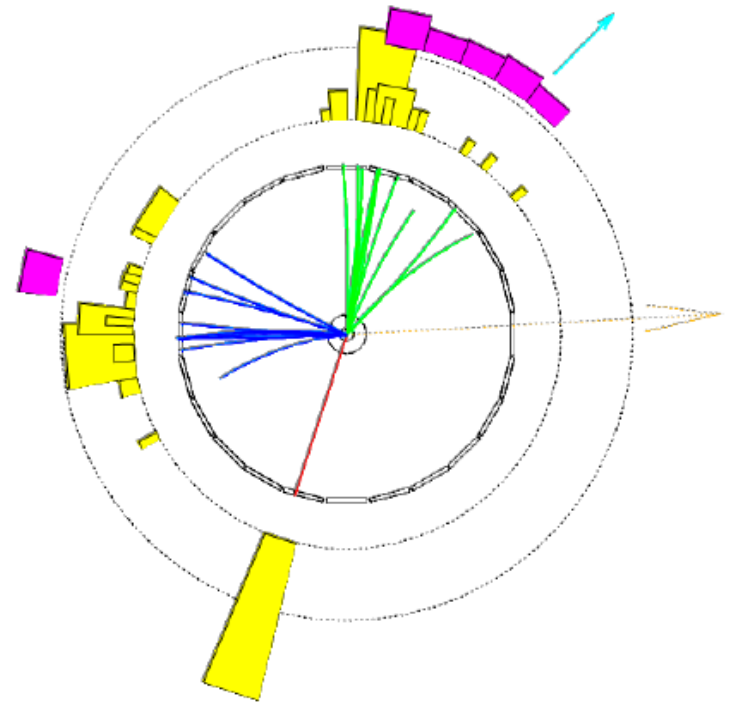


# Elementary Particle Physics: theory and experiments

## Electroweak Unification and the W and Z Bosons



Follow the course/slides from M. A. Thomson lectures at Cambridge University

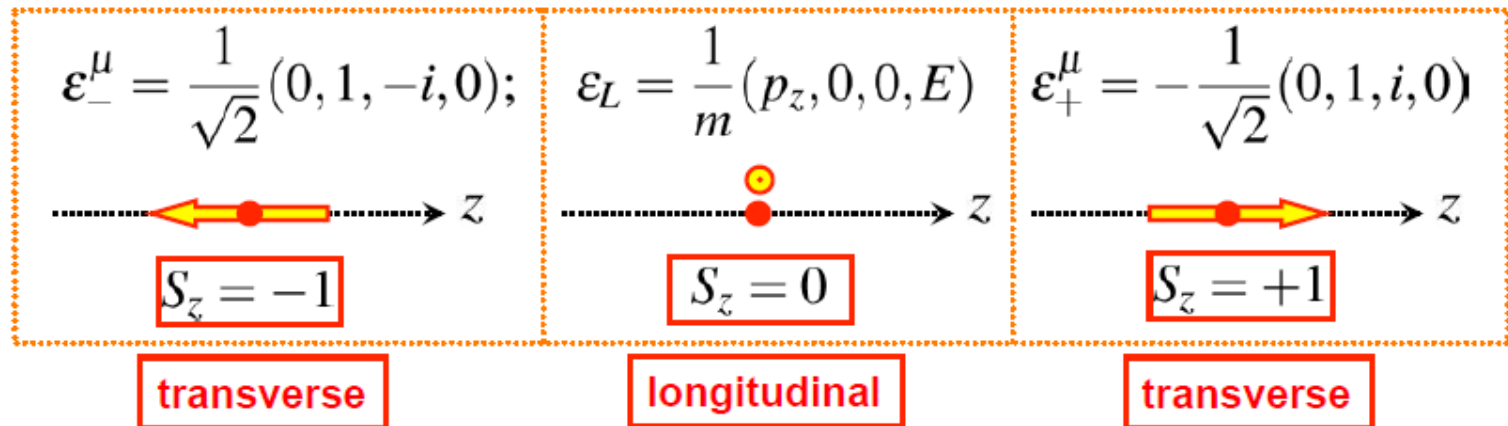
Prof. dr hab. Elżbieta Richter-Wąs

# Boson Polarization States

- ★ In this handout we are going to consider the decays of W and Z bosons, for this we will need to consider the polarization. Here simply quote results although the justification is given in **Appendices I and II**
- ★ A real (i.e. not virtual) **massless** spin-1 boson can exist in two **transverse** polarization states, a **massive** spin-1 boson also can be longitudinally **polarized**
- ★ Boson wave-functions are written in terms of the polarization four-vector  $\epsilon^\mu$

$$B^\mu = \epsilon^\mu e^{-ip \cdot x} = \epsilon^\mu e^{i(\vec{p} \cdot \vec{x} - Et)}$$

- ★ For a spin-1 boson **travelling along the z-axis**, the polarization four vectors are:

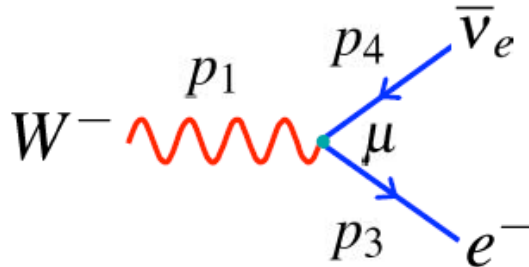


Longitudinal polarization isn't present for on-shell massless particles, the photon can exist in two helicity states  $h = \pm 1$  (**LH** and **RH** circularly polarized light)

# W-Boson Decay

★ To calculate the W-Boson decay rate first consider  $W^- \rightarrow e^- \bar{\nu}_e$

★ Want matrix element for :



Incoming W-boson :  $\epsilon_\mu(p_1)$   
 Out-going electron :  $\bar{u}(p_3)$   
 Out-going  $\bar{\nu}_e$  :  $v(p_4)$   
 Vertex factor :  $-i \frac{g_W}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1 - \gamma^5)$

$$-iM_{fi} = \epsilon_\mu(p_1) \cdot \bar{u}(p_3) \cdot -i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \cdot v(p_4)$$

Note, no propagator

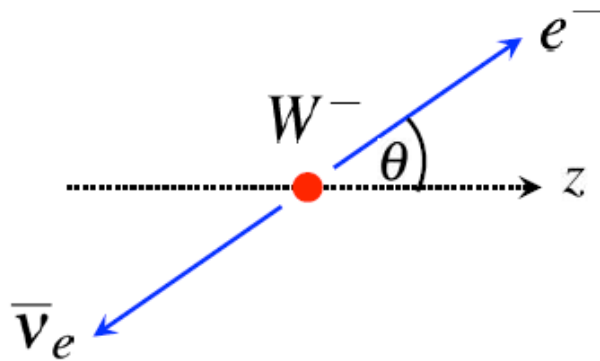
$$\Rightarrow M_{fi} = \frac{g_W}{\sqrt{2}} \epsilon_\mu(p_1) \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma^5) v(p_4)$$

