DATA SCIENCE WITH MACHINE LEARNING: RETRIEVAL

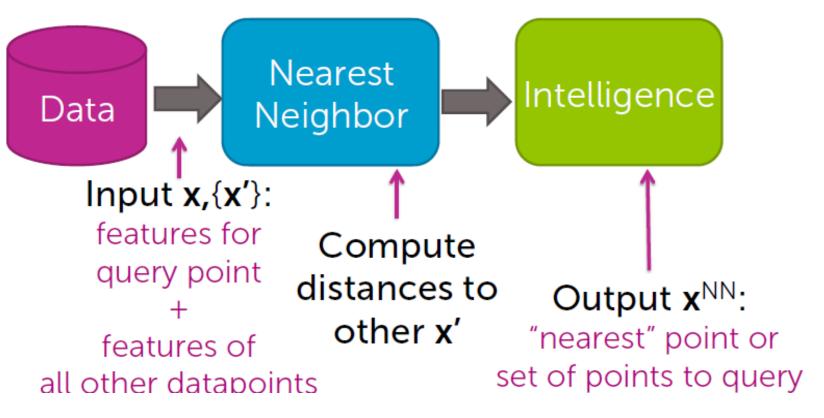
This lecture is based on course by E. Fox and C. Guestrin, Univ of Washington

25/01/2024

WFAiS UJ, Informatyka Stosowana I stopień studiów

What is retrieval?

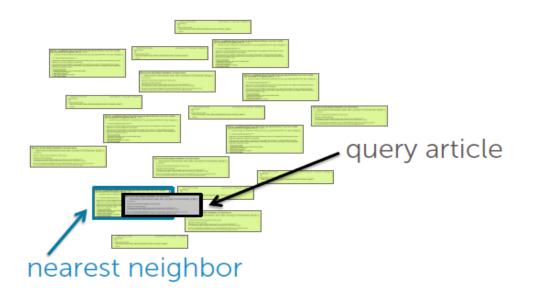
Search for related items



What is retrieval?

Retrieve "nearest neighbor" article

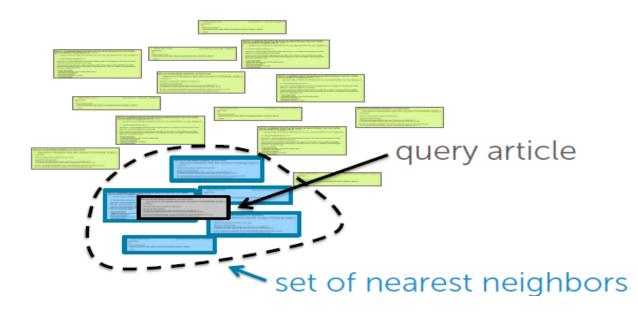
Space of all articles, organized by similarity of text



What is retrieval?

Or set of nearest neighbors

Space of all articles, organized by similarity of text



Retrieval applications

Just about everything...

Images



Products



Streaming content:

- Songs
- Movies

...

- TV shows



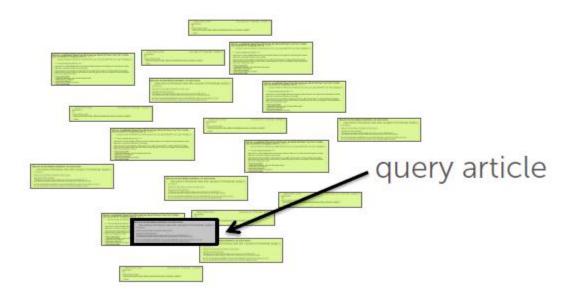
News articles

Social networks (people you might want to connect with)

Retrieval as k-nearest neighbor search

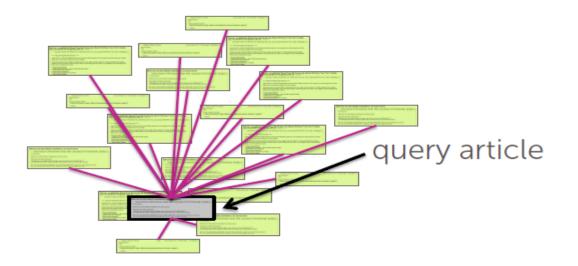
6

Space of all articles, organized by similarity of text



Compute distances to all docs

Space of all articles, organized by similarity of text



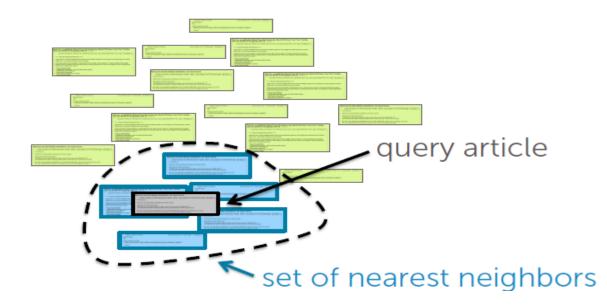
Retrieve "nearest neighbor"

Space of all articles, organized by similarity of text



Or set of nearest neighbors

Space of all articles, organized by similarity of text

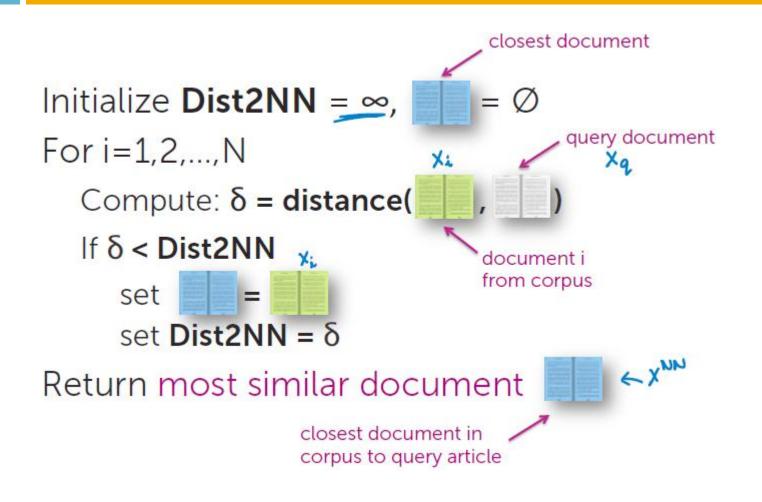


1-NN algorithm

1 – Nearest neighbor

 Input: Query article : X_q Corpus of documents (N docs)
 Most similar article : X₁, X₂, ..., X_N

1-NN algorithm



k-NN algorithm

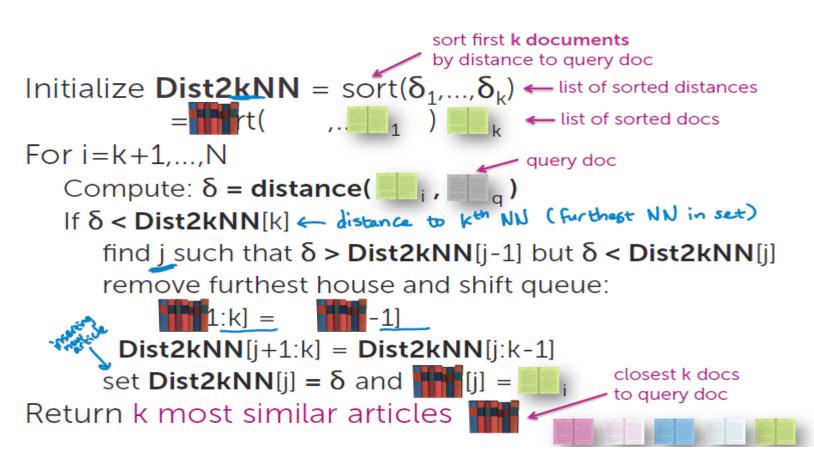
Input: Query article : x_q
 Corpus of documents

: **x**₁, **x**₂, ..., **x**_N

• **Output:** *List of k* similar articles

For all X: not in XNN, distance (X:, Xq) = max distance (XNNj, Xq) For all X: not in XNN, distance (X:, Xq) = max distance (XNNj, Xq)

k-NN algorithm



Critical elements of NN search

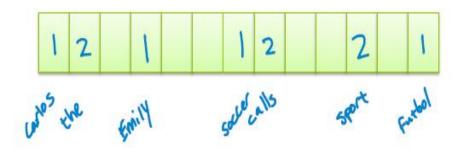
Item (e.g., doc) representation $\mathbf{x}_{q} \leftarrow$

