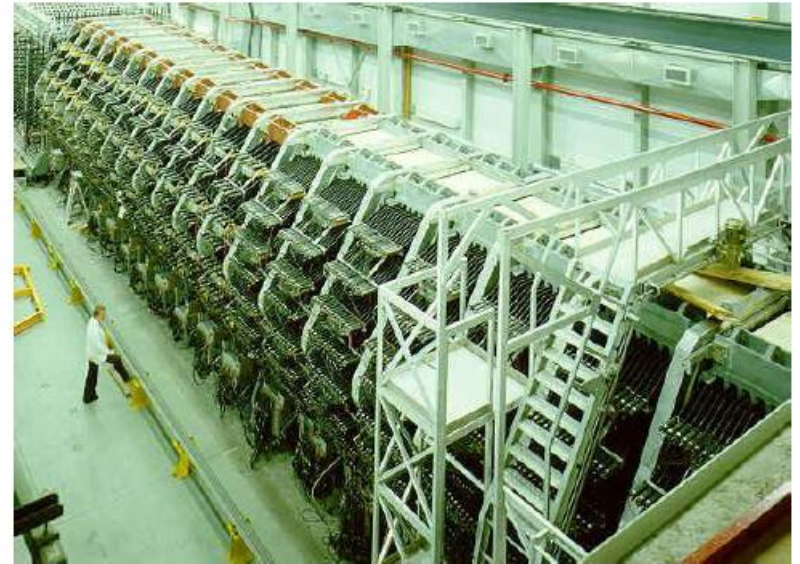


# Elementary Particle Physics: theory and experiments

**Leptonic Weak  
Interactions and Neutrino  
Deep Inelastic Scattering.**

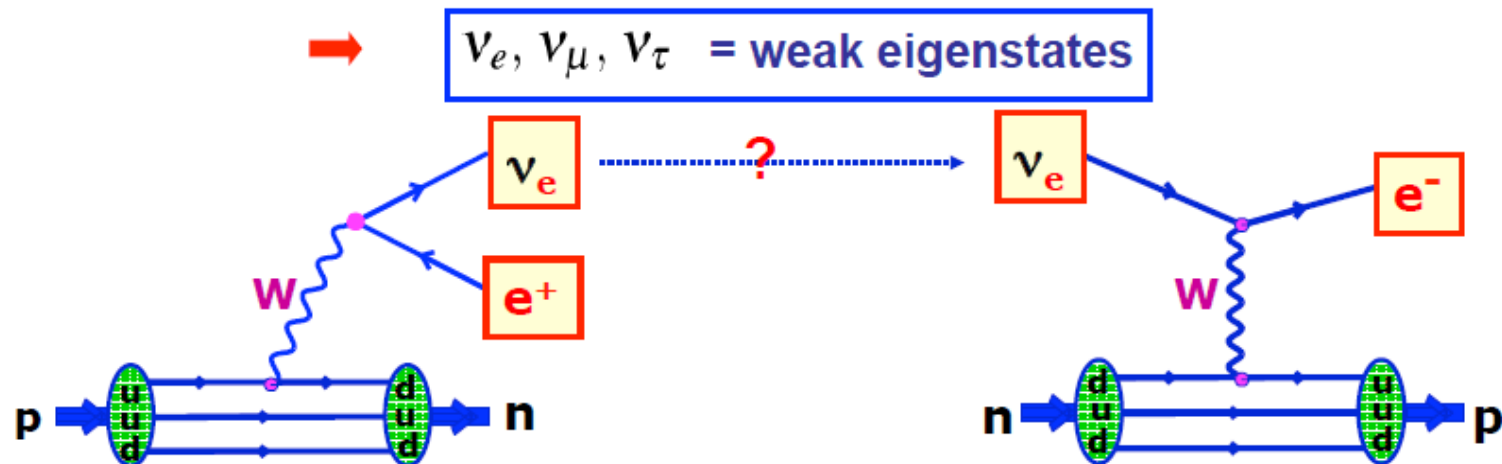


Follow the course/slides from M. A. Thomson lectures at Cambridge University

Prof. dr hab. Elżbieta Richter-Wąs

# Neutrino Flavours

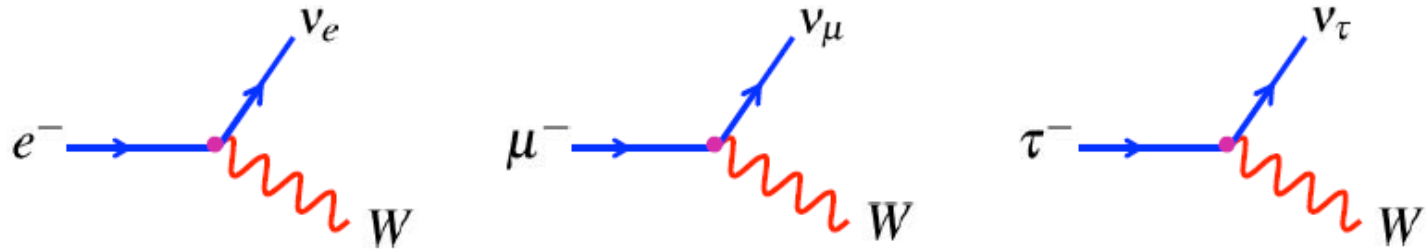
- ★ Recent experiments (→) neutrinos have mass (albeit very small)
- ★ The textbook neutrino states,  $\nu_e, \nu_\mu, \nu_\tau$ , are not the fundamental particles; these are  $\nu_1, \nu_2, \nu_3$
- ★ Concepts like “electron number” conservation are now known **not** to hold.
- ★ So what are  $\nu_e, \nu_\mu, \nu_\tau$  ?
- ★ Never **directly** observe neutrinos – can only detect them by their weak interactions. Hence by **definition**  $\nu_e$  is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state  $\nu_e$  produce an electron



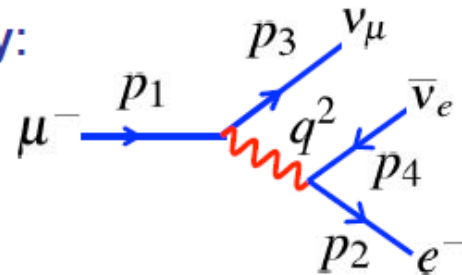
- ★ Unless dealing with very large distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the **weak interaction** continue to use  $\nu_e, \nu_\mu, \nu_\tau$  as if they were the fundamental particle states.

# Muon Decay and Lepton Universality

- ★ The leptonic **charged current** ( $W^\pm$ ) interaction vertices are:



- ★ Consider muon decay:



- It is straight-forward to write down the matrix element

$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2} [\bar{u}(p_3) \gamma^\mu (1 - \gamma^5) u(p_1)] g_{\mu\nu} [\bar{u}(p_2) \gamma^\nu (1 - \gamma^5) v(p_4)]$$

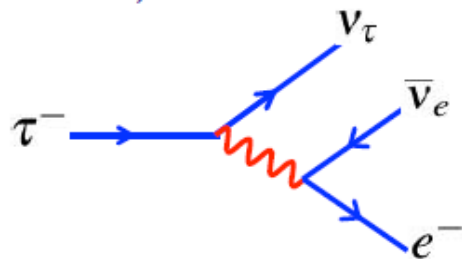
**Note:** for lepton decay  $q^2 \ll m_W^2$  so propagator is a constant  $1/m_W^2$   
i.e. in limit of Fermi theory

- Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

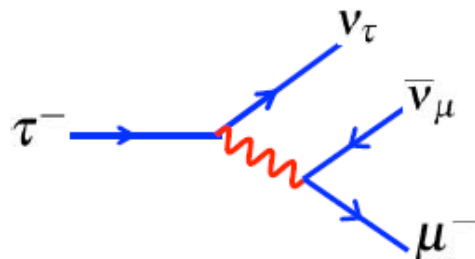
- The muon to electron rate  $\Gamma(\mu \rightarrow e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu}$  with  $G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}$

- Similarly for tau to electron  $\Gamma(\tau \rightarrow e\nu\nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$

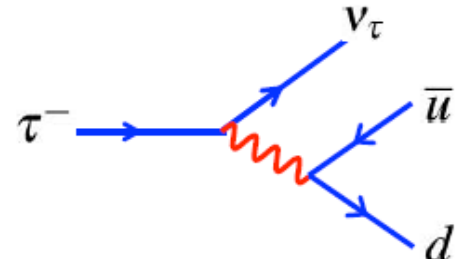
- However, the tau can decay to a number of final states:



$$Br(\tau \rightarrow e\nu\nu) = 0.1784(5)$$



$$Br(\tau \rightarrow \mu\nu\nu) = 0.1736(5)$$



- Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

- Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \rightarrow e\nu\nu) = \Gamma_\tau Br(\tau \rightarrow e\nu\nu) = Br(\tau \rightarrow e\nu\nu) / \tau_\tau$$

- Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \quad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \rightarrow e\nu\nu)$$

- All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94) \text{ GeV} \quad \tau_\mu = 2.19703(4) \times 10^{-6} \text{ s}$$

$$m_\tau = 1.77699(28) \text{ GeV} \quad \tau_\tau = 0.2906(10) \times 10^{-12} \text{ s}$$

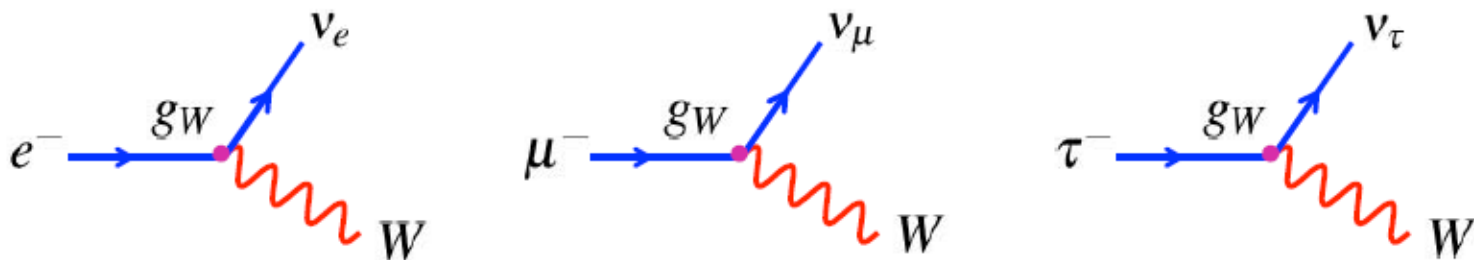
$$Br(\tau \rightarrow e\nu\nu) = 0.1784(5)$$

→ 
$$\frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \rightarrow e\nu\nu) = 1.0024 \pm 0.0033$$

- Similarly by comparing  $Br(\tau \rightarrow e\nu\nu)$  and  $Br(\tau \rightarrow \mu\nu\nu)$

$$\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004$$

- ★ Demonstrates the weak charged current is the same for all leptonic vertices



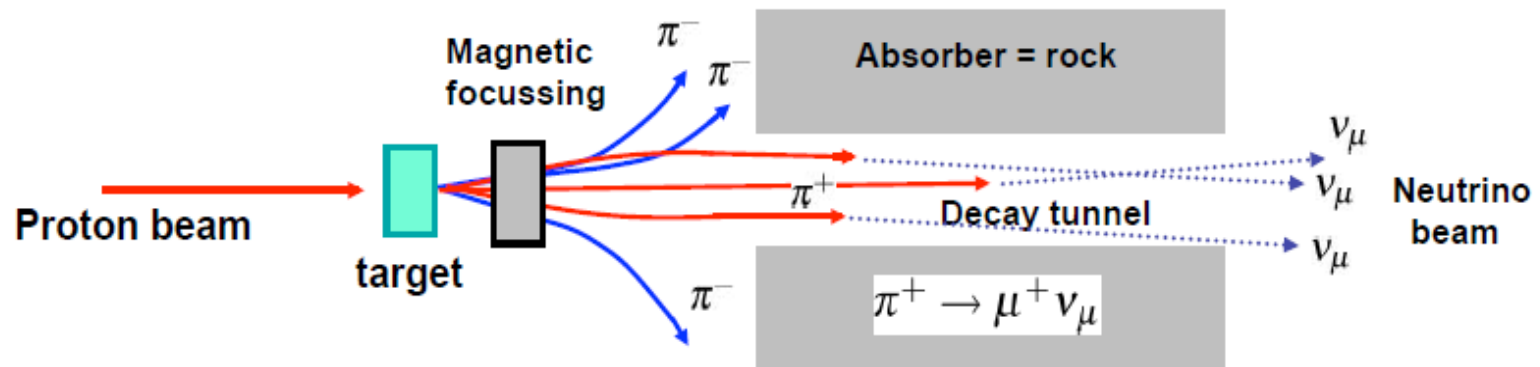
→ **Charged Current Lepton Universality**

# Neutrino Scattering

- Can also consider the weak interaction equivalent: **Neutrino Deep Inelastic Scattering** where a virtual W-boson probes the structure of nucleons
  - ➔ additional information about parton structure functions
- + provides a good example of calculations of weak interaction cross sections.

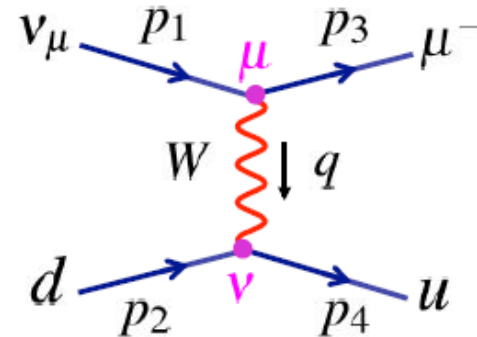
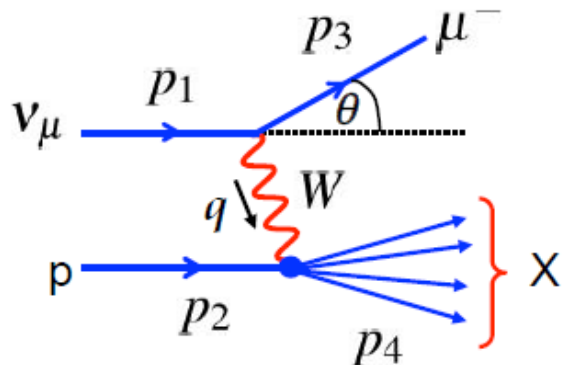
## ★ Neutrino Beams:

- Smash high energy protons into a fixed target ➔ hadrons
- Focus positive pions/kaons
- Allow them to decay  $\pi^+ \rightarrow \mu^+ \nu_\mu$  +  $K^+ \rightarrow \mu^+ \nu_\mu$  ( $BR \approx 64\%$ )
- Gives a beam of “collimated”  $\nu_\mu$
- Focus negative pions/kaons to give beam of  $\bar{\nu}_\mu$



# Neutrino – Quark Scattering

- ★ For  $\nu_\mu$ -proton Deep Inelastic Scattering the underlying process is  $\nu_\mu d \rightarrow \mu^- u$



- ★ In the limit  $q^2 \ll m_W^2$  the W-boson propagator is  $\approx ig_{\mu\nu}/m_W^2$

- The Feynman rules give:

$$-iM_{fi} = \left[ -i \frac{g_W}{\sqrt{2}} \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma^5) u(p_1) \right] \frac{ig_{\mu\nu}}{m_W^2} \left[ -i \frac{g_W}{\sqrt{2}} \bar{u}(p_4) \frac{1}{2} \gamma^\nu (1 - \gamma^5) u(p_2) \right]$$

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[ \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma^5) u(p_1) \right] \left[ \bar{u}(p_4) \frac{1}{2} \gamma^\nu (1 - \gamma^5) u(p_2) \right]$$

- Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected



- In this limit the helicity states are equivalent to the chiral states and

$$\frac{1}{2}(1 - \gamma^5)u_{\uparrow}(p_1) = 0 \quad \frac{1}{2}(1 - \gamma^5)u_{\downarrow}(p_1) = u_{\downarrow}(p_1)$$

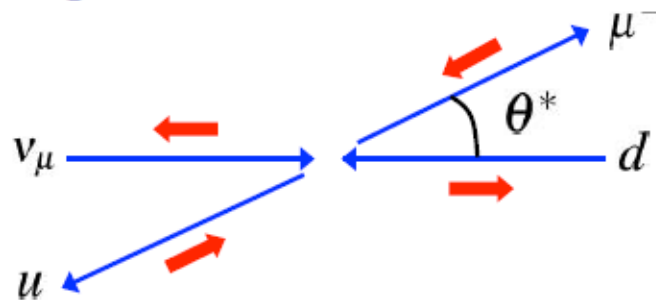
$$\Rightarrow M_{fi} = 0 \quad \text{for } u_{\uparrow}(p_1) \text{ and } u_{\uparrow}(p_2)$$

- Since the weak interaction “conserves the helicity”, the only helicity combination where the matrix element is non-zero is

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} [\bar{u}_{\downarrow}(p_3)\gamma^{\mu}u_{\downarrow}(p_1)] [\bar{u}_{\downarrow}(p_4)\gamma^{\nu}u_{\downarrow}(p_2)]$$

**NOTE:** we could have written this down straight away as in the ultra-relativistic limit only **LH helicity particle** states participate in the weak interaction.

