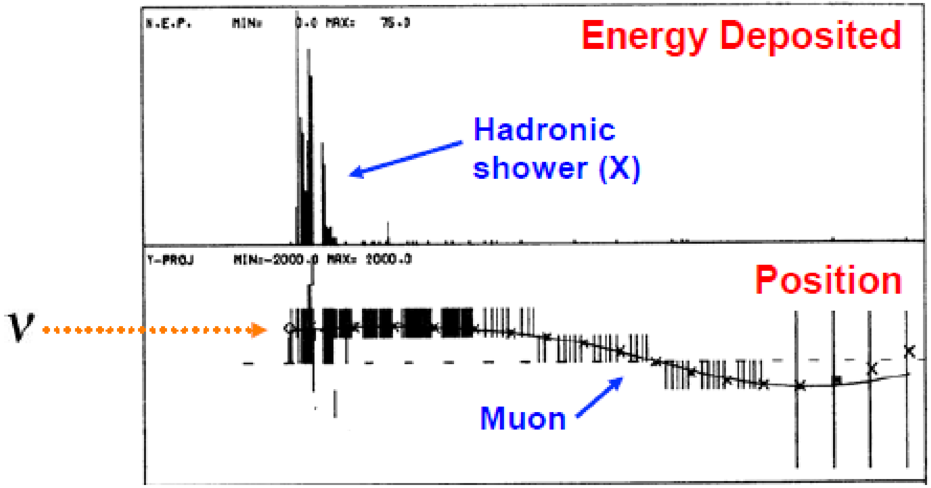
Experimental Signature:



e.g. CDHS Experiment (CERN 1976-1984)

Example Event:





- Measure energy of X

$$E_X$$

- Measure muon momentum from curvature in B-field

$$E_\mu$$

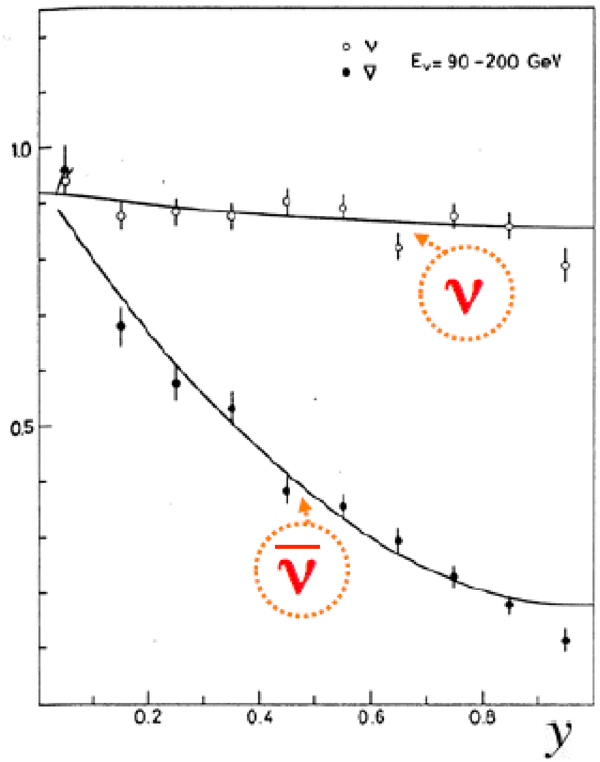
★ For each event can determine neutrino energy and y !

$$E_\nu = E_X + E_\mu$$

$$E_\mu = (1-y)E_\nu \quad \rightarrow \quad y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

