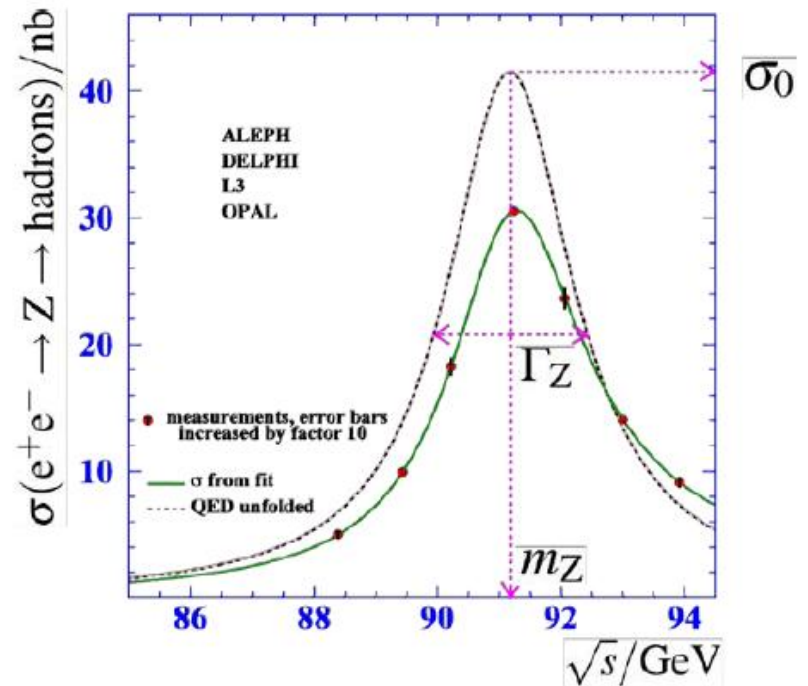


Elementary Particle Physics: theory and experiments

Precision Tests of the Standard Model

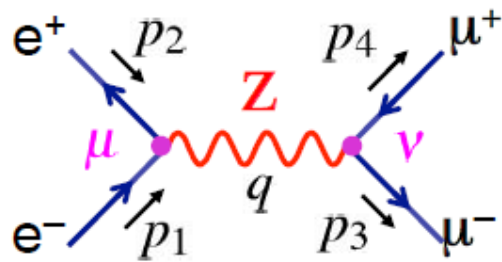


Follow the course/slides from M. A. Thomson lectures at Cambridge University

The Z Resonance

★ Want to calculate the cross-section for $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$

• Feynman rules for the diagram below give:



e^+e^- vertex: $\bar{v}(p_2) \cdot -ig_Z \gamma^\mu \frac{1}{2} (c_V^e - c_A^e \gamma^5) \cdot u(p_1)$

Z propagator: $\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

$\mu^+\mu^-$ vertex: $\bar{u}(p_3) \cdot -ig_Z \gamma^\nu \frac{1}{2} (c_V^\mu - c_A^\mu \gamma^5) \cdot v(p_4)$

→ $-iM_{fi} = [\bar{v}(p_2) \cdot -ig_Z \gamma^\mu \frac{1}{2} (c_V^e - c_A^e \gamma^5) \cdot u(p_1)] \cdot \frac{-ig_{\mu\nu}}{q^2 - m_Z^2} \cdot [\bar{u}(p_3) \cdot -ig_Z \gamma^\nu \frac{1}{2} (c_V^\mu - c_A^\mu \gamma^5) \cdot v(p_4)]$

→ $M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} [\bar{v}(p_2) \gamma^\mu \frac{1}{2} (c_V^e - c_A^e \gamma^5) \cdot u(p_1)] \cdot [\bar{u}(p_3) \gamma^\nu \frac{1}{2} (c_V^\mu - c_A^\mu \gamma^5) \cdot v(p_4)]$

★ Convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)

$$\frac{1}{2} (c_V - c_A \gamma^5) = c_L \frac{1}{2} (1 - \gamma^5) + c_R \frac{1}{2} (1 + \gamma^5)$$

LH and RH projections operators

hence $c_V = (c_L + c_R)$, $c_A = (c_L - c_R)$

$$\begin{aligned} \text{and } \frac{1}{2}(c_V - c_A \gamma^5) &= \frac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) \\ &= c_L \frac{1}{2}(1 - \gamma^5) + c_R \frac{1}{2}(1 + \gamma^5) \end{aligned}$$

with $c_L = \frac{1}{2}(c_V + c_A)$, $c_R = \frac{1}{2}(c_V - c_A)$

★ **Rewriting the matrix element in terms of LH and RH couplings:**

$$\begin{aligned} M_{fi} &= -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} [c_L^e \bar{v}(p_2) \gamma^\mu \frac{1}{2}(1 - \gamma^5) u(p_1) + c_R^e \bar{v}(p_2) \gamma^\mu \frac{1}{2}(1 + \gamma^5) u(p_1)] \\ &\quad \times [c_L^\mu \bar{u}(p_3) \gamma^\nu \frac{1}{2}(1 - \gamma^5) v(p_4) + c_R^\mu \bar{u}(p_3) \gamma^\nu \frac{1}{2}(1 + \gamma^5) v(p_4)] \end{aligned}$$

★ **Apply projection operators remembering that in the ultra-relativistic limit**

$$\frac{1}{2}(1 - \gamma^5)u = u_\downarrow; \quad \frac{1}{2}(1 + \gamma^5)u = u_\uparrow, \quad \frac{1}{2}(1 - \gamma^5)v = v_\uparrow, \quad \frac{1}{2}(1 + \gamma^5)v = v_\downarrow$$

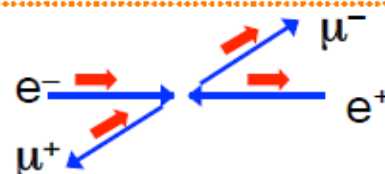
$$\begin{aligned} \Rightarrow M_{fi} &= -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} [c_L^e \bar{v}(p_2) \gamma^\mu u_\downarrow(p_1) + c_R^e \bar{v}(p_2) \gamma^\mu u_\uparrow(p_1)] \\ &\quad \times [c_L^\mu \bar{u}(p_3) \gamma^\nu v_\uparrow(p_4) + c_R^\mu \bar{u}(p_3) \gamma^\nu v_\downarrow(p_4)] \end{aligned}$$

★ **For a combination of V and A currents, $\bar{u}_\uparrow \gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions**

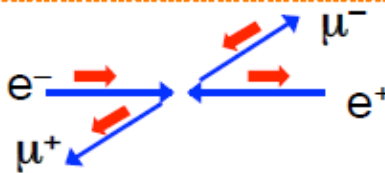
$$\begin{aligned} \Rightarrow & -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} [c_L^e \bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1) + c_R^e \bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] \\ & \times [c_L^\mu \bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4) + c_R^\mu \bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)] \end{aligned}$$

★ Sum of 4 terms

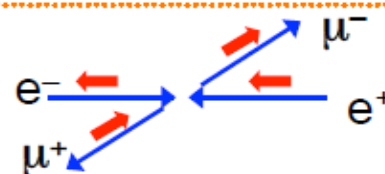
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$$



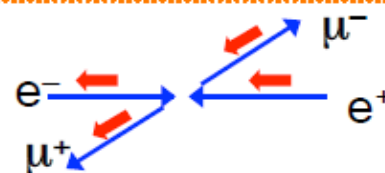
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state particles

★ Fortunately we have calculated these terms before when considering $e^+ e^- \rightarrow \gamma \rightarrow \mu^+ \mu^-$ giving:

$$[\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)] = s(1 + \cos \theta) \quad \text{etc.}$$

- ★ Applying the QED results to the Z exchange with gives:

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos \theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos \theta)^2$$

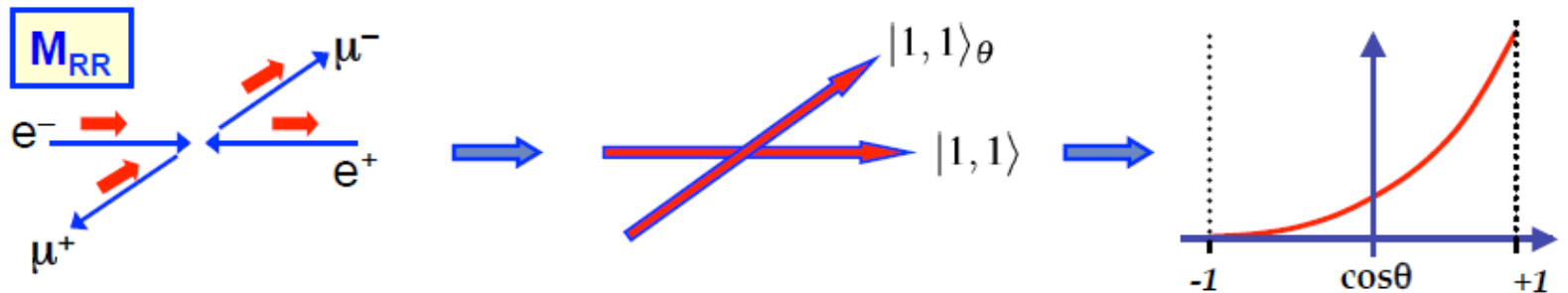
$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2$$

$$|M_{LL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos \theta)^2$$

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

- ★ As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.