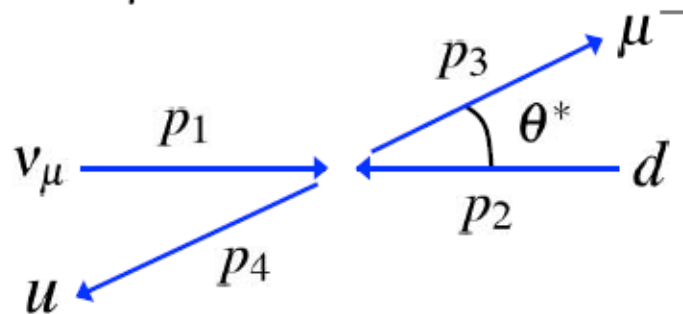
- ★ Consider the scattering in the C.o.M frame





# Evaluation of Neutrino-Quark Scattering ME

- Go through the calculation in gory detail (fortunately only one helicity combination)
- In the  $\nu_\mu d$  CMS frame, neglecting particle masses:



$$p_1 = (E, 0, 0, E),$$

$$p_2 = (E, 0, 0, -E)$$

$$p_3 = (E, E \sin \theta^*, 0, E \cos \theta^*)$$

$$p_4 = (E, -E \sin \theta^*, 0, -E \cos \theta^*)$$

- Dealing with LH helicity particle spinors. |  
for a massless particle travelling in direction  $(\theta, \phi)$ :

$$u_\downarrow = \sqrt{E} \begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}$$

$$c = \cos \frac{\theta}{2}; \quad s = \sin \frac{\theta}{2}$$

- Here  $(\theta_1, \phi_1) = (0, 0)$ ;  $(\theta_2, \phi_2) = (\pi, 0)$ ;  $(\theta_3, \phi_3) = (\theta^*, 0)$ ;  $(\theta_4, \phi_4) = (\pi - \theta^*, \pi)$   
giving:

$$u_\downarrow(p_1) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}; \quad u_\downarrow(p_2) = \sqrt{E} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad u_\downarrow(p_3) = \sqrt{E} \begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}; \quad u_\downarrow(p_4) = \sqrt{E} \begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

•To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} [\bar{u}_\downarrow(p_3) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\downarrow(p_4) \gamma^\nu u_\downarrow(p_2)]$$

need to evaluate two terms of form

$$\bar{\psi} \gamma^0 \phi = \psi^\dagger \gamma^0 \gamma^0 \phi = \psi_1^* \phi_1 + \psi_2^* \phi_2 + \psi_3^* \phi_3 + \psi_4^* \phi_4$$

$$\bar{\psi} \gamma^1 \phi = \psi^\dagger \gamma^0 \gamma^1 \phi = \psi_1^* \phi_4 + \psi_2^* \phi_3 + \psi_3^* \phi_2 + \psi_4^* \phi_1$$

$$\bar{\psi} \gamma^2 \phi = \psi^\dagger \gamma^0 \gamma^2 \phi = -i(\psi_1^* \phi_4 - \psi_2^* \phi_3 + \psi_3^* \phi_2 - \psi_4^* \phi_1)$$

$$\bar{\psi} \gamma^3 \phi = \psi^\dagger \gamma^0 \gamma^3 \phi = \psi_1^* \phi_3 - \psi_2^* \phi_4 + \psi_3^* \phi_1 - \psi_4^* \phi_2$$

•Using

$$u_\downarrow(p_1) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}; u_\downarrow(p_2) = \sqrt{E} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; u_\downarrow(p_3) = \sqrt{E} \begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}; u_\downarrow(p_4) = \sqrt{E} \begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$



$$\bar{u}_\downarrow(p_3) \gamma^\mu u_\downarrow(p_1) = 2E(c, s, -is, c)$$

$$\bar{u}_\downarrow(p_4) \gamma^\nu u_\downarrow(p_2) = 2E(c, -s, -is, -c)$$



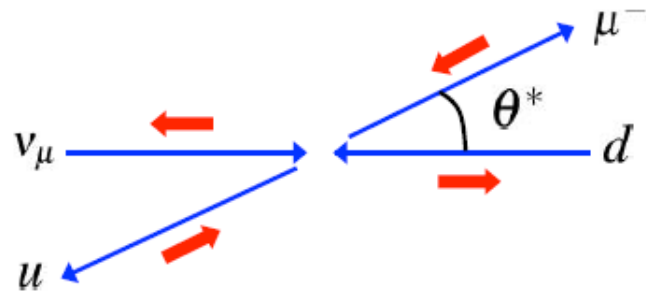
$$M_{fi} = \frac{g_W^2}{2m_W^2} 4E^2(c^2 + s^2 + s^2 + c^2) = \frac{g_W^2 \hat{s}}{m_W^2}$$

$$\hat{s} = (2E)^2$$

- ★ Note the Matrix Element is isotropic

$$M_{fi} = \frac{g_W^2}{m_W^2} \hat{s}$$

we could have anticipated this since the helicity combination (spins anti-parallel) has  $S_z = 0 \rightarrow$  no preferred polar angle



- ★ As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination (LL $\rightarrow$ LL) and **only 2 possible initial state combinations** (the neutrino is always produced in a LH helicity state)

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2} \cdot \left| \frac{g_W^2}{m_W^2} \hat{s} \right|^2$$

The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction

- ★ From handout 1, in the extreme relativistic limit, the cross section for any 2 $\rightarrow$ 2 body scattering process is

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2\hat{s}} \langle |M_{fi}|^2 \rangle = \frac{1}{64\pi\hat{s}} \frac{1}{2} \left( \frac{g_W^2 \hat{s}}{m_W^2} \right)^2 = \left( \frac{g_W^2}{8\sqrt{2}\pi m_W^2} \right)^2 \hat{s}$$

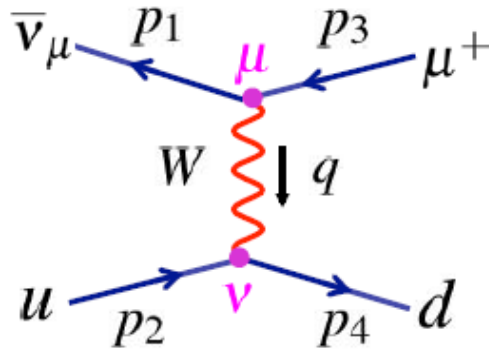
using  $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$   $\rightarrow$   $\boxed{\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}}$

★ Integrating this isotropic distribution over  $d\Omega^*$

$\rightarrow$   $\boxed{\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}}$  (1)

• cross section is a Lorentz invariant quantity so this is valid in any frame

# Antineutrino-Quark Scattering



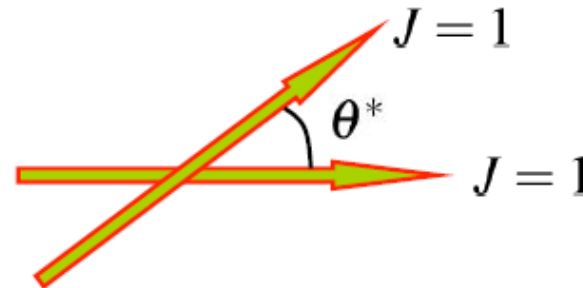
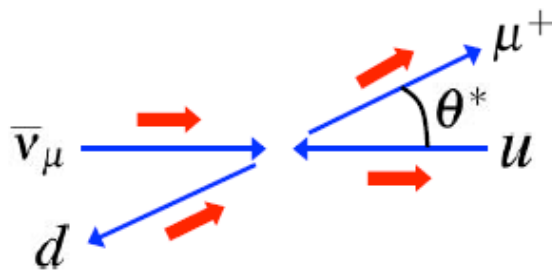
• In the ultra-relativistic limit, the charged-current interaction matrix element is:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[ \bar{v}(p_1) \gamma^\mu \frac{1}{2}(1 - \gamma^5) v(p_3) \right] \left[ \bar{u}(p_4) \gamma^\nu \frac{1}{2}(1 - \gamma^5) u(p_2) \right]$$

★ In the extreme relativistic limit only **LH Helicity particles** and **RH Helicity anti-particles** participate in the charged current weak interaction:

$$\Rightarrow M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[ \bar{v}_\uparrow(p_1) \gamma^\mu v_\uparrow(p_3) \right] \left[ \bar{u}_\downarrow(p_4) \gamma^\nu u_\downarrow(p_2) \right]$$