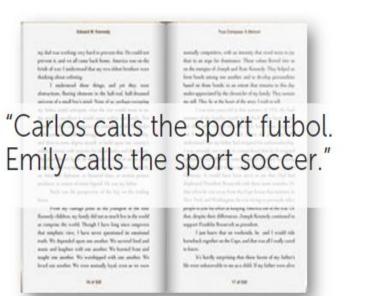
Measure of distance between items:

 $\delta = distance(\mathbf{x}_i, \mathbf{x}_q)$

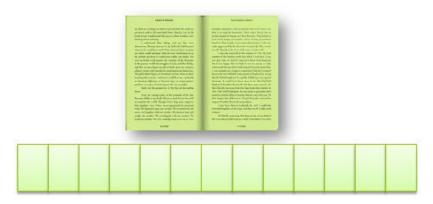
Bag of words model

- Ignore order of words
- Count # of instances of each word in vocabulary





Issues with word counts – Rare words



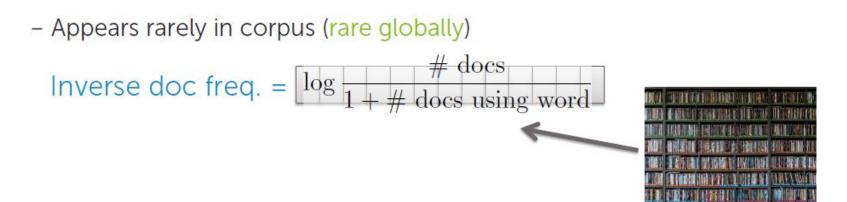
Common words in doc: "the", "player", "field", "goal" Dominate rare words like: "futbol", "Messi"

TF-IDF document representation

Emphasizes important words

- Appears frequently in document (common locally)

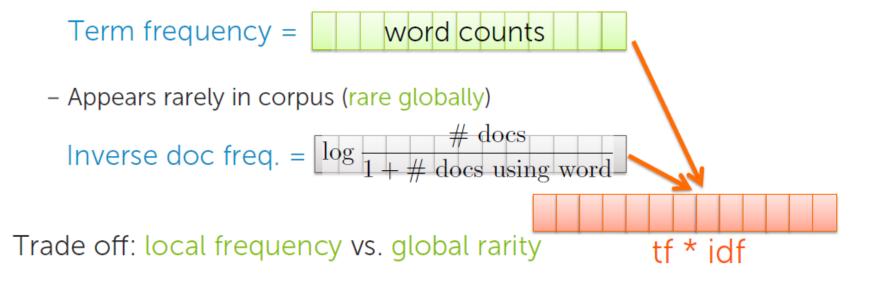
Term frequency = word counts



TF-IDF document representation

Emphasizes important words

- Appears frequently in document (common locally)



Distance metrics: Defining notion of "closest"

In 1D, just Euclidean distance:

distance
$$(x_i, x_q) = |x_i - x_q|$$

In multiple dimensions:

- can define many interesting distance functions
- most straightforwardly, might want to weight different dimensions differently

Weighting different features

Reasons:

- Some features are more relevant than others



bedrooms
bathrooms
sq.ft. living
sq.ft. lot
floors
year built
year renovated
waterfront



Weighting different features

Reasons:

- Some features are more relevant than others

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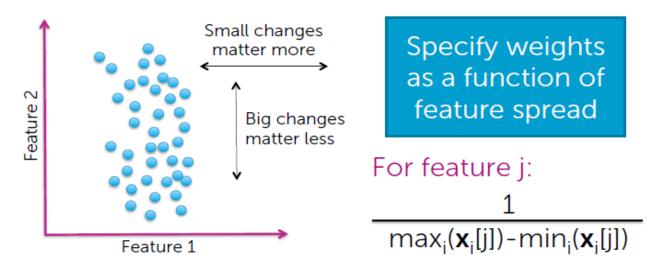
title abstract main body conclusion



Weighting different features

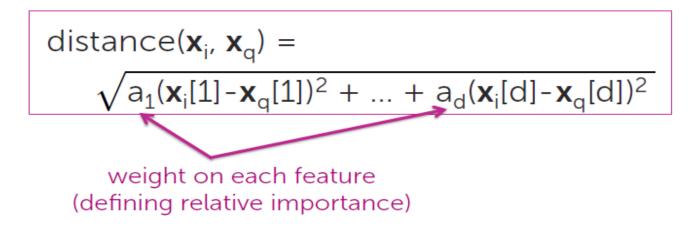
Reasons:

- Some features are more relevant than others
- Some features vary more than others

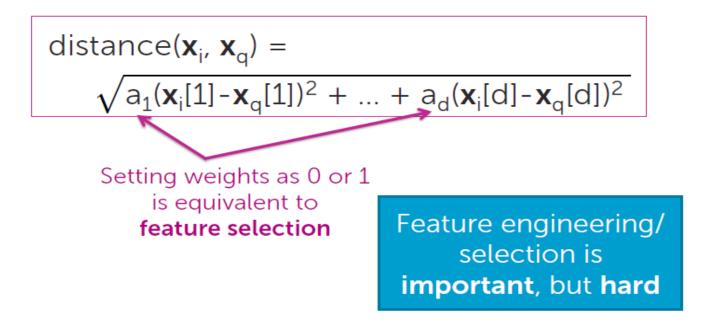


Scaled Euclidean distance

Formally, this is achieved via



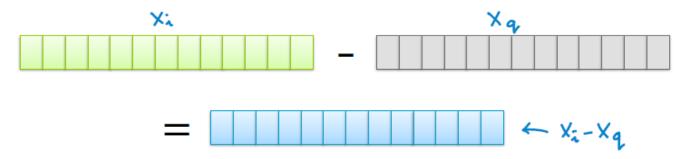
Effect of binary weights



(non-scaled) Euclidean distance

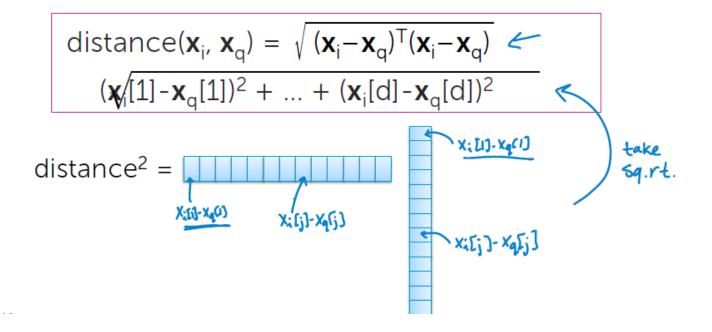
Defined in terms of inner product

distance(
$$\mathbf{x}_{i}, \mathbf{x}_{q}$$
) = $\sqrt{(\mathbf{x}_{i} - \mathbf{x}_{q})^{T}(\mathbf{x}_{i} - \mathbf{x}_{q})}$
(\mathbf{x}_{i} [1]- \mathbf{x}_{q} [1])² + ... + (\mathbf{x}_{i} [d]- \mathbf{x}_{q} [d])²



