Elementary Particle Physics: theory and experiments

Theory:

Electroweak unification and the W and Z boson physics

Precision tests of the Standard Model

The CKM matrix and CP violation

Slides taken from M. A. Thomson lectures at Cambridge University in 2011

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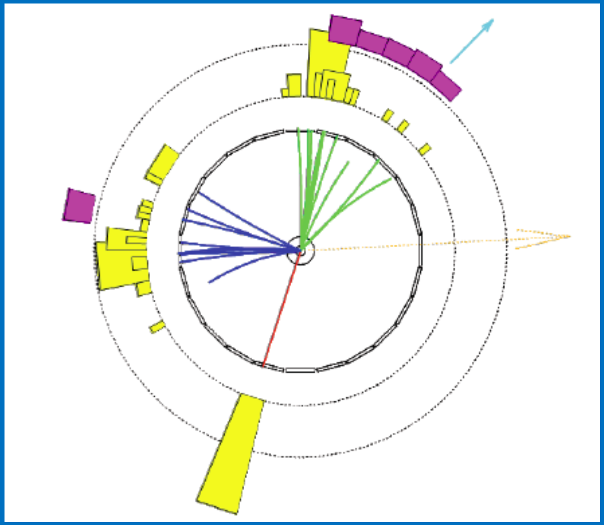
Electroweak unification and the W and Z boson physics

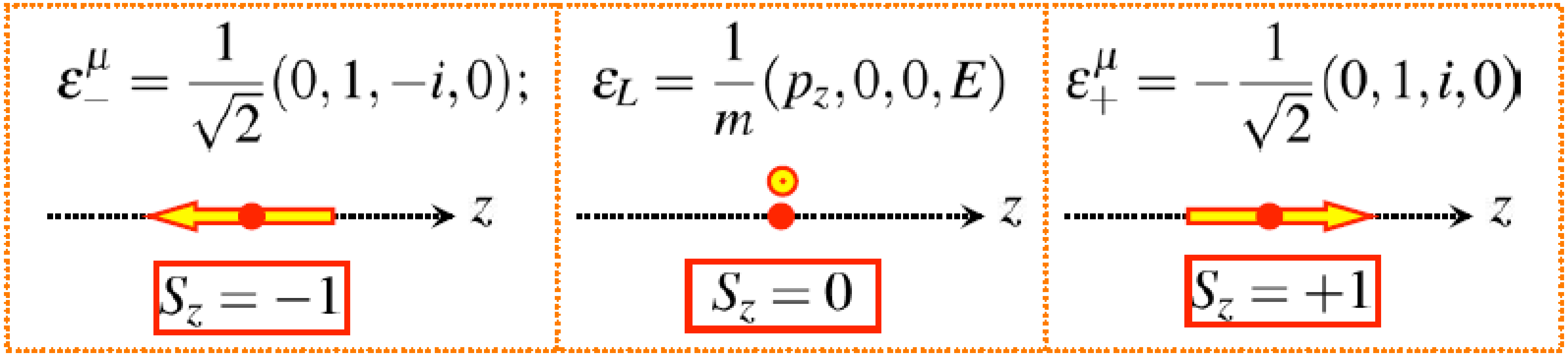


Boson polarisation states

- ★ A real (i.e. not virtual) massless spin-1 boson can exist in two transverse polarization states, a massive spin-1 boson also can be longitudinally polarized
- ★ Boson wave-functions are written in terms of the polarization four-vector ε^{μ}

$$\boxed{B^{\mu} = \varepsilon^{\mu} e^{-ip.x} = \varepsilon^{\mu} e^{i(\vec{p}.\vec{x}-Et)}}$$

- ★ For a spin-1 boson travelling along the z-axis, the polarization four vectors are:



$$\varepsilon^{\mu}_{-} = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = \frac{1}{m}(p_z,0,0,E) \quad \varepsilon^{\mu}_{+} = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

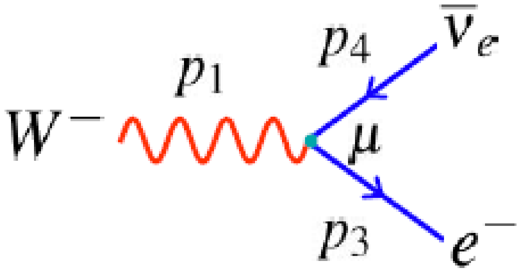
$\boxed{S_z = -1}$ $\boxed{S_z = 0}$ $\boxed{S_z = +1}$

transverse **longitudinal** **transverse**

Longitudinal polarization isn't present for on-shell massless particles, the photon can exist in two helicity states $h = \pm 1$ (LH and RH circularly polarized light)