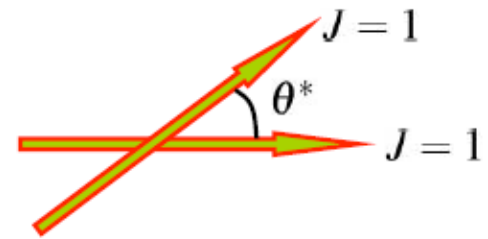
★ In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum 1 state



- ★ In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*} \frac{1}{4} (1 + \cos \theta^*)^2$$

- The factor  $\frac{1}{4} (1 + \cos \theta^*)^2$  can be understood in terms of the overlap of the initial and final angular momentum wave-functions



- ★ Similarly to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos \theta^*)^2 \hat{s}$$

- ★ Integrating over solid angle:

$$d\Omega = d\phi \sin \theta d\theta \rightarrow d\phi d(\cos \theta)$$

$$\int (1 + \cos \theta^*)^2 d\Omega^* = \int (1 + \cos \theta^*)^2 d(\cos \theta^*) d\phi = 2\pi \int_{-1}^{+1} (1 + \cos \theta^*)^2 d(\cos \theta^*) = \frac{16\pi}{3}$$

$$\rightarrow \sigma_{\bar{\nu}q} = \frac{G_F^2 \hat{s}}{3\pi}$$

- ★ This is a factor three smaller than the neutrino quark cross-section

$$\frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}$$

# (Anti)neutrino-(Anti)quark Scattering

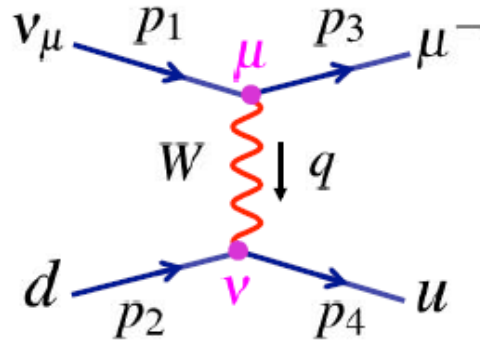
- Non-zero anti-quark component to the nucleon  $\Rightarrow$  also consider scattering from  $\bar{q}$
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include **LH particles** and **RH anti-particles**

$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{vq}}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}$	$\frac{d\sigma_{\bar{v}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos\theta^*)^2 \hat{s}$	$\frac{d\sigma_{v\bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos\theta^*)^2 \hat{s}$	$\frac{d\sigma_{\bar{v}\bar{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}$
$\sigma_{vq} = \frac{G_F^2 \hat{s}}{\pi}$	$\sigma_{\bar{v}q} = \frac{G_F^2 \hat{s}}{3\pi}$	$\sigma_{v\bar{q}} = \frac{G_F^2 \hat{s}}{3\pi}$	$\sigma_{\bar{v}\bar{q}} = \frac{G_F^2 \hat{s}}{\pi}$



# Differential Cross Section $d\sigma/dy$

- ★ Derived differential neutrino scattering cross sections in C.o.M frame, can convert to Lorentz invariant form



- As for DIS use Lorentz invariant  $y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$
- In relativistic limit  $y$  can be expressed in terms of the C.o.M. scattering angle  $y = \frac{1}{2}(1 - \cos \theta^*)$
- In lab. frame  $y = 1 - \frac{E_3}{E_1}$

- ★ Convert from  $\frac{d\sigma}{d\Omega^*} \rightarrow \frac{d\sigma}{dy}$  using

$$\frac{d\sigma}{dy} = \left| \frac{d\cos \theta^*}{dy} \right| \frac{d\sigma}{d\cos \theta^*} = \left| \frac{d\cos \theta^*}{dy} \right| 2\pi \frac{d\sigma}{d\Omega^*} = 4\pi \frac{d\sigma}{d\Omega^*}$$

- Already calculated (1)

$$\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}$$

- Hence:

$$\boxed{\frac{d\sigma_{\nu q}}{dy} = \frac{d\sigma_{\bar{\nu} \bar{q}}}{dy} = \frac{G_F^2}{\pi} \hat{s}}$$

and 
$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos\theta^*)^2 \hat{s}$$

becomes 
$$\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{4\pi} (1 + \cos\theta^*)^2 \hat{s}$$

from  $y = \frac{1}{2}(1 - \cos\theta^*) \rightarrow 1 - y = \frac{1}{2}(1 + \cos\theta^*)$

and hence 
$$\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{\pi} (1 - y)^2 \hat{s}$$

★ For comparison, the electro-magnetic  $e^\pm q \rightarrow e^\pm q$  cross section is:

**QED** 
$$\frac{d\sigma_{e^\pm q}}{dy} = \frac{2\pi\alpha^2}{q^4} e_q^2 [1 + (1 - y)^2] \hat{s}$$

DIFFERENCES:

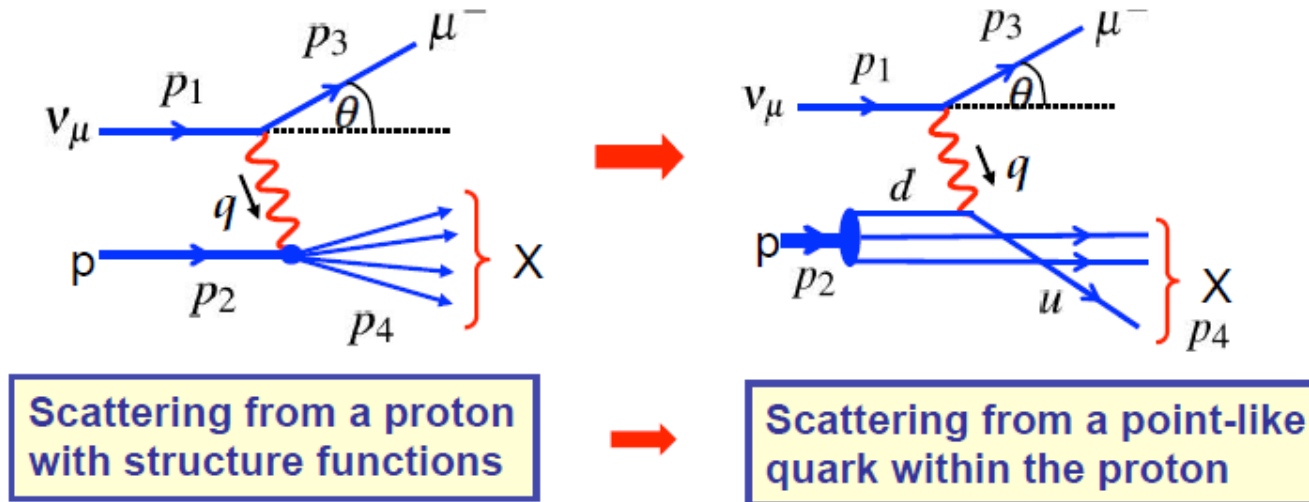
**Interaction  
+propagator**

**Helicity  
Structure**

**WEAK**

$$\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{\pi} (1 - y)^2 \hat{s}$$

# Parton Model For Neutrino Deep Inelastic Scattering



- ★ Neutrino-proton scattering can occur via scattering from a down-quark or from an anti-up quark
- In the parton model, number of down quarks within the proton in the momentum fraction range  $x \rightarrow x + dx$  is  $d^P(x)dx$ . Their contribution to the neutrino scattering cross-section is obtained by multiplying by the  $\nu_\mu d \rightarrow \mu^- u$  cross-section derived previously

$$\frac{d\sigma^{\nu P}}{dy} = \frac{G_F^2}{\pi} \hat{s} d^P(x) dx$$

where  $\hat{s}$  is the centre-of-mass energy of the  $\nu_\mu d$

- Similarly for the  $\bar{u}$  contribution

$$\frac{d\sigma^{\nu p}}{dy} = \frac{G_F^2}{\pi} \hat{s} (1-y)^2 \bar{u}^p(x) dx$$

- ★ Summing the two contributions and using  $\hat{s} = xs$

$$\rightarrow \frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} sx [d^p(x) + (1-y)^2 \bar{u}^p(x)]$$

- ★ The anti-neutrino proton differential cross section can be obtained in the same manner:

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2}{\pi} sx [(1-y)^2 u^p(x) + \bar{d}^p(x)]$$