★ This can be written in terms of the four-vector scalar product of the W-boson polarization  $\epsilon_\mu(p_1)$  and the weak charged current  $j^\mu$

$$M_{fi} = \frac{g_W}{\sqrt{2}} \epsilon_\mu(p_1) \cdot j^\mu \quad \text{with} \quad j^\mu = \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma^5) v(p_4)$$

# W-Decay: The Lepton Current

- ★ First consider the lepton current  $j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1 - \gamma^5)v(p_4)$
- ★ Work in Centre-of-Mass frame



$$p_1 = (m_W, 0, 0, 0);$$

$$p_3 = (E, E \sin \theta, 0, E \cos \theta)$$

$$p_4 = (E, -E \sin \theta, 0, -E \cos \theta)$$

with  $E = \frac{m_W}{2}$

- ★ In the ultra-relativistic limit only **LH particles** and **RH anti-particles** participate in the weak interaction so

$$j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1 - \gamma^5)v(p_4) = \bar{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

Note:  $\frac{1}{2}(1 - \gamma^5)v(p_4) = v_\uparrow(p_4)$

Chiral projection operator

$$\bar{u}(p_3)\gamma^\mu v_\uparrow(p_4) = \bar{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

“Helicity conservation”

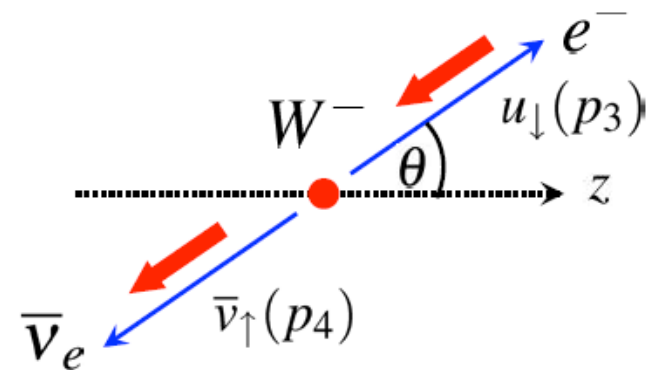
- We have already calculated the current

$$j^\mu = \bar{u}_\downarrow(p_3) \gamma^\mu v_\uparrow(p_4)$$

when considering  $e^+ e^- \rightarrow \mu^+ \mu^-$

- For  $\mu_L^- \mu_R^+$  we have

$$j_{\uparrow\downarrow}^\mu = 2E(0, -\cos\theta, -i, \sin\theta)$$

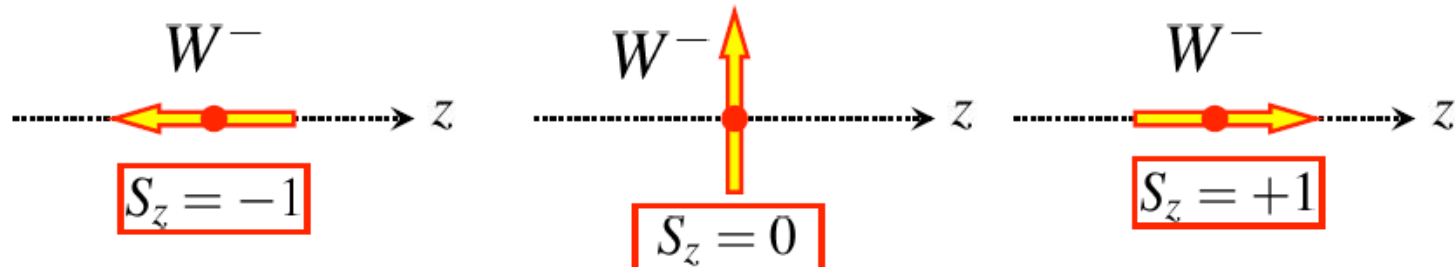


- For the charged current weak Interaction we only have to consider this **single** combination of helicities

$$j^\mu = \bar{u}(p_3) \gamma^\mu \frac{1}{2}(1 - \gamma^5)v(p_4) = \bar{u}_\downarrow(p_3) \gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

and the three possible W-Boson polarization states:

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \varepsilon_L = \frac{1}{m}(p_z, 0, 0, E) \quad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$



★ For a W-boson at rest these become:

$$\epsilon_-^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \epsilon_L = (0, 0, 0, 1) \quad \epsilon_+^\mu = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$

★ Can now calculate the matrix element for the different polarization states

$$M_{fi} = \frac{g_W}{\sqrt{2}} \epsilon_\mu(p_1) j^\mu \quad \text{with} \quad j^\mu = 2 \frac{m_W}{2} (0, -\cos \theta, -i, \sin \theta)$$

Decay at rest :  $E_e = E_\nu = m_W/2$

★ giving

$$\boxed{\epsilon_-} \quad M_- = \frac{g_W}{\sqrt{2}} \frac{1}{\sqrt{2}} (0, 1, -i, 0) \cdot m_W (0, -\cos \theta, -i, \sin \theta) = \frac{1}{2} g_W m_W (1 + \cos \theta)$$

$$\boxed{\epsilon_L} \quad M_L = \frac{g_W}{\sqrt{2}} (0, 0, 0, 1) \cdot m_W (0, -\cos \theta, -i, \sin \theta) = -\frac{1}{\sqrt{2}} g_W m_W \sin \theta$$

$$\boxed{\epsilon_+} \quad M_+ = -\frac{g_W}{\sqrt{2}} \frac{1}{\sqrt{2}} (0, 1, i, 0) \cdot m_W (0, -\cos \theta, -i, \sin \theta) = \frac{1}{2} g_W m_W (1 - \cos \theta)$$