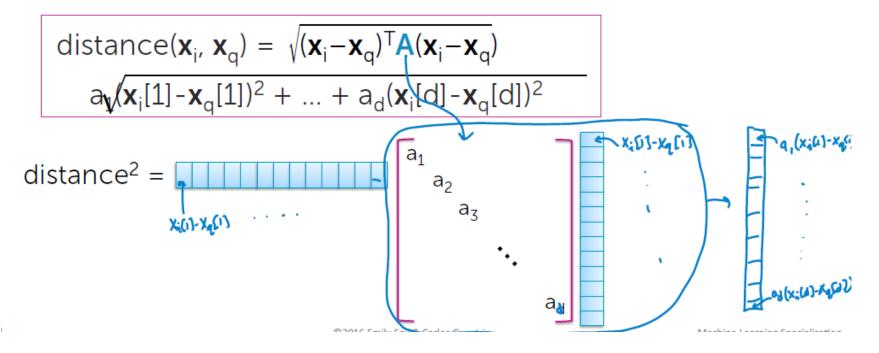
(non-scaled) Euclidean distance

Defined in terms of inner product

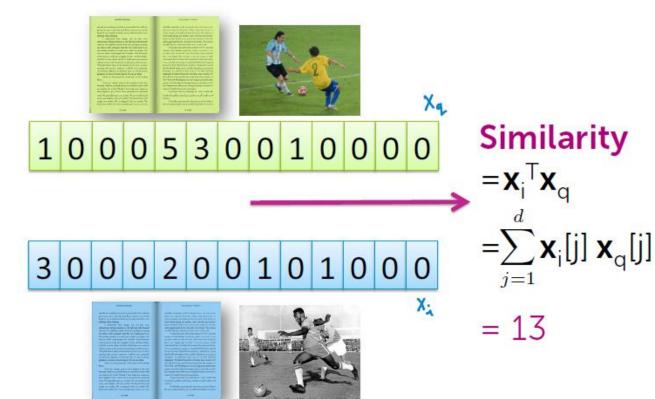


Scaled Euclidean distance

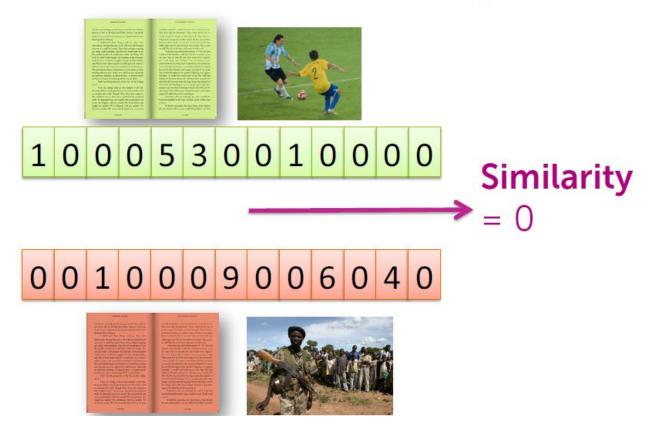
Defined in terms of inner product

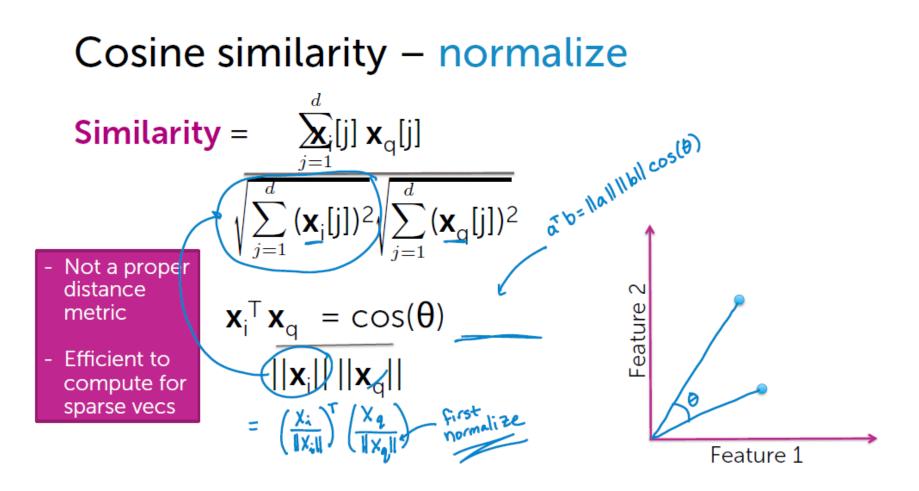


Another natural inner product measure



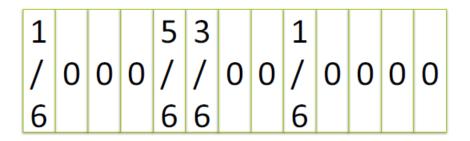
Another natural inner product measure

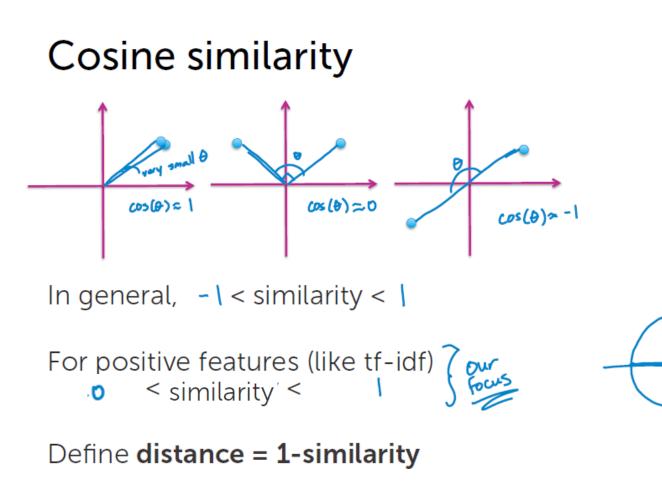




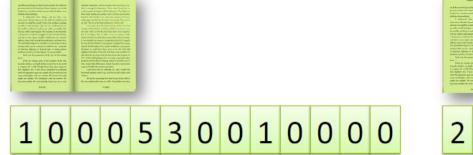
Normalize

$\frac{1 \ 0 \ 0 \ 5 \ 3 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ \leftarrow \times;}{\sqrt{(1^2 + 5^2 + 3^2 + 1^2)}} \leftarrow \|\chi_i\|_{=} \sum_{j=1}^{d} \chi_i L_j \int_{-\infty}^{2} |\chi_i|_{=} \sum_{j=1}^{d} \chi_j L_j \int_{-\infty}^{2} |\chi_i|_{=} \sum_{j=1}^{d} |\chi_j|_{=} \sum_{j=1}^{d} \chi_j L_j \int_{-\infty}^{2} |\chi_j|_{=} \sum_{j=1}^{d} |\chi_j|_{$

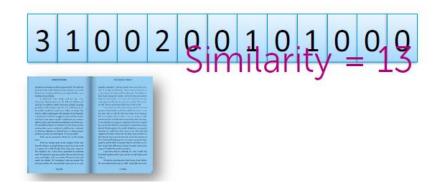


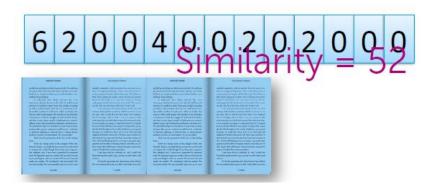


To normalize or not?

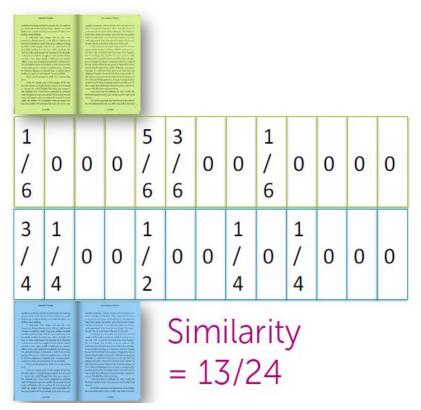


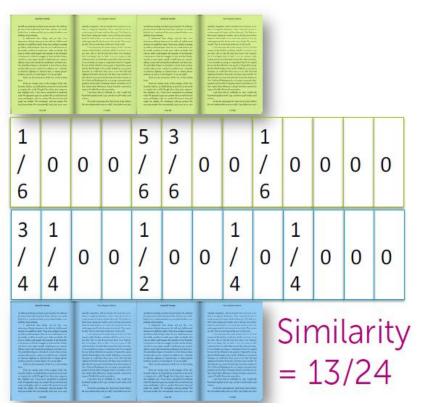




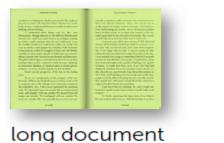


In the normalized case





But not always desired...



short tweet

Normalizing can make dissimilar objects appear more similar

<text><text><text><text><text><text><text><text><text><text>

long document



long document

Common compromise: Just cap maximum word counts

Distance metrics

Other distance metrics

- Mahalanobis
- rank-based
- correlation-based
- Manhattan
- Jaccard
- Hamming

- ...

