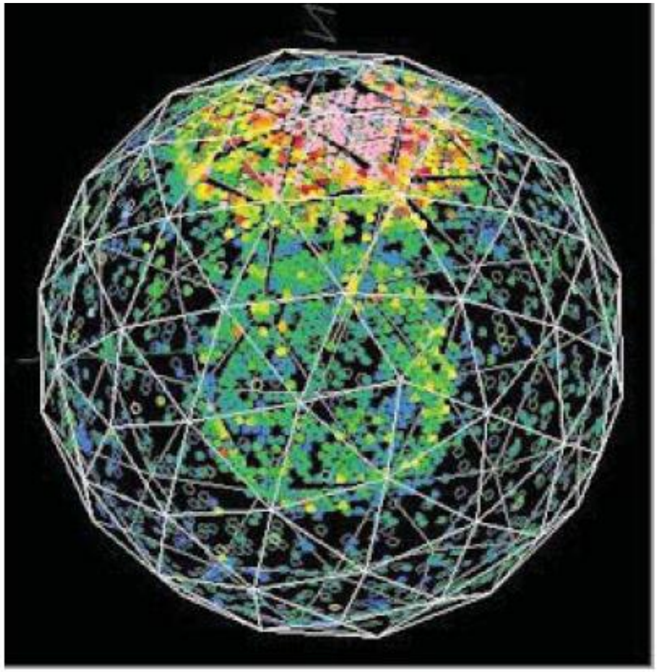
Elementary Particle Physics: theory and experiments

The Weak Interaction and V-A



Follow the course/slides from M. A. Thomson lectures at Cambridge University

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Parity

★ The parity operator performs spatial inversion through the origin:

$$\psi'(\vec{x},t) = \hat{P}\psi(\vec{x},t) = \psi(-\vec{x},t)$$

- applying $\hat{P}$ twice: $\hat{P}\hat{P}\psi(\vec{x},t) = \hat{P}\psi(-\vec{x},t) = \psi(\vec{x},t)$

  so $\hat{P}\hat{P} = I$ ➡ $\hat{P}^{-1} = \hat{P}$

- To preserve the normalisation of the wave-function

$$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle = \langle\psi|\hat{P}^\dagger\hat{P}|\psi\rangle$$

$\hat{P}^\dagger\hat{P} = I$ ➡ $\hat{P}$ Unitary

- But since $\hat{P}\hat{P} = I$ $\hat{P} = \hat{P}^\dagger$ ➡ $\hat{P}$ Hermitian

  which implies Parity is an observable quantity. If the interaction Hamiltonian commutes with $\hat{P}$, parity is an observable conserved quantity

- If $\psi(\vec{x},t)$ is an eigenfunction of the parity operator with eigenvalue P

  $\hat{P}\psi(\vec{x},t) = P\psi(\vec{x},t)$ ➡ $\hat{P}\hat{P}\psi(\vec{x},t) = P\hat{P}\psi(\vec{x},t) = P^2\psi(\vec{x},t)$

  since $\hat{P}\hat{P} = I$ $P^2 = 1$

  ➡ Parity has eigenvalues $P = \pm 1$

★ QED and QCD are invariant under parity

★ Experimentally observe that Weak Interactions do not conserve parity

Intrinsic Parities of fundamental particles:

Spin-1 Bosons

- From Gauge Field Theory can show that the gauge bosons have $P = -1$

$$P_\gamma = P_g = P_{W^+} = P_{W^-} = P_Z = -1$$

Spin-½ Fermions

- From the Dirac equation showed:

  Spin ½ particles have opposite parity to spin ½ anti-particles

- Conventional choice: spin ½ particles have $P = +1$

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_\nu = P_q = +1$$

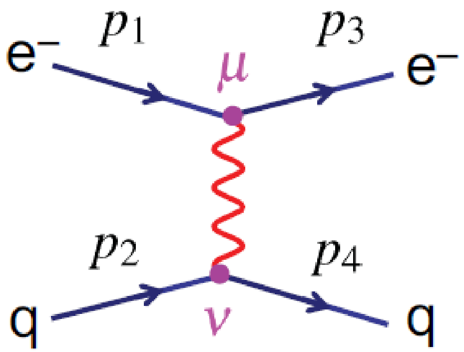
and anti-particles have opposite parity, i.e.

$$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\overline{\nu}} = P_{\overline{q}} = -1$$

★ For Dirac spinors it was shown that the parity operator is:

$$\boxed{\hat{P} = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}$$

Parity Conservation in QED and QCD



- Consider the QED process e⁻q → e⁻q
- The Feynman rules for QED give:

$$-iM = [\overline{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}_q(p_4)ie\gamma^\nu u_q(p_2)]$$

- Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2}g_{\mu\nu}j_e^\mu j_q^\nu = -\frac{e^2}{q^2}j_e.j_q$$

with $j_e = \overline{u}_e(p_3)\gamma^\mu u_e(p_1)$ and $j_q = \overline{u}_q(p_4)\gamma^\mu u_q(p_2)$

★ Consider the what happen to the matrix element under the parity transformation

- Spinors transform as $\boxed{u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u}$
- Adjoint spinors transform as

$$\overline{u} = u^\dagger\gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger\gamma^0 = u^\dagger\gamma^{0\dagger}\gamma^0 = u^\dagger\gamma^0\gamma^0 = \overline{u}\gamma^0$$

$$\boxed{\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0}$$

- Hence $j_e = \overline{u}_e(p_3)\gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \overline{u}_e(p_3)\gamma^0\gamma^\mu\gamma^0 u_e(p_1)$