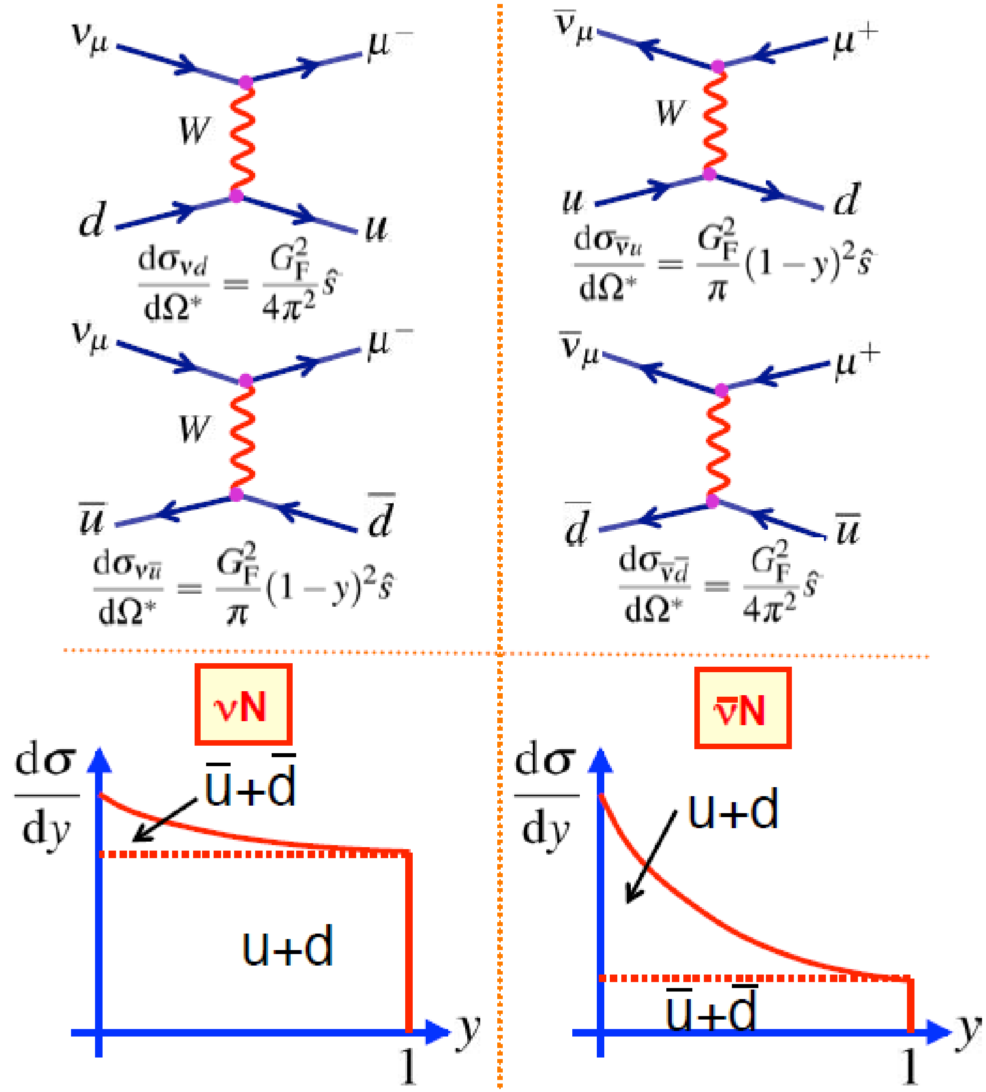
Measured y Distribution

- CDHS measured y distribution



J. de Groot et al., Z.Phys. C1 (1979) 143

- Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering



$$\frac{d\sigma_{\nu d}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s} \qquad \frac{d\sigma_{\overline{\nu} u}}{d\Omega^*} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}$$

$$\frac{d\sigma_{\nu \overline{u}}}{d\Omega^*} = \frac{G_F^2}{\pi}(1-y)^2\hat{s} \qquad \frac{d\sigma_{\overline{\nu}\overline{d}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$

Measured Total Cross Sections

★ Integrating the expressions for $\frac{d\sigma}{dy}$ (equations (6) and (7))

$$\sigma^{\nu N} = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3} f_{\overline{q}}\right] \qquad \sigma^{\overline{\nu} N} = \frac{G_F^2 s}{2\pi}\left[\frac{1}{3} f_q + f_{\overline{q}}\right]$$