Combining distance metrics

Example of document features:

- 1. Text of document
 - Distance metric: Cosine similarity
- 2. # of reads of doc
 - Distance metric: Euclidean distance

Add together with user-specified weights

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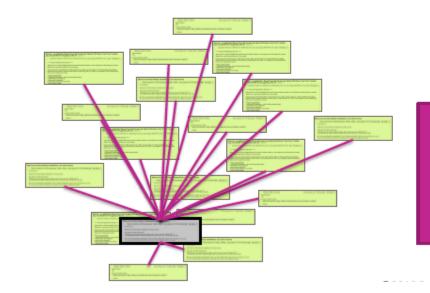
Scaling up k-NN search by storing data in a KD-tree

Complexity of brute-force search

40

Given a query point, scan through each point – O(N) distance computations per 1-NN query!

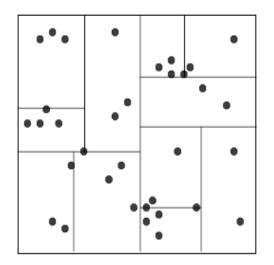
– O(Nlogk) per k-NN query!



What if *N* is huge??? (and many queries)

Structured organization of documents

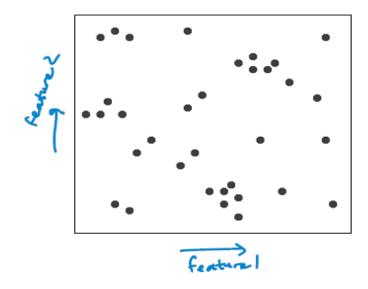
- Recursively partitions points into axis aligned boxes.
- Enables more efficient pruning of search space



Works "well" in "low-medium" dimensions

- We'll get back to this...

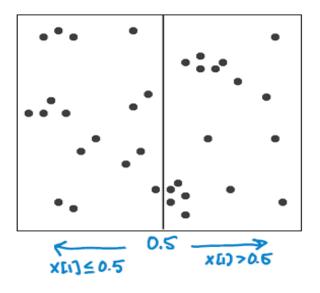
KD-tree construction

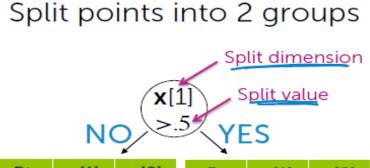


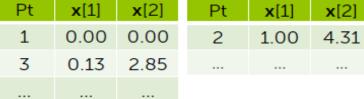
Start with a list of d-dimensional points.

Pt x[1] x[2] 1 0.00 0.00 2 1.00 4.31 3 0.13 2.85	obs. indices		feat. 1 (word 1)	Feat. The word	L 12)
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1 0.00 0.00		3	0.13	2.85	
		2	1.00	4.31	
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D1 (41 (01		Pt	x [1]	x [2]	

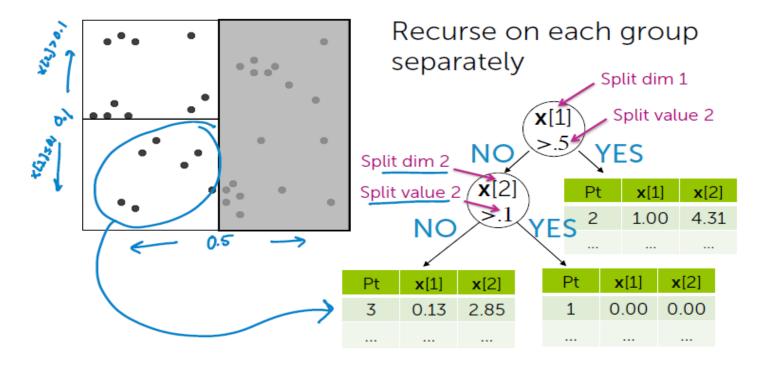
KD-tree construction



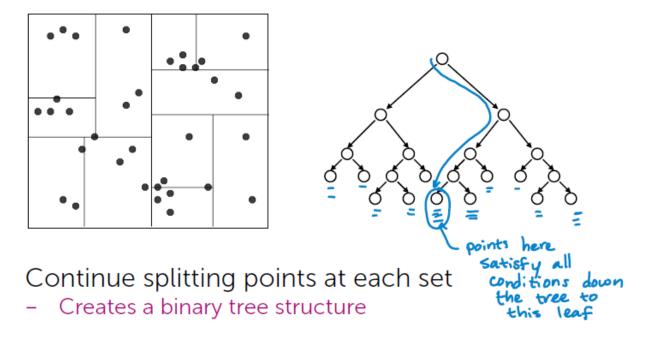




KD-tree construction

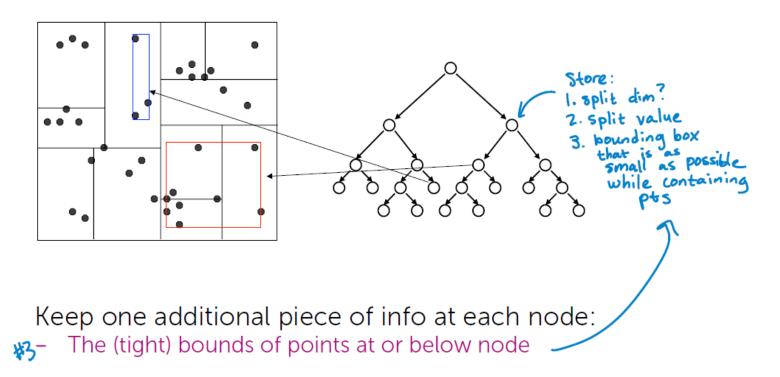


KD-tree construction



Each leaf node contains a list of points

KD-tree construction

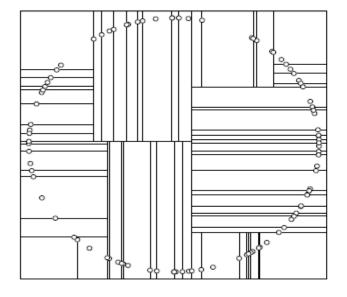


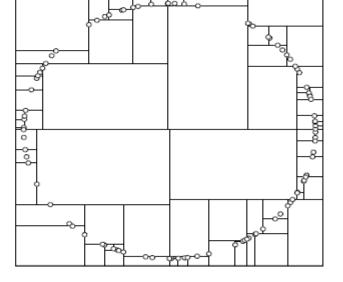
KD-tree construction choices

Use heuristics to make splitting decisions: – Which dimension do we split along? widest (or alternate) - Which value do we split at? median (or center point of box, ignoring data in box) - When do we stop? Fewer than m pts left or box hits minimum width



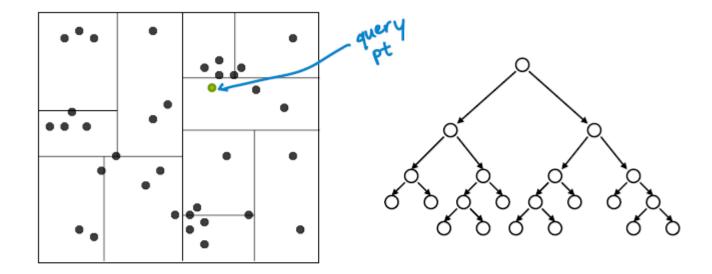
Many heuristics...