★ Consider the components of the four-vector current

0: $$j_e^0 \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^0\gamma^0 u = \overline{u}\gamma^0 u = j_e^0$$ since $\gamma^0\gamma^0 = 1$

k=1,2,3: $$j_e^k \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^k\gamma^0 u = -\overline{u}\gamma^k\gamma^0\gamma^0 u = -\overline{u}\gamma^k u = -j_e^k$$ since $\gamma^0\gamma^k = -\gamma^k\gamma^0$

- The time-like component remains unchanged and the space-like components change sign
- Similarly $\quad j_q^0 \xrightarrow{\hat{P}} j_q^0 \qquad j_q^k \xrightarrow{\hat{P}} -j_q^k \quad k = 1,2,3$

★ Consequently the four-vector scalar product

$$j_e . j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e . j_q \quad k = 1,3$$

or $\quad j^\mu \xrightarrow{\hat{P}} j_\mu$

$$j^\mu . j^\nu \xrightarrow{\hat{P}} j_\mu . j_\nu \xrightarrow{\hat{P}} j^\mu . j^\nu$$

QED Matrix Elements are Parity Invariant

➡ **Parity Conserved in QED**

★ The QCD vertex has the same form and, thus,

Parity Conserved in QCD

Parity Violation in β-Decay

★The parity operator $\hat{P}$ corresponds to a discrete transformation $x \rightarrow -x, etc.$

★Under the parity transformation:

Vectors change sign

$$\left\{\begin{array}{ll}\vec{r} \xrightarrow{\hat{P}} -\vec{r} & \\ \vec{p} \xrightarrow{\hat{P}} -\vec{p} & (p_x = \frac{\partial}{\partial x}, etc.)\end{array}\right.$$

Axial-Vectors unchanged

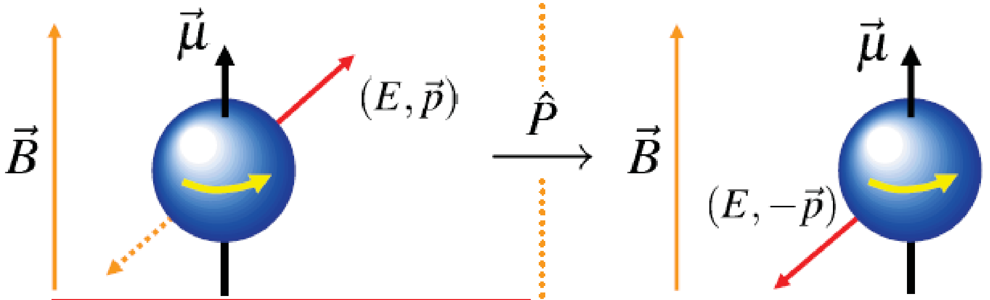
$$\left\{\begin{array}{ll}\vec{L} \xrightarrow{\hat{P}} \vec{L} & (\vec{L} = \vec{r} \wedge \vec{p}) \\ \vec{\mu} \xrightarrow{\hat{P}} \vec{\mu} & (\vec{\mu} \propto \vec{L})\end{array}\right.$$

Note B is an axial vector
$\mathrm{d}\vec{B} \propto \vec{J} \wedge \vec{r}\, \mathrm{d}^3\vec{r}$

★**1957:** C.S.Wu et al. studied beta decay of polarized cobalt-60 nuclei:

$$^{60}\mathrm{Co} \rightarrow ^{60}Ni^* + e^- + \overline{\nu}_e$$

★Observed electrons emitted preferentially in direction opposite to applied field



more e- in ↙ c.f. ↗

If parity were conserved: expect equal rate for producing e⁻ in directions along and opposite to the nuclear spin.

★Conclude parity is violated in WEAK INTERACTION
➡ that the WEAK interaction vertex is NOT of the form $\overline{u}_e \gamma^\mu u_\nu$

Bilinear Covariants

★The requirement of Lorentz invariance of the matrix element severely restricts the form of the interaction vertex. QED and QCD are "VECTOR" interactions:

$$j^{\mu} = \overline{\psi}\gamma^{\mu}\phi$$

★This combination transforms as a 4-vector (Handout 2 appendix V)

★ In general, there are only 5 possible combinations of two spinors and the gamma matrices that form Lorentz invariant currents, called "bilinear covariants":

Type	Form	Components	"Boson Spin"
♦ SCALAR	$\overline{\psi}\phi$	1	0
♦ PSEUDOSCALAR	$\overline{\psi}\gamma^{5}\phi$	1	0
♦ VECTOR	$\overline{\psi}\gamma^{\mu}\phi$	4	1
♦ AXIAL VECTOR	$\overline{\psi}\gamma^{\mu}\gamma^{5}\phi$	4	1
♦ TENSOR	$\overline{\psi}(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})\phi$	6	2

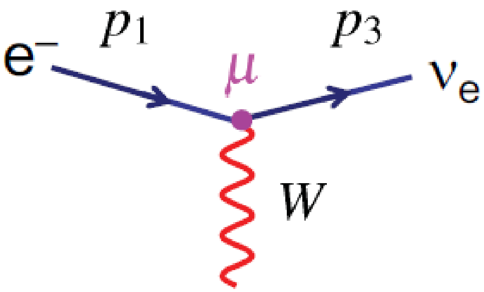
★ Note that in total the sixteen components correspond to the 16 elements of a general 4x4 matrix: "decomposition into Lorentz invariant combinations"