ν → p

$(E_\nu, 0, 0, +E_\nu)$ $(m_p, 0, 0, 0)$

$$s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$$

➡ **DIS cross section ∝ lab. frame neutrino energy**

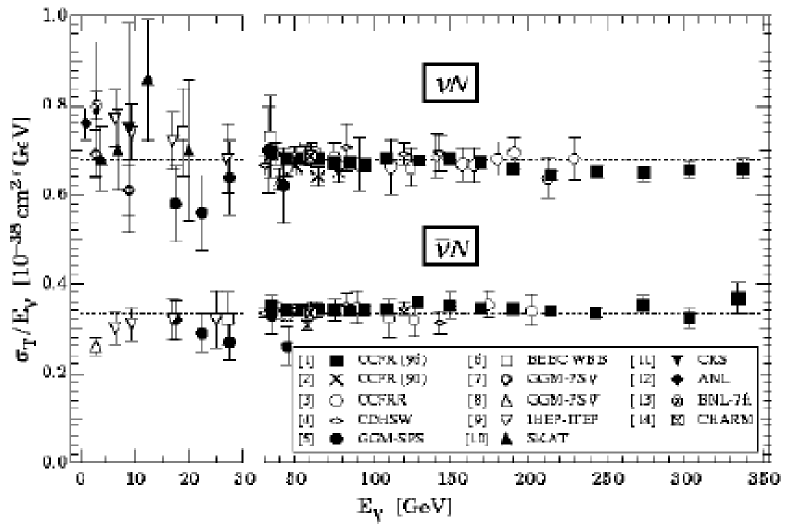
★ Measure cross sections can be used to determine fraction of protons momentum carried by quarks, f_q , and fraction carried by anti-quarks, $f_{\overline{q}}$

- Find: $f_q \approx 0.41; \quad f_{\overline{q}} \approx 0.08$
- ~50% of momentum carried by gluons (which don't interact with virtual W boson)
- If no anti-quarks in nucleons expect

$$\frac{\sigma^{\nu N}}{\sigma^{\overline{\nu} N}} = 3$$

- Including anti-quarks

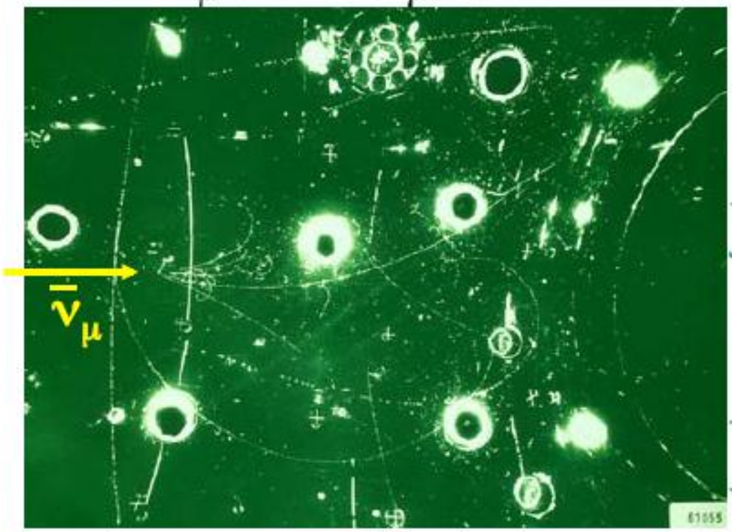
$$\frac{\sigma^{\nu N}}{\sigma^{\overline{\nu} N}} \approx 2$$



Weak Neutral Current

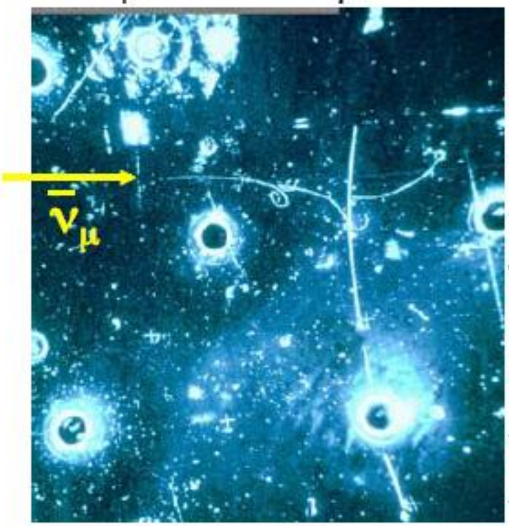
- Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973.