The Breit-Wigner Resonance

- ★ Need to consider carefully the propagator term $1/(s - m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson

- ★ To do this need to account for the fact that the Z boson is an unstable particle

- For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

- For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \quad \text{with} \quad \tau = \frac{1}{\Gamma_Z}$$

- Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

- ★ In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

- ★ Which gives:

$$(s - m_Z^2) \longrightarrow [s - (m_Z - i\Gamma_Z/2)^2] = s - m_Z^2 + im_Z\Gamma_Z + \frac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

- ★ Which gives

$$\left| \frac{1}{s - m_Z^2} \right|^2 \rightarrow \left| \frac{1}{s - m_Z^2 + im_Z\Gamma_Z} \right|^2 = \frac{1}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

- ★ And the Matrix elements become

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_R^e)^2 (c_R^\mu)^2 (1 + \cos \theta)^2 \quad \text{etc.}$$

- ★ In the limit where initial and final state particle mass can be neglected:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |M_{fi}|^2$$

- ★ Giving:

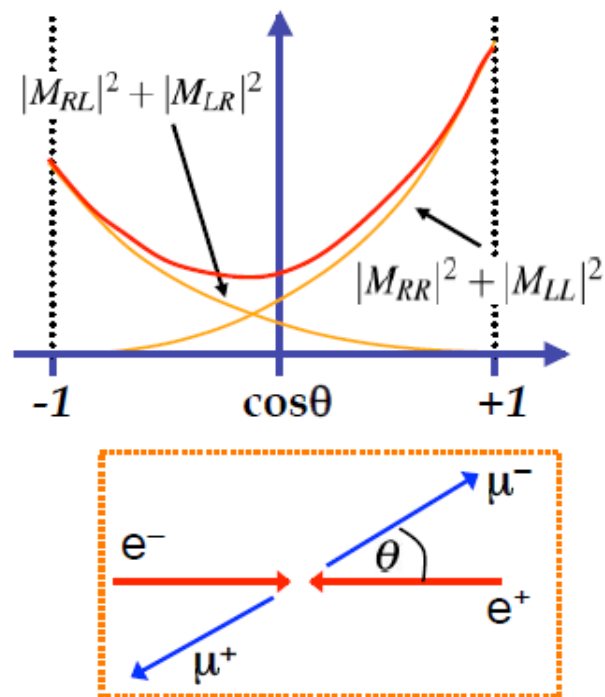
$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_R^e)^2 (c_R^\mu)^2 (1 + \cos \theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_L^e)^2 (c_L^\mu)^2 (1 + \cos \theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2$$

- ★ Because $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).



Cross section with unpolarised beams

- ★ To calculate the total cross section need to sum over all matrix elements and average over the initial spin states. Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there are four combinations of initial electron/positron spins, so

$$\begin{aligned} \langle |M_{fi}|^2 \rangle &= \frac{1}{2} \cdot \frac{1}{2} \cdot (|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2) \\ &= \frac{1}{2} \cdot \frac{1}{2} \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \times \left\{ [(c_R^e)^2 (c_R^\mu)^2 + (c_L^e)^2 (c_L^\mu)^2] (1 + \cos \theta)^2 \right. \\ &\quad \left. + [(c_L^e)^2 (c_R^\mu)^2 + (c_R^e)^2 (c_L^\mu)^2] (1 - \cos \theta)^2 \right\} \end{aligned}$$

- ★ The part of the expression $\{...\}$ can be rearranged:

$$\begin{aligned} \{...\} &= [(c_R^e)^2 + (c_L^e)^2][(c_R^\mu)^2 + (c_L^\mu)^2](1 + \cos^2 \theta) \\ &\quad + 2[(c_R^e)^2 - (c_L^e)^2][(c_R^\mu)^2 - (c_L^\mu)^2] \cos \theta \end{aligned} \tag{1}$$

and using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $c_V c_A = c_L^2 - c_R^2$

$$\{...\} = \frac{1}{4} [(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2](1 + \cos^2 \theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos \theta$$

★ Hence the complete expression for the unpolarized differential cross section is:

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s} \langle |M_{fi}|^2 \rangle \\ &= \frac{1}{64\pi^2} \cdot \frac{1}{4} \cdot \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \times \\ &\quad \left\{ \frac{1}{4} [(c_V^e)^2 + (c_A^e)^2] [(c_V^\mu)^2 + (c_A^\mu)^2] (1 + \cos^2 \theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos \theta \right\} \end{aligned}$$

★ Integrating over solid angle $d\Omega = d\phi d(\cos \theta) = 2\pi d(\cos \theta)$

$$\int_{-1}^{+1} (1 + \cos^2 \theta) d(\cos \theta) = \int_{-1}^{+1} (1 + x^2) dx = \frac{8}{3} \quad \text{and} \quad \int_{-1}^{+1} \cos \theta d(\cos \theta) = 0$$

$$\sigma_{e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-} = \frac{1}{192\pi} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} [(c_V^e)^2 + (c_A^e)^2] [(c_V^\mu)^2 + (c_A^\mu)^2]$$

★ Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions

$$(c_V^f)^2 + (c_A^f)^2$$

Connection to the Breit-Wigner Formula

- ★ Can write the total cross section

$$\sigma_{e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-} = \frac{1}{192\pi} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} [(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2]$$

in terms of the Z boson decay rates (partial widths)

$$\Gamma(Z \rightarrow e^+e^-) = \frac{g_Z^2 m_Z}{48\pi} [(c_V^e)^2 + (c_A^e)^2] \quad \text{and} \quad \Gamma(Z \rightarrow \mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi} [(c_V^\mu)^2 + (c_A^\mu)^2]$$

$$\Rightarrow \sigma = \frac{12\pi}{m_Z^2} \frac{s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \Gamma(Z \rightarrow e^+e^-) \Gamma(Z \rightarrow \mu^+\mu^-)$$

- ★ Writing the partial widths as $\Gamma_{ee} = \Gamma(Z \rightarrow e^+e^-)$ etc., the total cross section can be written

$$\sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) = \frac{12\pi}{m_Z^2} \frac{s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \Gamma_{ee} \Gamma_{ff} \quad (2)$$

where f is the final state fermion flavour:

Electroweak Measurements at LEP

- ★ The **L**arge **E**lectron **P**ositron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.



- 26 km circumference accelerator straddling French/Swiss border
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):

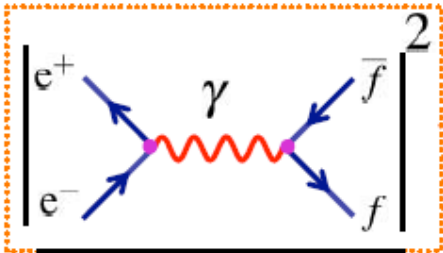
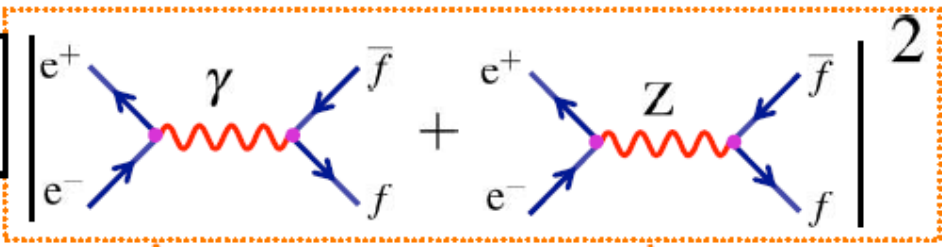
**ALEPH,
DELPHI,
L3,
OPAL**

Basically a large Z and W factory:

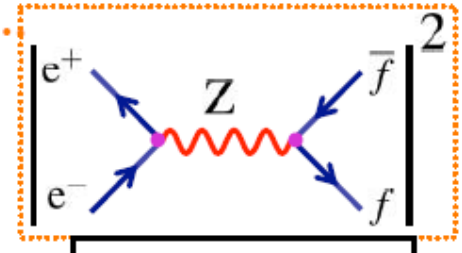
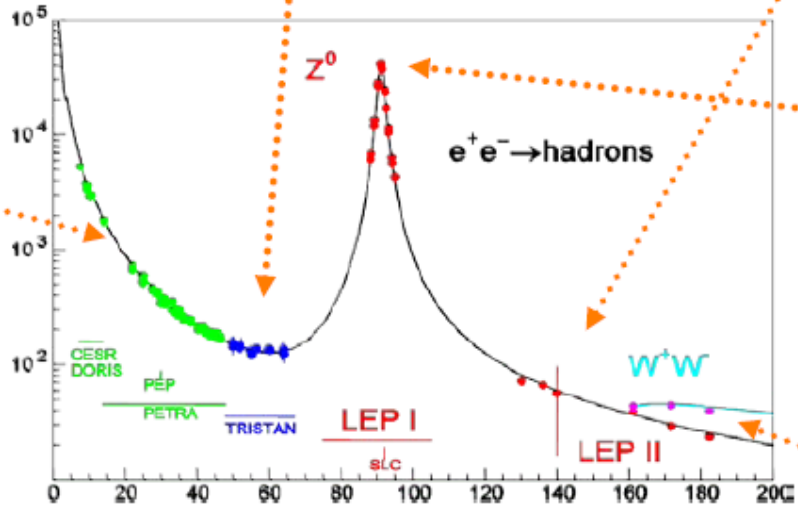
- ★ 1989-1995: Electron-Positron collisions at $\sqrt{s} = 91.2$ GeV
 - 17 Million **Z bosons** detected
- ★ 1996-2000: Electron-Positron collisions at $\sqrt{s} = 161$ -208 GeV
 - 30000 **W⁺W⁻** events detected

e^+e^- Annihilation in Feynman Diagrams

In general e^+e^- annihilation involves both photon and Z exchange : + interference

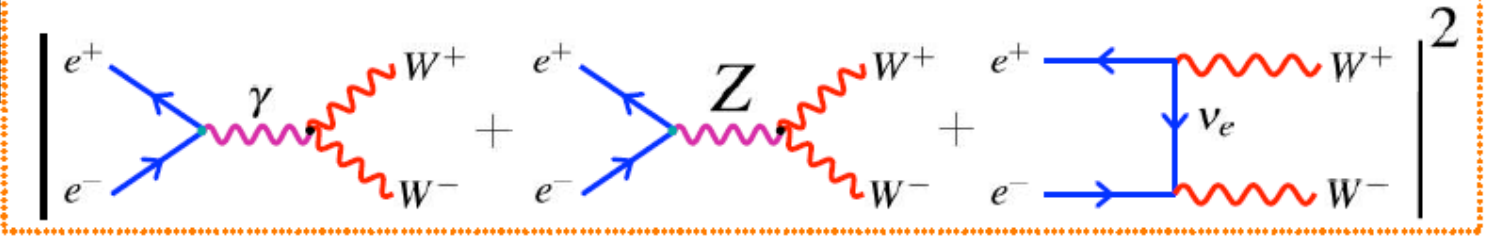


Well below Z: photon exchange dominant



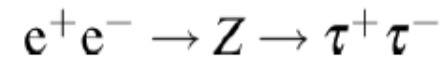
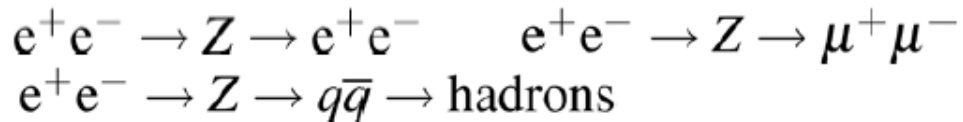
At Z resonance: Z exchange dominant

High energies: WW production

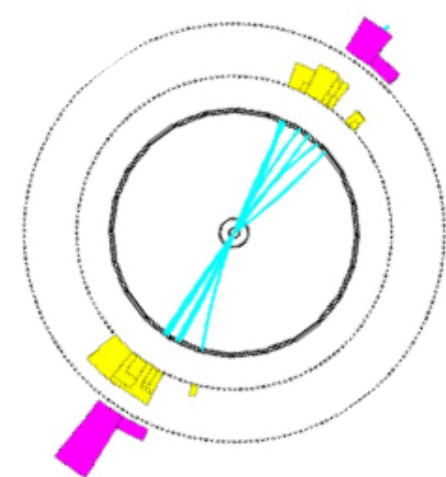
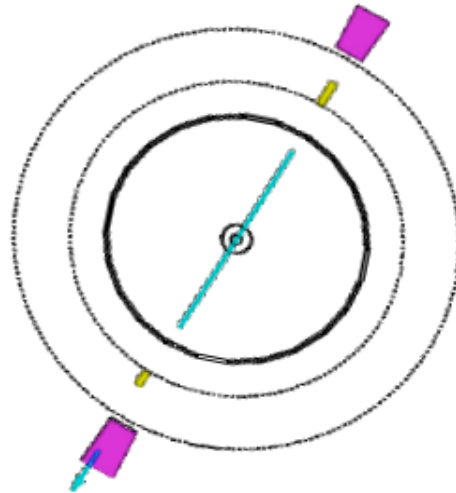
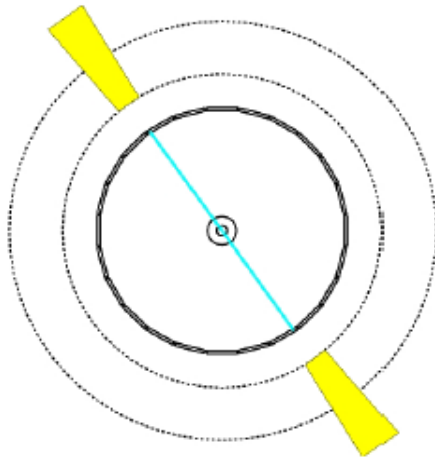
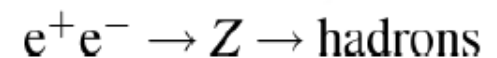
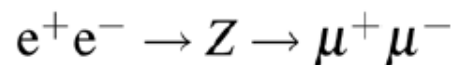
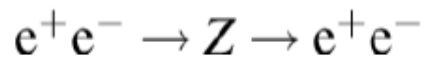


Cross Section Measurements

- ★ At Z resonance mainly observe four types of event:



- ★ Each has a distinct topology in the detectors, e.g.

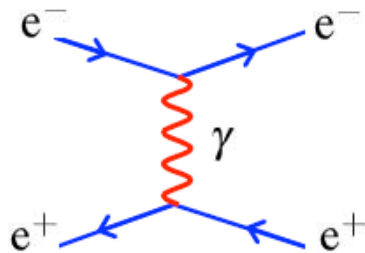


- ★ To work out cross sections, first count events of each type
- ★ Then need to know “integrated luminosity” of colliding beams, i.e. the relation between cross-section and expected number of interactions

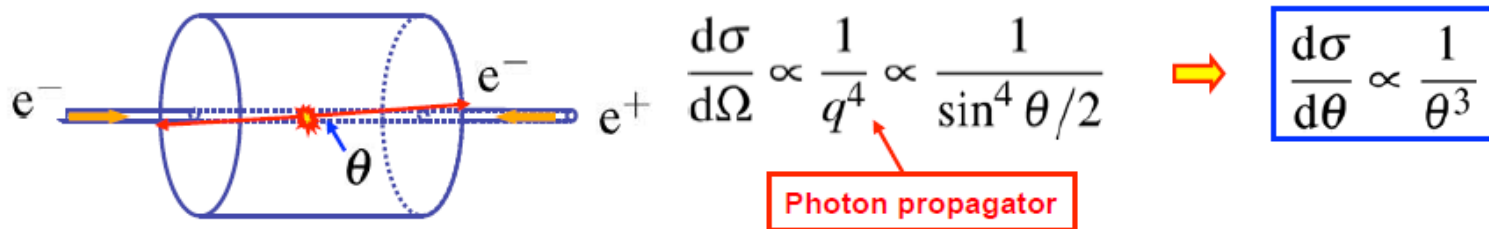
$$N_{\text{events}} = \mathcal{L} \sigma$$

- ★ To calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile
 - very difficult to achieve with precision of better than 10%

- ★ Instead “normalise” using another type of event:



- ◆ Use the QED Bhabha scattering process
- ◆ QED, so cross section can be calculated very precisely
- ◆ Very large cross section – small statistical errors
- ◆ Reaction is very forward peaked – i.e. the electron tends not to get deflected much



- ◆ Count events where the electron is scattered in the very forward direction

$$N_{\text{Bhabha}} = \mathcal{L} \sigma_{\text{Bhabha}} \Rightarrow \mathcal{L} \quad \boxed{\sigma_{\text{Bhabha}} \text{ known from QED calc.}}$$

- ★ Hence all other cross sections can be expressed as

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}} \sigma_{\text{Bhabha}}} \Rightarrow \boxed{\text{Cross section measurements involve just event counting !}}$$

Measurements of the Z line-shape

★ Measurements of the Z resonance lineshape determine:

- m_Z : peak of the resonance
- Γ_Z : FWHM of resonance
- Γ_f : Partial decay widths
- N_ν : Number of light neutrino generations

★ Measure cross sections to different final states versus C.o.M. energy \sqrt{s}

★ Starting from

$$\sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) = \frac{12\pi}{m_Z^2} \frac{s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \Gamma_{ee} \Gamma_{ff} \quad (3)$$

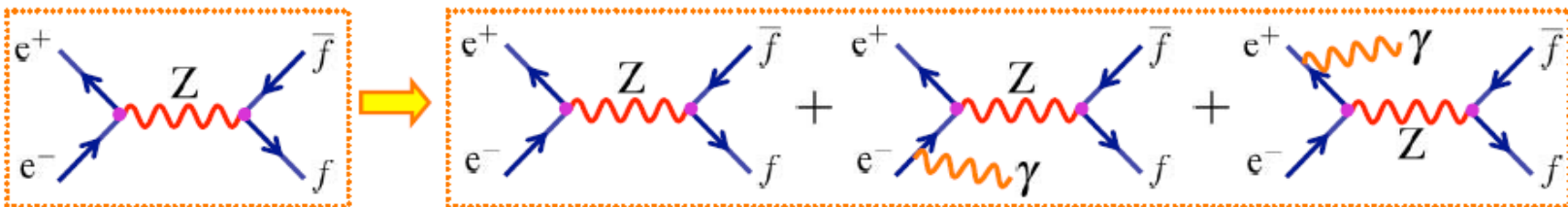
maximum cross section occurs at $\sqrt{s} = m_Z$ with peak cross section equal to

$$\sigma_{f\bar{f}}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee} \Gamma_{ff}}{\Gamma_Z^2}$$

★ Cross section falls to half peak value at $\sqrt{s} \approx m_Z \pm \frac{\Gamma_Z}{2}$ which can be seen immediately from eqn. (3)

★ Hence $\Gamma_Z = \frac{\hbar}{\tau_Z} = \text{FWHM of resonance}$

- ★ In practise, it is not that simple, QED corrections distort the measured line-shape
- ★ One particularly important correction: **initial state radiation (ISR)**



- ★ Initial state radiation reduces the centre-of-mass energy of the e^+e^- collision

$$e^+ \xrightarrow{E} \xleftarrow{E} e^- \quad \sqrt{s} = 2E$$

becomes

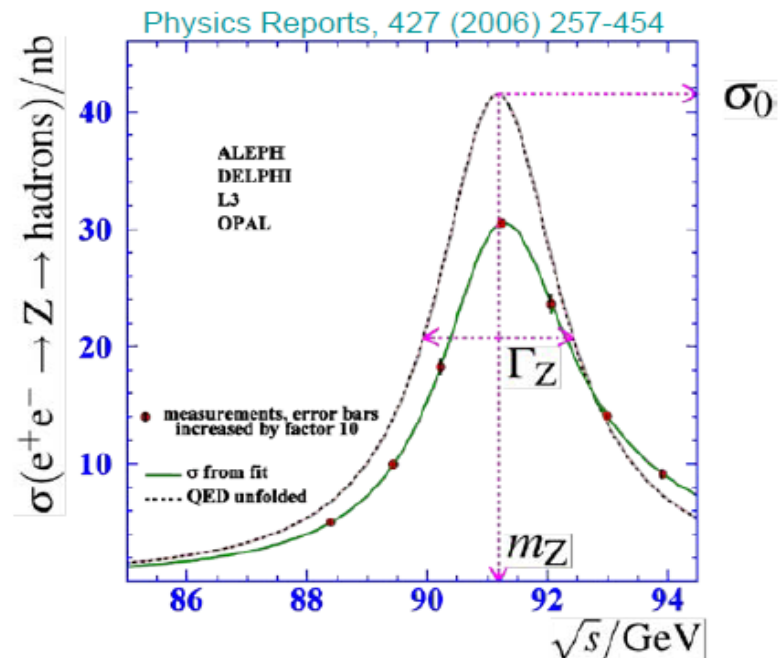
$$e^+ \xrightarrow{E} \xleftarrow{E-E_\gamma} e^- \quad \sqrt{s'} \approx 2E \left(1 - \frac{E_\gamma}{2E}\right)$$

- ★ Measured cross section can be written:

$$\sigma_{\text{meas}}(E) = \int \sigma(E') f(E', E) dE'$$

Probability of e^+e^- colliding with C.o.M. energy E' when C.o.M. energy before radiation is E

- ★ Fortunately can calculate $f(E', E)$ very precisely, just QED, and can then obtain Z line-shape from measured cross section



- ★ In principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

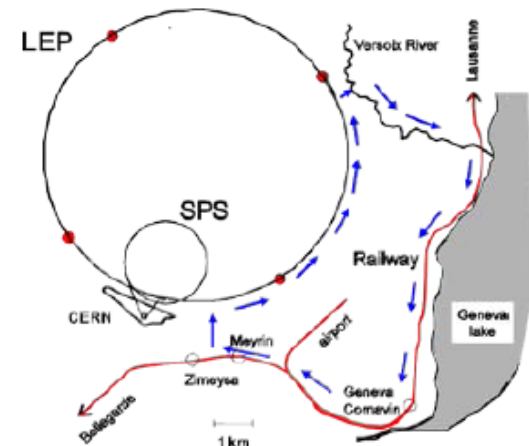
- ★ 0.002 % measurement of m_Z !
- ★ To achieve this level of precision – need to know energy of the colliding beams to better than 0.002 % : sensitive to unusual systematic effects...

Moon:

- ♦ As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- ♦ The nominal radius of the accelerator of 4.3 km varies by $\pm 0.15 \text{ mm}$
- ♦ Changes beam energy by $\sim 10 \text{ MeV}$: need to correct for tidal effects !

Trains:

- ♦ Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- ♦ Travelling via the Versoix river and using the LEP ring as a conductor.
- ♦ Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- ♦ LEP beam energy changes by $\sim 10 \text{ MeV}$



Number of generations

- ★ Total decay width measured from Z line-shape: $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$
- ★ If there were an additional 4th generation would expect $Z \rightarrow \nu_4 \bar{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. $> m_Z/2$)

★ Total decay width is the sum of the partial widths:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

★ Although don't observe neutrinos, $Z \rightarrow \nu\bar{\nu}$ decays affect the Z resonance shape for **all** final states

★ For all other final states can determine partial decay widths from peak cross sections:

$$\sigma_{ff}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

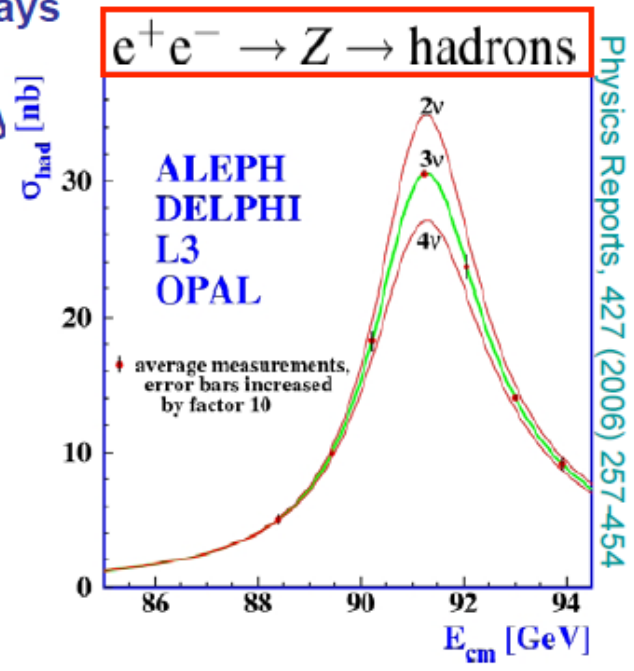
★ Assuming lepton universality:

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\nu}$$



→ $N_\nu = 2.9840 \pm 0.0082$

★ **ONLY 3 GENERATIONS** (unless a new 4th generation neutrino has very large mass)



Forward-Backward Asymmetry

- ★ The expression for the differential cross section:

$$\langle |M_{fi}| \rangle^2 \propto [(c_L^e)^2 + (c_R^e)^2][(c_L^\mu)^2 + (c_R^\mu)^2](1 + \cos^2 \theta) + [(c_L^e)^2 - (c_R^e)^2][(c_L^\mu)^2 - (c_R^\mu)^2] \cos \theta$$