★ In QED the factor $g_{\mu\nu}$ arose from the sum over polarization states of the virtual photon (2 transverse + 1 longitudinal, 1 scalar) = (2J+1) + 1

★ Associate SCALAR and PSEUDOSCALAR interactions with the exchange of a SPIN-0 boson, etc. – no spin degrees of freedom

V-A Structure of the Weak Interaction

- ★ The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants
- ★ For an interaction corresponding to the **exchange of a spin-1** particle the most general form is a linear combination of **VECTOR** and **AXIAL-VECTOR**
- ★ The form for WEAK interaction is <u>determined from experiment</u> to be **VECTOR – AXIAL-VECTOR (V – A)**



$$\boxed{\begin{array}{c} j^{\mu} \propto \overline{u}_{\nu_e}(\gamma^{\mu} - \gamma^{\mu}\gamma^{5})u_e \\ \textbf{V – A} \end{array}}$$

- ★ Can this account for parity violation?
- ★ First consider parity transformation of a **pure AXIAL-VECTOR** current

$$j_A = \overline{\psi}\gamma^{\mu}\gamma^{5}\phi \qquad \textbf{with} \qquad \gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}; \quad \gamma^{5}\gamma^{0} = -\gamma^{0}\gamma^{5}$$

$$j_A = \overline{\psi}\gamma^{\mu}\gamma^{5}\phi \xrightarrow{\hat{P}} \overline{\psi}\gamma^{0}\gamma^{\mu}\gamma^{5}\gamma^{0}\phi = -\overline{\psi}\gamma^{0}\gamma^{\mu}\gamma^{0}\gamma^{5}\phi$$

$$j_A^{0} = \xrightarrow{\hat{P}} -\overline{\psi}\gamma^{0}\gamma^{0}\gamma^{0}\gamma^{5}\phi = -\overline{\psi}\gamma^{0}\gamma^{5}\phi = -j_A^{0}$$

$$j_A^{k} = \xrightarrow{\hat{P}} -\overline{\psi}\gamma^{0}\gamma^{k}\gamma^{0}\gamma^{5}\phi = +\overline{\psi}\gamma^{k}\gamma^{5}\phi = +j_A^{k} \qquad k = 1,2,3$$

$$\boxed{\text{or} \quad j_A^{\mu} \xrightarrow{\hat{P}} -j_{A\mu}}$$

- The space-like components remain unchanged and the time-like components change sign (the opposite to the parity properties of a vector-current)

$$\boxed{j_A^0 \xrightarrow{\hat{P}} -j_A^0; \quad j_A^k \xrightarrow{\hat{P}} +j_A^k; \quad \Big| \quad j_V^0 \xrightarrow{\hat{P}} +j_V^0; \quad j_V^k \xrightarrow{\hat{P}} -j_V^k}$$

- Now consider the matrix elements

$$M \propto g_{\mu\nu} j_1^\mu j_2^\nu = j_1^0 j_2^0 - \sum_{k=1,3} j_1^k j_2^k$$

- For the combination of a two axial-vector currents

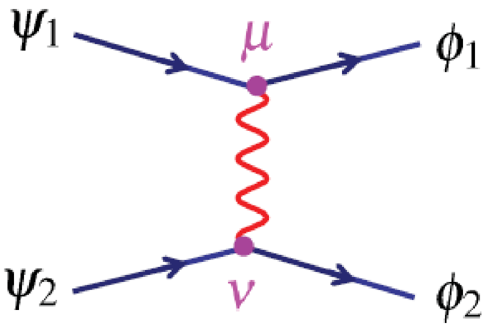
$$j_{A1} \cdot j_{A2} \xrightarrow{\hat{P}} (-j_1^0)(-j_2^0) - \sum_{k=1,3} (j_1^k)(j_2^k) = j_{A1} \cdot j_{A2}$$

- Consequently parity is conserved for both a pure vector and pure axial-vector interactions
- However the combination of a vector current and an axial vector current

$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} (j_1^0)(-j_2^0) - \sum_{k=1,3} (-j_1^k)(j_2^k) = -j_{V1} \cdot j_{A2}$$

changes sign under parity – can give parity violation !

★ Now consider a general linear combination of VECTOR and AXIAL-VECTOR (note this is relevant for the Z-boson vertex)



$$j_1 = \overline{\phi}_1(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)\psi_1 = g_V j_1^V + g_A j_1^A$$

$$\frac{g_{\mu\nu}}{q^2 - m^2}$$

$$j_2 = \overline{\phi}_2(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)\psi_2 = g_V j_2^V + g_A j_2^A$$

$$M_{fi} \propto j_1 . j_2 = g_V^2 j_1^V . j_2^V + g_A^2 j_1^A . j_2^A + g_V g_A (j_1^V . j_2^A + j_1^A . j_2^V)$$

- Consider the parity transformation of this scalar product

$$j_1 . j_2 \xrightarrow{\hat{P}} g_V^2 j_1^V . j_2^V + g_A^2 j_1^A . j_2^A - g_V g_A (j_1^V . j_2^A + j_1^A . j_2^V)$$

- If either g_A or g_V is zero, Parity is conserved, i.e. parity conserved in a pure VECTOR or pure AXIAL-VECTOR interaction

- Relative strength of parity violating part $\propto \dfrac{g_V g_A}{g_V^2 + g_A^2}$

Maximal Parity Violation for V-A (or V+A)

Chiral Structure of QED (Reminder)