$$\overline{\nu}_\mu + N \rightarrow \overline{\nu}_\mu + \text{hadrons}$$



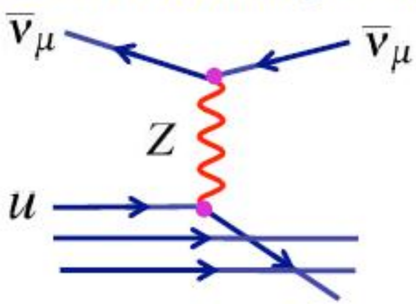
F.J. Hasert et al., Phys. Lett. 46B (1973) 138

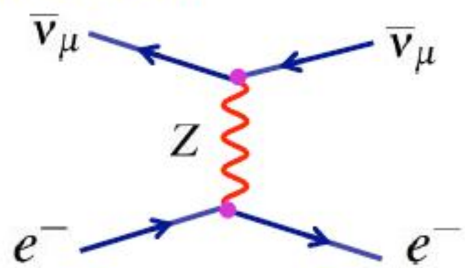
$$\overline{\nu}_\mu + e^- \rightarrow \overline{\nu}_\mu + e^-$$



F.J. Hasert et al., Phys. Lett. 46B (1973) 121

- Cannot be due to W exchange - first evidence for Z boson





Summary

- ★ Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections

- ★ Neutrino – nucleon scattering yields extra information about parton distributions functions:
  - ν couples to d and $\overline{u}$; $\overline{\nu}$ couples to u and $\overline{d}$
    ➡ investigate flavour content of nucleon
  - can measure anti-quark content of nucleon
    $\nu\overline{q}$ suppressed by factor $(1-y)^2$ compared with νq
    $\overline{\nu}q$ suppressed by factor $(1-y)^2$ compared with $\overline{\nu}\overline{q}$

- ★ Further aspects of neutrino deep-inelastic scattering (expressed in general structure functions) are covered in Appendix II

- ★ Finally observe that neutrinos interact via weak neutral currents (NC)

Appendix I

- For the adjoint spinors $\overline{u} = u^{\dagger}\gamma^{0}$ consider

$$\overline{\tfrac{1}{2}(1-\gamma^5)u} = [\tfrac{1}{2}(1-\gamma^5)u]^{\dagger}\gamma^0 = u^{\dagger}\tfrac{1}{2}(1-\gamma^5)\gamma^0 = u^{\dagger}\gamma^0\tfrac{1}{2}(1+\gamma^5) = \overline{u}\tfrac{1}{2}(1+\gamma^5)$$

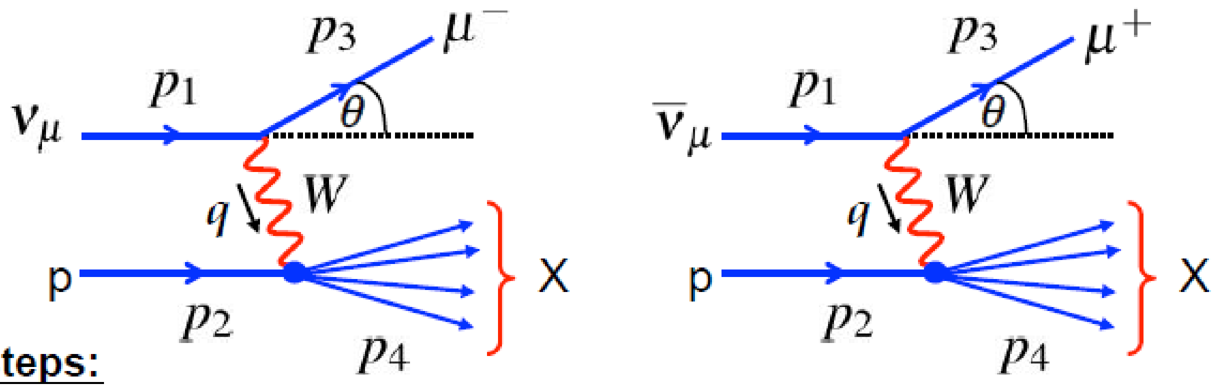
$$\tfrac{1}{2}(1-\gamma^5)u_{\uparrow} = 0 \quad \Rightarrow \quad \overline{u}\tfrac{1}{2}(1+\gamma^5) = 0$$