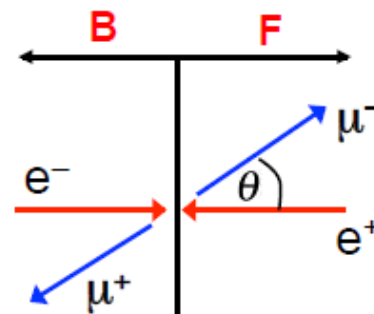
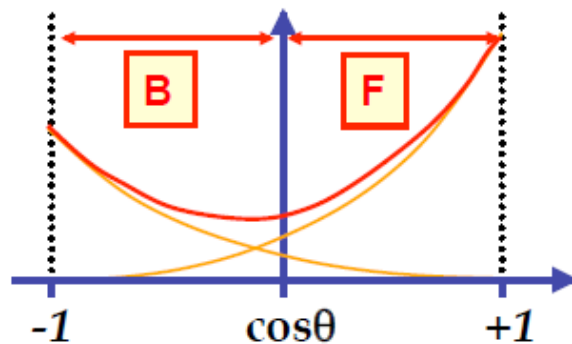
- ★ The differential cross sections is therefore of the form:

$$\frac{d\sigma}{d\Omega} = \kappa \times [A(1 + \cos^2 \theta) + B \cos \theta] \quad \begin{cases} A = [(c_L^e)^2 + (c_R^e)^2][(c_L^\mu)^2 + (c_R^\mu)^2] \\ B = [(c_L^e)^2 - (c_R^e)^2][(c_L^\mu)^2 - (c_R^\mu)^2] \end{cases}$$

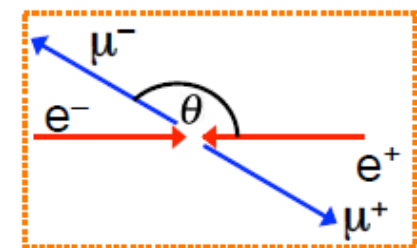
- ★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

$$\sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

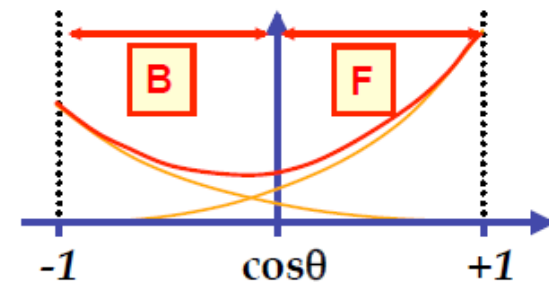


e.g. "backward hemisphere"



- ★ The level of asymmetry about $\cos\theta=0$ is expressed in terms of the Forward-Backward Asymmetry

$$A_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



- Integrating equation (1):

$$\sigma_F = \kappa \int_0^1 [A(1 + \cos^2 \theta) + B \cos \theta] d \cos \theta = \kappa \int_0^1 [A(1 + x^2) + Bx] dx = \kappa \left(\frac{4}{3}A + \frac{1}{2}B \right)$$

$$\sigma_B = \kappa \int_{-1}^0 [A(1 + \cos^2 \theta) + B \cos \theta] d \cos \theta = \kappa \int_{-1}^0 [A(1 + x^2) + Bx] dx = \kappa \left(\frac{4}{3}A - \frac{1}{2}B \right)$$

- ★ Which gives:

$$A_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4} \left[\frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} \right] \cdot \left[\frac{(c_L^\mu)^2 - (c_R^\mu)^2}{(c_L^\mu)^2 + (c_R^\mu)^2} \right]$$

- ★ This can be written as

$$A_{\text{FB}} = \frac{3}{4} A_e A_\mu$$

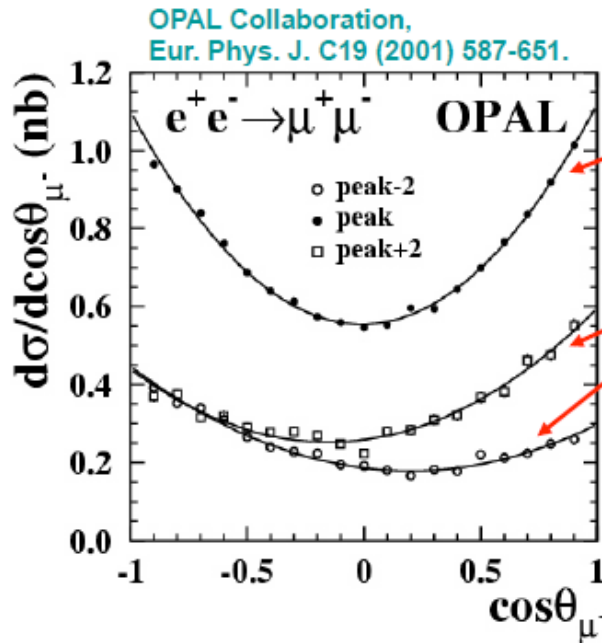
with

$$A_f \equiv \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} \quad (4)$$

- ★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

- ★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$



Because $\sin^2\theta_w \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ leads to a larger asymmetry

★ LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$

$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$

$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

- ★ To relate these measurements to the couplings uses $A_{FB} = \frac{3}{4}A_e A_\mu$
- ★ In all cases asymmetries depend on A_e
- ★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4}A_e^2$ (also see Appendix II for A_{LR})

Determination of the Weak Mixing Angle

- ★ From LEP : $A_{FB}^{0,f} = \frac{3}{4}A_e A_f$
 - ★ From SLC : $A_{LR} = A_e$
- $\left. \vphantom{\begin{matrix} A_{FB}^{0,f} \\ A_{LR} \end{matrix}} \right\} A_e, A_\mu, A_\tau, \dots$

Putting everything together →

$$\begin{aligned} A_e &= 0.1514 \pm 0.0019 \\ A_\mu &= 0.1456 \pm 0.0091 \\ A_\tau &= 0.1449 \pm 0.0040 \end{aligned}$$

includes results from other measurements

with $A_f \equiv \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2 \frac{c_V/c_A}{1 + (c_V/c_A)^2}$

- ★ Measured asymmetries give ratio of vector to axial-vector **Z** couplings.
- ★ In SM these are related to the weak mixing angle

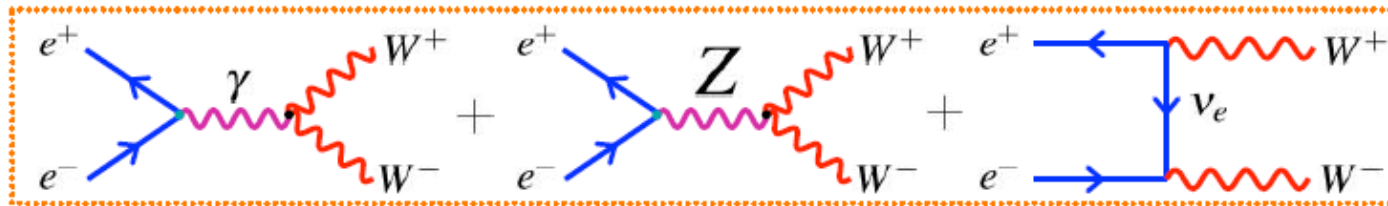
$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q \sin^2 \theta_W}{I_W^3} = 1 - \frac{2Q}{I_3} \sin^2 \theta_W = 1 - 4|Q| \sin^2 \theta_W$$

- ★ Asymmetry measurements give precise determination of $\sin^2 \theta_W$

$$\sin^2 \theta_W = 0.23154 \pm 0.00016$$

W⁺W⁻ Production

- ★ From 1995-2000 LEP operated above the threshold for W-pair production
- ★ Three diagrams “CC03” are involved

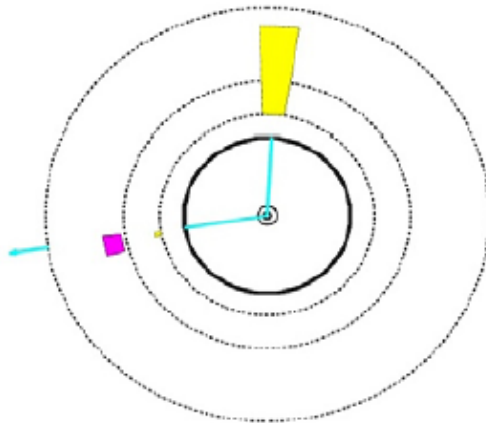


- ★ W bosons decay either to leptons or hadrons with branching fractions:

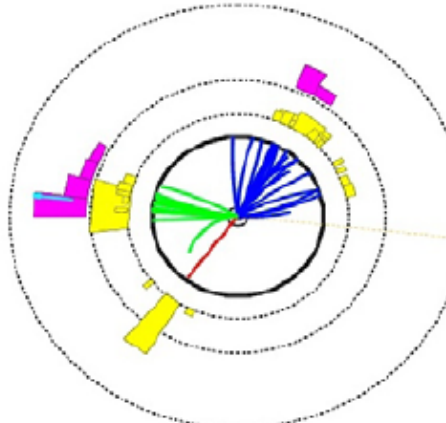
$$Br(W^- \rightarrow \text{hadrons}) \approx 0.67 \quad Br(W^- \rightarrow e^- \bar{\nu}_e) \approx 0.11$$

$$Br(W^- \rightarrow \mu^- \bar{\nu}_\mu) \approx 0.11 \quad Br(W^- \rightarrow \tau^- \bar{\nu}_\tau) \approx 0.11$$