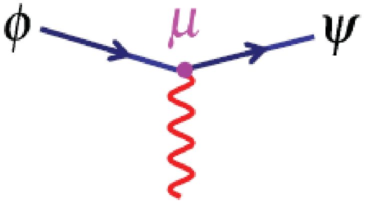
- ★ Recall introduced CHIRAL projections operators

$$P_R = \tfrac{1}{2}(1+\gamma^5); \qquad P_L = \tfrac{1}{2}(1-\gamma^5)$$

project out chiral right- and left- handed states

- ★ In the ultra-relativistic limit, chiral states correspond to helicity states
- ★ Any spinor can be expressed as:

$$\psi = \tfrac{1}{2}(1+\gamma^5)\psi + \tfrac{1}{2}(1-\gamma^5)\psi = P_R\psi + P_L\psi = \psi_R + \psi_L$$



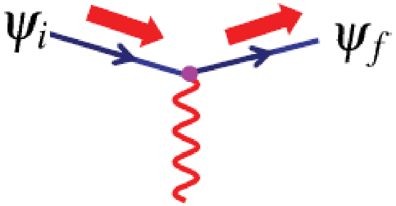
- **The QED vertex** $\overline{\psi}\gamma^\mu\phi$ in terms of chiral states:

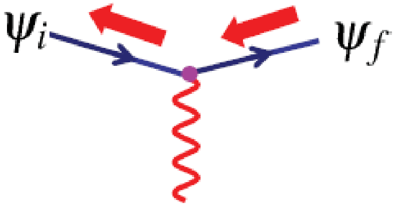
$$\overline{\psi}\gamma^\mu\phi = \overline{\psi}_R\gamma^\mu\phi_R + \overline{\psi}_R\gamma^\mu\phi_L + \overline{\psi}_L\gamma^\mu\phi_R + \overline{\psi}_L\gamma^\mu\phi_L$$

conserves chirality, e.g.

$$\begin{aligned}\overline{\psi}_R\gamma^\mu\phi_L &= \tfrac{1}{2}\psi^\dagger(1+\gamma^5)\gamma^0\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\phi \\ &= \tfrac{1}{4}\psi^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)\phi \\ &= \tfrac{1}{4}\overline{\psi}\gamma^\mu(1+\gamma^5)(1-\gamma^5)\phi = 0\end{aligned}$$

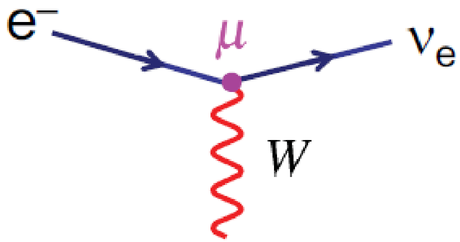
- ★ In the ultra-relativistic limit only two helicity combinations are non-zero





Helicity Structure of the Weak Interactions

★The charged current (W±) weak vertex is:



$$\boxed{\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)}$$

★Since $\frac{1}{2}(1-\gamma^5)$ projects out left-handed chiral particle states:

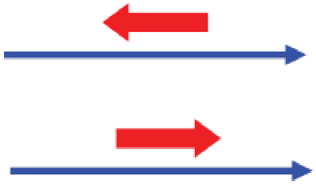
$$\overline{\psi}\tfrac{1}{2}\gamma^{\mu}(1-\gamma^5)\phi = \overline{\psi}\gamma^{\mu}\phi_L$$

★Writing $\overline{\psi} = \overline{\psi}_R + \overline{\psi}_L$ and from discussion of QED, $\overline{\psi}_R\gamma^{\mu}\phi_L = 0$ gives

$$\overline{\psi}\tfrac{1}{2}\gamma^{\mu}(1-\gamma^5)\phi = \overline{\psi}_L\gamma^{\mu}\phi_L$$

⇒ Only the left-handed chiral components of particle spinors and right-handed chiral components of anti-particle spinors participate in charged current weak interactions

★At very high energy $(E \gg m)$, the left-handed chiral components are helicity eigenstates :