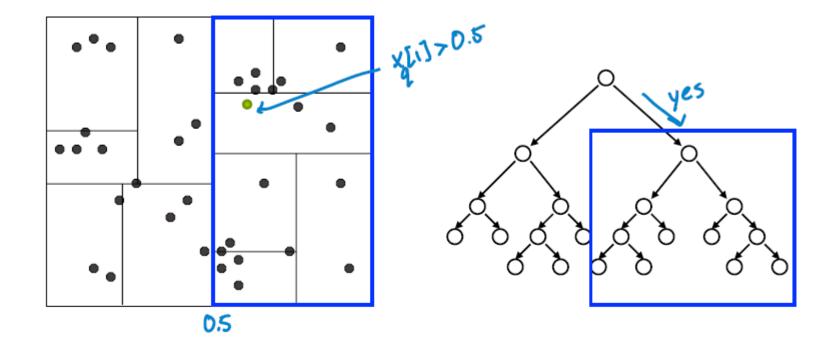


median heuristic

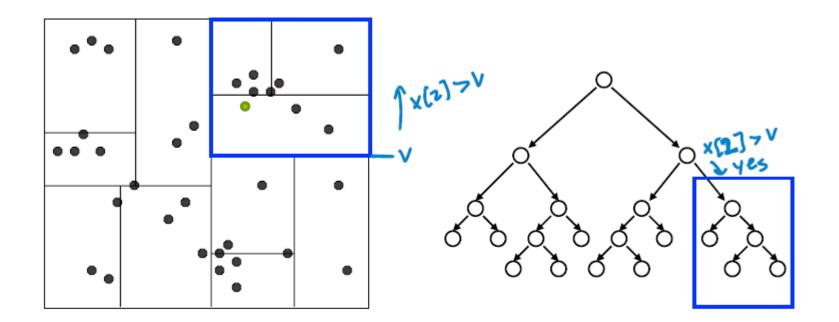
center-of-range heuristic



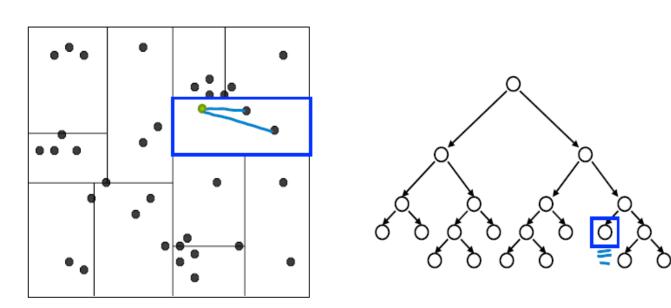
Traverse tree looking for nearest neighbor to query point



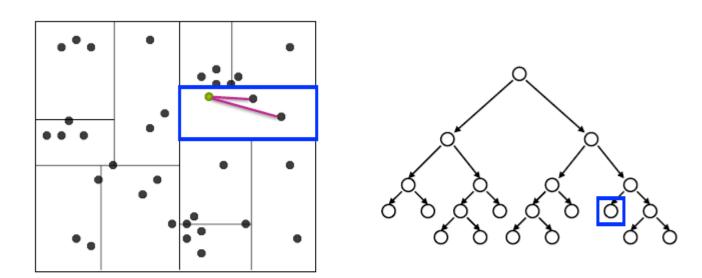
1. Start by exploring leaf node containing query point



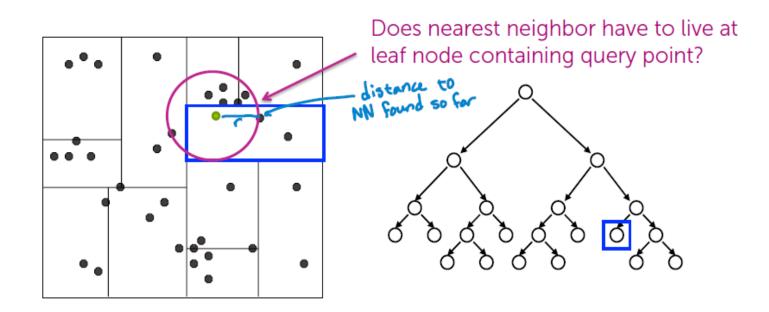
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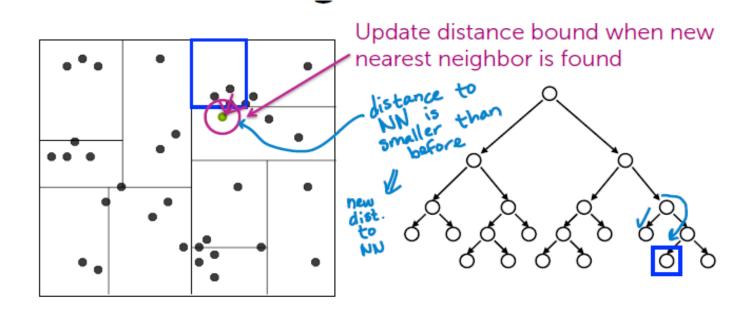


Start by exploring leaf node containing query point
 Compute distance to each other point at leaf node

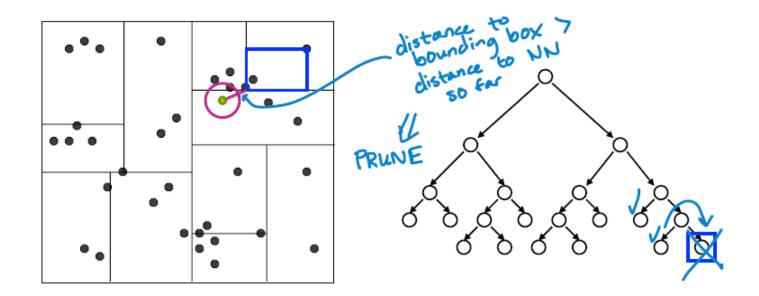


- 1. Start by exploring leaf node containing query point
- 2. Compute distance to each other point at leaf node

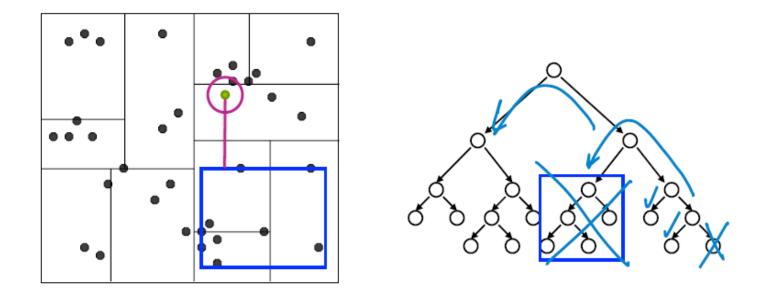




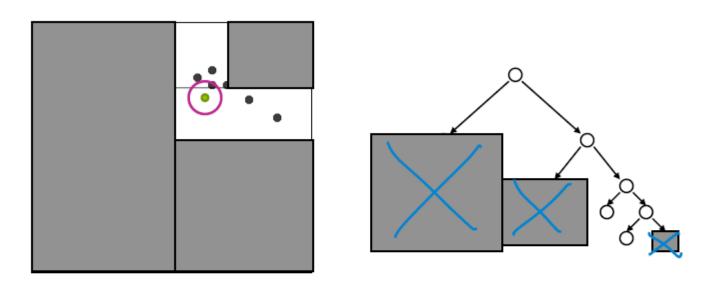
Start by exploring leaf node containing query point
 Compute distance to each other point at leaf node
 Backtrack and try other branch at each node visited



Use distance bound and bounding box of each node to prune parts of tree that cannot include nearest neighbor



Use distance bound and bounding box of each node to prune parts of tree that cannot include nearest neighbor



Use distance bound and bounding box of each node to prune parts of tree that cannot include nearest neighbor

Complexity

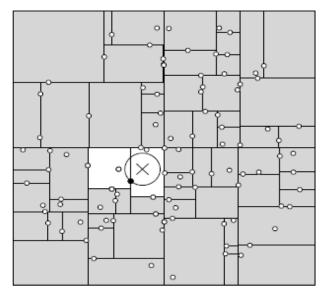


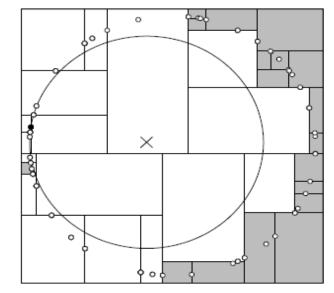
For (nearly) balanced, binary trees...