Using the fact that γ^5 and γ^{μ} anti-commute can rewrite ME:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{u}(p_3)\tfrac{1}{2}(1+\gamma^5)\gamma^{\mu}u(p_1)\right]\left[\overline{u}(p_4)\tfrac{1}{2}(1+\gamma^5)\gamma^{\nu}u(p_2)\right]$$

$\Rightarrow$ $M_{fi} = 0$ for $\overline{u}_{\uparrow}(p_3)$ and $\overline{u}_{\uparrow}(p_4)$

Appendix II: Deep-Inelastic Neutrino Scattering



Two steps:

- First write down most general cross section in terms of structure functions
- Then evaluate expressions in the quark-parton model

QED Revisited

★ In the limit $s \gg M^2$ the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written (Eq. 2 of handout 6)

$$\frac{d^2\sigma_{e^\pm p}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

- For neutrino scattering typically measure the energy of the produced muon $E_\mu = E_\nu(1-y)$ and differential cross-sections expressed in terms of $dx dy$

- Using $Q^2 = (s-M^2)xy \approx sxy$ $\Rightarrow$ $\frac{d^2\sigma}{dx dy} = \left|\frac{dQ^2}{dy}\right|\frac{d^2\sigma}{dx dQ^2} = sx\frac{d^2\sigma}{dx dQ^2}$

- In the limit $s \gg M^2$ the general Electro-magnetic DIS cross section can be written

$$\frac{d^2\sigma^{e^\pm p}}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)F_2(x,Q^2) + y^2 x F_1(x,Q^2)\right]$$

- **NOTE:** This is the most general **Lorentz Invariant** parity conserving expression

★ For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function $F_3(x,Q^2)$

$\nu_\mu p \rightarrow \mu^- X$

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x,Q^2) + y^2 x F_1^{\nu p}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x,Q^2)\right]$$

- For anti-neutrino scattering new structure function enters with opposite sign

$\overline{\nu}_\mu p \rightarrow \mu^+ X$

$$\frac{d^2\sigma^{\overline{\nu} p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\overline{\nu} p}(x,Q^2) + y^2 x F_1^{\overline{\nu} p}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\overline{\nu} p}(x,Q^2)\right]$$

- Similarly for neutrino-neutron scattering

$\nu_\mu n \rightarrow \mu^- X$

$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu n}(x,Q^2) + y^2 x F_1^{\nu n}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu n}(x,Q^2)\right]$$

$\overline{\nu}_\mu n \rightarrow \mu^+ X$

$$\frac{d^2\sigma^{\overline{\nu} n}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\overline{\nu} n}(x,Q^2) + y^2 x F_1^{\overline{\nu} n}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\overline{\nu} n}(x,Q^2)\right]$$

Neutrino Interaction Structure Functions

★ In terms of the parton distribution functions we found (2) :

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{\pi} sx\left[d(x) + (1-y)^2\overline{u}(x)\right]$$

- Compare coefficients of y with the general Lorentz Invariant form (p.321) and assume Bjorken scaling, i.e. $F(x,Q^2) \rightarrow F(x)$

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

- Re-writing (2) $\quad \frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{2\pi} s\left[2xd(x) + 2x\overline{u}(x) - 4xy\overline{u}(x) + 2xy^2\overline{u}(x)\right]$

and equating powers of y

$$\left.\begin{array}{rcl} 2xd + 2x\overline{u} & = & F_2 \\ -4x\overline{u} & = & -F_2 + xF_3 \\ 2\overline{u} & = & F_1 - xF_3/2 \end{array}\right\}$$

gives:

$$F_2^{\nu p} = 2xF_1^{\nu p} = 2x[d(x) + \overline{u}(x)]$$
$$xF_3^{\nu p} = 2x[d(x) - \overline{u}(x)]$$

