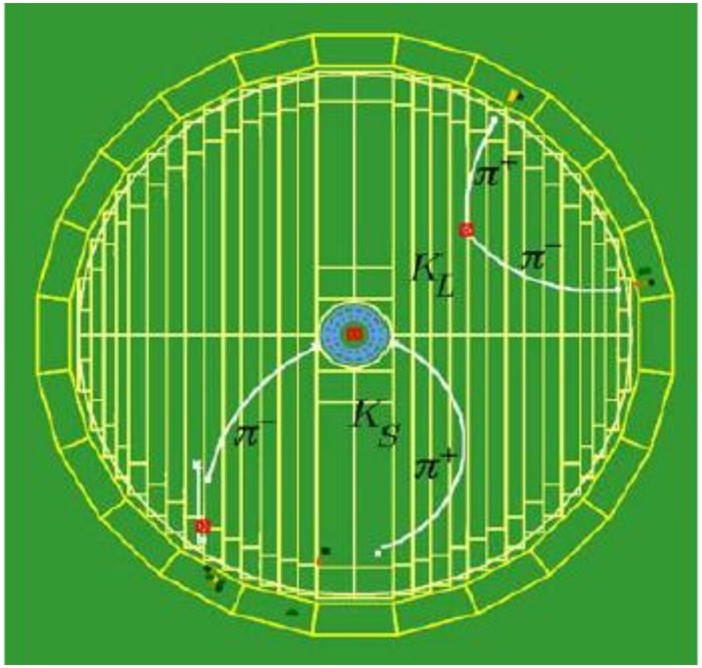
Elementary Particle Physics: theory and experiments

The CKM Matrix and CP Violation



Follow the course/slides from M. A. Thomson lectures at Cambridge University

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CP Violation in the Early Universe

- Very early in the universe might expect equal numbers of baryons and anti-baryons
- However, today the universe is matter dominated (no evidence for anti-galaxies, etc.)
- From "Big Bang Nucleosynthesis" obtain the matter/anti-matter asymmetry

$$\xi = \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every baryon in the universe today there are 10^9 photons

- How did this happen?

★ Early in the universe need to create a very small asymmetry between baryons and anti-baryons

e.g. for every 10^9 anti-baryons there were 10^9+1 baryons

baryons/anti-baryons annihilate ⇨

1 baryon + ~10^9 photons + no anti-baryons

★ To generate this initial asymmetry three conditions must be met (Sakharov, 1967):

❶ "Baryon number violation", i.e. $n_B - n_{\overline{B}}$ is not constant

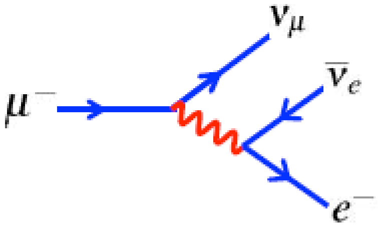
❷ "C and CP violation", if CP is conserved for a reaction which generates a net number of baryons over anti-baryons there would be a CP conjugate reaction generating a net number of anti-baryons

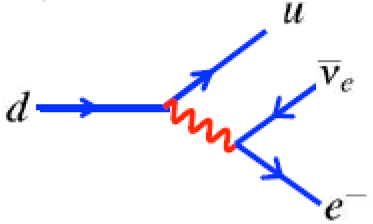
❸ "Departure from thermal equilibrium", in thermal equilibrium any baryon number violating process will be balanced by the inverse reaction

- CP Violation is an essential aspect of our understanding of the universe
- A natural question is whether the SM of particle physics can provide the necessary CP violation?
- There are two places in the SM where CP violation enters: the **PMNS matrix** and the **CKM matrix**
- **To date CP violation has been observed only in the quark sector**
- Because we are dealing with quarks, which are only observed as **bound states**, this is a fairly complicated subject. Here we will approach it in two steps:
    - i) Consider **particle – anti-particle oscillations** without CP violation
    - ii) Then discuss the effects of **CP violation**

The Weak Interaction of Quarks

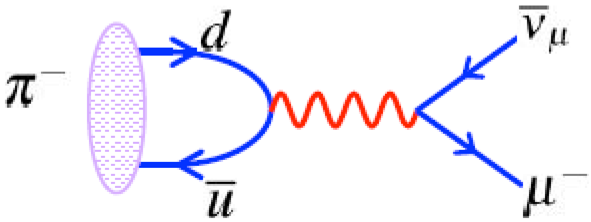
- ★ Slightly different values of G_F measured in μ decay and nuclear β decay:

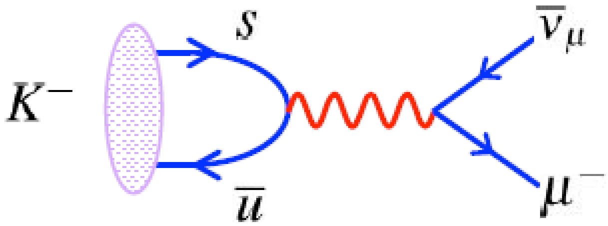




$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

- ★ In addition, certain hadronic decay modes are observed to be suppressed, e.g. compare $K^- \to \mu^- \overline{\nu}_\mu$ and $\pi^- \to \mu^- \overline{\nu}_\mu$. Kaon decay rate suppressed factor 20 compared to the expectation assuming a universal weak interaction for quarks.

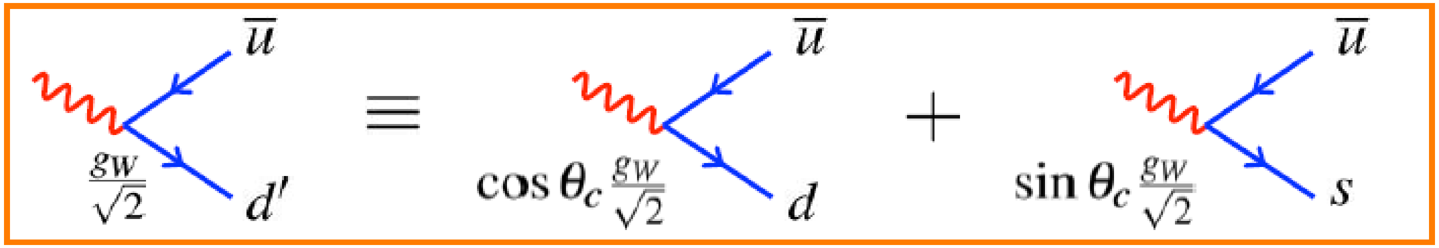




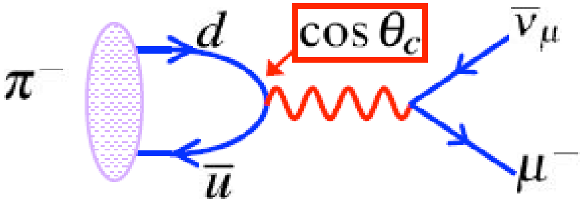
- Both observations explained by Cabibbo hypothesis (1963): weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d

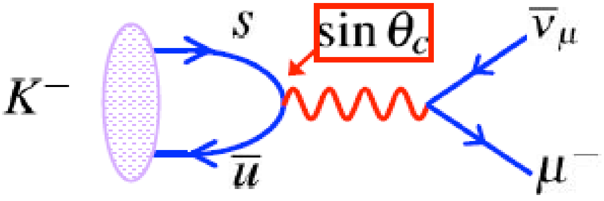
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

i.e. weak interaction couples different generations of quarks



- ★ Can explain the observations on the previous pages with $\theta_c = 13.1^\circ$
  - Kaon decay suppressed by a factor of $\tan^2\theta_c \approx 0.05$ relative to pion decay

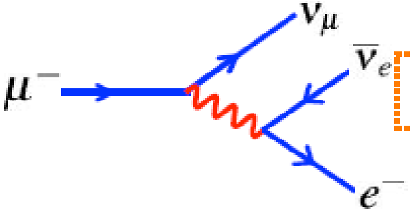




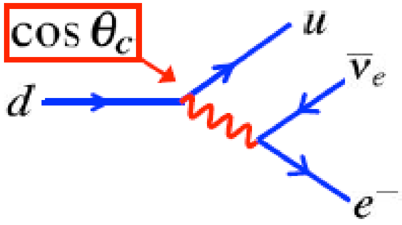
$$\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c$$

$$\Gamma(K^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$$

- Hence expect $G_F^\beta = G_F^\mu \cos^2\theta_c$



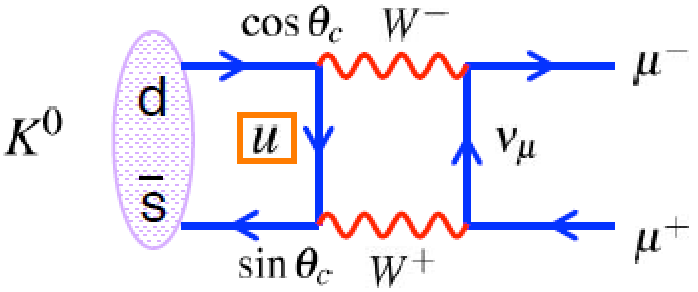
$\propto G_F.G_F$



$\propto G_F.(G_F\cos^2\theta_c)$

GIM Mechanism

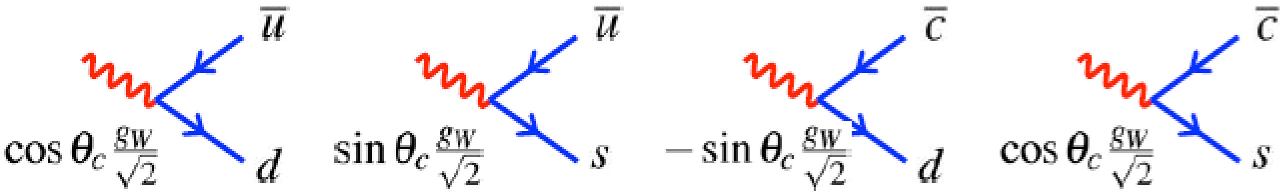
★ In the weak interaction have couplings between both ud and us which implies that neutral mesons can decay via box diagrams, e.g.