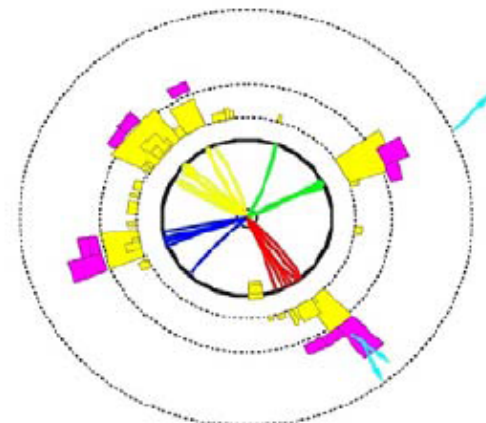
- ★ Gives rise to three **distinct topologies**



$$W^+W^- \rightarrow l^+ \nu l^- \bar{\nu}$$



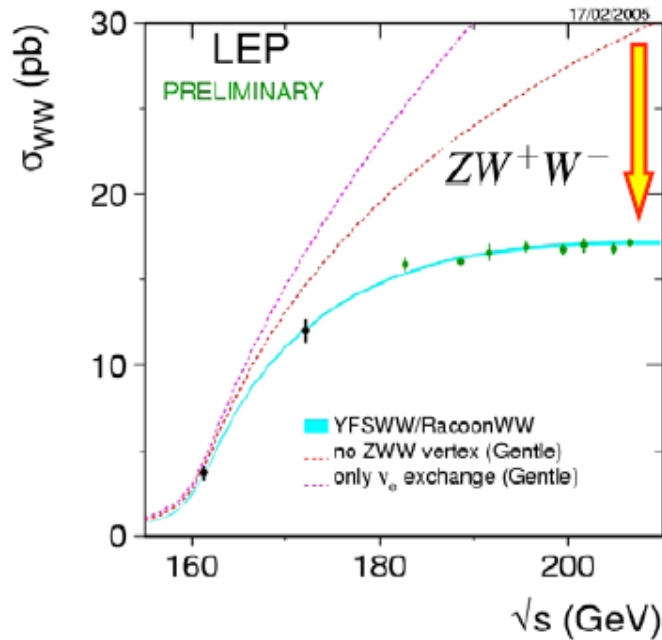
$$W^+W^- \rightarrow q\bar{q}l\nu$$



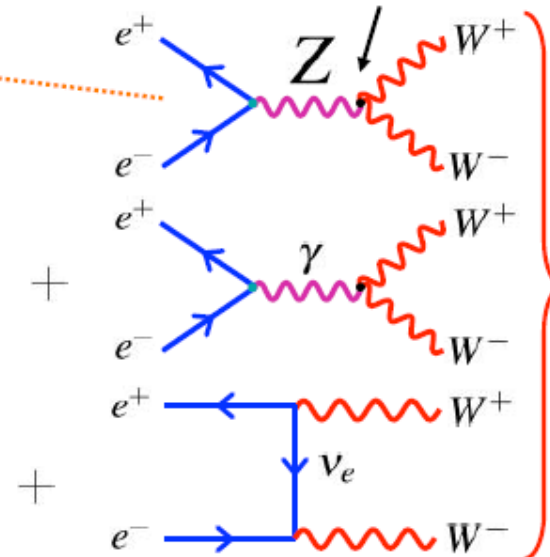
$$W^+W^- \rightarrow q\bar{q}q\bar{q}$$

$e^+e^- \rightarrow W^+ W^-$ Cross Section

- ★ Measure cross sections by counting events and normalising to low angle Bhabha scattering events



- ★ Data consistent with SM expectation
- ★ Provides a direct test of ZW^+W^- vertex

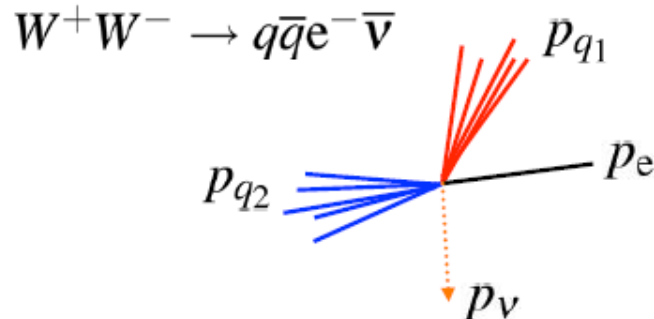


- ★ Recall that without the Z diagram the cross section violates unitarity
- ★ Presence of Z fixes this problem

W-mass and W-width

- ★ Unlike $e^+e^- \rightarrow Z$, the process $e^+e^- \rightarrow W^+W^-$ is not a resonant process
 \Rightarrow Different method to measure W-boson Mass

- Measure energy and momenta of particles produced in the W boson decays, e.g.



- Neutrino four-momentum from energy-momentum conservation !

$$p_{q1} + p_{q2} + p_e + p_\nu = (\sqrt{s}, 0)$$

- Reconstruct masses of two W bosons

$$M_+^2 = E^2 - \vec{p}^2 = (p_{q1} + p_{q2})^2$$

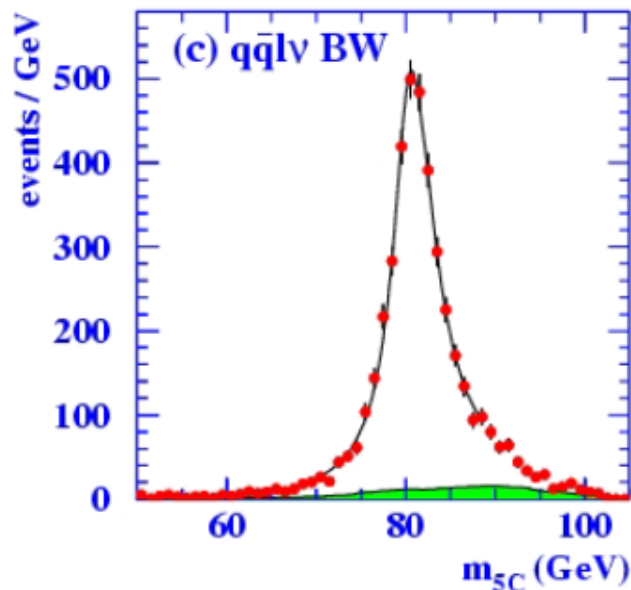
$$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$

- ★ Peak of reconstructed mass distribution gives

$$m_W = 80.376 \pm 0.033 \text{ GeV}$$

- ★ Width of reconstructed mass distribution gives:

$$\Gamma_W = 2.196 \pm 0.083 \text{ GeV}$$



$$\approx \frac{1}{2}(M_+ + M_-)$$

Does not include measurements from Tevatron at Fermilab

Does not include measurements ATLAS at LHC

The Higgs mechanism

- ★ The Higgs mechanism provides a way of giving the gauge bosons mass

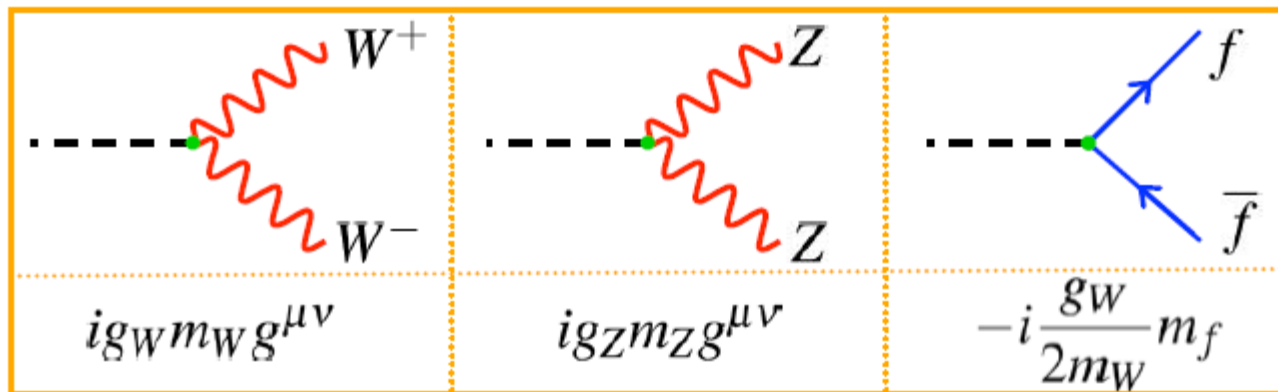
The Higgs Mechanism

- ★ Propose a scalar (spin 0) field with a **non-zero vacuum expectation value (VEV)**

Massless Gauge Bosons propagating through the vacuum with a non-zero Higgs VEV correspond to massive particles.