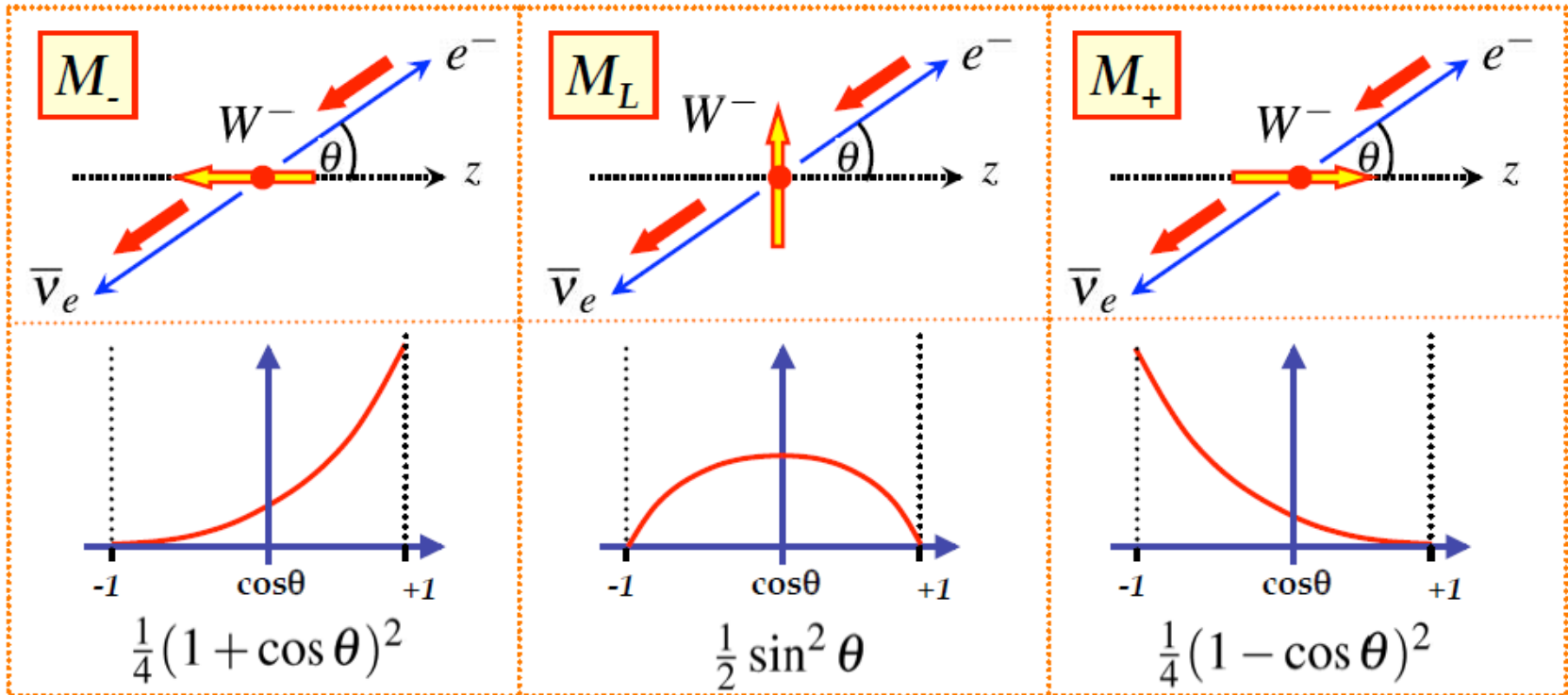


$$|M_-|^2 = g_W^2 m_W^2 \frac{1}{4} (1 + \cos \theta)^2$$

$$|M_L|^2 = g_W^2 m_W^2 \frac{1}{2} \sin^2 \theta$$

$$|M_+|^2 = g_W^2 m_W^2 \frac{1}{4} (1 - \cos \theta)^2$$

★ The angular distributions can be understood in terms of the spin of the particles



★ The differential decay rate can be found using:

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2} |M|^2$$

where  $p^*$  is the C.o.M momentum of the final state particles, here  $p^* = \frac{m_W}{2}$

- ★ Hence for the three different polarisations we obtain:

$$\frac{d\Gamma_-}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2} \frac{1}{4} (1 + \cos \theta)^2 \quad \frac{d\Gamma_L}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2} \frac{1}{2} \sin^2 \theta \quad \frac{d\Gamma_+}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2} \frac{1}{4} (1 - \cos \theta)^2$$

- ★ Integrating over all angles using

$$\int \frac{1}{4} (1 \pm \cos \theta)^2 d\phi d\cos \theta = \int \frac{1}{2} \sin^2 \theta d\phi d\cos \theta = \frac{4\pi}{3}$$

- ★ Gives

$$\Gamma_- = \Gamma_L = \Gamma_+ = \frac{g_W^2 m_W}{48\pi}$$

- ★ The total W-decay rate is independent of polarization; this has to be the case as the decay rate cannot depend on the arbitrary definition of the z-axis
- ★ For a sample of unpolarized W boson each polarization state is equally likely, for the **average matrix element** sum over all possible matrix elements and average over the three initial polarization states

$$\begin{aligned} \langle |M_{fi}|^2 \rangle &= \frac{1}{3} (|M_-|^2 + |M_L|^2 + |M_+|^2) \\ &= \frac{1}{3} g_W^2 m_W^2 \left[ \frac{1}{4} (1 + \cos \theta)^2 + \frac{1}{2} \sin^2 \theta + \frac{1}{4} (1 - \cos \theta)^2 \right] \\ &= \frac{1}{3} g_W^2 m_W^2 \end{aligned}$$

- ★ For a sample of unpolarized W-bosons, the decay is isotropic (as expected)



★ For this isotropic decay

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2} \langle |M|^2 \rangle \Rightarrow \Gamma = \frac{4\pi |p^*|}{32\pi^2 m_W^2} \langle |M|^2 \rangle$$

$$\Rightarrow \Gamma(W^- \rightarrow e^- \bar{\nu}) = \frac{g_W^2 m_W}{48\pi}$$

★ The calculation for the other decay modes (neglecting final state particle masses) is same. For quarks need to account for **colour** and **CKM matrix**. No decays to top – the top mass (175 GeV) is greater than the W-boson mass (80 GeV)

$W^- \rightarrow e^- \bar{\nu}_e$	$W^- \rightarrow d\bar{u}$	$\times 3  V_{ud} ^2$	$W^- \rightarrow d\bar{c}$	$\times 3  V_{cd} ^2$
$W^- \rightarrow \mu^- \bar{\nu}_\mu$	$W^- \rightarrow s\bar{u}$	$\times 3  V_{us} ^2$	$W^- \rightarrow s\bar{c}$	$\times 3  V_{cs} ^2$
$W^- \rightarrow \tau^- \bar{\nu}_\tau$	$W^- \rightarrow b\bar{u}$	$\times 3  V_{ub} ^2$	$W^- \rightarrow b\bar{c}$	$\times 3  V_{cb} ^2$

★ Unitarity of CKM matrix gives, e.g.  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

★ Hence  $BR(W \rightarrow qq') = 6BR(W \rightarrow e\nu)$

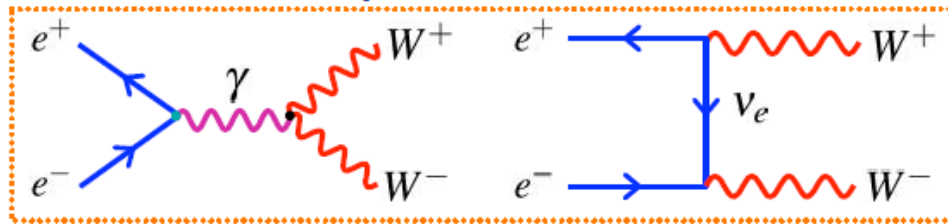
and thus the total decay rate :

$$\Gamma_W = 9\Gamma_{W \rightarrow e\nu} = \frac{3g_W^2 m_W}{16\pi} = 2.07 \text{ GeV}$$