NOTE: again we get the Callan-Gross relation $F_2 = 2xF_1$

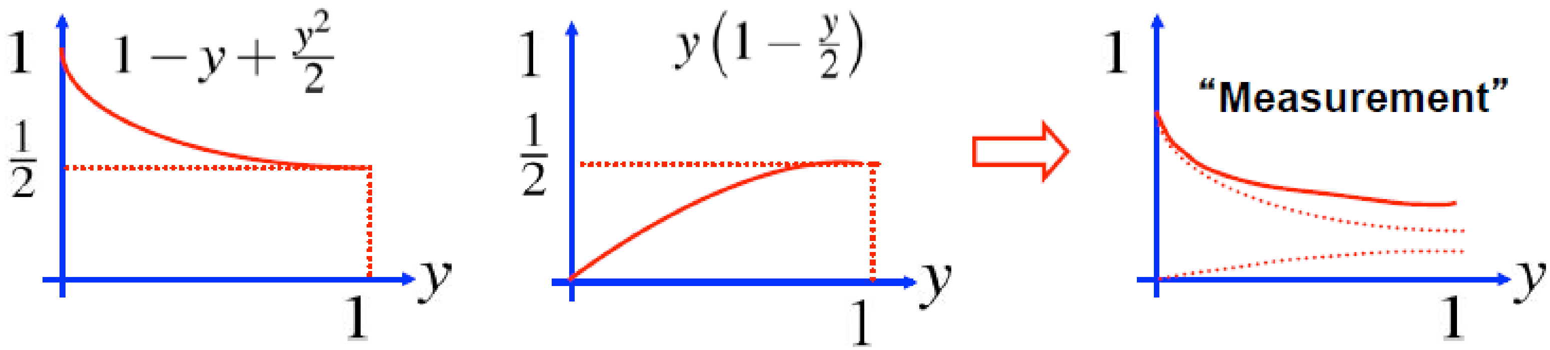
No surprise, underlying process is scattering from point-like spin-1/2 quarks

★ Substituting back in to expression for differential cross section:

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 s}{2\pi}\left[\left(1 - y + \frac{y^2}{2}\right)F_2^{\nu p}(x) + y\left(1 - \frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

★ Experimentally measure F_2 and F_3 from y distributions at fixed x

- Different **y** dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured



★ Then use $F_2^{\nu p} = 2x[d(x) + \overline{u}(x)]$ and $F_3^{\nu p} = 2[d(x) - \overline{u}(x)]$

➡ Determine $d(x)$ and $\overline{u}(x)$ separately

★ Neutrino experiments require large detectors (often iron) i.e. isoscalar target

$$F_2^{\nu N} = 2xF_1^{\nu N} = \tfrac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) = x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)]$$

$$xF_3^{\nu N} = \tfrac{1}{2}\left(xF_3^{\nu p} + xF_3^{\nu n}\right) = x[u(x) + d(x) - \overline{u}(x) - \overline{d}(x)]$$

★ For electron – nucleon scattering:

$$F_2^{ep} = 2xF_1^{ep} = x[\tfrac{4}{9}u(x) + \tfrac{1}{9}d(x) + \tfrac{4}{9}\overline{u}(x) + \tfrac{1}{9}\overline{d}(x)]$$

$$F_2^{en} = 2xF_1^{en} = x[\tfrac{4}{9}d(x) + \tfrac{1}{9}u(x) + \tfrac{4}{9}\overline{d}(x) + \tfrac{1}{9}\overline{u}(x)]$$

- For an isoscalar target

$$F_2^{eN} = \tfrac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \tfrac{5}{18}x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)]$$