- ★ For (anti)neutrino – neutron scattering:

$$\frac{d^2\sigma^{\nu n}}{dx dy} = \frac{G_F^2}{\pi} sx [d^n(x) + (1-y)^2 \bar{u}^n(x)]$$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dx dy} = \frac{G_F^2}{\pi} sx [(1-y)^2 u^n(x) + \bar{d}^n(x)]$$

- As before, define neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^p(x) = d^n(x); \quad d(x) \equiv d^p(x) = u^n(x)$$

$$\bar{u}(x) \equiv \bar{u}^p(x) = \bar{d}^n(x); \quad \bar{d}(x) \equiv \bar{d}^p(x) = \bar{u}^n(x)$$

$$\frac{d^2 \sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} s x [d(x) + (1-y)^2 \bar{u}(x)] \quad (2)$$

$$\frac{d^2 \sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2}{\pi} s x [(1-y)^2 u(x) + \bar{d}(x)] \quad (3)$$

$$\frac{d^2 \sigma^{\nu n}}{dx dy} = \frac{G_F^2}{\pi} s x [u(x) + (1-y)^2 \bar{d}(x)] \quad (4)$$

$$\frac{d^2 \sigma^{\bar{\nu} n}}{dx dy} = \frac{G_F^2}{\pi} s x [(1-y)^2 d(x) + \bar{u}(x)] \quad (5)$$

- ★ Because neutrino cross sections are very small, need massive detectors. These are usually made of Iron, hence, experimentally measure a combination of proton/neutron scattering cross sections

- ★ For an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon:

$$\frac{d^2\sigma^{\nu N}}{dx dy} = \frac{1}{2} \left( \frac{d^2\sigma^{\nu p}}{dx dy} + \frac{d^2\sigma^{\nu n}}{dx dy} \right)$$

→ 
$$\frac{d^2\sigma^{\nu N}}{dx dy} = \frac{G_F^2}{2\pi} s x [u(x) + d(x) + (1-y)^2(\bar{u}(x) + \bar{d}(x))]$$

- Integrate over momentum fraction  $x$

$$\frac{d\sigma^{\nu N}}{dy} = \frac{G_F^2}{2\pi} s [f_q + (1-y)^2 f_{\bar{q}}] \quad (6)$$

where  $f_q$  and  $f_{\bar{q}}$  are the total momentum fractions carried by the quarks and by the anti-quarks within a nucleon

$$f_q \equiv f_d + f_u = \int_0^1 x [u(x) + d(x)] dx; \quad f_{\bar{q}} \equiv f_{\bar{d}} + f_{\bar{u}} = \int_0^1 x [\bar{u}(x) + \bar{d}(x)] dx$$

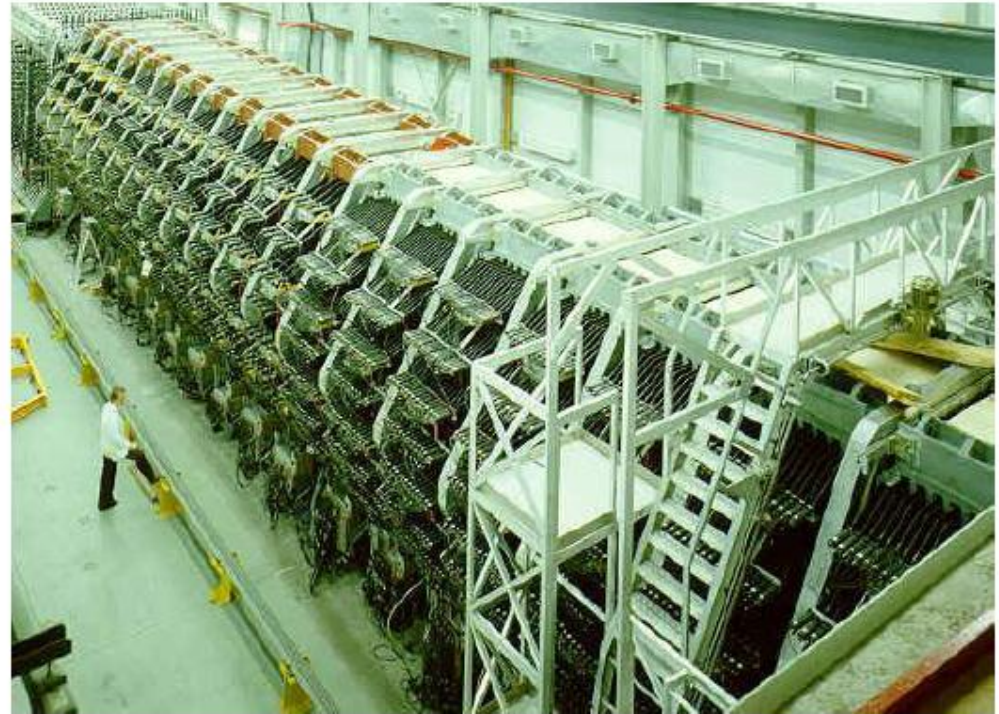
- Similarly

$$\frac{d\sigma^{\bar{\nu} N}}{dy} = \frac{G_F^2}{2\pi} s [(1-y)^2 f_q + f_{\bar{q}}] \quad (7)$$

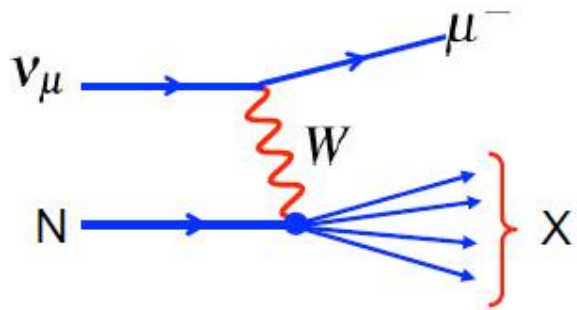


# e.g. CDHS Experiment (CERN 1976-1984)

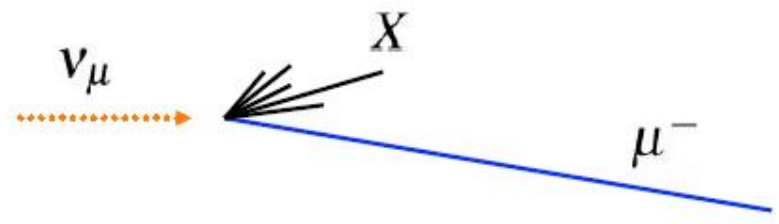
- 1250 tons
- Magnetized iron modules
- Separated by drift chambers



**Study Neutrino Deep Inelastic Scattering**



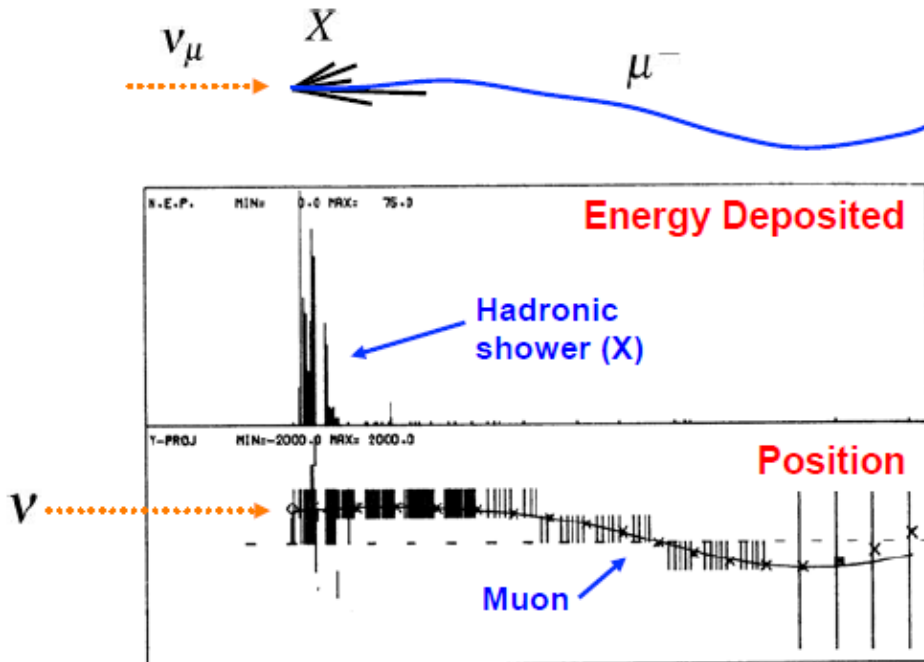
Experimental Signature:





# e.g. CDHS Experiment (CERN 1976-1984)

## Example Event:



- Measure energy of X  
 $E_X$

- Measure muon momentum from curvature in B-field  
 $E_\mu$

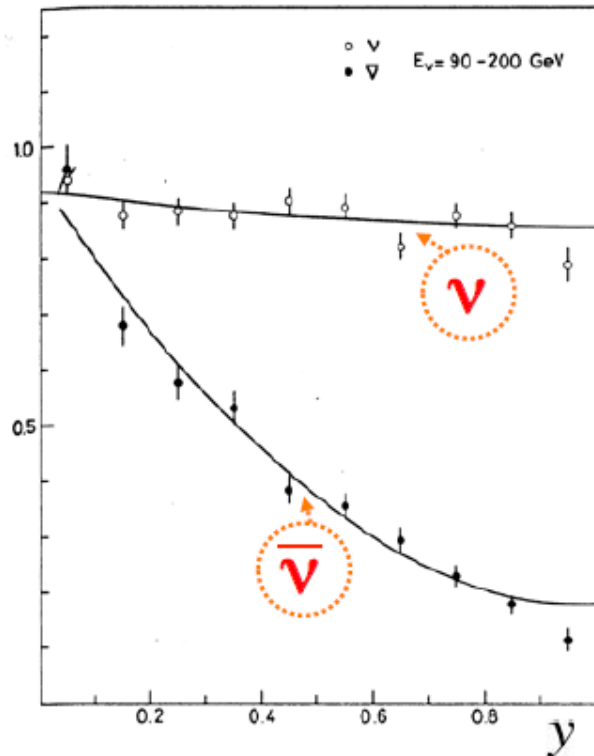
★ For each event can determine neutrino energy and  $y$  !

$$E_\nu = E_X + E_\mu$$

$$E_\mu = (1 - y)E_\nu \quad \rightarrow \quad y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

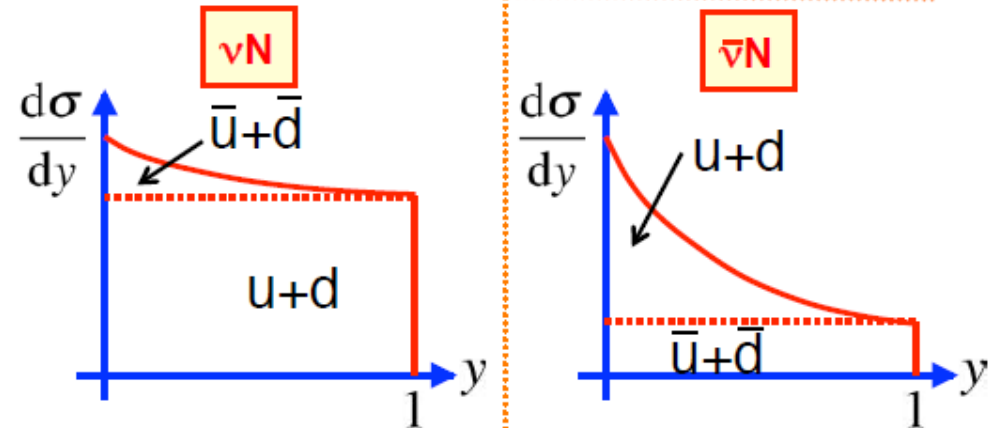
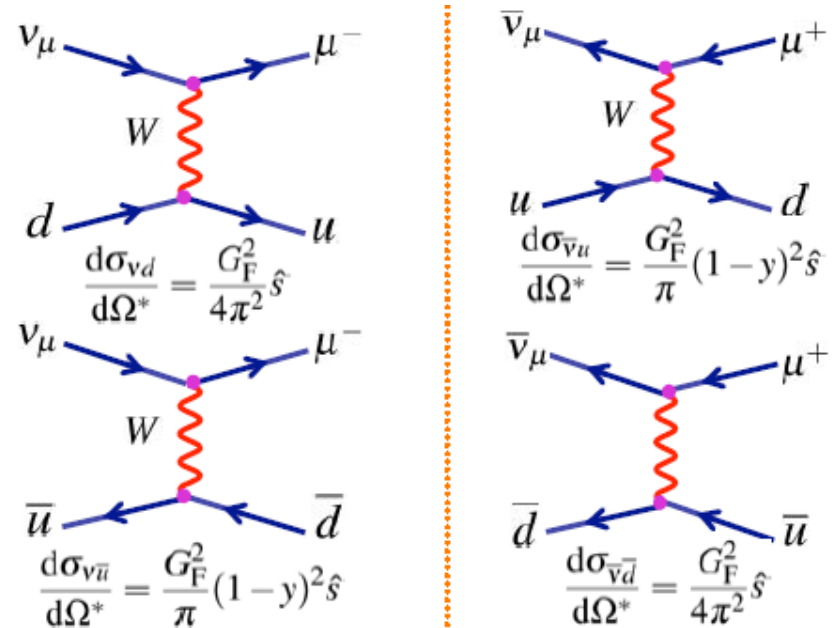
# Measured $y$ Distribution

- CDHS measured  $y$  distribution



J. de Groot et al., Z.Phys. C1 (1979) 143

- Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering



# Measured Total Cross Sections

- ★ Integrating the expressions for  $\frac{d\sigma}{dy}$  (equations (6) and (7))

$$\sigma^{\nu N} = \frac{G_{FS}^2}{2\pi} \left[ f_q + \frac{1}{3} f_{\bar{q}} \right]$$

$$\sigma^{\bar{\nu} N} = \frac{G_{FS}^2}{2\pi} \left[ \frac{1}{3} f_q + f_{\bar{q}} \right]$$

$$\begin{array}{ccc} \nu & \longrightarrow & p \\ (E_\nu, 0, 0, +E_\nu) & & (m_p, 0, 0, 0) \end{array}$$

$$s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$$

→ **DIS cross section  $\propto$  lab. frame neutrino energy**

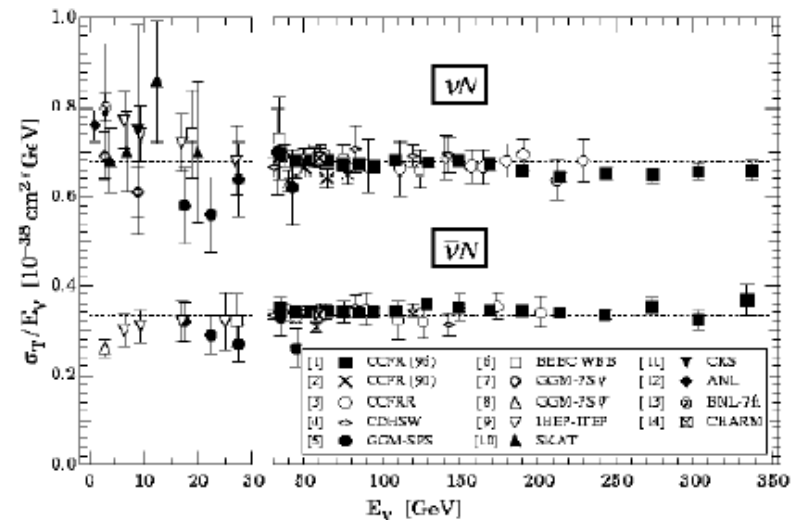
- ★ Measure cross sections can be used to determine fraction of protons momentum carried by quarks,  $f_q$ , and fraction carried by anti-quarks,  $f_{\bar{q}}$

- Find:  $f_q \approx 0.41$ ;  $f_{\bar{q}} \approx 0.08$
- ~50% of momentum carried by gluons (which don't interact with virtual W boson)
- If no anti-quarks in nucleons expect

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = 3$$

- Including anti-quarks

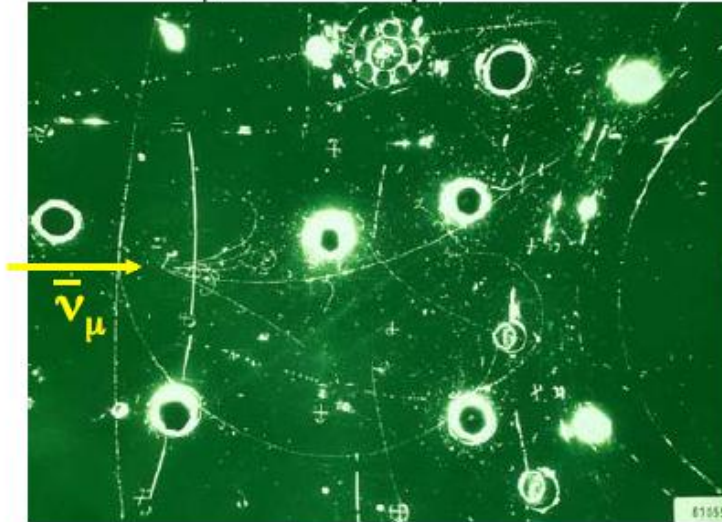
$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} \approx 2$$