**Experiment: 2.14±0.04 GeV**  
(our calculation neglected a 3% QCD correction to decays to quarks)

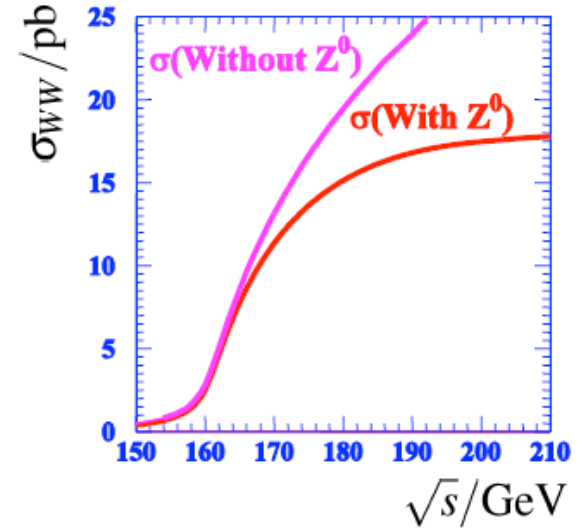
# From W to Z

- ★ The  $W^\pm$  bosons carry the EM charge - suggestive Weak and EM forces are related.
- ★ W bosons can be produced in  $e^+e^-$  annihilation

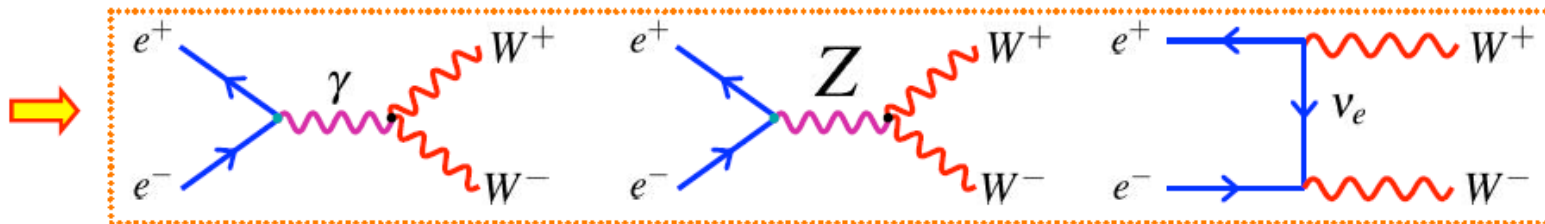


- ★ With just these two diagrams there is a problem: the cross section increases with C.o.M energy and at some point violates **QM unitarity**

**UNITARITY VIOLATION:** when QM calculation gives larger flux of W bosons than incoming flux of electrons/positrons



- ★ Problem can be “fixed” by introducing a new boson, the Z. The new diagram interferes negatively with the above two diagrams fixing the unitarity problem



$$|M_{\gamma WW} + M_{Z WW} + M_{\nu WW}|^2 < |M_{\gamma WW} + M_{\nu WW}|^2$$

- ★ Only works if **Z,  $\gamma$ , W** couplings are related: need **ELECTROWEAK UNIFICATION**

# SU(2)<sub>L</sub> : The Weak Interaction

- ★ The Weak Interaction arises from **SU(2)** local phase transformations

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$$\psi \rightarrow \psi' = \psi e^{i\vec{\alpha}(x) \cdot \frac{\vec{\sigma}}{2}}$$

where the  $\vec{\sigma}$  are the generators of the SU(2) symmetry, i.e the **three** Pauli spin matrices

→ **3 Gauge Bosons**  $W_1^\mu, W_2^\mu, W_3^\mu$

- ★ The wave-functions have two components which, in analogy with isospin, are represented by **“weak isospin”**
- ★ The fermions are placed in isospin doublets and the local phase transformation corresponds to

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}' = e^{i\vec{\alpha}(x) \cdot \frac{\vec{\sigma}}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

- ★ Weak Interaction only couples to **LH particles/RH anti-particles**, hence only place **LH particles/RH anti-particles** in weak isospin doublets:  $I_W = \frac{1}{2}$   
**RH particles/LH anti-particles** placed in weak isospin singlets:  $I_W = 0$

**Weak Isospin**

$I_W = \frac{1}{2}$	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$	$I_W^3 = +\frac{1}{2}$ $I_W^3 = -\frac{1}{2}$
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$I_W = 0$	$(\nu_e)_R, (e^-)_R, \dots (u)_R, (d)_R, \dots$	<b>Note: RH/LH refer to chiral states</b>
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- ★ For simplicity only consider  $\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ .
- The gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) – [note: here include interaction strength in current]

$$j_\mu^1 = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_1 \chi_L \quad j_\mu^2 = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_2 \chi_L \quad j_\mu^3 = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_3 \chi_L$$

- ★ The charged current  $W^+/W^-$  interaction enters as a linear combinations of  $W_1, W_2$

$$W^{\pm\mu} = \frac{1}{\sqrt{2}} (W_1^\mu \pm W_2^\mu)$$

- ★ The  $W^\pm$  interaction terms

$$j_\pm^\mu = \frac{g_W}{\sqrt{2}} (j_1^\mu \pm i j_2^\mu) = \frac{g_W}{\sqrt{2}} \bar{\chi}_L \gamma^\mu \frac{1}{2} (\sigma_1 \pm i \sigma_2) \chi_L$$

- ★ Express in terms of the weak isospin ladder operators  $\sigma_\pm = \frac{1}{2} (\sigma_1 \pm i \sigma_2)$

$$j_\pm^\mu = \frac{g_W}{\sqrt{2}} \bar{\chi}_L \gamma^\mu \sigma_\pm \chi_L \quad \left. \vphantom{j_\pm^\mu} \right\} \text{Origin of } \frac{1}{\sqrt{2}} \text{ in Weak CC}$$



which can be understood in terms of the weak isospin doublet

$$j_+^\mu = \frac{g_W}{\sqrt{2}} \bar{\chi}_L \gamma^\mu \sigma_+ \chi_L = \frac{g_W}{\sqrt{2}} (\bar{\nu}_L, \bar{e}_L) \gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{g_W}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L = \frac{g_W}{\sqrt{2}} \bar{\nu} \gamma^\mu \frac{1}{2} (1 - \gamma^5) e$$