- Construction
 - Size: 2N-1 nodes if I datapt at each leaf -> O(N)
 - Depth: 0(10gN)
 - Median + send points left right: O(N) at every level of the tree
 - Construction time: O(Nlog N)
- 1-NN query
 - Traverse down tree to starting point: $O(\log N)$
 - Maximum backtrack and traverse: ((N) in worst case
 - Complexity range: $O(\log N) \rightarrow O(N)$

Under some assumptions on distribution of points, we get O(logN) but exponential in d

Complexity





pruned many (closer to O(log N))

pruned few (closer to O(N))

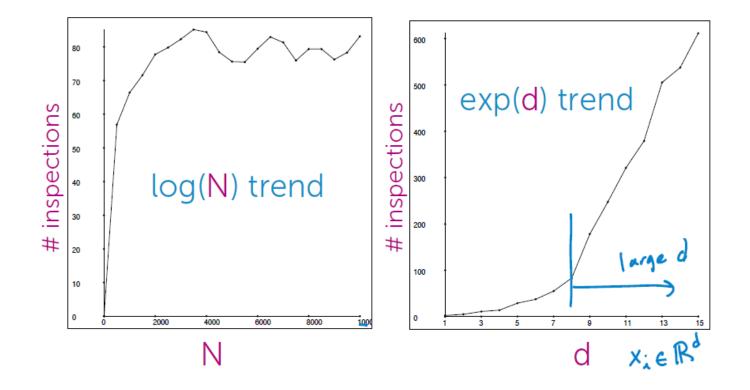
Complexity for N queries

- Ask for nearest neighbor to each doc
 N queries
- Brute force 1-NN:
 〇(い)
- kd-trees:

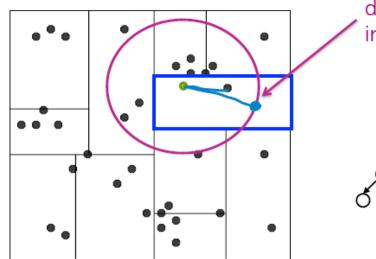
O(NlogN) -> O(N²) Potencially Very large Very large Savings For ! Jarge N!

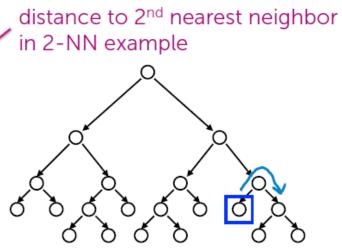
Complexity for N queries

Inspections vs. N and d



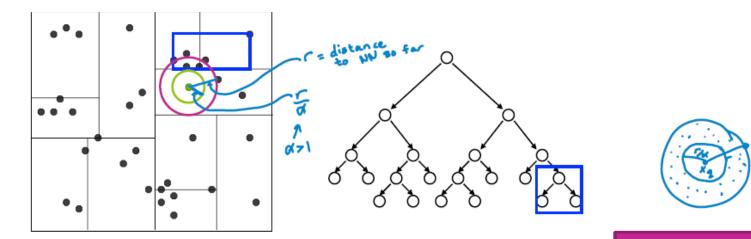
k-NN with KD-trees





Exactly same algorithm, but maintain distance to furthest of current *k* nearest neighbors

Approximate k-NN with KD-trees



Before: Prune when distance to bounding box > r **Now:** Prune when distance to bounding box > r/α

Prunes more than allowed, but can guarantee that if we return a neighbor at distance r, then there is no neighbor closer than r/α

Saves lots of search time at little cost in quality of NN!

Bound loose...In practice, often closer to optimal.

Closing remarks on KD-trees

Tons of variants of kd-trees

- On construction of trees (heuristics for splitting, stopping, representing branches...)
- Other representational data structures for fast NN search (e.g., ball trees,...)

Nearest Neighbor Search

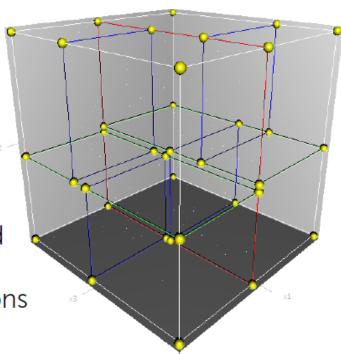
- Distance metric and data representation crucial to answer returned

For both, high-dim spaces are hard!

- Number of kd-tree searches can be exponential in dimension
 - Rule of thumb... $N >> 2^d$... Typically useless for large d.
- Distances sensitive to irrelevant features
 - Most dimensions are just noise \rightarrow everything is far away
 - Need technique to learn which features are important to given task

KD-tree in high dimmensions

- 66
- Unlikely to have any data points close to query point
- Once "nearby" point is found, the search radius is likely to intersect many hypercubes in at least one dim
- Not many nodes can be pruned
- Can show under some conditions that you visit at least 2^d nodes



Moving away from exact NN search

- Approximate neighbor finding...
 - Don't find exact neighbor, but that's okay for many applications

Out of millions of articles, do we need the closest article or just one that's pretty similar? Do we even fully trust our measure of similarity???

 Focus on methods that provide good probabilistic guarantees on approximation

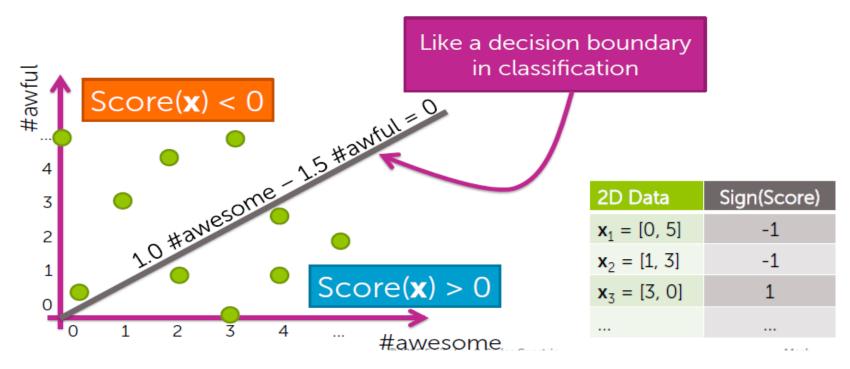
68

Locality Sensitive Hashing (LHS) as alternative to KD-trees

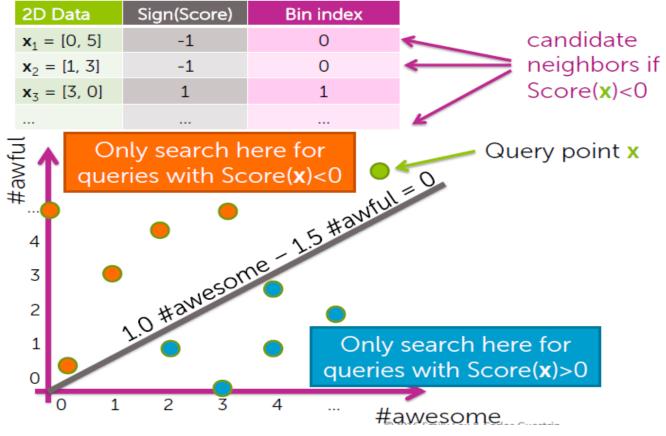
69

Simple "binning" of data into 2 bins

 $Score(\mathbf{x}) = 1.0$ #awesome -1.5#awful

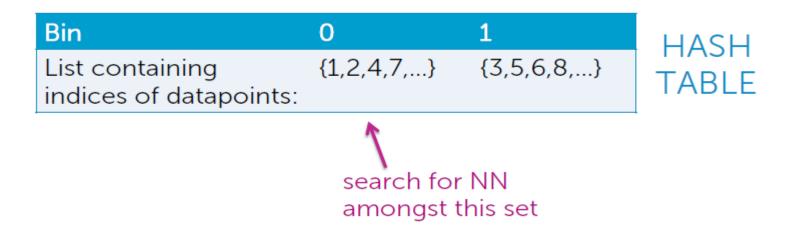


Using bins for NN search

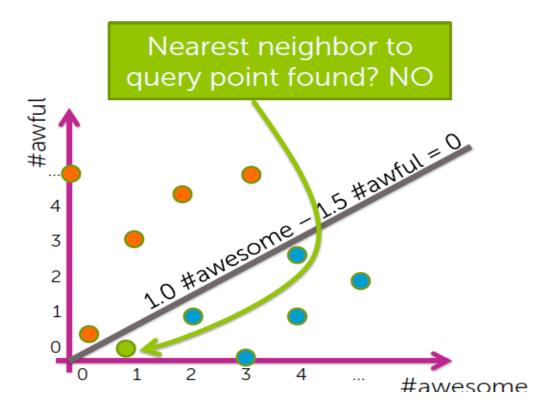


Using score for NN search

2D Data	Sign(Score)	Bin index		
x ₁ = [0, 5]	-1	0	5	candidate
x ₂ = [1, 3]	-1	0	\leftarrow	neighbors if
x ₃ = [3, 0]	1	1		Score(x)<0