- ★ The Higgs is **electrically neutral** but carries **weak hypercharge of $1/2$**
- ★ The photon does not couple to the Higgs field and remains massless
- ★ The W bosons and the Z couple to weak hypercharge and become massive
- ★ The Higgs mechanism results in absolute predictions for masses of gauge bosons
- ★ In the SM, fermion masses are also ascribed to interactions with the Higgs field
 - however, here no prediction of the masses – just put in by hand

Feynman Vertex factors:



★ Within the SM of Electroweak unification with the Higgs mechanism:



Relations between standard model parameters

$$m_W = \left(\frac{\pi \alpha_{em}}{\sqrt{2} G_F} \right)^{\frac{1}{2}} \frac{1}{\sin \theta_W}$$

$$m_Z = \frac{m_W}{\cos \theta_W}$$

★ Hence, if you know any three of : $\alpha_{em}, G_F, m_W, m_Z, \sin \theta_W$ predict the other two.

Precision Tests of the Standard Model

- ★ From LEP and elsewhere have precise measurements – can test predictions of the Standard Model !

•e.g. predict: $m_W = m_Z \cos \theta_W$

measure

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\sin^2 \theta_W = 0.23154 \pm 0.00016$$

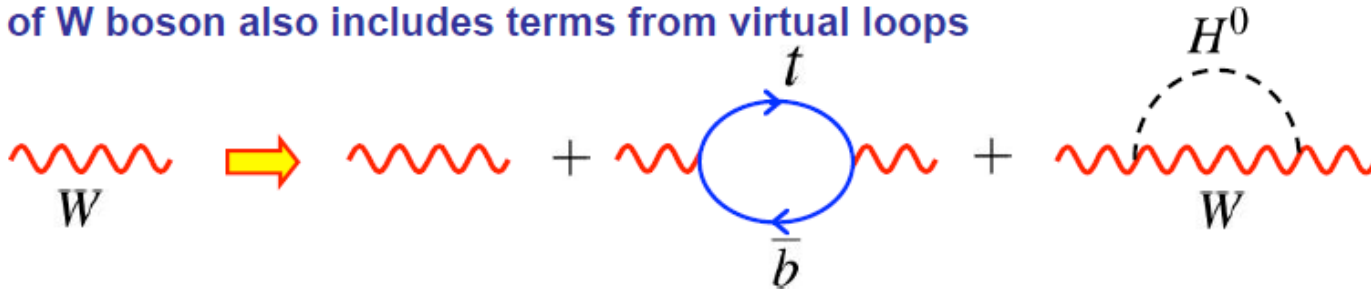
- Therefore expect:

$$m_W = 79.946 \pm 0.008 \text{ GeV}$$

but
measure

$$m_W = 80.376 \pm 0.033 \text{ GeV}$$

- ★ Close, but not quite right – but have only considered lowest order diagrams
- ★ Mass of W boson also includes terms from virtual loops



$$m_W = m_W^0 + am_t^2 + b \ln \left(\frac{m_H}{m_W} \right)$$

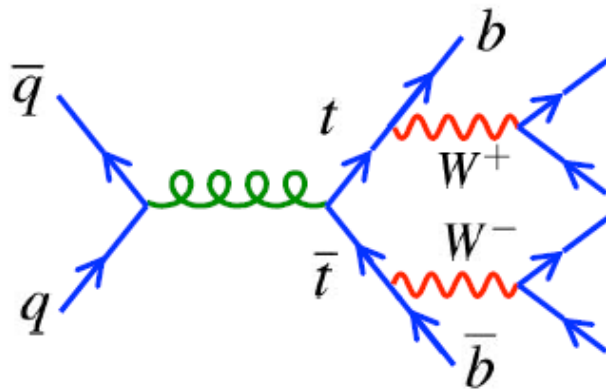
- ★ Above “discrepancy” due to these virtual loops, i.e. by making very high precision measurements become sensitive to the masses of particles inside the virtual loops !

The Top quark

- ★ From virtual loop corrections and precise LEP data can predict the top quark mass:

$$m_t^{\text{loop}} = 173 \pm 11 \text{ GeV}$$

- ★ In 1994 top quark observed at the Tevatron proton anti-proton collider at Fermilab
– with the predicted mass !



- ★ The top quark almost exclusively decays to a bottom quark since

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

- ★ Complicated final state topologies:

$$t\bar{t} \rightarrow b\bar{b}q\bar{q}q\bar{q} \rightarrow 6 \text{ jets}$$

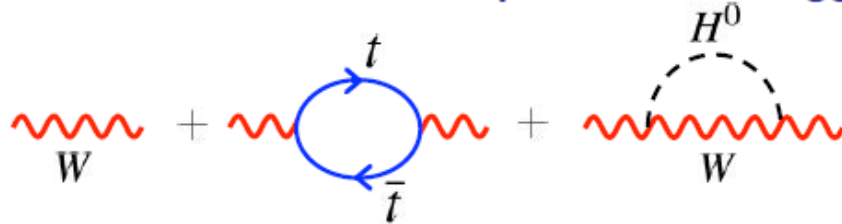
$$t\bar{t} \rightarrow b\bar{b}q\bar{q}\ell\nu \rightarrow 4 \text{ jets} + \ell + \nu$$

$$t\bar{t} \rightarrow b\bar{b}\ell\nu\ell\nu \rightarrow 2 \text{ jets} + 2\ell + 2\nu$$

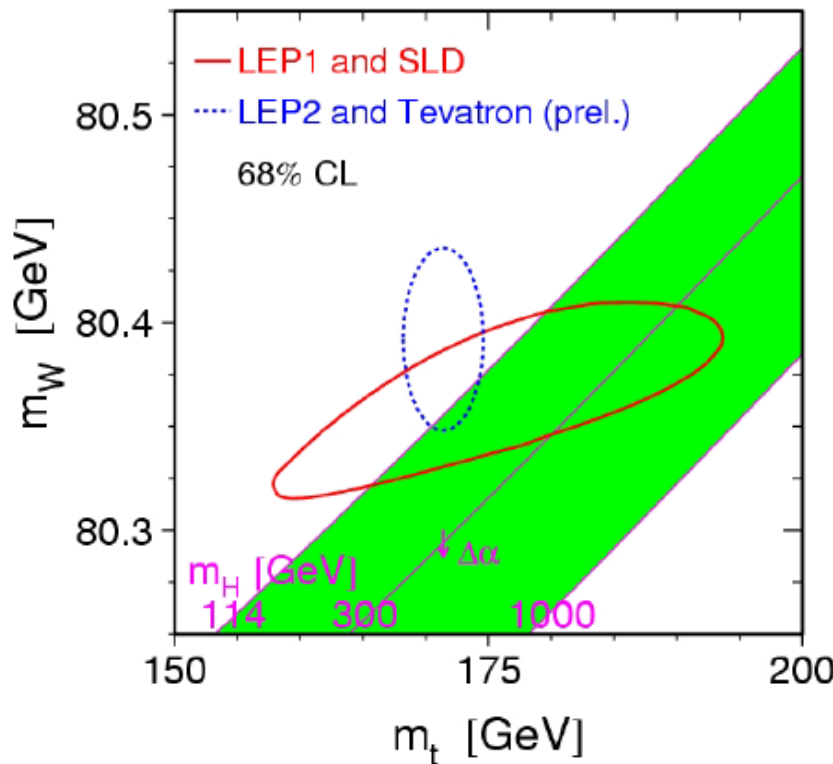
- ★ Mass determined by direct reconstruction (see W boson mass)

$$m_t^{\text{meas}} = 174.2 \pm 3.3 \text{ GeV}$$

★ But the W mass also depends on the Higgs mass (albeit only logarithmically)



$$m_W = m_W^0 + am_t^2 + b \ln \left(\frac{m_H}{m_W} \right)$$



★ Measurements are sufficiently precise to have some sensitivity to the Higgs mass

★ Direct and indirect values of the top and W mass can be compared to prediction for different Higgs mass

- Direct: W and top masses from direct reconstruction
- Indirect: from SM interpretation of Z mass, θ_W etc. and