★ Similarly

**W<sup>-</sup>**  corresponds to  $j_-^\mu = \frac{g_W}{\sqrt{2}} \bar{\chi}_L \gamma^\mu \sigma_- \chi_L$

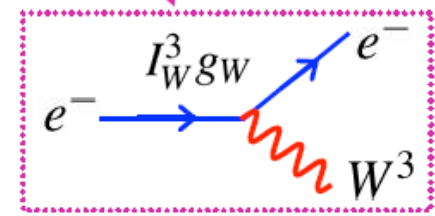
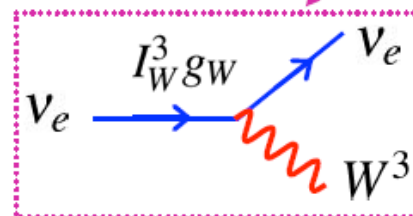
$$j_-^\mu = \frac{g_W}{\sqrt{2}} \bar{\chi}_L \gamma^\mu \sigma_- \chi_L = \frac{g_W}{\sqrt{2}} (\bar{\nu}_L, \bar{e}_L) \gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{g_W}{\sqrt{2}} \bar{e}_L \gamma^\mu \nu_L = \frac{g_W}{\sqrt{2}} \bar{e} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \nu$$

★ However have an additional interaction due to **W<sup>3</sup>**

$$j_3^\mu = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_3 \chi_L$$

expanding this:

$$j_3^\mu = g_W \frac{1}{2} (\bar{\nu}_L, \bar{e}_L) \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = g_W \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - g_W \frac{1}{2} \bar{e}_L \gamma^\mu e_L$$



**NEUTRAL CURRENT INTERACTIONS !**

# Electroweak Unification

- ★ Tempting to identify the  $W^3$  as the  $Z$
- ★ However this is not the case, have two physical neutral spin-1 gauge bosons,  $\gamma, Z$  and the  $W^3$  is a mixture of the two,
- ★ Equivalently write the photon and  $Z$  in terms of the  $W^3$  and a new neutral spin-1 boson the  $B$
- ★ The **physical** bosons (the  $Z$  and photon field,  $A$ ) are:

$$A_\mu = B_\mu \cos \theta_W + W_\mu^3 \sin \theta_W$$

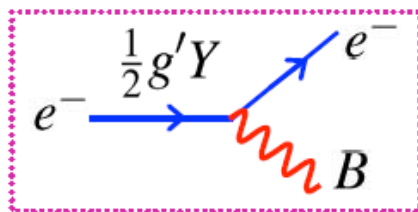
$$Z_\mu = -B_\mu \sin \theta_W + W_\mu^3 \cos \theta_W$$

$\theta_W$  is the weak mixing angle

- ★ The new boson is associated with a new gauge symmetry similar to that of electromagnetism :  $U(1)_Y$
- ★ The charge of this symmetry is called **WEAK HYPERCHARGE**  $Y$

$$Y = 2Q - 2I_W^3$$

$Q$  is the EM charge of a particle  
 $I_W^3$  is the third comp. of weak isospin



- By convention the coupling to the  $B_\mu$  is  $\frac{1}{2}g'Y$
- |  |                  |
|--|------------------|
| $e_L : Y = 2(-1) - 2(-\frac{1}{2}) = -1$ | $\nu_L : Y = +1$ |
| $e_R : Y = 2(-1) - 2(0) = -2$            | $\nu_R : Y = 0$  |

(this identification of hypercharge in terms of  $Q$  and  $I_3$  makes all of the following work out)



- ★ For this to work the coupling constants of the  $W^3$ ,  $B$ , and photon must be related e.g. consider contributions involving the neutral interactions of electrons:

$$\boxed{\gamma} \quad j_{\mu}^{em} = e\bar{\psi}Q_e\gamma_{\mu}\psi = e\bar{e}_L Q_e \gamma_{\mu} e_L + e\bar{e}_R Q_e \gamma_{\mu} e_R$$

$$\boxed{W^3} \quad j_{\mu}^{W^3} = -\frac{g_W}{2}\bar{e}_L\gamma_{\mu}e_L$$

$$\boxed{B} \quad j_{\mu}^Y = \frac{g'}{2}\bar{\psi}Y_e\gamma_{\mu}\psi = \frac{g'}{2}\bar{e}_L Y_{e_L} \gamma_{\mu} e_L + \frac{g'}{2}\bar{e}_R Y_{e_R} \gamma_{\mu} e_R$$

- ★ The relation  $A_{\mu} = B_{\mu} \cos \theta_W + W_{\mu}^3 \sin \theta_W$  is equivalent to requiring

$$\boxed{j_{\mu}^{em} = j_{\mu}^Y \cos \theta_W + j_{\mu}^{W^3} \sin \theta_W}$$

- Writing this in full:

$$e\bar{e}_L Q_e \gamma_{\mu} e_L + e\bar{e}_R Q_e \gamma_{\mu} e_R = \frac{1}{2}g' \cos \theta_W [\bar{e}_L Y_{e_L} \gamma_{\mu} e_L + \bar{e}_R Y_{e_R} \gamma_{\mu} e_R] - \frac{1}{2}g_W \sin \theta_W [\bar{e}_L \gamma_{\mu} e_L]$$

$$-e\bar{e}_L \gamma_{\mu} e_L - e\bar{e}_R \gamma_{\mu} e_R = \frac{1}{2}g' \cos \theta_W [-\bar{e}_L \gamma_{\mu} e_L - 2\bar{e}_R \gamma_{\mu} e_R] - \frac{1}{2}g_W \sin \theta_W [\bar{e}_L \gamma_{\mu} e_L]$$

which works if:  $\boxed{e = g_W \sin \theta_W = g' \cos \theta_W}$  (i.e. equate coefficients of L and R terms)

- ★ Couplings of electromagnetism, the weak interaction and the interaction of the  $U(1)_Y$  symmetry are therefore related.