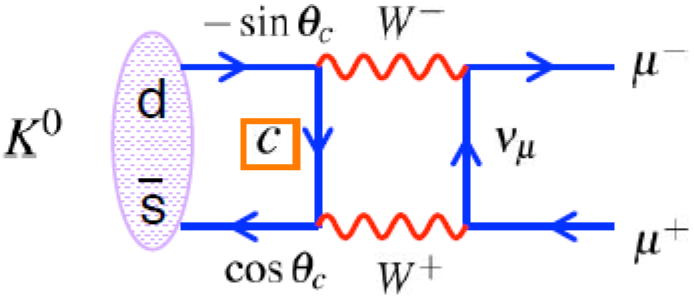
$$\boxed{M_1 \propto g_W^4 \cos\theta_c \sin\theta_c}$$

•Historically, the observed branching was **much smaller** than predicted

★ Led **Glashow, Illiopoulos** and **Maiani** to postulate existence of an extra quark
- **before** discovery of charm quark in **1974**. Weak interaction couplings become



★ Gives another box diagram for $K^0 \to \mu^+\mu^-$



$$\boxed{M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c}$$

•Same final state so sum amplitudes

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

•Cancellation not exact because $m_u \neq m_c$

CKM Matrix

- ★ Extend ideas to three quark flavours

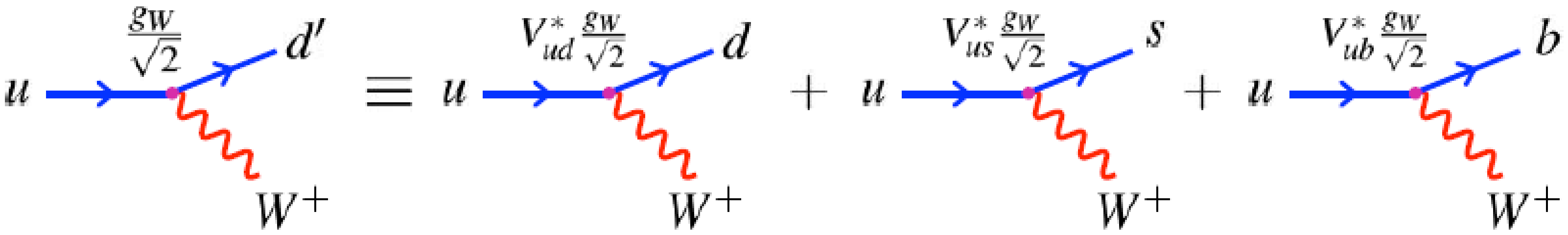
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$

Weak eigenstates **CKM Matrix** **Mass Eigenstates**

(Cabibbo, Kobayashi, Maskawa)

- ★ e.g. Weak eigenstate d' is produced in weak decay of an up quark:

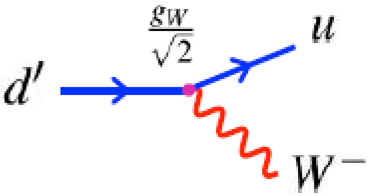


- The CKM matrix elements V_{ij} are complex constants
- The CKM matrix is unitary
- The V_{ij} are not predicted by the SM – have to determined from experiment

Feynman Rules

- Depending on the order of the interaction, $u \to d$ or $d \to u$, the CKM matrix enters as either V_{ud} or V_{ud}^*
- Writing the interaction in terms of the WEAK eigenstates

NOTE: $\bar{u}$ is the adjoint spinor not the anti-up quark

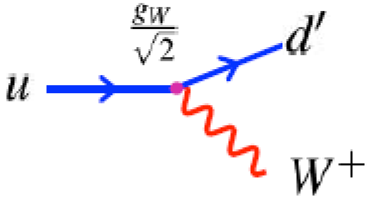


$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] d'$$

- Giving the $\boxed{d \to u}$ weak current:

$$\boxed{j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] V_{ud} d}$$

- For $u \to d'$ the weak current is:



$$j_{ud'} = \overline{d'}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u$$

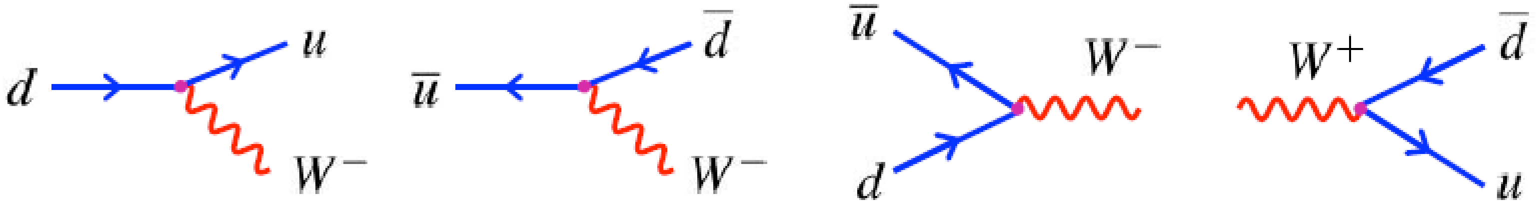
- In terms of the mass eigenstates $\overline{d'} = d'^\dagger\gamma^0 \to (V_{ud}d)^\dagger\gamma^0 = V_{ud}^* d^\dagger\gamma^0 = V_{ud}^*\overline{d}$

- Giving the $\boxed{u \to d}$ weak current:

$$\boxed{j_{ud} = \overline{d}V_{ud}^*\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u}$$

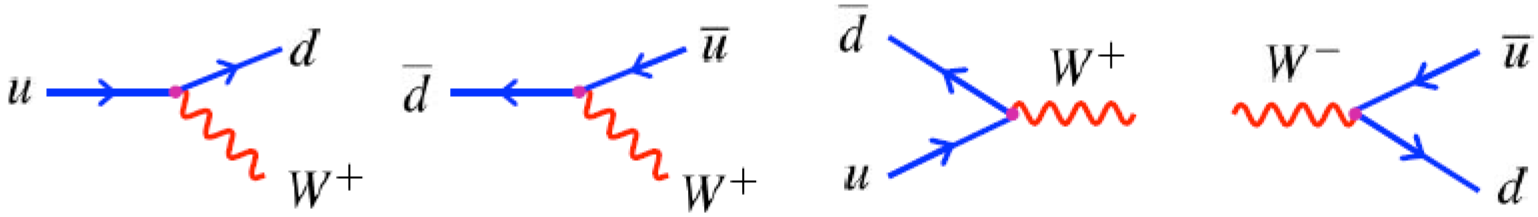
- Hence, when the charge $-\frac{1}{3}$ quark enters as the adjoint spinor, the complex conjugate of the CKM matrix is used

★ The vertex factor the following diagrams:



is $$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^\mu\tfrac{1}{2}(1-\gamma^5)}$$

★ Whereas, the vertex factor for:



is $$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}^*\gamma^\mu\tfrac{1}{2}(1-\gamma^5)}$$

- Experimentally (see Appendix I) determine

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

- Currently little direct experimental information on V_{td}, V_{ts}, V_{tb}
- Assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$
  gives:

Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

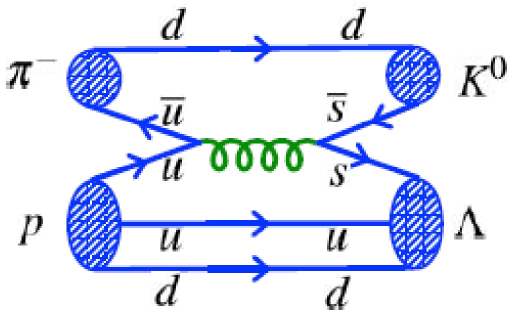
Near diagonal

- **NOTE:** within the SM, the charged current, $W^{\pm}$, weak interaction:
  ① Provides the only way to change flavour !
  ② only way to change from one generation of quarks or leptons to another !

- **However,** the off-diagonal elements of the CKM matrix are relatively small.
  - Weak interaction largest between quarks of the same generation.
  - Coupling between first and third generation quarks is very small !

- **the CKM matrix allows CP violation in the SM**

The Neutral Kaon System

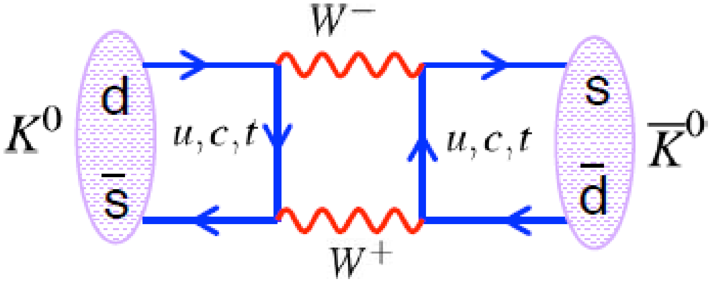


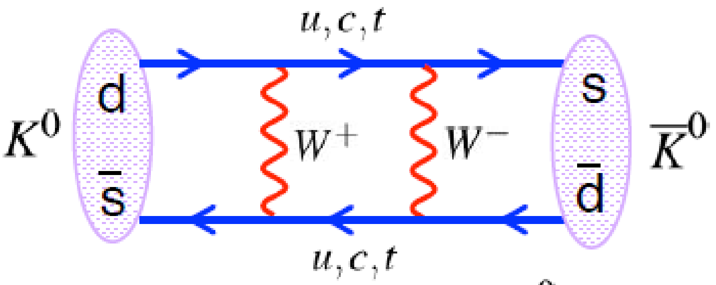
- **Neutral Kaons** are produced copiously in strong interactions, e.g.

$$\pi^-(d\bar{u}) + p(uud) \to \Lambda(uds) + K^0(d\bar{s})$$

$$\pi^+(u\bar{d}) + p(uud) \to K^+(u\bar{s}) + \overline{K}^0(s\bar{d}) + p(uud)$$

- **Neutral Kaons** decay via the **weak interaction**
- The Weak Interaction also allows **mixing** of neutral kaons via **"box diagrams"**





- This allows **transitions** between the strong eigenstates states $K^0, \overline{K}^0$
- Consequently, the neutral kaons propagate as eigenstates of the overall strong + weak interaction (Appendix II); i.e. as linear combinations of $K^0, \overline{K}^0$
- These neutral kaon states are called the **"K-short"** K_S and the **"K-long"** K_L
- These states have approximately the same mass $m(K_S) \approx m(K_L) \approx 498\,\mathrm{MeV}$
- But very different lifetimes: $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\mathrm{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\mathrm{s}}$

CP Eigenstates

★The K_S and K_L are closely related to eigenstates of the combined charge conjugation and parity operators: CP

- The strong eigenstates $K^0(d\bar{s})$ and $\overline{K}^0(s\bar{d})$ have $J^P = 0^-$

with

$$\hat{P}|K^0\rangle = -|K^0\rangle, \quad \hat{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

- The charge conjugation operator changes particle into anti-particle and *vice versa*

$$\hat{C}|K^0\rangle = \hat{C}|d\bar{s}\rangle = +|s\bar{d}\rangle = |\overline{K}^0\rangle$$

similarly

$$\hat{C}|\overline{K}^0\rangle = |K^0\rangle$$

The + sign is purely conventional, could have used a - with no physical consequences