W boson decay

- ★ To calculate the W-Boson decay rate first consider $W^- \rightarrow e^- \overline{\nu}_e$
- ★ Want matrix element for :



Incoming W-boson : $\varepsilon_\mu(p_1)$

Out-going electron : $\overline{u}(p_3)$

Out-going $\overline{\nu}_e$: $v(p_4)$

Vertex factor : $-i\frac{g_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$

$$-iM_{fi} = \varepsilon_\mu(p_1).\overline{u}(p_3).-i\frac{g_W}{\sqrt{2}}\gamma^\mu\tfrac{1}{2}(1-\gamma^5).v(p_4)$$

Note, no propagator

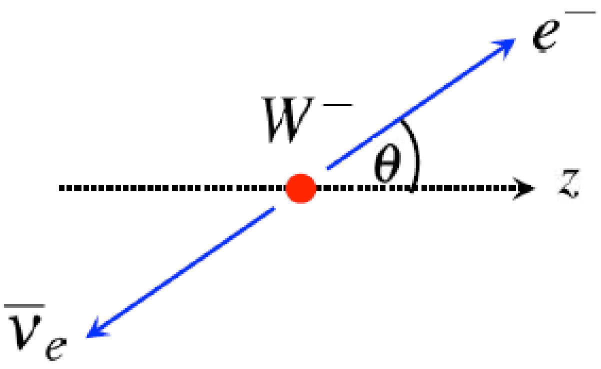
$$\Rightarrow \quad \boxed{M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)\overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)}$$

- ★ This can be written in terms of the four-vector scalar product of the W-boson polarization $\varepsilon_\mu(p_1)$ and the weak charged current j^μ

$$\boxed{M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1).j^\mu} \quad \text{with} \quad \boxed{j^\mu = \overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)}$$

W decay – the lepton current

- First consider the lepton current $j^{\mu} = \overline{u}(p_3)\gamma^{\mu}\frac{1}{2}(1-\gamma^5)v(p_4)$
- Work in Centre-of-Mass frame



$$p_1 = (m_W, 0, 0, 0);$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta)$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

with $E = \frac{m_W}{2}$

- In the ultra-relativistic limit only LH particles and RH anti-particles participate in the weak interaction so

$$j^{\mu} = \overline{u}(p_3)\gamma^{\mu}\frac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_{\downarrow}(p_3)\gamma^{\mu}v_{\uparrow}(p_4)$$

Note: $\frac{1}{2}(1-\gamma^5)v(p_4) = v_{\uparrow}(p_4)$ $\overline{u}(p_3)\gamma^{\mu}v_{\uparrow}(p_4) = \overline{u}_{\downarrow}(p_3)\gamma^{\mu}v_{\uparrow}(p_4)$

Chiral projection operator,

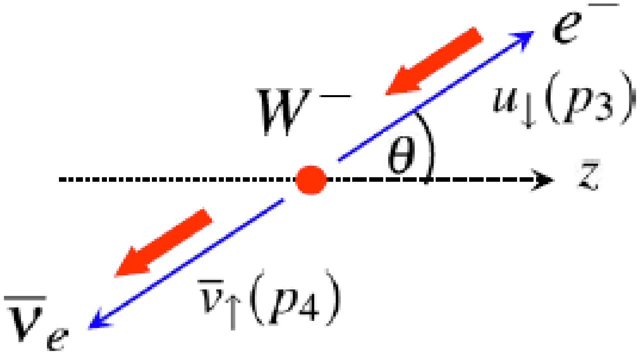
“Helicity conservation”, e.g.