# The Z Boson

- ★ In this model we can now derive the couplings of the Z Boson

$$Z_\mu = -B_\mu \sin \theta_W + W_\mu^3 \cos \theta_W \quad \boxed{I_W^3} \quad \text{for the electron } I_W^3 = \frac{1}{2}$$

$$j_\mu^Z = -\frac{1}{2} g' \sin \theta_W [\bar{e}_L Y_{eL} \gamma_\mu e_L + \bar{e}_R Y_{eR} \gamma_\mu e_R] - \frac{1}{2} g_W \cos \theta_W [e_L \gamma_\mu e_L]$$

- Writing this in terms of weak isospin and charge:

$$j_\mu^Z = -\frac{1}{2} g' \sin \theta_W [\bar{e}_L (2Q - 2I_W^3) \gamma_\mu e_L + \bar{e}_R (2Q) \gamma_\mu e_R] + \boxed{I_W^3} g_W \cos \theta_W [e_L \gamma_\mu e_L]$$

For RH chiral states  $I_3=0$

- Gathering up the terms for LH and RH chiral states:

$$j_\mu^Z = [g' I_W^3 \sin \theta_W - g' Q \sin \theta_W + g_W I_W^3 \cos \theta_W] \bar{e}_L \gamma_\mu e_L - [g' Q \sin \theta_W] e_R \gamma_\mu e_R$$

- Using:  $e = g_W \sin \theta_W = g' \cos \theta_W$  gives

$$j_\mu^Z = \left[ g' \frac{(I_W^3 - Q \sin^2 \theta_W)}{\sin \theta_W} \right] \bar{e}_L \gamma_\mu e_L - \left[ g' \frac{Q \sin^2 \theta_W}{\sin \theta_W} \right] e_R \gamma_\mu e_R$$

$$j_\mu^Z = g_Z (I_W^3 - Q \sin^2 \theta_W) [\bar{e}_L \gamma_\mu e_L] - g_Z Q \sin^2 \theta_W [e_R \gamma_\mu e_R]$$

with

$$e = g_Z \cos \theta_W \sin \theta_W$$

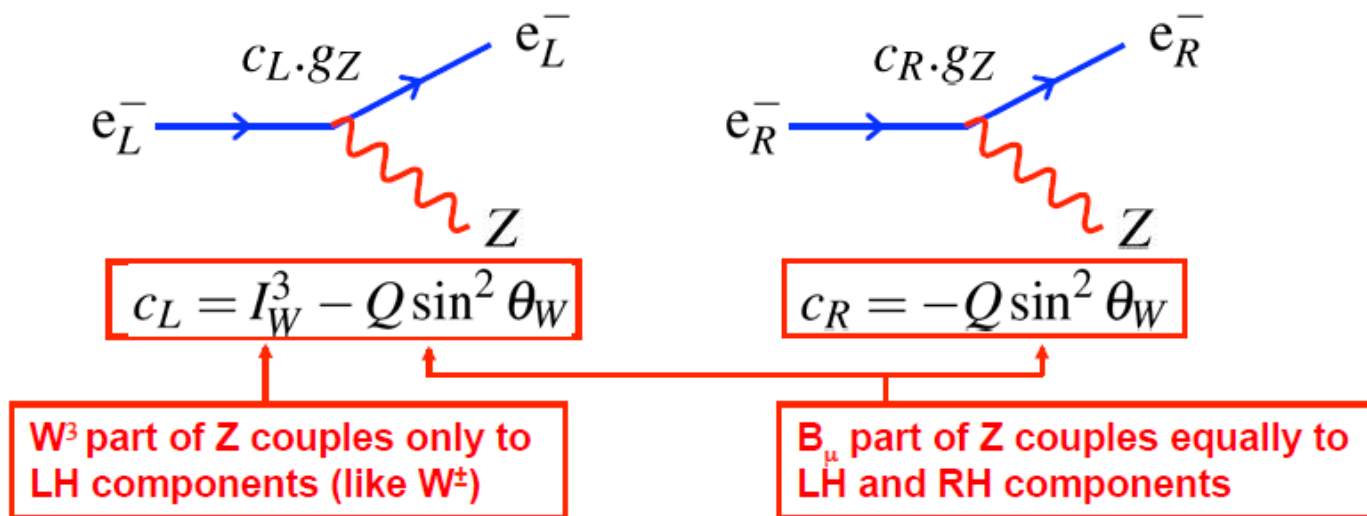
i.e.

$$g_Z = \frac{g_W}{\cos \theta_W}$$



- ★ Unlike for the Charged Current Weak interaction (W) the Z Boson couples to both LH and RH chiral components, but not equally...

$$\begin{aligned}
 j_\mu^Z &= g_Z(I_W^3 - Q \sin^2 \theta_W) [\bar{e}_L \gamma_\mu e_L] - g_Z Q \sin^2 \theta_W [e_R \gamma_\mu e_R] \\
 &= g_Z c_L [\bar{e}_L \gamma_\mu e_L] + g_Z c_R [e_R \gamma_\mu e_R]
 \end{aligned}$$



- ★ Use projection operators to obtain vector and axial vector couplings

$$\bar{u}_L \gamma_\mu u_L = \bar{u} \gamma_\mu \frac{1}{2} (1 - \gamma_5) u \quad \bar{u}_R \gamma_\mu u_R = \bar{u} \gamma_\mu \frac{1}{2} (1 + \gamma_5) u$$

$$j_\mu^Z = g_Z \bar{u} \gamma_\mu \left[ c_L \frac{1}{2} (1 - \gamma_5) + c_R \frac{1}{2} (1 + \gamma_5) \right] u$$

$$j_\mu^Z = \frac{g_Z}{2} \bar{u} \gamma_\mu [(c_L + c_R) + (c_R - c_L) \gamma_5] u$$

★ Which in terms of **V** and **A** components gives:  $j_\mu^Z = \frac{g_Z}{2} \bar{u} \gamma_\mu [c_V - c_A \gamma_5] u$

with  $c_V = c_L + c_R = I_W^3 - 2Q \sin^2 \theta_W$

$$c_A = c_L - c_R = I_W^3$$

★ Hence the vertex factor for the Z boson is:

$$-ig_Z \frac{1}{2} \gamma_\mu [c_V - c_A \gamma_5]$$



★ Using the experimentally determined value of the weak mixing angle:

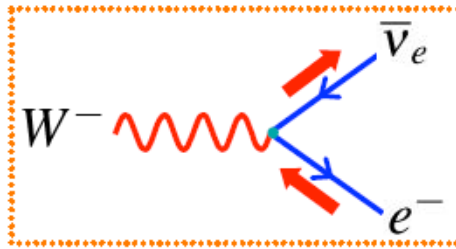
$$\sin^2 \theta_W \approx 0.23$$



Fermion	$Q$	$I_W^3$	$c_L$	$c_R$	$c_V$	$c_A$
$\nu_e, \nu_\mu, \nu_\tau$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
$e^-, \mu^-, \tau^-$	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
$u, c, t$	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	+0.19	$+\frac{1}{2}$
$d, s, b$	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

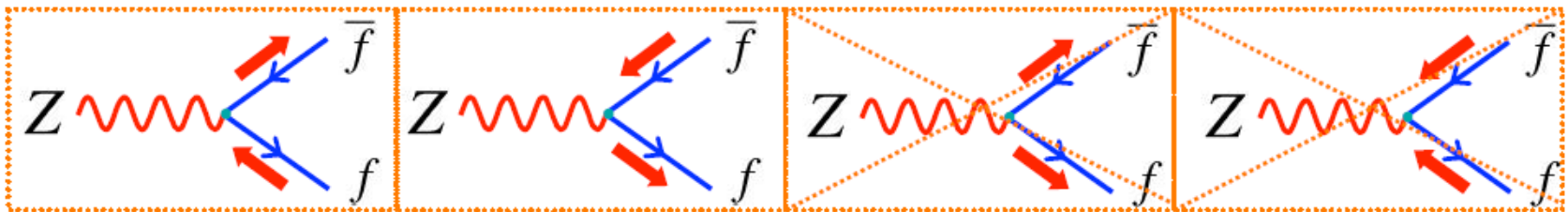
# Z Boson Decay: $\Gamma_Z$

- ★ In W-boson decay only had to consider one helicity combination of (assuming we can neglect final state masses: helicity states = chiral states)