- Consequently

$$\hat{C}\hat{P}|K^0\rangle = -|\overline{K}^0\rangle \qquad \hat{C}\hat{P}|\overline{K}^0\rangle = -|K^0\rangle$$

i.e. neither K^0 or $\overline{K}^0$ are eigenstates of CP

- Form CP eigenstates from linear combinations:

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

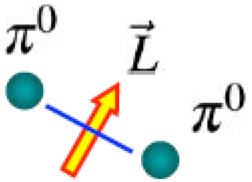
$$\hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$$

$$\hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$$

Decays of CP Eigenstates

- Neutral kaons often decay to pions (the lightest hadrons)
- The kaon masses are approximately **498 MeV** and the pion masses are approximately **140 MeV**. Hence neutral kaons can decay to either 2 or 3 pions

Decays to Two Pions:



☆ $K^0 \to \pi^0\pi^0 \qquad J^P: \quad 0^- \to 0^- + 0^-$

- Conservation of angular momentum → $\vec{L} = 0$

$$\Rightarrow \hat{P}(\pi^0\pi^0) = -1.-1.(-1)^L = +1$$

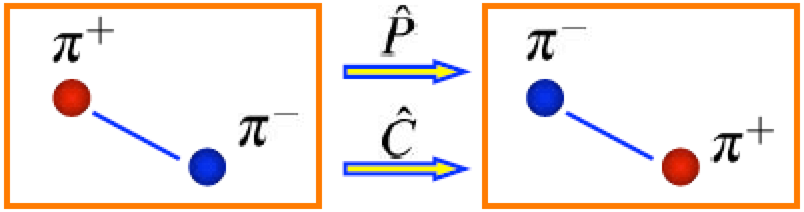
- The $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ is an eigenstate of $\hat{C}$

$$C(\pi^0\pi^0) = C\pi^0.C\pi^0 = +1.+1 = +1$$

$$\Rightarrow \boxed{CP(\pi^0\pi^0) = +1}$$

☆ $K^0 \to \pi^+\pi^-$ as before $\hat{P}(\pi^+\pi^-) = +1$

★ Here the **C** and **P** operations have the identical effect



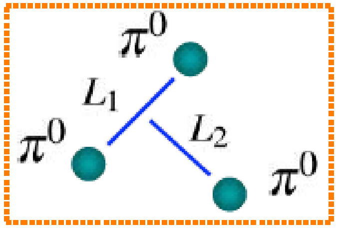
Hence the combined effect of $\hat{C}\hat{P}$ is to leave the system unchanged

$$\boxed{\hat{C}\hat{P}(\pi^+\pi^-) = +1}$$

Neutral kaon decays to two pions occur in CP even (i.e. +1) eigenstates

Decays to Three Pions:

- $K^0 \to \pi^0\pi^0\pi^0$ $\quad J^P: \quad 0^- \to 0^- + 0^- + 0^-$



Remember L is magnitude of angular momentum vector

- Conservation of angular momentum:

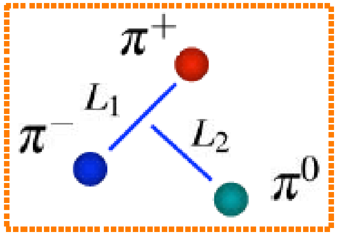
$$L_1 \oplus L_2 = 0 \quad \Rightarrow \quad L_1 = L_2$$

$$P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0\pi^0\pi^0) = +1.+1.+1$$

$$\Rightarrow \quad \boxed{CP(\pi^0\pi^0\pi^0) = -1}$$

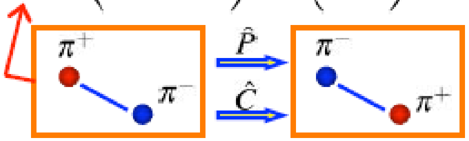
- $K^0 \to \pi^+\pi^-\pi^0$



- Again $L_1 = L_2$

$$P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^{L_1}$$



Hence: $\boxed{CP(\pi^+\pi^-\pi^0) = -1.(-1)^{L_1}}$

- The small amount of energy available in the decay, $m(K) - 3m(\pi) \approx 70\,\text{MeV}$ means that the L>0 decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)

Neutral kaon decays to three pions occur in CP odd (i.e. -1) eigenstates

- ★ If CP were conserved in the Weak decays of neutral kaons, would expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1 , K_2)

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \qquad K_1 \rightarrow \pi\pi$$

CP EVEN

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \qquad K_2 \rightarrow \pi\pi\pi$$

CP ODD

- ★ Expect lifetimes of CP eigenstates to be very different
  - For two pion decay energy available: $m_K - 2m_\pi \approx 220\,\mathrm{MeV}$
  - For three pion decay energy available: $m_K - 3m_\pi \approx 80\,\mathrm{MeV}$
- ★ Expect decays to two pions to be more rapid than decays to three pions due to increased phase space
- ★ This is exactly what is observed: a short-lived state "K-short" which decays to (mainly) to two pions and a long-lived state "K-long" which decays to three pions
- ★ In the **absence of CP violation** we can identify

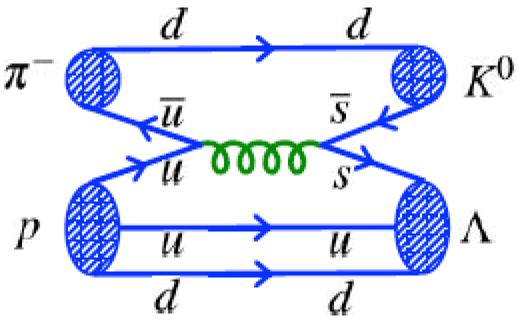
$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \text{with decays:} \quad K_S \rightarrow \pi\pi$$

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \text{with decays:} \quad K_L \rightarrow \pi\pi\pi$$

Neutral Kaon Decays to pions

- Consider the decays of a beam of K^0
- The decays to pions occur in states of definite CP
- If CP is conserved in the decay, need to express K^0 in terms of K_S and K_L

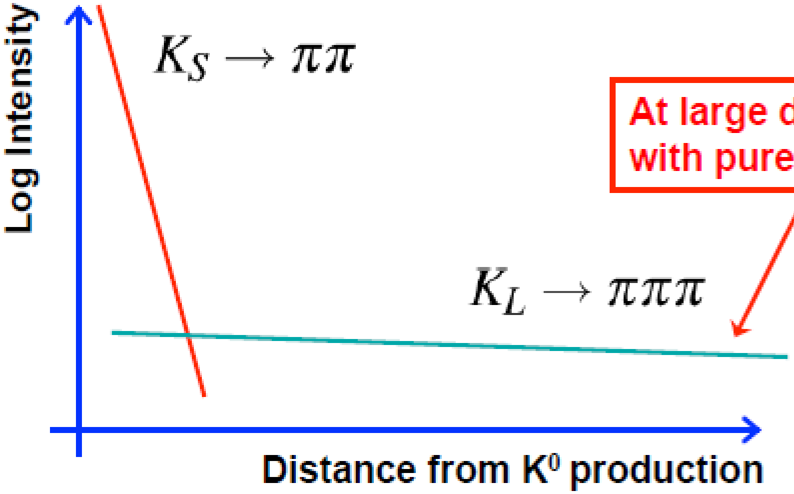
$$|K_0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$



- Hence from the point of view of decays to pions, a K^0 beam is a linear combination of CP eigenstates:

  a rapidly decaying CP-even component and a long-lived CP-odd component

- Therefore, expect to see predominantly two-pion decays near start of beam and predominantly three pion decays further downstream



At large distance left with pure K_L beam

★ To see how this works algebraically:

- Suppose at time t=0 make a beam of pure K^0

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

- Put in the time dependence of wave-function

$$|K_S(t)\rangle = |K_S\rangle e^{-im_S t - \Gamma_S t/2}$$

K$_S$ mass: m_S
K$_S$ decay rate: $\Gamma_S = 1/\tau_S$

NOTE the term $e^{-\Gamma_S t/2}$ ensures the K$_S$ probability density decays exponentially

i.e. $|\psi_S|^2 = \langle K_S(t)|K_S(t)\rangle = e^{-\Gamma_S t} = e^{-t/\tau_S}$

- Hence wave-function evolves as

$$|\psi(t)\rangle \quad = \quad \frac{1}{\sqrt{2}}\left[|K_S\rangle e^{-(im_S+\frac{\Gamma_S}{2})t} + |K_L\rangle e^{-(im_L+\frac{\Gamma_L}{2})t}\right]$$

- Writing $\theta_S(t) = e^{-(im_S+\frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L+\frac{\Gamma_L}{2})t}$

$$|\psi(t)\rangle \quad = \quad \frac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

- **The decay rate to two pions** for a state which was produced as K^0:

$$\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_S|\psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}$$

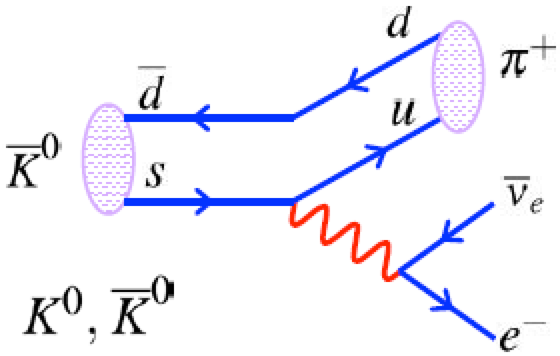
which is as anticipated, i.e. decays of the short lifetime component K$_S$

Neutral Kaon Decays to Leptons

- Neutral kaons can also decay to leptons

$$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e \qquad \bar{K}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu$$

$$K^0 \rightarrow \pi^- e^+ \nu_e \qquad K^0 \rightarrow \pi^- \mu^+ \nu_\mu$$



- **Note:** the final states are not CP eigenstates which is why we express these decays in terms of $K^0, \bar{K}^0$

- Neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L. The main decay modes/branching fractions are:

$$\boxed{\begin{aligned}
K_S \;&\rightarrow\; \pi^+\pi^- && BR = 69.2\% \\
&\rightarrow\; \pi^0\pi^0 && BR = 30.7\% \\
&\rightarrow\; \pi^- e^+ \nu_e && BR = 0.03\% \\
&\rightarrow\; \pi^+ e^- \bar{\nu}_e && BR = 0.03\% \\
&\rightarrow\; \pi^- \mu^+ \nu_\mu && BR = 0.02\% \\
&\rightarrow\; \pi^+ \mu^- \bar{\nu}_\mu && BR = 0.02\%
\end{aligned}}
\qquad
\boxed{\begin{aligned}
K_L \;&\rightarrow\; \pi^+\pi^-\pi^0 && BR = 12.6\% \\
&\rightarrow\; \pi^0\pi^0\pi^0 && BR = 19.6\% \\
&\rightarrow\; \pi^- e^+ \nu_e && BR = 20.2\% \\
&\rightarrow\; \pi^+ e^- \bar{\nu}_e && BR = 20.2\% \\
&\rightarrow\; \pi^- \mu^+ \nu_\mu && BR = 13.5\% \\
&\rightarrow\; \pi^+ \mu^- \bar{\nu}_\mu && BR = 13.5\%
\end{aligned}}$$