$\frac{1}{2}(1-\gamma^5)u \Rightarrow$

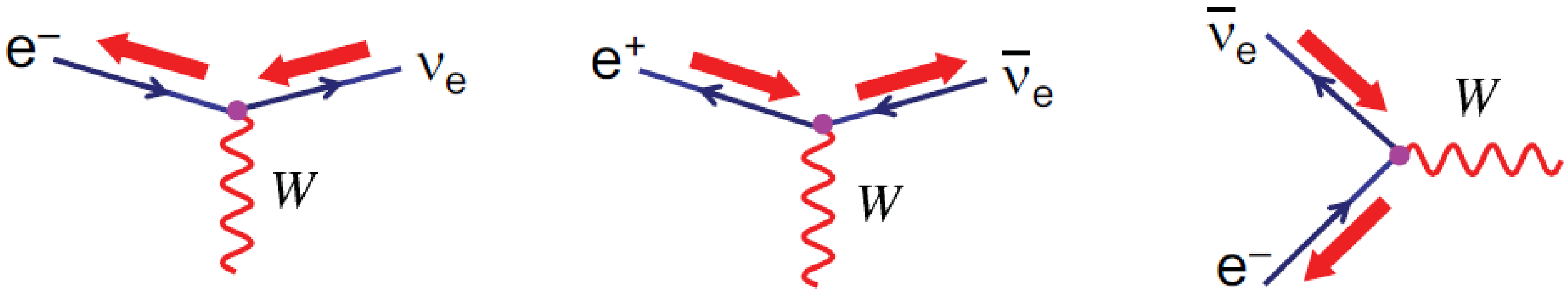
LEFT-HANDED PARTICLES
Helicity = -1

$\frac{1}{2}(1-\gamma^5)v \Rightarrow$

RIGHT-HANDED ANTI-PARTICLES
Helicity = +1

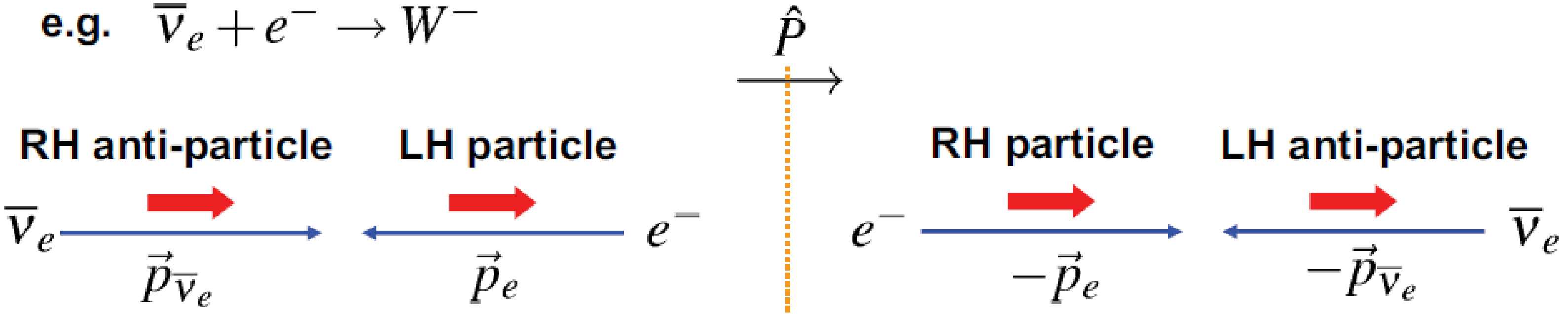
In the ultra-relativistic limit only left-handed particles and right-handed antiparticles participate in charged current weak interactions

e.g. In the relativistic limit, the only possible electron – neutrino interactions are:



★ The helicity dependence of the weak interaction ↔ parity violation

e.g. $\bar{\nu}_e + e^- \rightarrow W^-$

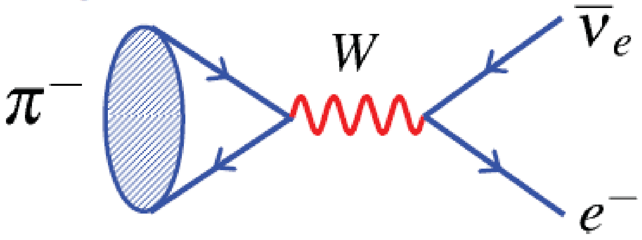


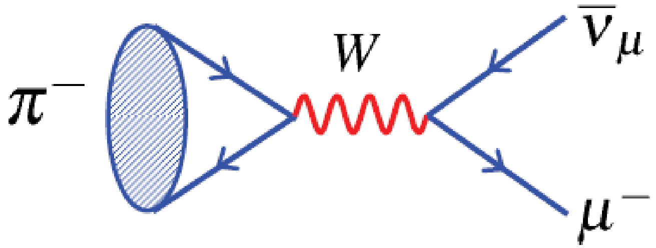
Valid weak interaction

Does not occur

Helicity in Pion Decay

★ The decays of charged pions provide a good demonstration of the role of helicity in the weak interaction





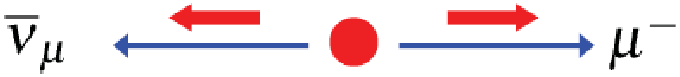
<u>EXPERIMENTALLY:</u>

$$\frac{\Gamma(\pi^- \rightarrow e^-\overline{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^-\overline{\nu}_\mu)} = 1.23\times10^{-4}$$

- Might expect the decay to electrons to dominate – due to increased phase space…. The opposite happens, the electron decay is helicity suppressed

★ Consider decay in pion rest frame.

- Pion is spin zero: so the spins of the $\overline{\nu}$ and μ are opposite
- Weak interaction only couples to RH chiral anti-particle states. Since neutrinos are (almost) massless, must be in RH Helicity state
- Therefore, to conserve angular mom. muon is emitted in a RH **HELICITY** state



- But only left-handed **CHIRAL** particle states participate in weak interaction

★ The general right-handed helicity solution to the Dirac equation is

$$u_\uparrow = N\begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\vec{p}|}{E+m}c \\ \frac{|\vec{p}|}{E+m}e^{i\phi}s \end{pmatrix} \quad \text{with} \quad c=\cos\frac{\theta}{2} \quad \text{and} \quad s=\sin\frac{\theta}{2}$$

- project out the left-handed chiral part of the wave-function using

$$P_L = \tfrac{1}{2}(1-\gamma^5) = \tfrac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

giving

$$P_L u_\uparrow = \tfrac{1}{2}N\left(1-\frac{|\vec{p}|}{E+m}\right)\begin{pmatrix} c \\ e^{i\phi}s \\ -c \\ -e^{i\phi}s \end{pmatrix} = \tfrac{1}{2}N\left(1-\frac{|\vec{p}|}{E+m}\right)u_L$$

In the limit $m \ll E$ this tends to zero

- similarly

$$P_R u_\uparrow = \tfrac{1}{2}N\left(1+\frac{|\vec{p}|}{E+m}\right)\begin{pmatrix} c \\ e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix} = \tfrac{1}{2}N\left(1+\frac{|\vec{p}|}{E+m}\right)u_R$$