- We have already calculated the current

$$j^\mu = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

when considering $e^+e^- \to \mu^+\mu^-$

$$\boxed{j^\mu_{\uparrow\downarrow} = 2E(0, -\cos\theta, -i, \sin\theta)}$$

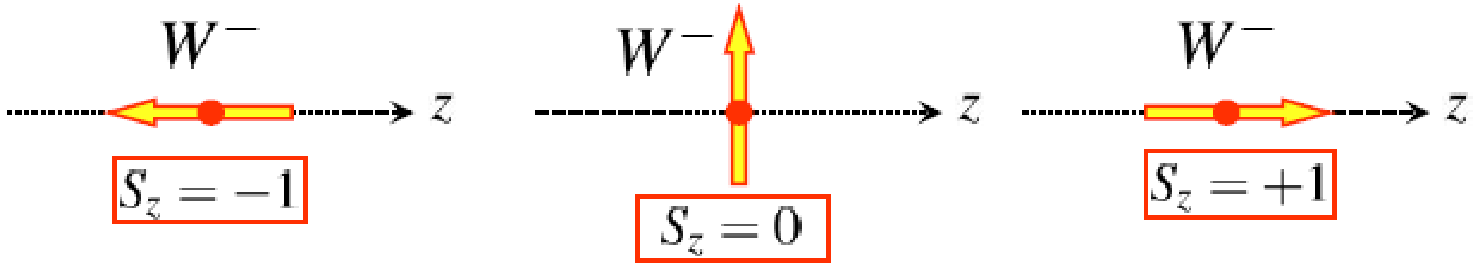


- For the charged current weak Interaction we only have to consider this single combination of helicities

$$j^\mu = \overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

and the three possible W-Boson polarization states:

$$\varepsilon^\mu_- = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \varepsilon_L = \frac{1}{m}(p_z, 0, 0, E) \quad \varepsilon^\mu_+ = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$



- ★ **For a W-boson at rest these become:**

$$\varepsilon^{\mu}_{-} = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = (0,0,0,1) \quad \varepsilon^{\mu}_{+} = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

- ★ **Can now calculate the matrix element for the different polarization states**

$$M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)j^\mu \quad \text{with} \quad j^\mu = 2\frac{m_W}{2}(0,-\cos\theta,-i,\sin\theta)$$

Decay at rest : $E_e = E_\nu = m_W/2$

- ★ giving

$$\boxed{\varepsilon_-} \quad M_- = \frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,-i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \tfrac{1}{2}g_W m_W(1+\cos\theta)$$

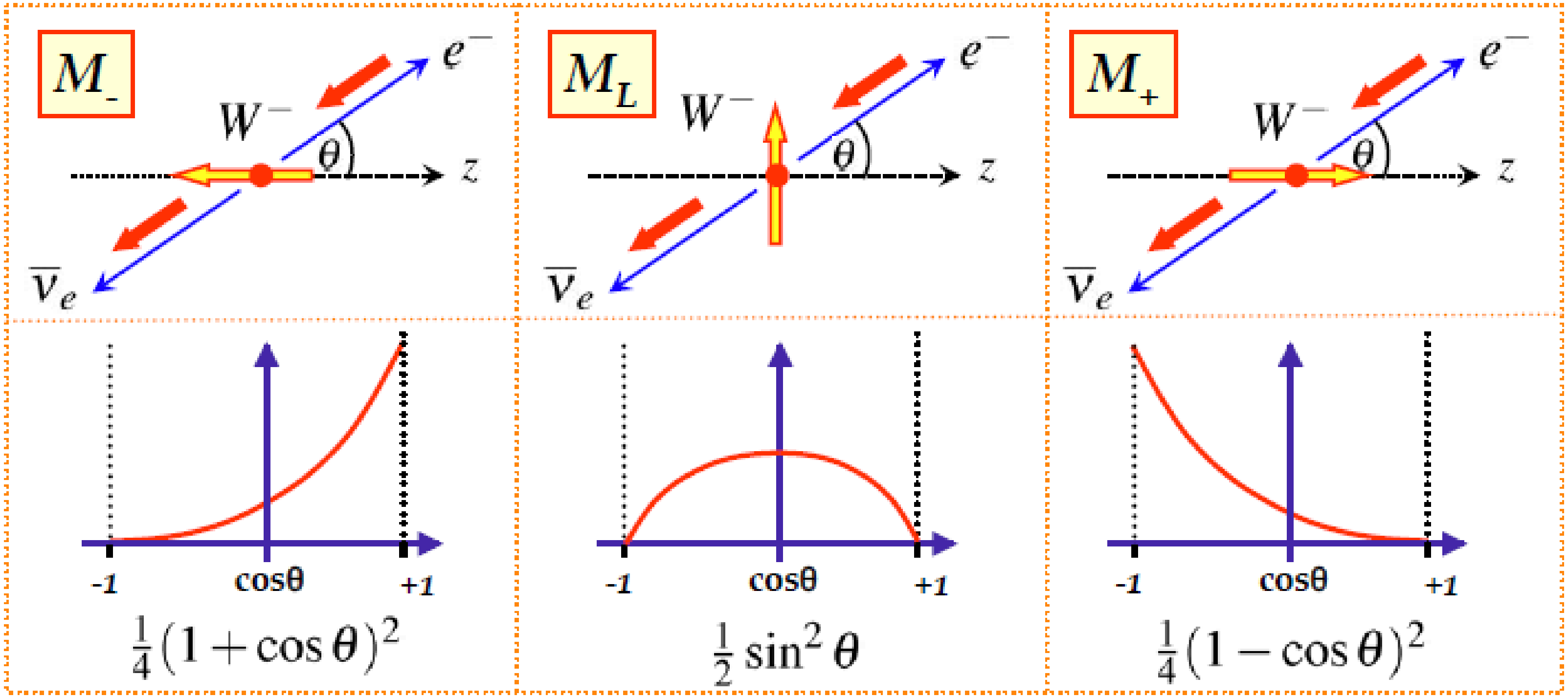
$$\boxed{\varepsilon_L} \quad M_L = \frac{g_W}{\sqrt{2}}(0,0,0,1).m_W(0,-\cos\theta,-i,\sin\theta) = -\frac{1}{\sqrt{2}}g_W m_W \sin\theta$$