W-boson couples:  
to LH particles  
and RH anti-particles

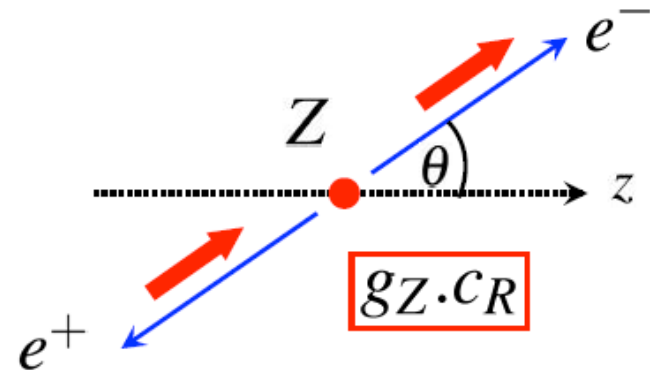
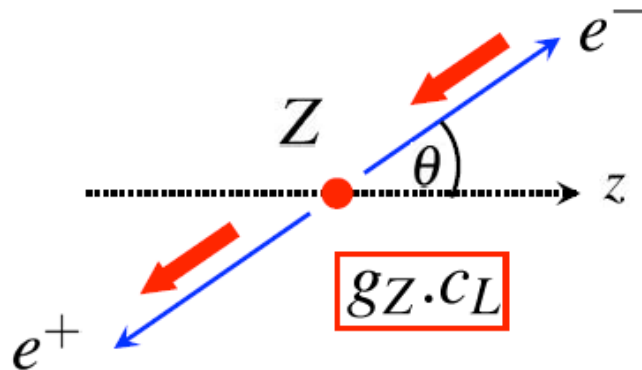
- ★ But Z-boson couples to LH and RH particles (with different strengths)
- ★ Need to consider **only two** helicity (or more correctly chiral) combinations:



This can be seen by considering either of the combinations which give zero

$$\begin{aligned}
 \text{e.g. } \bar{u}_R \gamma^\mu (c_V + c_A \gamma^5) v_R &= u^\dagger \frac{1}{2} (1 + \gamma^5) \gamma^0 \gamma^\mu (c_V + c_A \gamma^5) \frac{1}{2} (1 - \gamma^5) v \\
 &= \frac{1}{4} u^\dagger \gamma^0 (1 - \gamma^5) \gamma^\mu (1 - \gamma^5) (c_V + c_A \gamma^5) v \\
 &= \frac{1}{4} \bar{u} \gamma^\mu (1 + \gamma^5) (1 - \gamma^5) (c_V + c_A \gamma^5) v = 0
 \end{aligned}$$

- ★ In terms of left and right-handed combinations need to calculate:



- ★ For unpolarized Z bosons:

$$\langle |M_{fi}|^2 \rangle = \frac{1}{3} [2c_L^2 g_Z^2 m_Z^2 + 2c_R^2 g_Z^2 m_Z^2] = \frac{2}{3} g_Z^2 m_Z^2 (c_L^2 + c_R^2)$$

average over polarization

★ Using  $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$  and  $\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2} |M|^2$



$$\Gamma(Z \rightarrow e^+ e^-) = \frac{g_Z^2 m_Z}{48\pi} (c_V^2 + c_A^2)$$

# Z Branching Ratios

- ★ (Neglecting fermion masses) obtain the same expression for the other decays

$$\Gamma(Z \rightarrow f\bar{f}) = \frac{g_Z^2 m_Z}{48\pi} (c_V^2 + c_A^2)$$

- Using values for  $c_V$  and  $c_A$  obtain:

$$Br(Z \rightarrow e^+e^-) = Br(Z \rightarrow \mu^+\mu^-) = Br(Z \rightarrow \tau^+\tau^-) \approx 3.5\%$$

$$Br(Z \rightarrow \nu_1\bar{\nu}_1) = Br(Z \rightarrow \nu_2\bar{\nu}_2) = Br(Z \rightarrow \nu_3\bar{\nu}_3) \approx 6.9\%$$

$$Br(Z \rightarrow d\bar{d}) = Br(Z \rightarrow s\bar{s}) = Br(Z \rightarrow b\bar{b}) \approx 15\%$$

$$Br(Z \rightarrow u\bar{u}) = Br(Z \rightarrow c\bar{c}) \approx 12\%$$

- The Z Boson therefore predominantly decays to hadrons

$$Br(Z \rightarrow \text{hadrons}) \approx 69\%$$

Mainly due to factor 3 from colour

- Also predict total decay rate (total width)

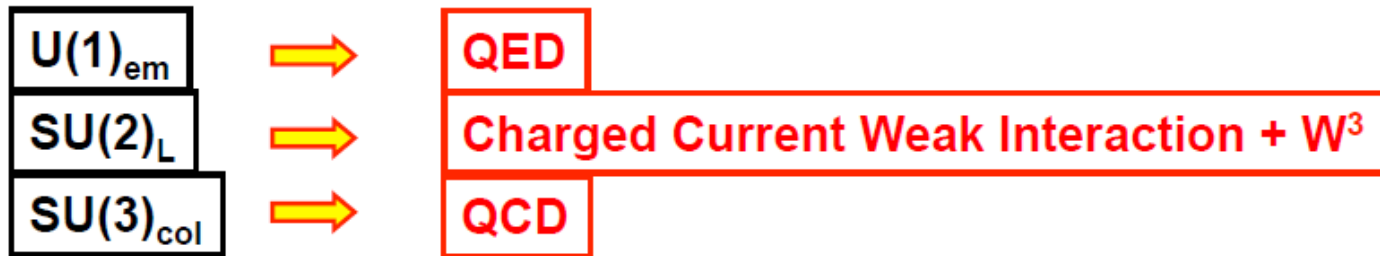
$$\Gamma_Z = \sum_i \Gamma_i = 2.5 \text{ GeV}$$

Experiment:

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

# Summary

- ★ The Standard Model interactions are mediated by spin-1 **gauge bosons**
- ★ The form of the interactions are completely specified by the assuming an underlying local phase transformation → **GAUGE INVARIANCE**



- ★ In order to “unify” the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry :  $U(1)$  hypercharge