- Leptonic decays are more likely for the K-long because the three pion decay modes have a lower decay rate than the two pion modes of the K-short

Strangeness Oscillations (neglecting CP violation)

- The "semi-leptonic" decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state. Hence to calculate the expected decay rate, need to know the K^0 component of the wave-function. For example, for a beam which was initially K^0 we have (1)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

- Writing K_S, K_L in terms of $K^0, \overline{K}^0$

$$\begin{aligned} |\psi(t)\rangle &= \tfrac{1}{2}\left[\theta_S(t)(|K^0\rangle - |\overline{K}^0\rangle) + \theta_L(t)(|K^0\rangle + |\overline{K}^0\rangle)\right] \\ &= \tfrac{1}{2}(\theta_S + \theta_L)|K^0\rangle + \tfrac{1}{2}(\theta_L - \theta_S)|\overline{K}^0\rangle \end{aligned}$$

- Because $\theta_S(t) \neq \theta_L(t)$ a state that was initially a K^0 evolves with time into a mixture of K^0 and $\overline{K}^0$ - "strangeness oscillations"

- The K^0 intensity (i.e. K^0 fraction):

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S + \theta_L|^2 \quad (2)$$

- Similarly $\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S - \theta_L|^2$ (3)

- Using the identity $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$

$$\begin{aligned} |\theta_S \pm \theta_L|^2 &= |e^{-(im_S+\frac{1}{2}\Gamma_S)t} \pm e^{-(im_L+\frac{1}{2}\Gamma_L)t}|^2 \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\Re\{e^{-im_S t} e^{-\frac{1}{2}\Gamma_S t}.e^{+im_L t} e^{-\frac{1}{2}\Gamma_L t}\} \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\Re\{e^{-i(m_S-m_L)t}\} \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos(m_S-m_L)t \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t \end{aligned}$$

- Oscillations between neutral kaon states with frequency given by the mass splitting $\Delta m = m(K_L) - m(K_S)$
- Reminiscent of neutrino oscillations ! Only this time we have decaying states.
- Using equations (2) and (3):

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right] \qquad (4)$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right] \qquad (5)$$

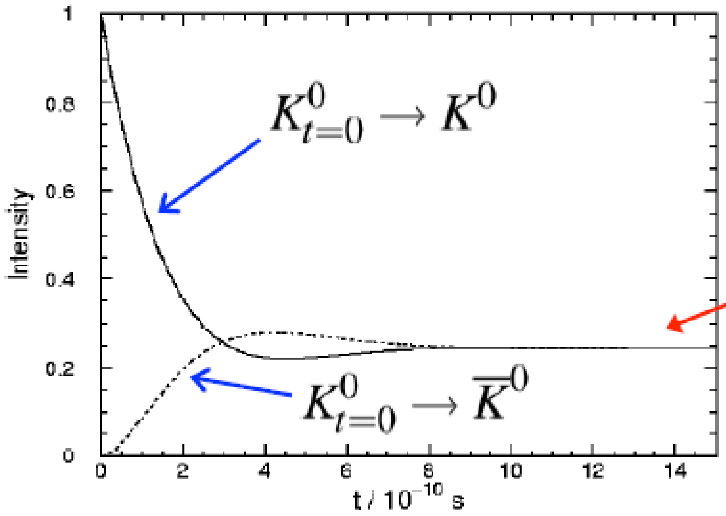
- Experimentally we find: $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\mathrm{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\mathrm{s}}$

  and $\boxed{\Delta m = (3.506 \pm 0.006) \times 10^{-15}\,\mathrm{GeV}}$

  i.e. the K-long mass is greater than the K-short by 1 part in 10^{16}
- The mass difference corresponds to an oscillation period of

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\,\mathrm{s}$$

- The oscillation period is relatively long compared to the K_S lifetime and consequently, do not observe very pronounced oscillations

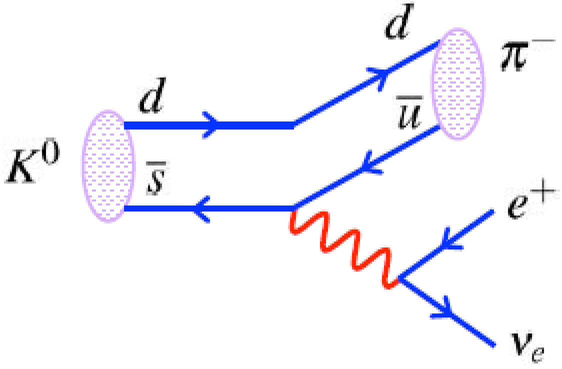


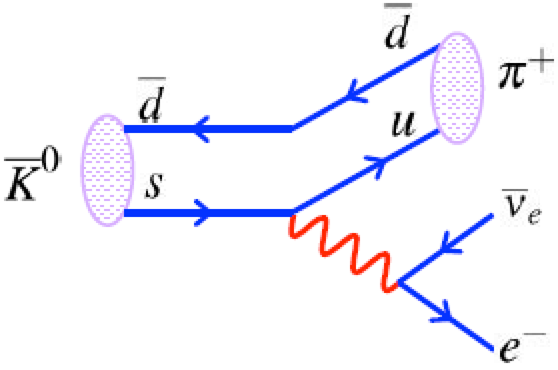
$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

★ Strangeness oscillations can be studied by looking at semi-leptonic decays





★ The charge of the observed pion (or lepton) tags the decay as from either a $\overline{K}^0$ or K^0 because

$$K^0 \rightarrow \pi^- e^+ \nu_e \qquad \overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}_e$$

but

$$\left.\begin{array}{l} \overline{K}^0 \not\rightarrow \pi^- e^+ \nu_e \\ K^0 \not\rightarrow \pi^+ e^- \overline{\nu}_e \end{array}\right\}$$

NOT ALLOWED

•So for an initial K^0 beam, observe the decays to both charge combinations:

$$K^0_{t=0} \rightarrow K^0 \hookrightarrow \pi^- e^+ \nu_e \qquad K^0_{t=0} \rightarrow \overline{K}^0 \hookrightarrow \pi^+ e^- \overline{\nu}_e$$

which provides a way of measuring strangeness oscillations

The CPLEAR Experiment

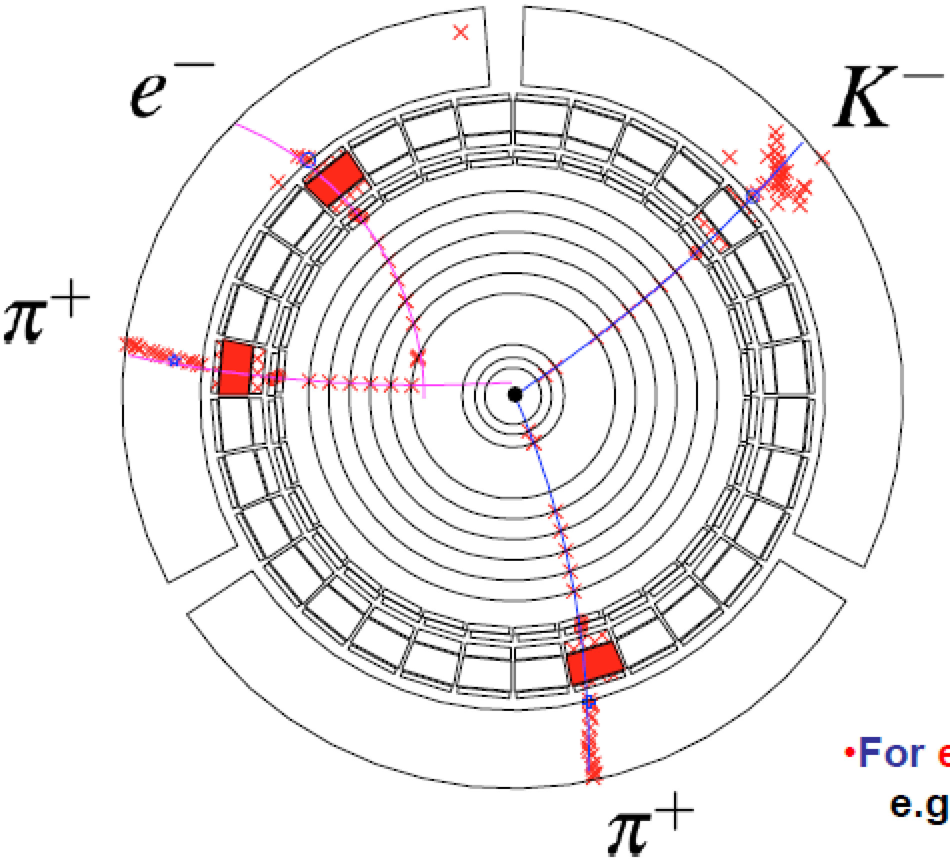
- CERN : 1990-1996
- Used a low energy anti-proton beam
- Neutral kaons produced in reactions

$$\bar{p}p \to K^- \pi^+ K^0$$
$$\bar{p}p \to K^+ \pi^- \bar{K}^0$$

- Low energy, so particles produced almost at rest
- Observe production process and decay in the same detector
- Charge of $K^{\pm}\pi^{\mp}$ in the production process tags the initial neutral kaon as either K^0 or $\bar{K}^0$

- Charge of decay products tags the decay as either as being either K^0 or $\bar{K}^0$
- Provides a direct probe of strangeness oscillations

An example of a CPLEAR event



$K^-(s\bar{u})$
$K^0(d\bar{s})$
$\bar{K}^0(s\bar{d})$

Production:

$$\bar{p}p \longrightarrow \boxed{K^-\pi^+}K^0$$

Mixing

Decay:

$$\bar{K}^0 \longrightarrow \boxed{\pi^+ e^-}\bar{\nu}_e$$

- For each event know initial wave-function, e.g. here: $|\psi(t=0)\rangle = |K^0\rangle$

- Can measure decay rates as a function of time for all combinations:

  e.g. $R^{+} = \Gamma(K^{0}_{t=0} \rightarrow \pi^{-} e^{+} \overline{\nu}_{e}) \propto \Gamma(K^{0}_{t=0} \rightarrow K^{0})$