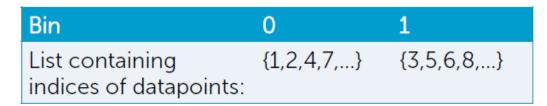


Provides approximate NN

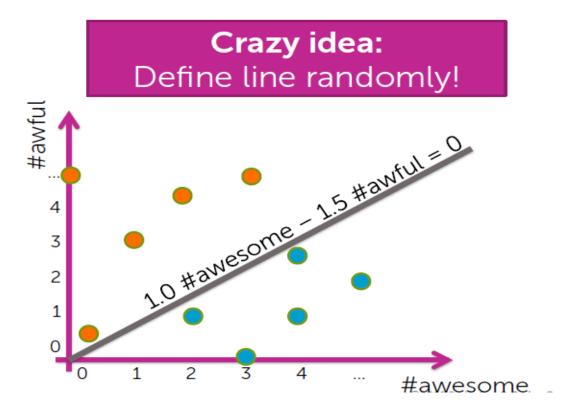


Three potential issues with simple approach

- 1. Challenging to find good line
- 2. Poor quality solution:
 - Points close together get split into separate bins
- 3. Large computational cost:
 - Bins might contain many points, so still searching over large set for each NN query

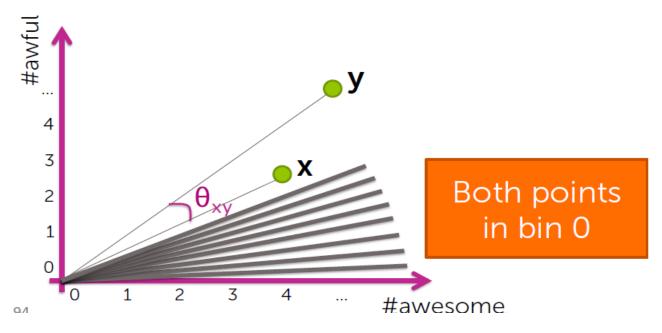


How to define the line?



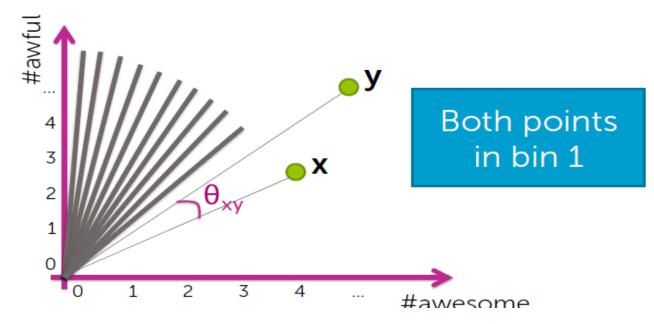
How bad can a random line be?

Goal: If **x**,**y** are close (according to cosine similarity), want binned values to be the same.



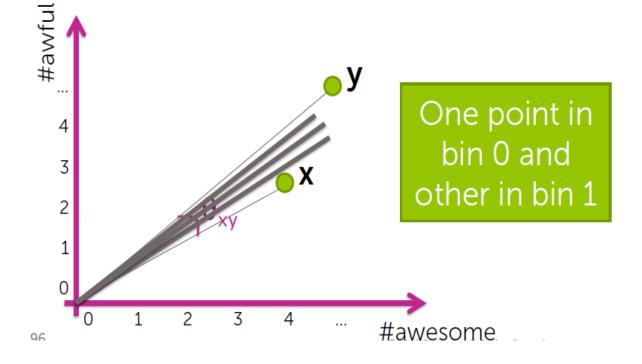
How bad can a random line be?

Goal: If **x**,**y** are close (according to cosine similarity), want binned values to be the same.



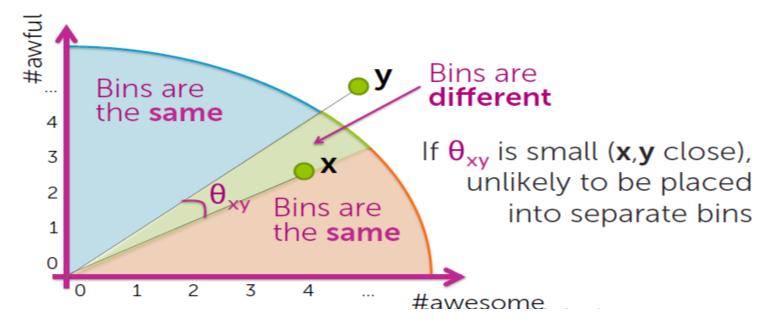
77

Goal: If **x**,**y** are close (according to cosine similarity), want binned values to be the same.



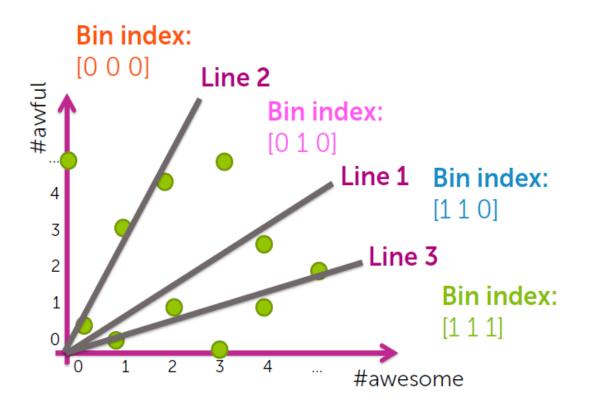
How bad can a random line be?

Goal: If **x**,**y** are close (according to cosine similarity), want binned values to be the same.



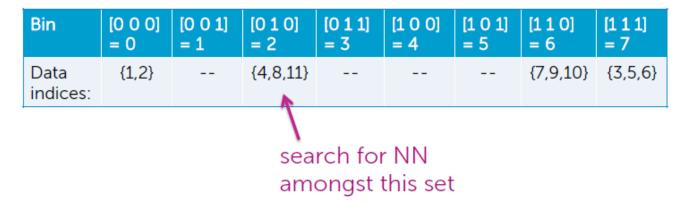
78

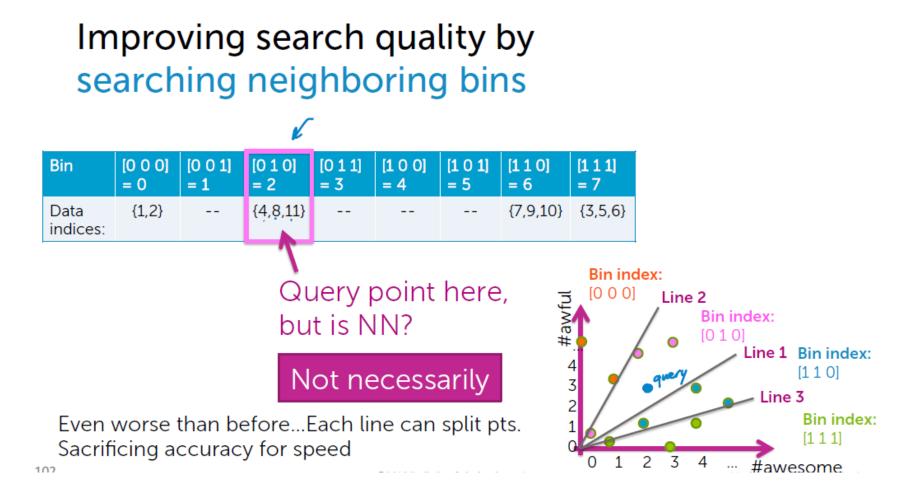
Reducing search cost through more bins



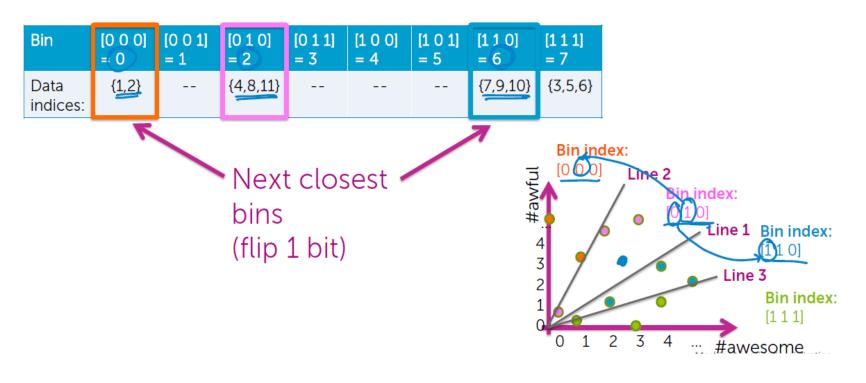
Using score for NN search

2D Data	Sign (Score ₁)	Bin 1 index	Sign (Score ₂)	Bin 2 index	Sign (Score ₃)	Bin 3 index
x ₁ = [0, 5]	-1	0	-1	0	-1	0
x ₂ = [1, 3]	-1	0	-1	0	-1	0
x ₃ = [3, 0]	1	1	1	1	1	1