$$\boxed{\varepsilon_+} \quad M_+ = -\frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \tfrac{1}{2}g_W m_W(1-\cos\theta)$$

➡

$$\boxed{\begin{aligned} |M_-|^2 &= g_W^2 m_W^2 \tfrac{1}{4}(1+\cos\theta)^2 \\ |M_L|^2 &= g_W^2 m_W^2 \tfrac{1}{2}\sin^2\theta \\ |M_+|^2 &= g_W^2 m_W^2 \tfrac{1}{4}(1-\cos\theta)^2 \end{aligned}}$$

★ The angular distributions can be understood in terms of the spin of the particles



★ The differential decay rate can be found using:

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

where p* is the C.o.M momentum of the final state particles, here $p^* = \frac{m_W}{2}$

- Hence for the three different polarisations we obtain:

$$\frac{d\Gamma_-}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1+\cos\theta)^2 \qquad \frac{d\Gamma_L}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{2}\sin^2\theta \qquad \frac{d\Gamma_+}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1-\cos\theta)^2$$

- Integrating over all angles using

$$\int \tfrac{1}{4}(1\pm\cos\theta)^2 d\phi\, d\cos\theta = \int \tfrac{1}{2}\sin^2\theta\, d\phi\, d\cos\theta = \frac{4\pi}{3}$$

- Gives

$$\boxed{\Gamma_- = \Gamma_L = \Gamma_+ = \frac{g_W^2 m_W}{48\pi}}$$

- The total W-decay rate is independent of polarization; this has to be the case as the decay rate cannot depend on the arbitrary definition of the z-axis
- For a sample of unpolarized W boson each polarization state is equally likely, for the average matrix element sum over all possible matrix elements and average over the three initial polarization states

$$\begin{aligned}\langle|M_{fi}|^2\rangle &= \tfrac{1}{3}(|M_-|^2+|M_L|^2+|M_+|^2)\\ &= \tfrac{1}{3}g_W^2 m_W^2\left[\tfrac{1}{4}(1+\cos\theta)^2+\tfrac{1}{2}\sin^2\theta+\tfrac{1}{4}(1-\cos\theta)^2\right]\\ &= \tfrac{1}{3}g_W^2 m_W^2\end{aligned}$$

- For a sample of unpolarized W-bosons, the decay is isotropic (as expected)

★ For this isotropic decay

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega}=\frac{|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle \Rightarrow \Gamma=\frac{4\pi|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle$$

$$\Rightarrow \boxed{\Gamma(W^-\to e^-\overline{\nu})=\frac{g_W^2 m_W}{48\pi}}$$

★ The calculation for the other decay modes (neglecting final state particle masses) is same. For quarks need to account for colour and CKM matrix. No decays to top – the top mass (175 GeV) is greater than the W-boson mass (80 GeV)

$$\begin{array}{lll}
W^- \to e^-\overline{\nu}_e & W^- \to d\overline{u} & \boxed{\begin{array}{l}\times 3|V_{ud}|^2\\ \times 3|V_{us}|^2\\ \times 3|V_{ub}|^2\end{array}} \\
W^- \to \mu^-\overline{\nu}_\mu & W^- \to s\overline{u} & \\
W^- \to \tau^-\overline{\nu}_\tau & W^- \to b\overline{u} &
\end{array}
\qquad
\begin{array}{ll}
W^- \to d\overline{c} & \boxed{\begin{array}{l}\times 3|V_{cd}|^2\\ \times 3|V_{cs}|^2\\ \times 3|V_{cb}|^2\end{array}} \\
W^- \to s\overline{c} & \\
W^- \to b\overline{c} &
\end{array}$$