- From equations (4), (5) and similar relations:

$$R_{+} \equiv \Gamma(K^{0}_{t=0} \rightarrow \pi^{-} e^{+} \nu_{e}) = N_{\pi e \nu} \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$R_{-} \equiv \Gamma(K^{0}_{t=0} \rightarrow \pi^{+} e^{-} \overline{\nu}_{e}) = N_{\pi e \nu} \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\overline{R}_{-} \equiv \Gamma(\overline{K}^{0}_{t=0} \rightarrow \pi^{+} e^{-} \overline{\nu}_{e}) = N_{\pi e \nu} \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\overline{R}_{+} \equiv \Gamma(\overline{K}^{0}_{t=0} \rightarrow \pi^{-} e^{+} \nu_{e}) = N_{\pi e \nu} \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

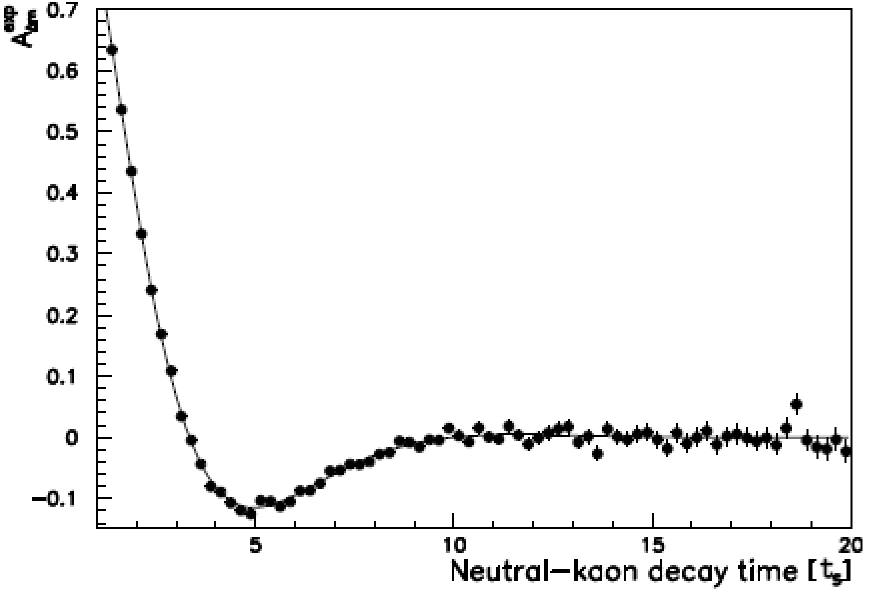
where $N_{\pi e \nu}$ is some overall normalisation factor

- Express measurements as an “asymmetry” to remove dependence on $N_{\pi e \nu}$

$$A_{\Delta m} = \frac{(R_{+} + \overline{R}_{-}) - (R_{-} + \overline{R}_{+})}{(R_{+} + \overline{R}_{-}) + (R_{-} + \overline{R}_{+})}$$

- Using the above expressions for R_+ etc., obtain

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta mt}{e^{-\Gamma_S t}+e^{-\Gamma_L t}}$$



A. Angelopoulos et al., Eur. Phys. J. C22 (2001) 55

★ Points show the data

★ The line shows the theoretical prediction for the value of Δm most consistent with the **CPLEAR** data:

$$\boxed{\Delta m = 3.485\times10^{-15}\,\mathrm{GeV}}$$

- The sign of Δm is not determined here but is known from other experiments
- When the CPLEAR results are combined with experiments at FermiLab obtain:

$$\boxed{\Delta m = m(K_L)-m(K_S) = (3.506\pm0.006)\times10^{-15}\,\mathrm{GeV}}$$

CP Violation in the Kaon System

- So far we have ignored CP violation in the neutral kaon system
- Identified the K-short as the CP-even state and the K-long as the CP-odd state

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$ with decays: $K_S \to \pi\pi$ **CP = +1**

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$ with decays: $K_L \to \pi\pi\pi$ **CP = -1**

- At a long distance from the production point a beam of neutral kaons will be 100% K-long (the K-short component will have decayed away). Hence, if CP is conserved, would expect to see only three-pion decays.
- In 1964 Fitch & Cronin (joint Nobel prize) observed 45 $K_L \to \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point

⇨ **Weak interactions violate CP**

- CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000

K_L to pion BRs:

K_L	$\rightarrow$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$	$CP = -1$
	$\rightarrow$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$	$CP = -1$
	$\rightarrow$	$\pi^+\pi^-$	$BR = 0.20\%$	$CP = +1$
	$\rightarrow$	$\pi^0\pi^0$	$BR = 0.08\%$	$CP = +1$

★ Two possible explanations of CP violation in the kaon system:

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$\boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle]} \quad \boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]}$$

with $|\varepsilon| \sim 2 \times 10^{-3}$

- In this case the observation of $K_L \rightarrow \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

$\rightarrow \pi\pi$ **CP = +1**

$\rightarrow \pi\pi\pi$ **CP = -1**

ii) and/or CP is violated in the decay

$|K_L\rangle = |K_2\rangle$ **CP = -1**

$\rightarrow \pi\pi\pi$ **CP = -1**

$\rightarrow \pi\pi$ **CP = +1**

Parameterised by ε'

★ Experimentally both known to contribute to the mechanism for CP violation in the kaon system but i) dominates: $\varepsilon'/\varepsilon = (1.7 \pm 0.3) \times 10^{-3}$ { NA48 (CERN)
KTeV (FermiLab)

★ The dominant mechanism is discussed in Appendix III

CP Violation in Semi-leptonic decays

- If observe a neutral kaon beam a long time after production (i.e. a large distances) it will consist of a pure K$_L$ component

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\underset{\hookrightarrow\, \pi^+e^-\overline{\nu}_e}{|K_0\rangle} + (1-\varepsilon)\underset{\hookrightarrow\, \pi^-e^+\nu_e}{|\overline{K}^0\rangle}\right]$$

- Decays to $\pi^- e^+ \nu_e$ must come from the $\overline{K}^0$ component, and decays to $\pi^+ e^- \overline{\nu}_e$ must come from the K^0 component

$$\Gamma(K_L \to \pi^+ e^- \overline{\nu}_e) \propto |\langle \overline{K}^0 | K_L \rangle|^2 \propto |1-\varepsilon|^2 \approx 1 - 2\Re\{\varepsilon\}$$

$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1+\varepsilon|^2 \approx 1 + 2\Re\{\varepsilon\}$$

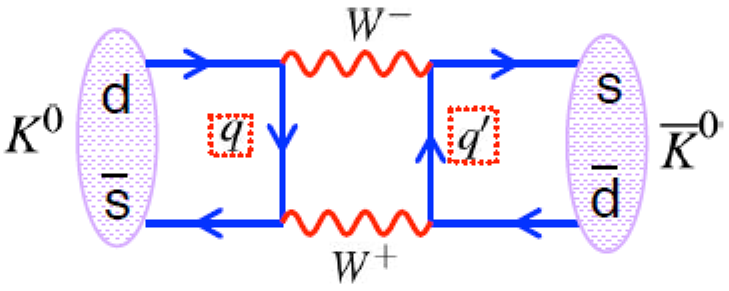
- Results in a small difference in decay rates: the decay to $\pi^- e^+ \nu_e$ is 0.7 % more likely than the decay to $\pi^+ e^- \overline{\nu}_e$
  - This difference has been observed and thus provides the first direct evidence for an absolute difference between matter and anti-matter.

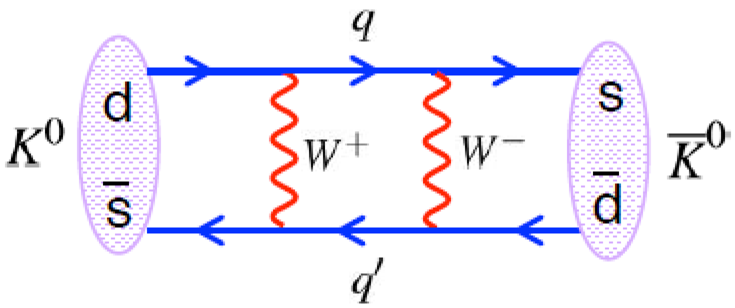
- It also provides an unambiguous definition of matter which could, for example, be transmitted to aliens in a distant galaxy

> "The electrons in our atoms have the same charge as those emitted least often in the decays of the long-lived neutral kaon"

CP Violation and the CKM Matrix

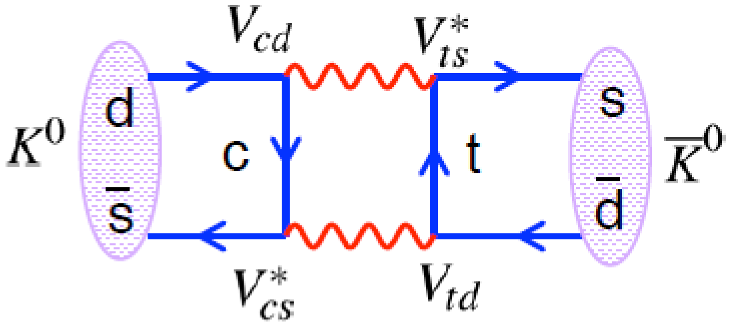
- How can we explain $\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)$ in terms of the CKM matrix ?
  - Consider the box diagrams responsible for mixing, i.e.





where $q = \{u, c, t\}, \quad q' = \{u, c, t\}$

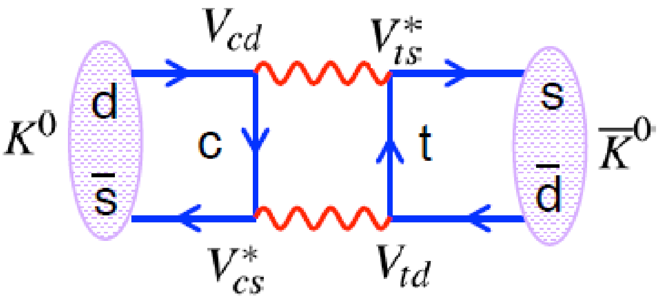
- Have to sum over all possible quark exchanges in the box. For simplicity consider just one diagram