In the limit $m \ll E$, $P_R u_\uparrow \rightarrow u_R$

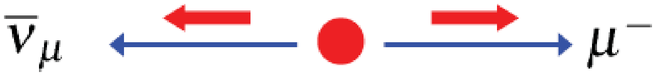
★Hence
$$\boxed{u_\uparrow} = P_R u_\uparrow + P_L u_\uparrow = \frac{1}{2}\left(1+\frac{|\vec{p}|}{E+m}\right)\boxed{u_R} + \frac{1}{2}\left(1-\frac{|\vec{p}|}{E+m}\right)\boxed{u_L}$$

RH Helicity

RH Chiral

LH Chiral

- In the limit $E \gg m$, as expected, the RH chiral and helicity states are identical
- Although only LH chiral particles participate in the weak interaction the contribution from RH Helicity states is not necessarily zero !



$m_\nu \approx 0$: RH Helicity ≡ RH Chiral

$m_\mu \neq 0$: RH Helicity has LH Chiral Component

★ Expect matrix element to be proportional to LH chiral component of RH Helicity electron/muon spinor

$$M_{fi} \propto \frac{1}{2}\left(1-\frac{|\vec{p}|}{E+m}\right) = \boxed{\frac{m_\mu}{m_\pi + m_\mu}}$$

from the kinematics of pion decay at rest

★ Hence because the electron mass is much smaller than the pion mass the decay $\pi^- \to e^- \overline{\nu}_e$ is heavily suppressed.

Evidence for V-A

★ The V-A nature of the charged current weak interaction vertex fits with experiment

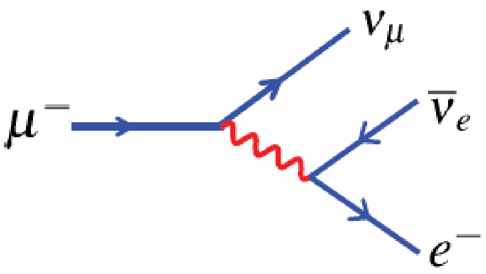
EXAMPLE charged pion decay

- Experimentally measure: $\dfrac{\Gamma(\pi^- \to e^-\overline{\nu}_e)}{\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu)} = (1.230 \pm 0.004) \times 10^{-4}$
- Theoretical predictions (depend on Lorentz Structure of the interaction)

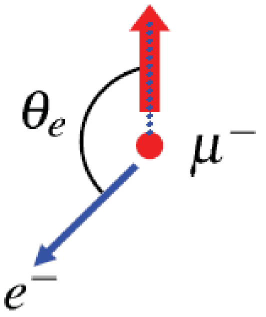
V-A $(\overline{\psi}\gamma^\mu(1-\gamma^5)\phi)$ or V+A $(\overline{\psi}\gamma^\mu(1+\gamma^5)\phi)$ ➡ $\dfrac{\Gamma(\pi^- \to e^-\overline{\nu}_e)}{\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu)} \approx 1.3 \times 10^{-4}$

Scalar $(\overline{\psi}\phi)$ or Pseudo-Scalar $(\overline{\psi}\gamma^5\phi)$ ➡ $\dfrac{\Gamma(\pi^- \to e^-\overline{\nu}_e)}{\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu)} = 5.5$

EXAMPLE muon decay



Measure electron energy and angular distributions relative to muon spin direction. Results expressed in terms of general S+P+V+A+T form in "Michel Parameters"



e.g. TWIST expt: 6x10⁹ μ decays
Phys. Rev. Lett. 95 (2005) 101805

$$\rho = 0.75080 \pm 0.00105$$

V-A Prediction: $\rho = 0.75$

TWIST expt.,
Phys.Rev.Lett 106 (2011) 041804 $\rho = 0.74977 \pm 0.00012\ (\text{stat}) \pm\ 0.00023\ (\text{syst})$

Weak Charged Current Propagator

★ The charged-current Weak interaction is different from QED and QCD in that it is mediated by massive W-bosons (80.3 GeV)

★ This results in a more complicated form for the propagator:

- showed that for the exchange of a massive particle:

massless massive

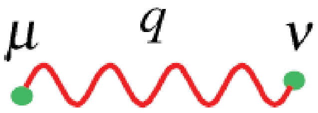
$$\frac{1}{q^2} \longrightarrow \frac{1}{q^2 - m^2}$$

- In addition the sum over W boson polarization states modifies the numerator

● **W-boson propagator**

spin 1 $W^\pm$

$$\frac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2}$$



★ However in the limit where q^2 is small compared with $m_W = 80.3\,\mathrm{GeV}$ the interaction takes a simpler form.

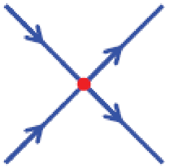
● **W-boson propagator** ($q^2 \ll m_W^2$)

$$\frac{i g_{\mu\nu}}{m_W^2}$$

- The interaction appears point-like (i.e no q^2 dependence)

Connection to Fermi Theory

- In 1934, before the discovery of parity violation, Fermi proposed, in analogy with QED, that the invariant matrix element for β-decay was of the form:



$$M_{fi} = G_F\, g_{\mu\nu}[\overline{\psi}\gamma^{\mu}\psi][\overline{\psi}\gamma^{\nu}\psi]$$

where $G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$

  - Note the absence of a propagator : i.e. this represents an interaction at a point

- After the discovery of parity violation in 1957 this was modified to

$$M_{fi} = \frac{G_F}{\sqrt{2}} g_{\mu\nu}[\overline{\psi}\gamma^{\mu}(1-\gamma^5)\psi][\overline{\psi}\gamma^{\nu}(1-\gamma^5)\psi]$$