★ Unitarity of CKM matrix gives, e.g. $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$

★ Hence $BR(\mathrm{W}\to qq')=6BR(\mathrm{W}\to e\nu)$

and thus the total decay rate :

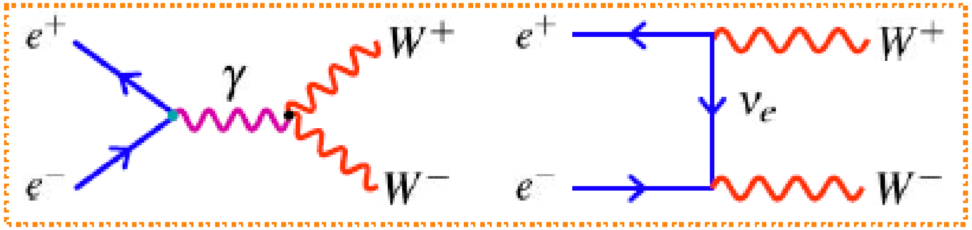
$$\boxed{\Gamma_W=9\Gamma_{W\to e\nu}=\frac{3g_W^2 m_W}{16\pi}=2.07\,\mathrm{GeV}}$$

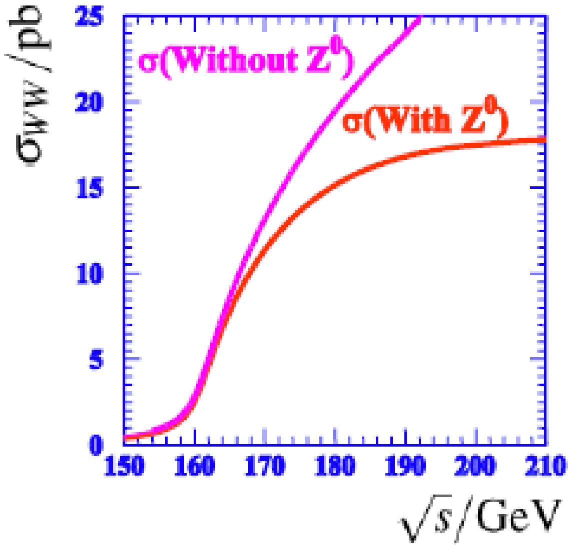
Experiment: 2.14±0.04 GeV
(our calculation neglected a 3% QCD correction to decays to quarks)

From W to Z

★ The $W^{\pm}$ bosons carry the **EM** charge - suggesting Weak are **EM** forces are related.

★ W bosons can be produced in e^+e^- annihilation

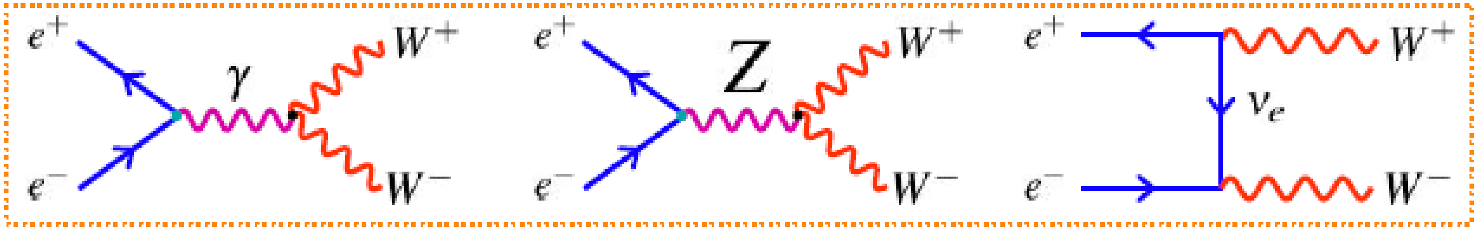




★ With just these two diagrams there is a problem: the cross section increases with C.o.M energy and at some point violates QM unitarity

UNITARITY VIOLATION: when QM calculation gives larger flux of W bosons than incoming flux of electrons/positrons