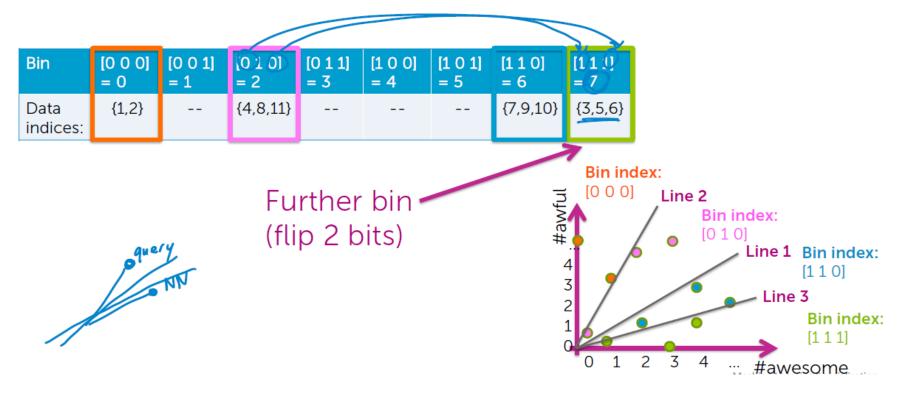




Improving search quality by searching neighboring bins



Improving search quality by searching neighboring bins



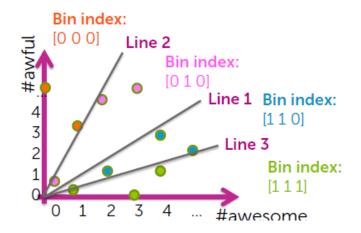
Improving search quality by searching neighboring bins

Bin	[0 0 0]	[0 0 1]	[0 1 0]	[0 1 1]	[1 0 0]	[1 0 1]	[1 1 0]	[1 1 1]
	= 0	= 1	= 2	= 3	= 4	= 5	= 6	= 7
Data indices:	{1,2}		{4,8,11}				{7,9,10}	{3,5,6}

Quality of retrieved NN can only improve with searching more bins

Algorithm:

Continue searching until computational budget is reached or quality of NN good enough



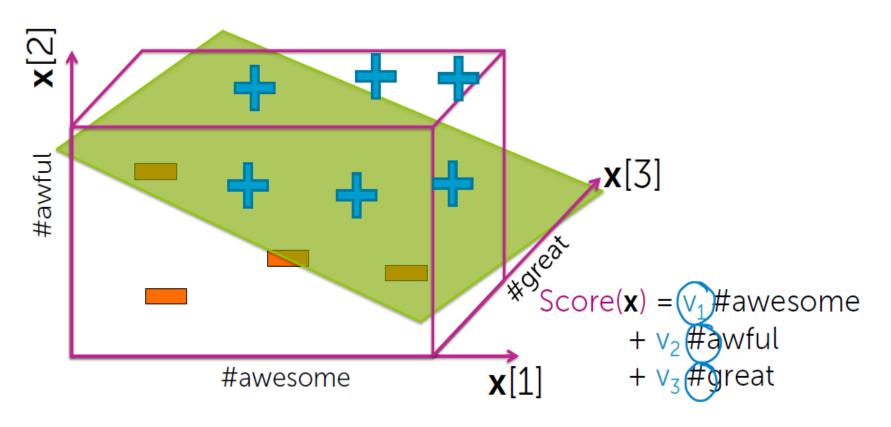
LSH recap



- Draw h random lines
- Compute "score" for each point under each line and translate to binary index
- Use h-bit binary vector per data point as bin index
- Create hash table
- For each query point x, search bin(x), then neighboring bins until time limit

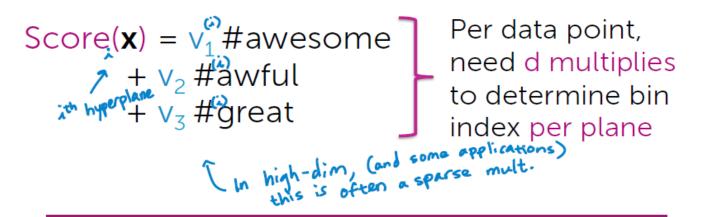
LSH: moving to higher dimmensions d

Draw random planes



LSH: moving to higher dimmensions d

Cost of binning points in d-dim



One-time cost offset if many queries of fixed dataset

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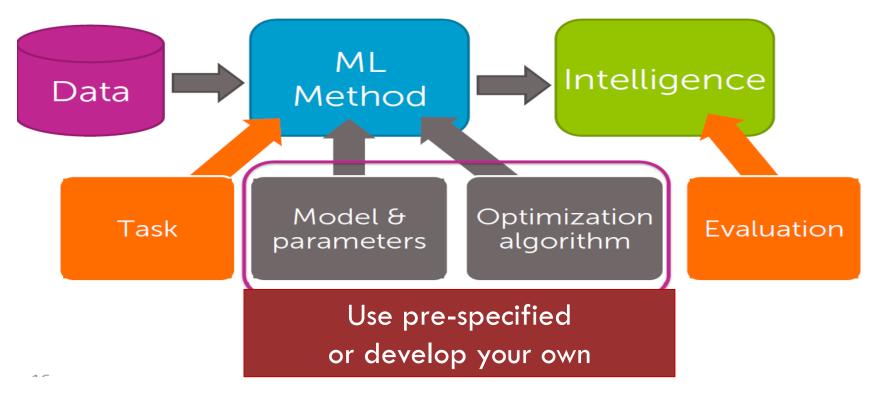
Wrapping up

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Deploing inteligence module

89

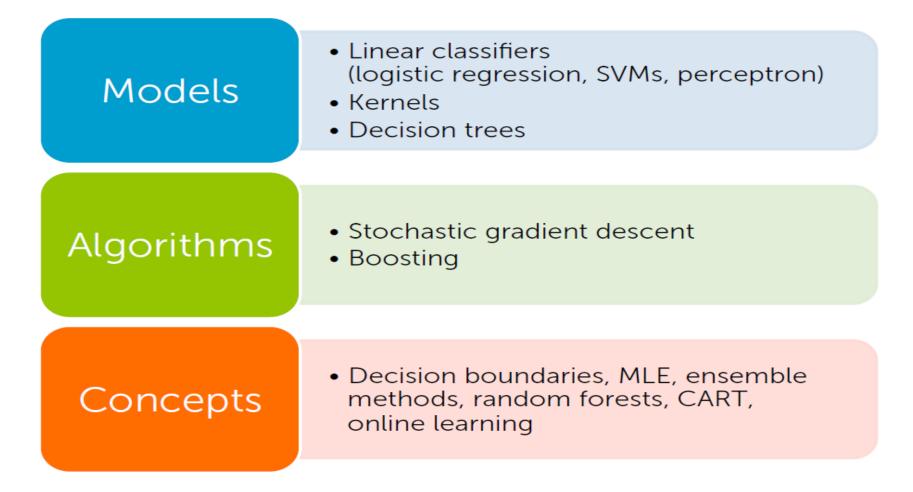
Case studied are about building, evaluating, deploying inteligence in data analysis.



Prediction: Predicting house prices

Models	 Linear regression Regularization: Ridge (L2), Lasso (L1) 			
Algorithms	Gradient descentCoordinate descent			
Concepts	 Loss functions, bias-variance tradeoff, cross-validation, sparsity, overfitting, model selection 			

Classification: Sentiment analysis



Clustering& Retrieval: Finding documents

Models	 Nearest neighbors Clustering, mixtures of Gaussians Latent Dirichlet allocation (LDA)
Algorithms	 KD-trees, locality-sensitive hashing (LSH) K-means Expectation-maximization (EM)
Concepts	 Distance metrics, approximation algorithms, hashing, sampling algorithms, scaling up with map-reduce