(the factor of √2 was included so the numerical value of G_F did not need to be changed)

- Compare to the prediction for W-boson exchange

$$M_{fi} = [\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^{\mu}(1-\gamma^5)\psi]\frac{g_{\mu\nu}-q_{\mu}q_{\nu}/m_W^2}{q^2-m_W^2}[\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^{\nu}(1-\gamma^5)\psi]$$

which for $q^2 \ll m_W^2$ becomes:

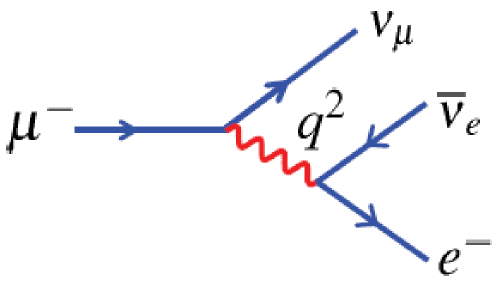
$$M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu}[\overline{\psi}\gamma^{\mu}(1-\gamma^5)\psi][\overline{\psi}\gamma^{\nu}(1-\gamma^5)\psi]$$

$$\Rightarrow \boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$$

Still usually use G_F to express strength of weak interaction as the is the quantity that is precisely determined in muon decay

Strength of Weak Interaction

★ **Strength of weak interaction most precisely measured in muon decay**



- **Here** $q^2 < m_\mu \; (0.106\,\mathrm{GeV})$
- **To a very good approximation the W-boson propagator can be written**

$$\frac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2} \approx \frac{i g_{\mu\nu}}{m_W^2}$$

- **In muon decay measure** g_W^2 / m_W^2
- **Muon decay** ➡ $G_F = 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}}$$

★ **To obtain the intrinsic strength of weak interaction need to know mass of W-boson:** $m_W = 80.403 \pm 0.029\,\mathrm{GeV}$

➡ $$\alpha_W = \frac{g_W^2}{4\pi} = \frac{8 m_W^2 G_F}{4\sqrt{2}\pi} = \frac{1}{30}$$

★ **The intrinsic strength of the weak interaction is similar to, but greater than, the EM interaction ! It is the massive W-boson in the propagator which makes it appear weak. For $q^2 \gg m_W^2$ weak interactions are more likely than EM.**

Summary

- ★ Weak interaction is of form **Vector – Axial-vector (V-A)**

$$\boxed{\frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)}$$

- ★ Consequently only **left-handed chiral particle states** and **right-handed chiral anti-particle states** participate in the weak interaction

➡ **MAXIMAL PARITY VIOLATION**

- ★ Weak interaction also violates Charge Conjugation symmetry

- ★ At low q^2 weak interaction is only weak because of the large W-boson mass

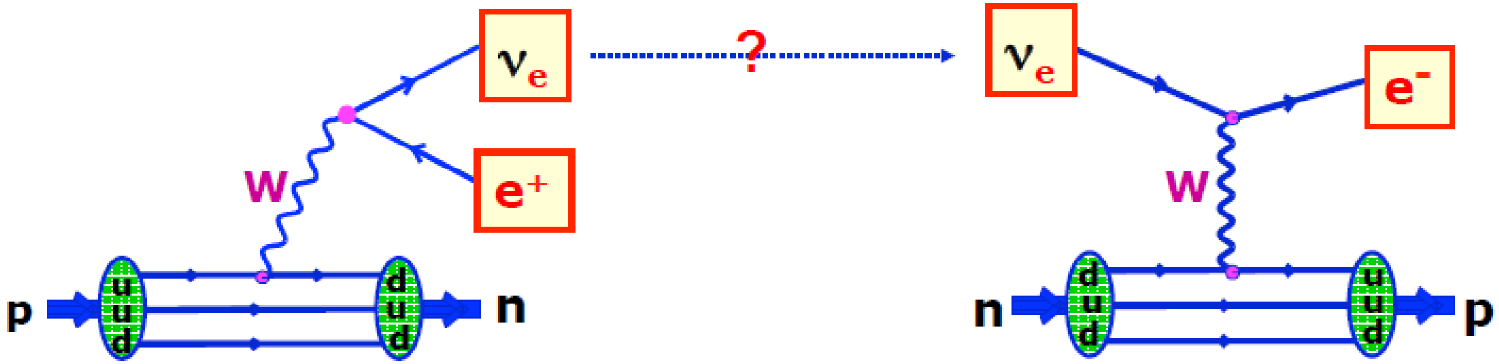
$$\boxed{\frac{G_\mathrm{F}}{\sqrt{2}}=\frac{g_W^2}{8m_W^2}}$$

- ★ Intrinsic strength of weak interaction is similar to that of QED

Neutrino Flavours

- Recent experiments (→ neutrinos have mass (albeit very small)
- The textbook neutrino states, ν_e, ν_μ, ν_τ, are not the fundamental particles; these are ν_1, ν_2, ν_3
- Concepts like "electron number" conservation are now known not to hold.
- So what are ν_e, ν_μ, ν_τ ?
- Never directly observe neutrinos – can only detect them by their weak interactions. Hence by definition ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