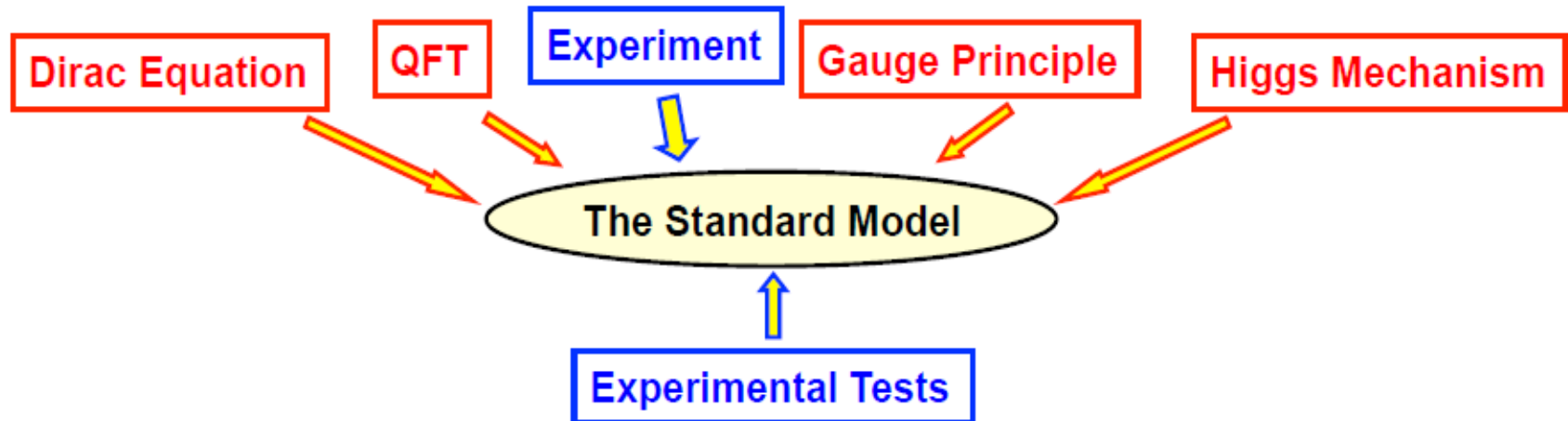
★ Data favour a light Higgs:



$$m_H < 200 \text{ GeV}$$

Summary

- ★ The Standard Model of Particle Physics is one of the great scientific triumphs of the late 20th century
- ★ Developed through close interplay of experiment and theory



- ★ Modern experimental particle physics provides many precise measurements. and the **Standard Model** **successfully describes all current data !**
- ★ Despite its great success, we should not forget that it is just a model; a collection of beautiful theoretical ideas cobbled together to fit with experimental data.
- ★ There are many issues / open questions...

Appendix I: Non-relativistic Breit-Wigner

★ For energies close to the peak of the resonance, can write $\sqrt{s} = m_Z + \Delta$

$$s = m_Z^2 + 2m_Z\Delta + \Delta^2 \approx m_Z^2 + 2m_Z\Delta \quad \text{for } \Delta \ll m_Z$$

so with this approximation

$$\begin{aligned} (s - m_Z^2)^2 + m_Z^2\Gamma_Z^2 &\approx (2m_Z\Delta)^2 + m_Z^2\Gamma_Z^2 = 4m_Z^2(\Delta + \frac{1}{4}\Gamma_Z^2) \\ &= 4m_Z^2[(\sqrt{s} - m_Z)^2 + \frac{1}{4}\Gamma_Z^2] \end{aligned}$$

★ Giving: $\sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) \approx \frac{3\pi}{m_Z^4} \frac{s}{(\sqrt{s} - m_Z)^2 + \frac{1}{4}\Gamma_Z^2} \Gamma_e\Gamma_f$

★ Which can be written:

$$\sigma(E) = \frac{g\lambda_e^2}{4\pi} \frac{\Gamma_i\Gamma_f}{(E - E_0)^2 + \frac{1}{4}\Gamma^2} \quad (3)$$

Γ_i, Γ_f : are the partial decay widths of the initial and final states

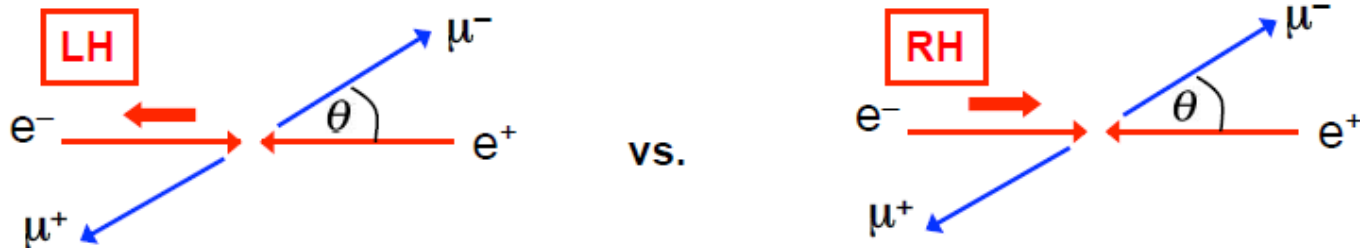
E, E_0 : are the centre-of-mass energy and the energy of the resonance

$g = \frac{(2J_Z+1)}{(2S_e+1)(2S_e+1)}$ is the spin counting factor $g = \frac{3}{2 \times 2}$

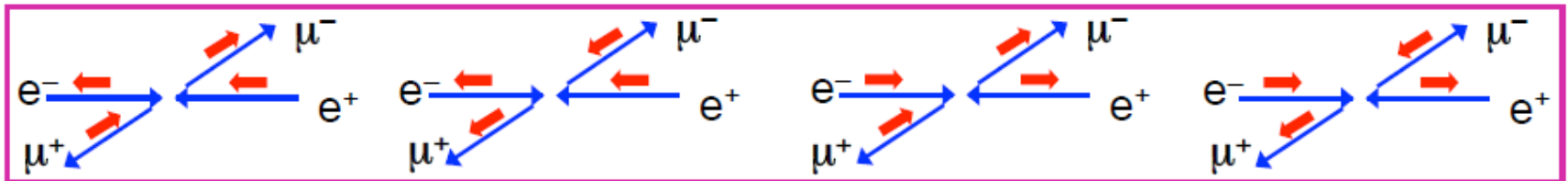
$\lambda_e = \frac{2\pi}{E}$: is the Compton wavelength (natural units) in the C.o.M of either initial particle

Appendix II: Left-Right Asymmetry, A_L , A_R

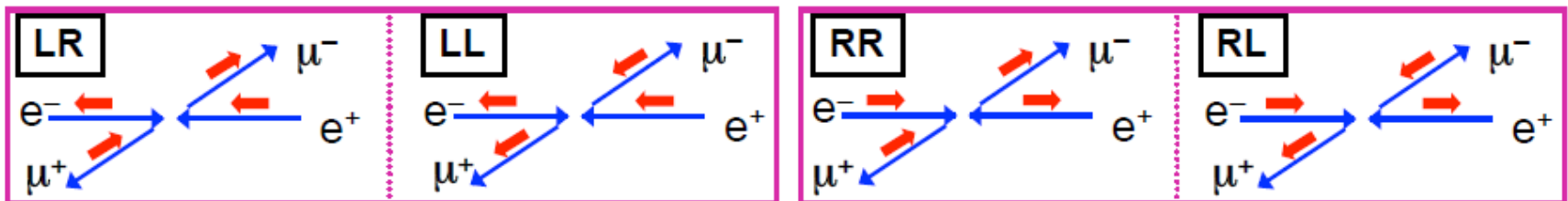
- ★ At an e^+e^- linear collider it is possible to produce polarized electron beams
e.g. SLC linear collider at SLAC (California), 1989-2000
- ★ Measure cross section for any process for **LH** and **RH** electrons separately



- At LEP measure total cross section: sum of 4 helicity combinations:



- At SLC, by choosing the polarization of the electron beam are able to measure cross sections separately for **LH / RH** electrons



- ★ Averaging over the two possible polarization states of the positron for a given electron polarization:

$$\langle |M_L|^2 \rangle = \frac{1}{2}(|M_{LL}|^2 + |M_{LR}|^2) \quad \langle |M_R|^2 \rangle = \frac{1}{2}(|M_{RL}|^2 + |M_{RR}|^2)$$

⇒ $\sigma_L = \frac{1}{2}(\sigma_{LR} + \sigma_{LL})$

$\sigma_R = \frac{1}{2}(\sigma_{RR} + \sigma_{RL})$

- ★ Define cross section asymmetry:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

- ★ Integrating the expressions on page 494 gives:

$$\sigma_{LL} \propto (c_L^e)^2 (c_L^\mu)^2 \quad \sigma_{LR} \propto (c_L^e)^2 (c_R^\mu)^2 \quad \sigma_{RL} \propto (c_R^e)^2 (c_L^\mu)^2 \quad \sigma_{RR} \propto (c_R^e)^2 (c_R^\mu)^2$$

⇒ $\sigma_L \propto (c_L^e)^2 [(c_L^\mu)^2 + (c_R^\mu)^2] \quad \sigma_R \propto (c_R^e)^2 [(c_L^\mu)^2 + (c_R^\mu)^2]$

$$A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e$$

- ★ Hence the Left-Right asymmetry for any cross section depends only on the couplings of the electron