- ★ The physical  $Z$  boson and the photon are mixtures of the neutral  $W$  boson and  $B$  determined by the **Weak Mixing angle**

$$\sin \theta_W \approx 0.23$$

- ★ Have we really unified the EM and Weak interactions ? Well not really...
  - Started with two independent theories with coupling constants  $g_W, e$
  - Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model  $\theta_W$
  - Interactions not unified from any higher theoretical principle... **but it works!**

# Appendix I: Photon Polarization

- For a free photon (i.e.  $j^\mu = 0$ ) equation (A7) becomes

$$\boxed{\square^2 A^\mu = 0} \quad (\text{B1})$$


(note have chosen a gauge where the Lorentz condition is satisfied)

- ★ Equation (A8) has solutions (i.e. the wave-function for a free photon)

$$A^\mu = \varepsilon^\mu(q) e^{-iq \cdot x}$$

where  $\varepsilon^\mu$  is the four-component polarization vector and  $q$  is the photon four-momentum

$$0 = \square^2 A^\mu = -q^2 \varepsilon^\mu e^{-iq \cdot x}$$

  $q^2 = 0$

- ★ Hence equation (B1) describes a massless particle.
- ★ But the solution has four components – might ask how it can describe a spin-1 particle which has three polarization states?
- ★ But for (A8) to hold we must satisfy the Lorentz condition:

$$0 = \partial_\mu A^\mu = \partial_\mu (\varepsilon^\mu e^{-iq \cdot x}) = \varepsilon^\mu \partial_\nu (e^{-iq \cdot x}) = -i \varepsilon^\mu q_\mu e^{-iq \cdot x}$$

Hence the Lorentz condition gives  $\boxed{q_\mu \varepsilon^\mu = 0}$  (B2)

i.e. only 3 independent components.

★ However, in addition to the Lorentz condition still have the additional gauge freedom of  $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda$  with (A8)  $\square^2 \Lambda = 0$

• Choosing  $\Lambda = iae^{-iq \cdot x}$  which has  $\square^2 \Lambda = q^2 \Lambda = 0$

$$\begin{aligned} A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda &= \epsilon_\mu e^{-iq \cdot x} + ia \partial_\mu e^{-iq \cdot x} \\ &= \epsilon_\mu e^{-iq \cdot x} + ia(-iq_\mu) e^{-iq \cdot x} \\ &= (\epsilon_\mu + aq_\mu) e^{-iq \cdot x} \end{aligned}$$

★ Hence the electromagnetic field is left unchanged by

$$\epsilon_\mu \rightarrow \epsilon'_\mu = \epsilon_\mu + aq_\mu$$

★ Hence the two polarization vectors which differ by a multiple of the photon four-momentum describe the same photon. Choose  $a$  such that the time-like component of  $\epsilon_\mu$  is zero, i.e.  $\epsilon_0 \equiv 0$

★ With this choice of gauge, which is known as the **COULOMB GAUGE**, the Lorentz condition (B2) gives

$$\boxed{\vec{\epsilon} \cdot \vec{q} = 0} \quad (\text{B3})$$

i.e. only 2 independent components, both transverse to the photons momentum



- ★ A massless photon has two transverse polarisation states. For a photon travelling in the z direction these can be expressed as the transversely polarized states:

$$\boldsymbol{\varepsilon}_1^\mu = (0, 1, 0, 0); \quad \boldsymbol{\varepsilon}_2^\mu = (0, 0, 1, 0)$$

- ★ Alternatively take linear combinations to get the circularly polarized states

$$\boldsymbol{\varepsilon}_-^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \boldsymbol{\varepsilon}_+^\mu = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$

- ★ It can be shown that the  $\boldsymbol{\varepsilon}_+$  state corresponds the state in which the photon spin is directed in the +z direction, i.e.  $S_z = +1$

# Appendix II: Massive Spin-1 particles

- For a massless photon we had (before imposing the Lorentz condition) we had from equation (A5)

$$\square^2 A^\mu - \partial^\mu (\partial_\nu A^\nu) = j^\mu$$

- ★ The Klein-Gordon equation for a spin-0 particle of mass  $m$  is

$$(\square^2 + m^2)\phi = 0$$

suggestive that the appropriate equations for a massive spin-1 particle can be obtained by replacing  $\square^2 \rightarrow \square^2 + m^2$

- ★ This is indeed the case, and from QFT it can be shown that for a massive spin 1 particle equation (A5) becomes

$$(\square^2 + m^2)B^\mu - \partial^\mu (\partial_\nu B^\nu) = j^\mu$$

- ★ Therefore a free particle must satisfy

$$(\square^2 + m^2)B^\mu - \partial^\mu (\partial_\nu B^\nu) = 0 \tag{B4}$$

- Acting on equation (B4) with  $\partial_\nu$  gives

$$\begin{aligned}(\square^2 + m^2)\partial_\mu B^\mu - \partial_\mu \partial^\mu (\partial_\nu B^\nu) &= 0 \\(\square^2 + m^2)\partial_\mu B^\mu - \square^2 (\partial_\nu B^\nu) &= 0 \\m^2 \partial_\mu B^\mu &= 0\end{aligned}\tag{B5}$$

- ★ Hence, for a massive spin-1 particle, unavoidably have  $\partial_\mu B^\mu = 0$  ; note this is not a relation that reflects to choice of gauge.

- Equation (B4) becomes

$$\boxed{(\square^2 + m^2)B^\mu = 0}\tag{B6}$$

- ★ For a free spin-1 particle with 4-momentum,  $p^\mu$ , equation (B6) admits solutions

$$B_\mu = \varepsilon_\mu e^{-ip \cdot x}$$

- ★ Substituting into equation (B5) gives

$$\boxed{p_\mu \varepsilon^\mu = 0}$$

- ★ The four degrees of freedom in  $\varepsilon^\mu$  are reduced to three, but for a massive particle, equation (B6) **does not** allow a choice of gauge and we can not reduce the number of degrees of freedom any further.

- ★ Hence we need to find three orthogonal polarisation states satisfying

$$\boxed{p_\mu \varepsilon^\mu = 0} \quad (\text{B7})$$

- ★ For a particle travelling in the z direction, can still admit the circularly polarized states.

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$

- ★ Writing the third state as

$$\varepsilon_L^\mu = \frac{1}{\sqrt{\alpha^2 + \beta^2}}(\alpha, 0, 0, \beta)$$

equation (B7) gives  $\alpha E - \beta p_z = 0$

$$\Rightarrow \varepsilon_L^\mu = \frac{1}{m}(p_z, 0, 0, E)$$

- ★ This **longitudinal polarisation state** is only present for massive spin-1 particles, i.e. there is no analogous state for a free photon (although off-mass shell virtual photons can be longitudinally polarized – a fact that was alluded to on page 114).