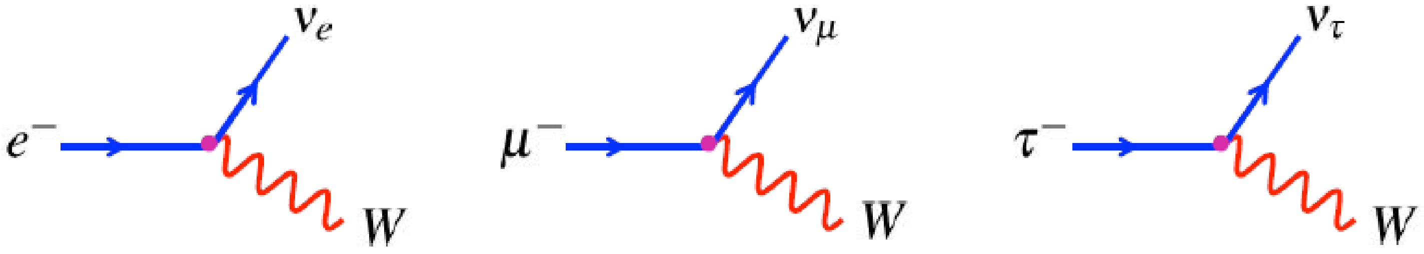
→ ν_e, ν_μ, ν_τ = weak eigenstates



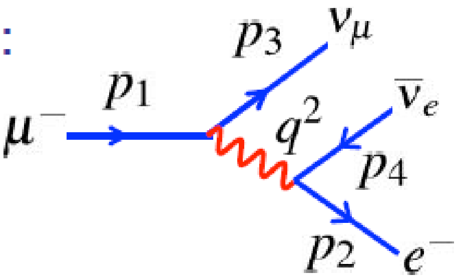
- Unless dealing with very large distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the weak interaction continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

Muon Decay and Lepton Universality

★The leptonic **charged current** (W±) interaction vertices are:



★Consider muon decay:



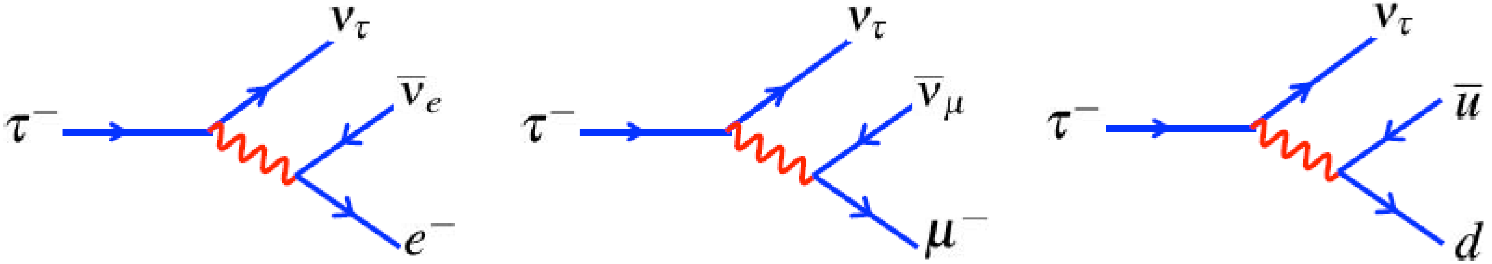
- It is straight-forward to write down the matrix element

$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2}[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)]g_{\mu\nu}[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)]$$

Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$
i.e. in limit of Fermi theory

- Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

- The muon to electron rate $\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu}$ with $G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}$
- Similarly for tau to electron $\Gamma(\tau \to e\nu\nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$
- However, the tau can decay to a number of final states:



$Br(\tau \to e\nu\nu) = 0.1784(5)$ $\quad$ $Br(\tau \to \mu\nu\nu) = 0.1736(5)$

- Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

- Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

- Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$$

- All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94)\,\mathrm{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\mathrm{s}$$

$$m_\tau = 1.77699(28)\,\mathrm{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\mathrm{s}$$

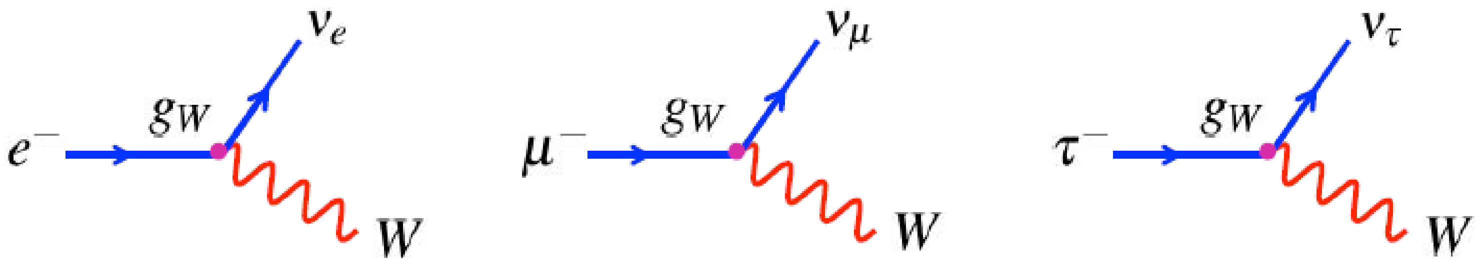
$$Br(\tau \to e\nu\nu) = 0.1784(5)$$

➡ $$\boxed{\frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \to e\nu\nu) = 1.0024 \pm 0.0033}$$

- Similarly by comparing $Br(\tau \to e\nu\nu)$ and $Br(\tau \to \mu\nu\nu)$

$$\boxed{\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004}$$

★ Demonstrates the weak charged current is the same for all leptonic vertices



➡ **Charged Current Lepton Universality**