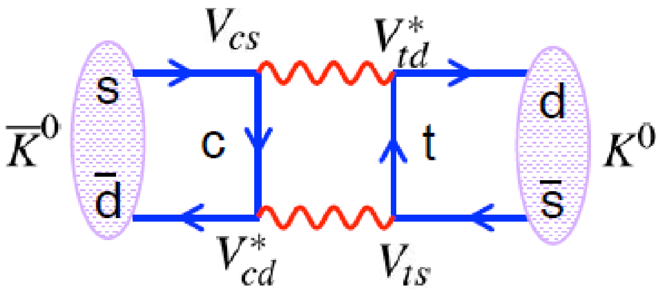


$$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$$

A constant related to integrating over virtual momenta

- Compare the equivalent box diagrams for $K^0 \rightarrow \overline{K}^0$ and $\overline{K}^0 \rightarrow K^0$





$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$

$$M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

- Therefore difference in rates

$$\Gamma(K^0 \rightarrow \overline{K}^0) - \Gamma(\overline{K}^0 \rightarrow K^0) \propto M_{fi} - M_{fi}^* = 2\Im\{M_{fi}\}$$

- Hence the rates can only be different if the CKM matrix has imaginary component

$$|\varepsilon| \propto \Im\{M_{fi}\}$$

- A more formal derivation is given in **Appendix IV**

- In the kaon system we can show

$$|\varepsilon| \propto A_{ut}.\Im\{V_{ud} V_{us}^* V_{td} V_{ts}^*\} + A_{ct}.\Im\{V_{cd} V_{cs}^* V_{td} V_{ts}^*\} + A_{tt}.\Im\{V_{td} V_{ts}^* V_{td} V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

Summary

★ The weak interactions of quarks are described by the **CKM** matrix

★ Similar structure to the lepton sector, although unlike the **PMNS** matrix, the **CKM** matrix is nearly diagonal

★ CP violation enters through via a complex phase in the **CKM** matrix

★ A great deal of experimental evidence for CP violation in the weak interactions of quarks

★ CP violation is needed to explain matter – anti-matter asymmetry in the Universe

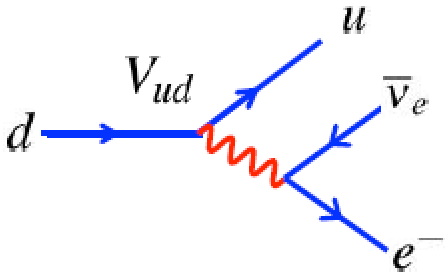
★ **HOWEVER**, CP violation in the SM is **not sufficient** to explain the matter – anti-matter asymmetry. There is probably another mechanism.

Appendix I: Determination of the CKM Matrix

- The experimental determination of the CKM matrix elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision

❶ $\boxed{|V_{ud}|}$ **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



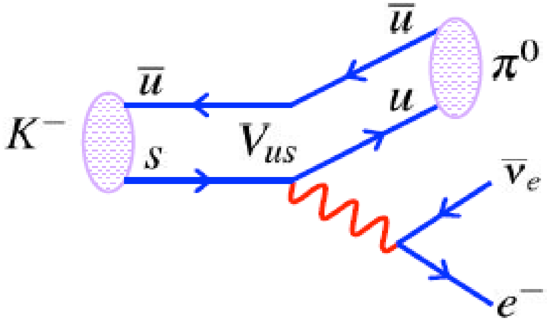
Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$\boxed{|V_{ud}| = 0.97377 \pm 0.00027} \qquad (\approx \cos\theta_c)$$

❷ **$|V_{us}|$** **from semi-leptonic kaon decays**

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



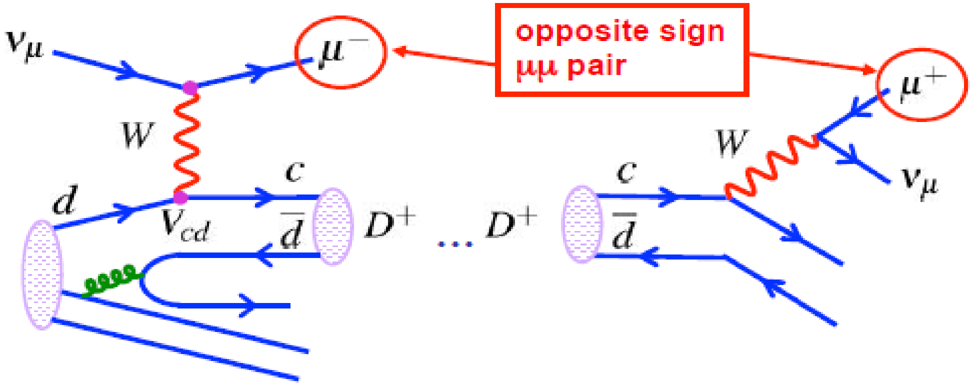
$$\Gamma \propto |V_{us}|^2$$

$$\boxed{|V_{us}| = 0.2257 \pm 0.0021} \qquad (\approx \sin\theta_c)$$

❸ **$|V_{cd}|$** **from neutrino scattering** $\nu_\mu + N \to \mu^+\mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson



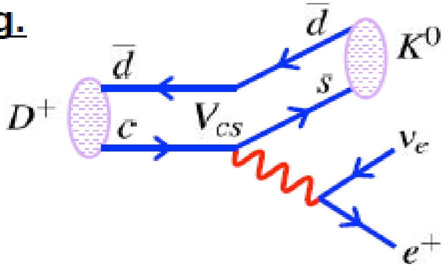
$$\text{Rate} \propto |V_{cd}|^2 \underbrace{\text{Br}(D^+ \to X\mu^+\nu_\mu)}_{\text{Measured in various collider experiments}}$$

⇨ $\boxed{|V_{cd}| = 0.230 \pm 0.011}$

❹ **$|V_{cs}|$** from semi-leptonic charmed meson decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cs}|^2$$

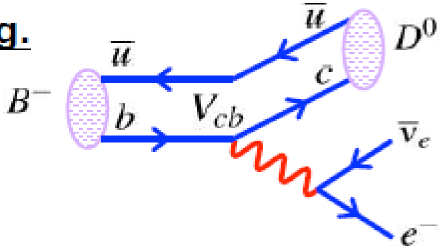
- Precision limited by theoretical uncertainties

$$\boxed{|V_{cs}| = 0.957 \pm \underbrace{0.017}_{\text{experimental error}} \pm \underbrace{0.093}_{\text{theory uncertainty}}}$$

❺ **$|V_{cb}|$** from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



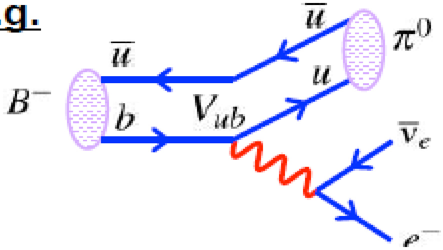
$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

❻ **$|V_{ub}|$** from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$$

Appendix II: Particle-Anti-Particle Mixing

- The wave-function for a single particle with lifetime $\tau = 1/\Gamma$ evolves with time as:

$$\psi(t) = N e^{-\Gamma t/2} e^{-iMt}$$

which gives the appropriate exponential decay of

$$\langle \psi(t) | \psi(t) \rangle = \langle \psi(0) | \psi(0) \rangle e^{-t/\tau}$$

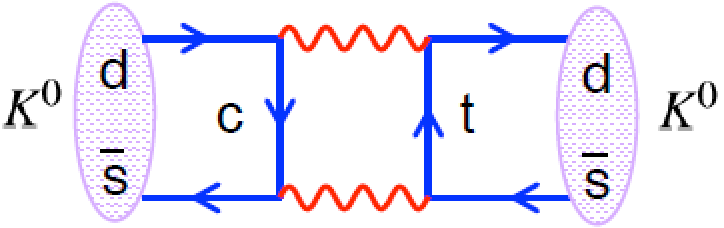
- The wave-function satisfies the time-dependent wave equation:

$$\hat{H} |\psi(t)\rangle = \boxed{(M - \tfrac{1}{2} i\Gamma)} |\psi(t)\rangle = i \frac{\partial}{\partial t} |\psi(t)\rangle \qquad \text{(A1)}$$

- For a bound state such as a K^0 the mass term includes the "mass" from the weak interaction "potential" $\hat{H}_{\text{weak}}$

$$M = m_{K^0} + \langle K^0 | \hat{H}_{\text{weak}} | K^0 \rangle + \sum_j \frac{|\langle K^0 | \hat{H}_{\text{weak}} | j \rangle|^2}{m_{K^0} - E_j}$$

Sum over intermediate states j



The third term is the 2nd order term in the perturbation expansion corresponding to box diagrams resulting in $K^0 \rightarrow \overline{K}^0$

- The total decay rate is the sum over all possible decays $K^0 \rightarrow f$

$$\Gamma = 2\pi \sum_f |\langle f|\hat{H}_{weak}|K^0\rangle|^2 \rho_F$$

Density of final states

★ Because there are also diagrams which allow $K^0 \leftrightarrow \overline{K}^0$ mixing need to consider the time evolution of a mixed stated

$$\psi(t) = a(t)K^0 + b(t)\overline{K}^0 \qquad \text{(A2)}$$

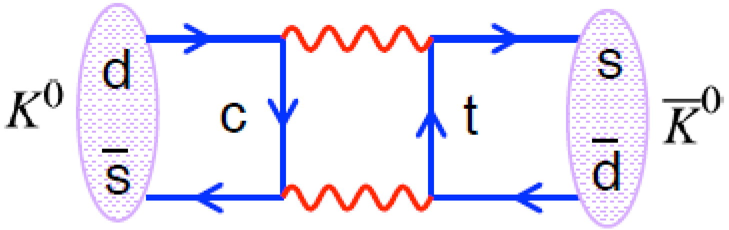
★ The time dependent wave-equation of (A1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} \qquad \text{(A3)}$$

the diagonal terms are as before, and the off-diagonal terms are due to mixing.

$$M_{11} = m_{K^0} + \langle K^0|\hat{H}_{\text{weak}}|K^0\rangle + \sum_n \frac{|\langle K^0|\hat{H}_{\text{weak}}|K^0\rangle|^2}{m_{K^0} - E_n}$$

$$M_{12} = \sum_j \frac{\langle K^0|\hat{H}_{\text{weak}}|j\rangle^* \langle j|\hat{H}_{\text{weak}}|\overline{K}^0\rangle}{m_{K^0} - E_j}$$



- The off-diagonal decay terms include the effects of interference between decays to a common final state

$$\Gamma_{12} = 2\pi \sum_f \langle f|\hat{H}_{weak}|K^0\rangle^* \langle f|\hat{H}_{weak}|\overline{K}^0\rangle \rho_F$$

- In terms of the time dependent coefficients for the kaon states, (A3) becomes

$$[\mathbf{M} - i\tfrac{1}{2}\Gamma] \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix}$$

where the Hamiltonian can be written:

$$\mathbf{H} = \mathbf{M} - i\tfrac{1}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \tfrac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