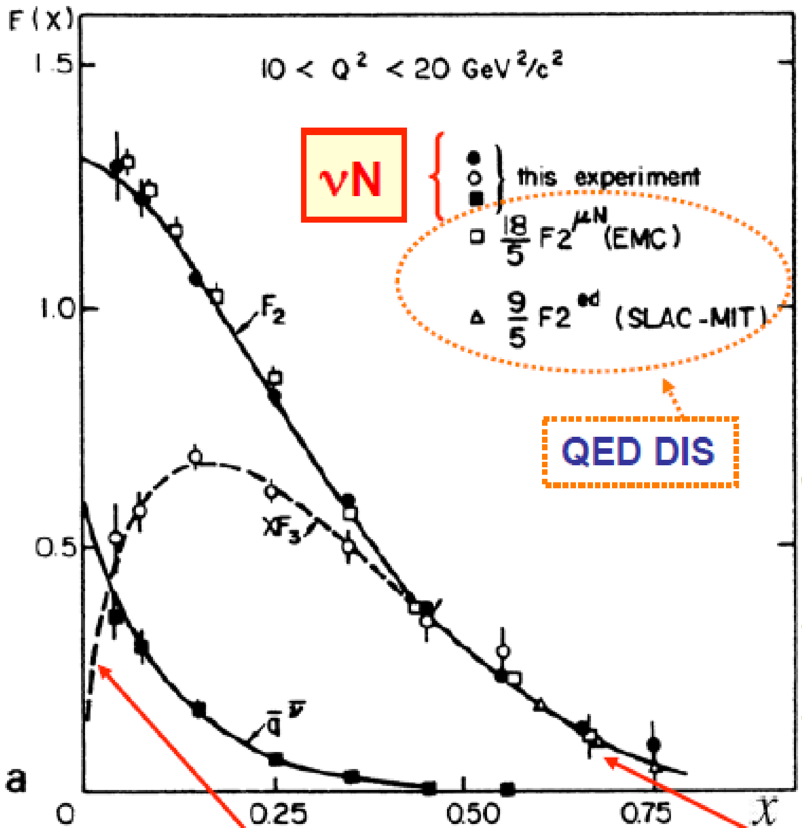
➡ $$F_2^{\nu N} = \tfrac{18}{5}F_2^{eN}$$

- Note that the factor $\frac{5}{18} = \frac{1}{2}\left(q_u^2 + q_d^2\right)$ and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

Experiment: 0.29 ± 0.02

Measurements of $F_2(x)$ and $F_3(x)$

•CDHS Experiment $\nu_\mu + \mathrm{Fe} \rightarrow \mu^- + X$



H. Abramowicz et al., Z.Phys. C17 (1983) 283

$$F_2^{\nu N} = x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)]$$
$$xF_3^{\nu N} = x[u(x) + d(x) - \overline{u}(x) - \overline{d}(x)]$$

➡ $F_2^{\nu N} - xF_3^{\nu N} = 2x[\overline{u} + \overline{d}]$

✶ Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions

Sea dominates so expect xF_3 to go to zero as q(x) = $\overline{\text{q}}$(x)

Sea contribution goes to zero

Valence Contribution

★ Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$
$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$
$$\overline{u}(x) = \overline{u}_S(x) = S(x)$$
$$\overline{d}(x) = \overline{d}_S(x) = S(x)$$

➡ $$F_3^{\nu N} = [u(x) + d(x) - \overline{u}(x) - \overline{d}(x)] = u_V(x) + d_V(x)$$

➡ $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = \int_0^1 (u_V(x) + d_V(x))\,\mathrm{d}x = N_u^V + N_d^V$$

★ Area under measured function $F_3^{\nu N}(x)$ gives a measurement of the total number of valence quarks in a nucleon !

expect $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = 3$$

"Gross – Llewellyn-Smith sum rule"

Experiment: 3.0±0.2

•Note: $F_2^{\overline{\nu}p} = F_2^{\nu n}$; $F_2^{\overline{\nu}n} = F_2^{\nu p}$; $F_3^{\overline{\nu}p} = F_3^{\nu n}$; $F_3^{\overline{\nu}n} = F_3^{\nu p}$ and anti-neutrino structure functions contain same pdf information