# Weak Neutral Current

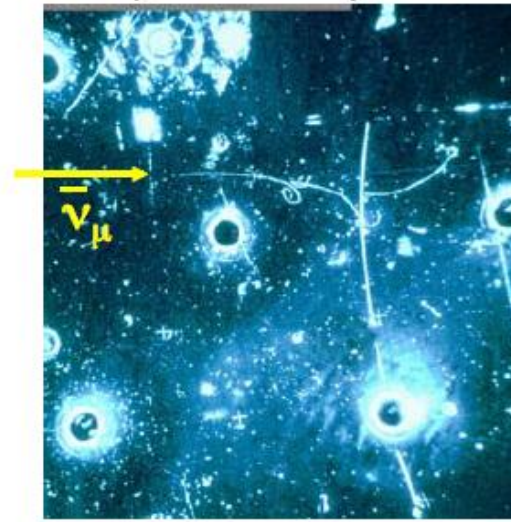
- ★ Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973.

$$\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + \text{hadrons}$$



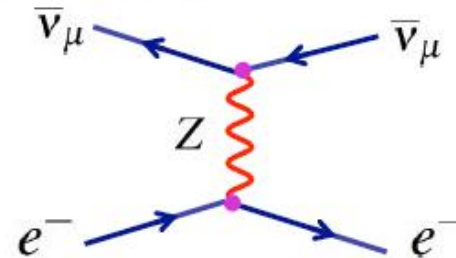
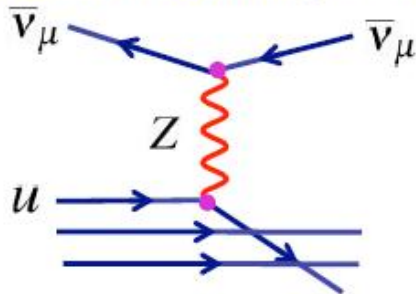
F.J. Hasert et al., Phys. Lett. 46B (1973) 138

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$



F.J. Hasert et al., Phys. Lett. 46B (1973) 121

- ★ Cannot be due to W exchange - first evidence for Z boson



# Summary

- ★ Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections
- ★ Neutrino – nucleon scattering yields extra information about parton distributions functions:
  - $\nu$  couples to  $d$  and  $\bar{u}$  ;  $\bar{\nu}$  couples to  $u$  and  $\bar{d}$ 
    - ➔ investigate flavour content of nucleon
  - can measure anti-quark content of nucleon
    - $\nu\bar{q}$  suppressed by factor  $(1-y)^2$  compared with  $\nu q$
    - $\bar{\nu}q$  suppressed by factor  $(1-y)^2$  compared with  $\bar{\nu}\bar{q}$
- ★ Further aspects of neutrino deep-inelastic scattering (expressed in general structure functions) are covered in **Appendix II**
- ★ Finally observe that neutrinos interact via weak neutral currents (NC)

# Appendix I

• For the adjoint spinors  $\bar{u} = u^\dagger \gamma^0$  consider

$$\overline{\frac{1}{2}(1 - \gamma^5)u} = \left[\frac{1}{2}(1 - \gamma^5)u\right]^\dagger \gamma^0 = u^\dagger \frac{1}{2}(1 - \gamma^5) \gamma^0 = u^\dagger \gamma^0 \frac{1}{2}(1 + \gamma^5) = \bar{u} \frac{1}{2}(1 + \gamma^5)$$

$$\frac{1}{2}(1 - \gamma^5)u_\uparrow = 0 \quad \rightarrow \quad \bar{u} \frac{1}{2}(1 + \gamma^5) = 0$$

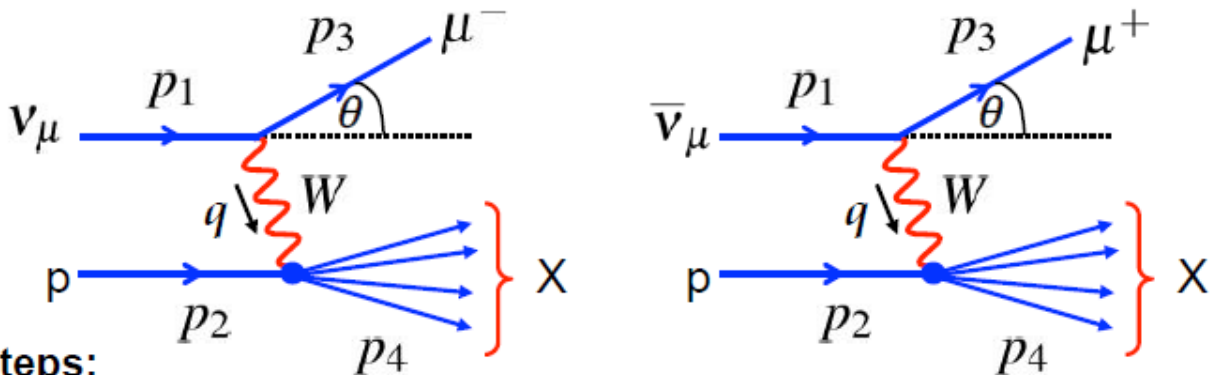
Using the fact that  $\gamma^5$  and  $\gamma^\mu$  anti-commute can rewrite ME:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[ \bar{u}(p_3) \frac{1}{2}(1 + \gamma^5) \gamma^\mu u(p_1) \right] \left[ \bar{u}(p_4) \frac{1}{2}(1 + \gamma^5) \gamma^\nu u(p_2) \right]$$

$$\rightarrow M_{fi} = 0 \quad \text{for} \quad \bar{u}_\uparrow(p_3) \quad \text{and} \quad \bar{u}_\uparrow(p_4)$$



# Appendix II: Deep-Inelastic Neutrino Scattering



Two steps:

- First write down most general cross section in terms of structure functions
- Then evaluate expressions in the quark-parton model

## QED Revisited

- ★ In the limit  $s \gg M^2$  the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written (Eq. 2 of handout 6)

$$\frac{d^2\sigma_{e^\pm p}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ (1-y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

- For neutrino scattering typically measure the energy of the produced muon  $E_\mu = E_\nu(1-y)$  and differential cross-sections expressed in terms of  $dx dy$

- Using  $Q^2 = (s - M^2)xy \approx sxy \implies \frac{d^2\sigma}{dx dy} = \left| \frac{dQ^2}{dy} \right| \frac{d^2\sigma}{dx dQ^2} = sx \frac{d^2\sigma}{dx dQ^2}$



- In the limit  $s \gg M^2$  the general Electro-magnetic DIS cross section can be written

$$\frac{d^2 \sigma^{e^\pm p}}{dx dy} = \frac{4\pi\alpha^2 s}{Q^4} [(1-y)F_2(x, Q^2) + y^2 x F_1(x, Q^2)]$$

- **NOTE:** This is the most general Lorentz Invariant **parity conserving** expression
- ★ For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function  $F_3(x, Q^2)$

$$\nu_\mu p \rightarrow \mu^- X \quad \frac{d^2 \sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[ (1-y)F_2^{\nu p}(x, Q^2) + y^2 x F_1^{\nu p}(x, Q^2) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x, Q^2) \right]$$

- For anti-neutrino scattering new structure function enters with opposite sign

$$\bar{\nu}_\mu p \rightarrow \mu^+ X \quad \frac{d^2 \sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[ (1-y)F_2^{\bar{\nu} p}(x, Q^2) + y^2 x F_1^{\bar{\nu} p}(x, Q^2) - y \left(1 - \frac{y}{2}\right) x F_3^{\bar{\nu} p}(x, Q^2) \right]$$

- Similarly for neutrino-neutron scattering

$$\nu_\mu n \rightarrow \mu^- X \quad \frac{d^2 \sigma^{\nu n}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[ (1-y)F_2^{\nu n}(x, Q^2) + y^2 x F_1^{\nu n}(x, Q^2) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu n}(x, Q^2) \right]$$

$$\bar{\nu}_\mu n \rightarrow \mu^+ X \quad \frac{d^2 \sigma^{\bar{\nu} n}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[ (1-y)F_2^{\bar{\nu} n}(x, Q^2) + y^2 x F_1^{\bar{\nu} n}(x, Q^2) - y \left(1 - \frac{y}{2}\right) x F_3^{\bar{\nu} n}(x, Q^2) \right]$$

# Neutrino Interaction Structure Functions

- ★ In terms of the parton distribution functions we found (2) :