- Both the mass and decay matrices represent observable quantities and are Hermitian

$$M_{11} = M_{11}^*, \quad M_{22} = M_{22}^*, \quad M_{12} = M_{21}^*$$

$$\Gamma_{11} = \Gamma_{11}^*, \quad \Gamma_{22} = \Gamma_{22}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

- Furthermore, if CPT is conserved then the masses and decay rates of the $\overline{K}^0$ and K^0 are identical:

$$M_{11} = M_{22} = M; \qquad \Gamma_{11} = \Gamma_{22} = \Gamma$$

- Hence the time evolution of the system can be written:

$$\boxed{\begin{pmatrix} M-\frac{1}{2}i\Gamma & M_{12}-\frac{1}{2}i\Gamma_{12} \\ M_{12}^{*}-\frac{1}{2}i\Gamma_{12}^{*} & M-\frac{1}{2}i\Gamma \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix}} \qquad \textbf{(A4)}$$

- To solve the coupled differential equations for a(t) and b(t), first find the eigenstates of the Hamiltonian (the K_L and K_S) and then transform into this basis. The eigenvalue equation is:

$$\begin{pmatrix} M-\frac{1}{2}i\Gamma & M_{12}-\frac{1}{2}i\Gamma_{12} \\ M_{12}^{*}-\frac{1}{2}i\Gamma_{12}^{*} & M-\frac{1}{2}i\Gamma \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \textbf{(A5)}$$

- Which has non-trivial solutions for

$$|\mathbf{H}-\lambda I| = 0$$

$$\Rightarrow \quad (M-\tfrac{1}{2}i\Gamma-\lambda)^2-(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12}) = 0$$

with eigenvalues

$$\lambda = M-\tfrac{1}{2}i\Gamma \pm \sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}$$

- The eigenstates can be obtained by substituting back into **(A5)**

$$(M-\tfrac{1}{2}i\Gamma)x_1+(M_{12}-\tfrac{1}{2}i\Gamma_{12})=(M-\tfrac{1}{2}i\Gamma\pm\sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})})x_1$$

⇨
$$\frac{x_2}{x_1}=\pm\sqrt{\frac{M_{12}^*-\frac{1}{2}i\Gamma_{12}^*}{M_{12}-\frac{1}{2}i\Gamma_{12}}}$$

★ **Define**

$$\eta=\sqrt{\frac{M_{12}^*-\frac{1}{2}i\Gamma_{12}^*}{M_{12}-\frac{1}{2}i\Gamma_{12}}}$$

★ **Hence the normalised eigenstates are**

$$|K_\pm\rangle=\frac{1}{\sqrt{1+|\eta|^2}}\begin{pmatrix}1\\ \pm\eta\end{pmatrix}=\frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle\pm\eta|\overline{K}^0\rangle)$$

★ **Note, in the limit where M_{12},Γ_{12} are real, the eigenstates correspond to the CP eigenstates K$_1$ and K$_2$. Hence we can identify the general eigenstates as as the long and short lived neutral kaons:**

$$\boxed{|K_L\rangle=\frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle+\eta|\overline{K}^0\rangle)}\quad\boxed{|K_S\rangle=\frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle-\eta|\overline{K}^0\rangle)}$$

- Substituting these states back into (A2):

$$\begin{aligned}
|\psi(t)\rangle &= a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle \\
&= \sqrt{1+|\eta|^2}\left[\frac{a(t)}{2}(K_L+K_S) + \frac{b(t)}{2\eta}(K_L-K_S)\right] \\
&= \sqrt{1+|\eta|^2}\left[\left(\frac{a(t)}{2}+\frac{b(t)}{2\eta}\right)K_L + \left(\frac{a(t)}{2}-\frac{b(t)}{2\eta}\right)K_S\right] \\
&= \frac{\sqrt{1+|\eta|^2}}{2}\left[a_L(t)K_L + a_S(t)K_S\right]
\end{aligned}$$

with

$$\boxed{a_L(t) \equiv a(t) + \frac{b(t)}{\eta}} \qquad \boxed{a_S(t) \equiv a(t) - \frac{b(t)}{\eta}}$$

- Now consider the time evolution of $a_L(t)$

$$i\frac{\partial a_L}{\partial t} = i\frac{\partial a}{\partial t} + \frac{i}{\eta}\frac{\partial b}{\partial t}$$

- Which can be evaluated using (A4) for the time evolution of a(t) and b(t):

$$\begin{aligned}
i\frac{\partial a_L}{\partial t} &= \left[(M-\tfrac{1}{2}i\Gamma_{12})a+(M_{12}-\tfrac{1}{2}i\Gamma_{12})b\right]+\frac{1}{\eta}\left[(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})a+(M-\tfrac{1}{2}i\Gamma)b\right]\\
&= (M-\tfrac{1}{2}i\Gamma)\left(a+\frac{b}{\eta}\right)+(M_{12}-\tfrac{1}{2}i\Gamma_{12})b+\frac{1}{\eta}(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})a\\
&= (M-\tfrac{1}{2}i\Gamma)a_L+(M_{12}-\tfrac{1}{2}i\Gamma_{12})b+\left(\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)a\\
&= (M-\tfrac{1}{2}i\Gamma)a_L+\left(\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)\left(a+\frac{b}{\eta}\right)\\
&= (M-\tfrac{1}{2}i\Gamma)a_L+\left(\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)a_L\\
&= (m_L-\tfrac{1}{2}i\Gamma_L)a_L
\end{aligned}$$

★ **Hence:**

$$\boxed{i\frac{\partial a_L}{\partial t}=(m_L-\tfrac{1}{2}i\Gamma_L)a_L}$$

with $m_L = M+\Re\left\{\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right\}$

and $\Gamma_L = \Gamma-2\Im\left\{\sqrt{(M_{12}^{*}-\tfrac{1}{2}i\Gamma_{12}^{*})(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right\}$

- Following the same procedure obtain:

$$\boxed{i\frac{\partial a_S}{\partial t} = (m_S - \tfrac{1}{2}i\Gamma_S)a_S}$$

with $$m_S = M - \Re\left\{\sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}$$

and $$\Gamma_S = \Gamma + 2\Im\left\{\sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}$$

- In matrix notation we have

$$\begin{pmatrix} M_L - \frac{1}{2}i\Gamma_L & 0 \\ 0 & M_S - \frac{1}{2}i\Gamma_S \end{pmatrix} \begin{pmatrix} a_L \\ a_S \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a_L \\ a_S \end{pmatrix}$$

- Solving we obtain

$$a_L(t) \propto e^{-im_Lt - \Gamma_Lt/2} \qquad a_S(t) \propto e^{-im_St - \Gamma_St/2}$$

- Hence in terms of the K_L and K_S basis the states propagate as independent particles with definite masses and lifetimes (the mass eigenstates). The time evolution of the neutral kaon system can be written

$$|\psi(t)\rangle = A_Le^{-im_Lt - \Gamma_Lt/2}|K_L\rangle + A_Se^{-im_St - \Gamma_St/2}|K_S\rangle$$

where A_L and A_S are constants

Appendix III: CP Violation: $\pi\pi$ decays

★ Consider the development of the $K^0 - \overline{K}^0$ system now including CP violation

★ Repeat previous derivation using

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

- Writing the CP eigenstates in terms of $K^0, \overline{K}^0$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle - (1-\varepsilon)|\overline{K}^0\rangle\right]$$

- Inverting these expressions obtain

$$\boxed{|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}(|K_L\rangle + |K_S\rangle)} \qquad \boxed{|\overline{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}(|K_L\rangle - |K_S\rangle)}$$

- Hence a state that was produced as a K^0 evolves with time as:

$$|\psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}(\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle)$$

where as before $\theta_S(t) = e^{-(im_S+\frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L+\frac{\Gamma_L}{2})t}$

- If we are considering the decay rate to ππ need to express the wave-function in terms of the CP eigenstates (remember we are neglecting CP violation in the decay)

$$
\begin{aligned}
|\psi(t)\rangle &= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[(|K_2\rangle + \varepsilon|K_1\rangle)\theta_L(t) + (|K_1\rangle + \varepsilon|K_2\rangle)\theta_S(t)\right] \\
&= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[(\theta_S + \varepsilon\theta_L)|K_1\rangle + (\theta_L + \varepsilon\theta_S)|K_2\rangle\right]
\end{aligned}
$$

CP Eigenstates

- Two pion decays occur with CP = +1 and therefore arise from decay of the CP = +1 kaon eigenstate, i.e. K_1

$$\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_1|\psi(t)\rangle|^2 = \frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2 |\theta_S + \varepsilon\theta_L|^2$$

- Since $|\varepsilon| \ll 1$

$$\left|\frac{1}{1+\varepsilon}\right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\Re\{\varepsilon\}} \approx 1 - 2\Re\{\varepsilon\}$$

- Now evaluate the $|\theta_S + \varepsilon\theta_L|^2$ term again using

$$|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$$

$$
\begin{aligned}
|\theta_S + \varepsilon\theta_L|^2 &= |e^{-im_S t - \frac{\Gamma_S}{2}t} + \varepsilon e^{-im_L t - \frac{\Gamma_L}{2}t}|^2 \\
&= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2\Re\{e^{-im_S t - \frac{\Gamma_S}{2}t}.\varepsilon^* e^{+im_L t - \frac{\Gamma_L}{2}t}\}
\end{aligned}
$$

- Writing $\varepsilon = |\varepsilon|e^{i\phi}$

$$
\begin{aligned}
|\theta_S + \varepsilon\theta_L|^2 &= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\Re\{e^{i(m_L - m_S)t - \phi}\} \\
&= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m.t - \phi)
\end{aligned}
$$

- Putting this together we obtain:

$$
\boxed{\Gamma(K^0_{t=0} \to \pi\pi) = \frac{1}{2}(1 - 2\Re\{\varepsilon\})N_{\pi\pi}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m.t - \phi)\right]}
$$

Short lifetime component $K_S \to \pi\pi$

CP violating long lifetime component $K_L \to \pi\pi$

Interference term

- In exactly the same manner obtain for a beam which was produced as $\overline{K}^0$

$$
\boxed{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) = \frac{1}{2}(1 + 2\Re\{\varepsilon\})N_{\pi\pi}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m.t - \phi)\right]}
$$