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} s x [d(x) + (1-y)^2 \bar{u}(x)]$$

- Compare coefficients of  $y$  with the general Lorentz Invariant form (p.321) and assume Bjorken scaling, i.e.  $F(x, Q^2) \rightarrow F(x)$

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[ (1-y) F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x) \right]$$

- Re-writing (2)  $\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{2\pi} s [2x d(x) + 2x \bar{u}(x) - 4xy \bar{u}(x) + 2xy^2 \bar{u}(x)]$

and equating powers of  $y$

$$\left. \begin{aligned} 2xd + 2x\bar{u} &= F_2 \\ -4x\bar{u} &= -F_2 + xF_3 \\ 2\bar{u} &= F_1 - xF_3/2 \end{aligned} \right\}$$

gives:

$$\boxed{\begin{aligned} F_2^{\nu p} &= 2xF_1^{\nu p} = 2x[d(x) + \bar{u}(x)] \\ xF_3^{\nu p} &= 2x[d(x) - \bar{u}(x)] \end{aligned}}$$

**NOTE:** again we get the **Callan-Gross** relation  $F_2 = 2xF_1$

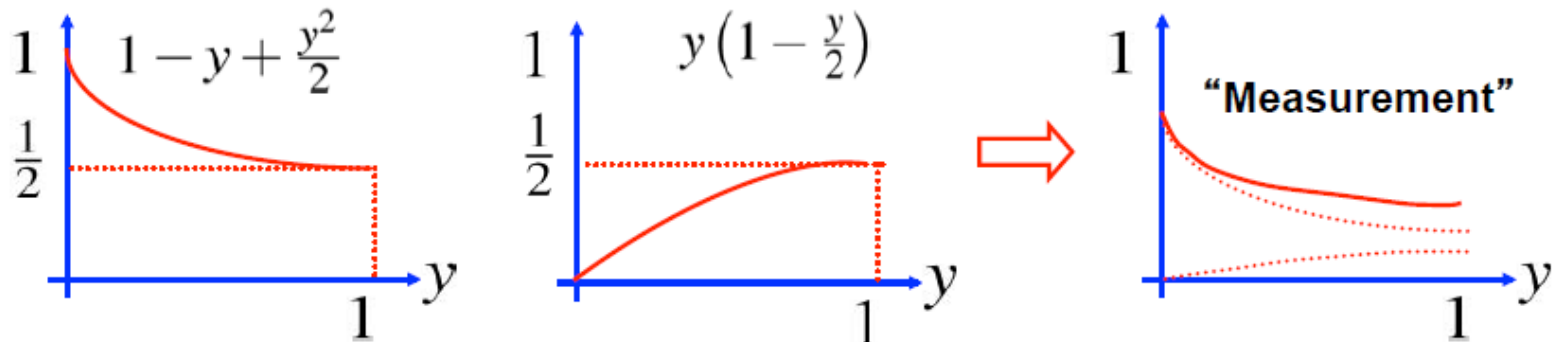
No surprise, underlying process is scattering from point-like spin-1/2 quarks

★ Substituting back in to expression for differential cross section:

$$\frac{d^2\sigma^{vp}}{dx dy} = \frac{G_{FS}^2}{2\pi} \left[ \left(1 - y + \frac{y^2}{2}\right) F_2^{vp}(x) + y \left(1 - \frac{y}{2}\right) x F_3^{vp}(x) \right]$$

★ Experimentally measure  $F_2$  and  $F_3$  from  $y$  distributions at fixed  $x$

- ♦ Different  $y$  dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured



★ Then use  $F_2^{vp} = 2x[d(x) + \bar{u}(x)]$  and  $F_3^{vp} = 2[d(x) - \bar{u}(x)]$

➡ Determine  $d(x)$  and  $\bar{u}(x)$  separately

- ★ Neutrino experiments require large detectors (often iron) i.e. isoscalar target

$$F_2^{\nu N} = 2xF_1^{\nu N} = \frac{1}{2} (F_2^{\nu p} + F_2^{\nu n}) = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

$$xF_3^{\nu N} = \frac{1}{2} (xF_3^{\nu p} + xF_3^{\nu n}) = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$$

- ★ For electron – nucleon scattering:

$$F_2^{ep} = 2xF_1^{ep} = x\left[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right]$$

$$F_2^{en} = 2xF_1^{en} = x\left[\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)\right]$$

- For an isoscalar target

$$F_2^{eN} = \frac{1}{2} (F_2^{ep} + F_2^{en}) = \frac{5}{18}x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

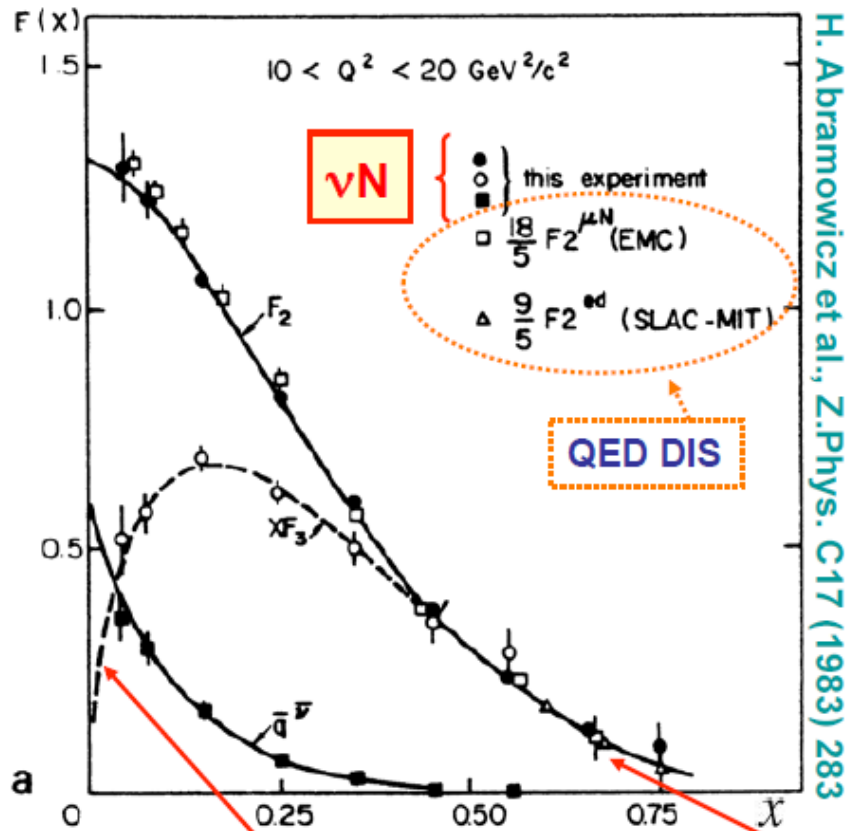
$$\rightarrow F_2^{\nu N} = \frac{18}{5}F_2^{eN}$$

- Note that the factor  $\frac{5}{18} = \frac{1}{2} (q_u^2 + q_d^2)$  and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

**Experiment:  $0.29 \pm 0.02$**

# Measurements of $F_2(x)$ and $F_3(x)$

• CDHS Experiment  $\nu_\mu + \text{Fe} \rightarrow \mu^- + X$



H. Abramowicz et al., Z.Phys. C17 (1983) 283

$$F_2^{\nu N} = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

$$xF_3^{\nu N} = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$$

$$\rightarrow F_2^{\nu N} - xF_3^{\nu N} = 2x[\bar{u} + \bar{d}]$$

\* Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions

Sea dominates so expect  $xF_3$  to go to zero as  $q(x) = \bar{q}(x)$

Sea contribution goes to zero

# Valence Contribution

- ★ Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$

$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) = S(x)$$

$$\bar{d}(x) = \bar{d}_S(x) = S(x)$$

→  $F_3^{vN} = [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)] = u_V(x) + d_V(x)$

→  $\int_0^1 F_3^{vN}(x) dx = \int_0^1 (u_V(x) + d_V(x)) dx = N_u^V + N_d^V$

- ★ Area under measured function  $F_3^{vN}(x)$  gives a measurement of the total number of valence quarks in a nucleon !

expect  $\int_0^1 F_3^{vN}(x) dx = 3$  “Gross – Llewellyn-Smith sum rule”

**Experiment:  $3.0 \pm 0.2$**

•Note:  $F_2^{\bar{v}P} = F_2^{vN}$ ;  $F_2^{\bar{v}n} = F_2^{vP}$ ;  $F_3^{\bar{v}P} = F_3^{vN}$ ;  $F_3^{\bar{v}n} = F_3^{vP}$  and anti-neutrino structure functions contain same pdf information