Interference term changes sign

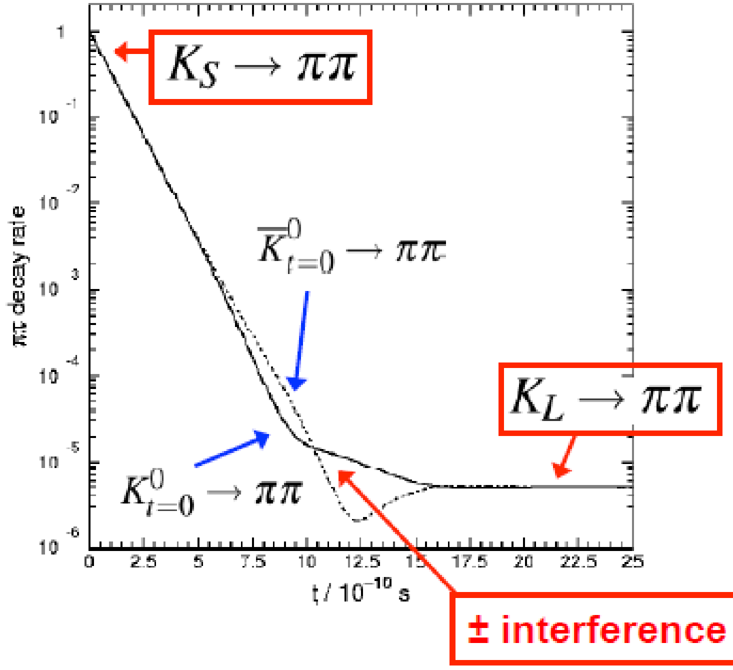
- At large proper times only the long lifetime component remains :

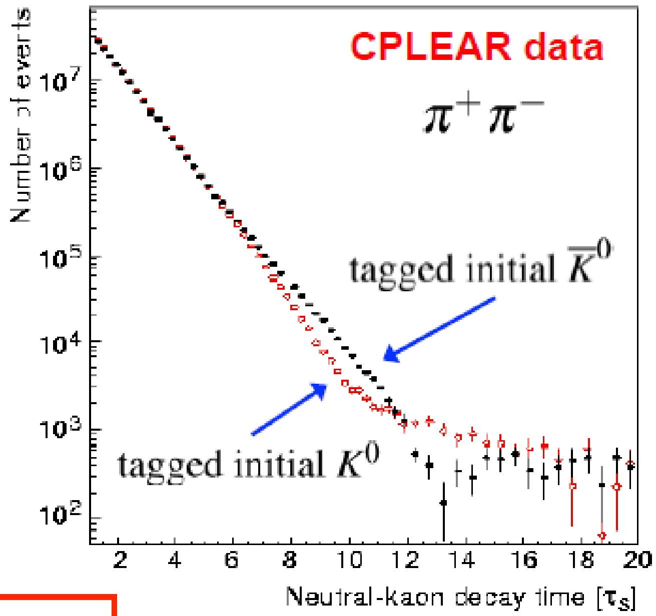
$$\Gamma(K^0_{t=0} \to \pi\pi) \quad \to \quad \frac{1}{2}(1-2\Re\{\varepsilon\})N_{\pi\pi}.|\varepsilon|^2 e^{-\Gamma_L t}$$

i.e. CP violating $K_L \to \pi\pi$ decays

- Since CPLEAR can identify whether a K^0 or $\overline{K}^0$ was produced, able to measure $\Gamma(K^0_{t=0} \to \pi\pi)$ and $\Gamma(\overline{K}^0_{t=0} \to \pi\pi)$

Prediction with CP violation





★ **The CPLEAR data shown previously can be used to measure** $\varepsilon = |\varepsilon| e^{i\phi}$

- **Define the asymmetry:**

$$A_{+-} = \frac{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) - \Gamma(K^0_{t=0} \to \pi\pi)}{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) + \Gamma(K^0_{t=0} \to \pi\pi)}$$

- **Using expressions on page 443**

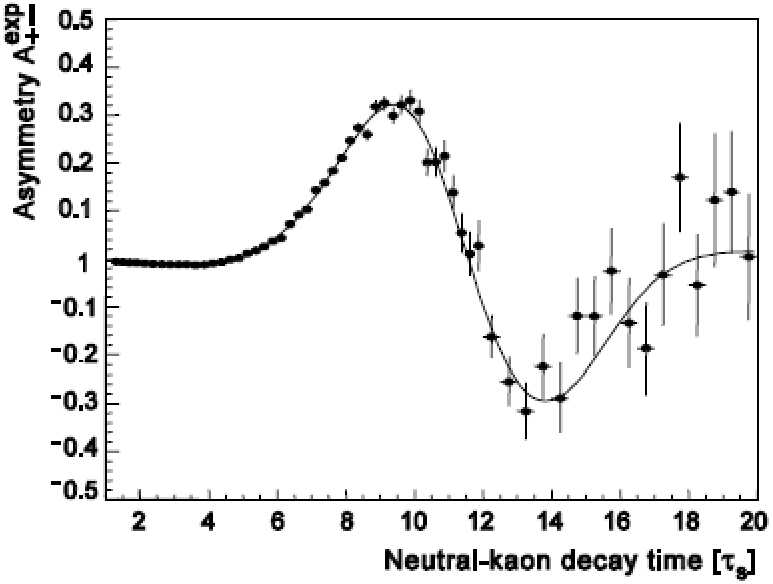
$$A_{+-} = \frac{4\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 4|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{2\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - \underbrace{8\Re\{\varepsilon\}|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}_{\boxed{\propto |\varepsilon|\Re\{\varepsilon\}\ \text{i.e. two small quantities and can safely be neglected}}}}$$

$$A_{+-} \approx \frac{2\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 2|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon| e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon| e^{(\Gamma_S-\Gamma_L)t/2}\cos(\Delta m.t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

A.Apostolakis et al., Eur. Phys. J. C18 (2000) 41



Best fit to the data:

$$\boxed{\begin{aligned} |\varepsilon| &= (2.264 \pm 0.035) \times 10^{-3} \\ \phi &= (43.19 \pm 0.73)^\circ \end{aligned}}$$

Appendix IV: CP Violation via Mixing

- ★ A full description of the SM origin of CP violation in the kaon system is beyond the level of this course, nevertheless, the relation to the box diagrams is illustrated below
- ★ The K-long and K-short wave-functions depend on η

$$\boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle + \eta|\overline{K}^0\rangle)} \quad \boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle - \eta|\overline{K}^0\rangle)}$$

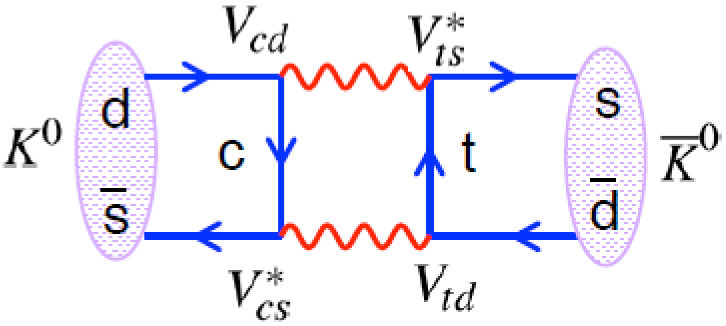
with $$\eta = \sqrt{\frac{M^*_{12} - \frac{1}{2}i\Gamma^*_{12}}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

- ★ If $M^*_{12} = M_{12}; \quad \Gamma^*_{12} = \Gamma_{12}$ then the K-long and K-short correspond to the CP eigenstates K_1 and K_2

- CP violation is therefore associated with imaginary off-diagonal mass and decay elements for the neutral kaon system
- Experimentally, CP violation is small and $\eta \approx 1$
- Define: $\varepsilon = \frac{1-\eta}{1+\eta}$ ⇨ $\eta = \frac{1-\varepsilon}{1+\varepsilon}$

- Consider the mixing term M_{12} which arises from the sum over all possible intermediate states in the mixing box diagrams

e.g.



$$M_{12} = A_{ct} V_{cd} V_{cs}^* V_{ts}^* V_{td} + \ldots$$

- Therefore it can be seen that, in the Standard Model, CP violation is associated with the imaginary components of the CKM matrix

- It can be shown that mixing leads to CP violation with

$$|\varepsilon| \propto \Im\{M_{12}\}$$

- The differences in masses of the mass eigenstates can be shown to be:

$$\Delta m_K = m_{K_L} - m_{K_S} \approx \sum_{q,q'} \frac{G_F^2}{3\pi^2} f_K^2 m_K |V_{qd} V_{qs}^* V_{q'd} V_{q's}^*| m_q m_{q'}$$

where q and q' are the quarks in the loops and f_K is a constant

- In terms of the small parameter ε

$$|K_L\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_S\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1-\varepsilon)|K^0\rangle + (1+\varepsilon)|\overline{K}^0\rangle\right]$$

★ If epsilon is non-zero we have CP violation in the neutral kaon system

Writing $\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}} = \sqrt{\frac{z^*}{z}}$ and $z = ae^{i\phi}$

gives $\eta = e^{-i\phi}$

★ From which we can find an expression for ε

$$\varepsilon.\varepsilon^* = \frac{1-e^{-i\phi}}{1+e^{-i\phi}}\cdot\frac{1-e^{+i\phi}}{1+e^{i\phi}} = \frac{2-\cos\phi}{2+\cos\phi} = \tan^2\frac{\phi}{2}$$

$$|\varepsilon| = |\tan\tfrac{\phi}{2}|$$

★ Experimentally we know ε is small, hence ϕ is small

$$|\varepsilon| \approx \frac{1}{2}\phi = \frac{1}{2}\arg z \approx \frac{1}{2}\frac{\Im\{M_{12} - \frac{1}{2}i\Gamma_{12}\}}{|M_{12} - \frac{1}{2}i\Gamma_{12}|}$$

Appendix V: Time Reversal Violation

- Previously, equations (4) and (5), obtained expressions for strangeness oscillations in the absence of CP violation, e.g.

$$\Gamma(K^0_{t=0} \rightarrow K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

- This analysis can be extended to include the effects of CP violation to give the following rates (

$$\Gamma(K^0_{t=0} \rightarrow K^0) \propto \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(\overline{K}^0_{t=0} \rightarrow \overline{K}^0) \propto \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(\overline{K}^0_{t=0} \rightarrow K^0) \propto \boxed{\tfrac{1}{4}(1+4\Re\{\varepsilon\})}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \rightarrow \overline{K}^0) \propto \boxed{\tfrac{1}{4}(1-4\Re\{\varepsilon\})}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

★ Including the effects of CP violation find that

$$\boxed{\Gamma(\overline{K}^0_{t=0} \rightarrow K^0) \neq \Gamma(K^0_{t=0} \rightarrow \overline{K}^0)}$$

Violation of time reversal symmetry !

★ No surprise, as CPT is conserved, CP violation implies T violation