★ Problem can be "fixed" by introducing a new boson, the **Z**. The new diagram interferes negatively with the above two diagrams fixing the unitarity problem



$$|M_{\gamma WW}+M_{ZWW}+M_{\nu WW}|^2 < |M_{\gamma WW}+M_{\nu WW}|^2$$

★ Only works if Z, γ, W couplings are related: need ELECTROWEAK UNIFICATION

SU(2)$_L$: the weak interaction

- The Weak Interaction arises from **SU(2)** local phase transformations

$$\psi \rightarrow \psi' = \psi e^{i\vec{\alpha}(x)\cdot\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e the three Pauli spin matrices

⇒ **3 Gauge Bosons** $W_1^\mu,\ W_2^\mu,\ W_3^\mu$

- The wave-functions have two components which, in analogy with isospin, are represented by "weak isospin"
- The fermions are placed in isospin doublets and the local phase transformation corresponds to

$$\begin{pmatrix}\nu_e\\e^-\end{pmatrix} \rightarrow \begin{pmatrix}\nu_e\\e^-\end{pmatrix}' = e^{i\vec{\alpha}(x)\cdot\frac{\vec{\sigma}}{2}}\begin{pmatrix}\nu_e\\e^-\end{pmatrix}$$

- Weak Interaction only couples to LH particles/RH anti-particles. hence only place LH particles/RH anti-particles in weak isospin doublets: $I_W = \frac{1}{2}$
  RH particles/LH anti-particles placed in weak isospin singlets: $I_W = 0$

Weak Isospin

$I_W = \frac{1}{2}$ $$\begin{pmatrix}\nu_e\\e^-\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\\mu^-\end{pmatrix}_L, \begin{pmatrix}\nu_\tau\\\tau^-\end{pmatrix}_L, \begin{pmatrix}u\\d'\end{pmatrix}_L, \begin{pmatrix}c\\s'\end{pmatrix}_L, \begin{pmatrix}t\\b'\end{pmatrix}_L$$

← $I_W^3 = +\frac{1}{2}$

← $I_W^3 = -\frac{1}{2}$

$I_W = 0$ $$(\nu_e)_R,\ (e^-)_R, \ldots (u)_R,\ (d)_R, \ldots$$

Note: RH/LH refer to chiral states

★ For simplicity only consider $\chi_L = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L$

•The gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) – [note: here include interaction strength in current]

$$j^1_\mu = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_1 \chi_L \qquad j^2_\mu = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_2 \chi_L \qquad j^3_\mu = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_3 \chi_L$$

★The charged current W^+/W^- interaction enters as a linear combinations of W_1, W_2

$$W^{\pm\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \pm W_2^\mu)$$

★ The $W^\pm$ interaction terms

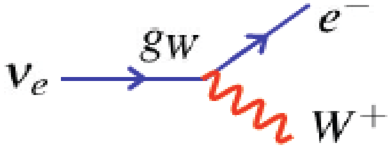
$$j^\mu_\pm = \frac{g_W}{\sqrt{2}}(j_1^\mu \pm i j_2^\mu) = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)\chi_L$$

★ Express in terms of the weak isospin ladder operators $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$\left.\boxed{j^\mu_\pm = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_\pm\chi_L}\;\right\}$$

Origin of $\frac{1}{\sqrt{2}}$ in Weak CC

W^+



corresponds to

$$\boxed{j^\mu_+ = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_+\chi_L}$$

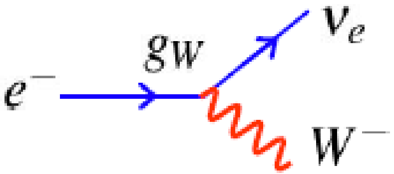
Bars indicates adjoint spinors

which can be understood in terms of the weak isospin doublet

$$j^\mu_+ = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_+\chi_L = \frac{g_W}{\sqrt{2}}(\overline{\nu}_L,\overline{e}_L)\gamma^\mu\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L = \frac{g_W}{\sqrt{2}}\overline{\nu}_L\gamma^\mu e_L = \boxed{\frac{g_W}{\sqrt{2}}\overline{\nu}\gamma^\mu\tfrac{1}{2}(1-\gamma^5)e}$$

★ Similarly

W⁻



corresponds to $\boxed{j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_-\chi_L}$

$$j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_-\chi_L = \frac{g_W}{\sqrt{2}}(\overline{\nu}_L,\overline{e}_L)\gamma^\mu\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}\nu\\e\end{pmatrix}_L = \frac{g_W}{\sqrt{2}}\overline{e}_L\gamma^\mu\nu_L = \frac{g_W}{\sqrt{2}}\overline{e}\gamma^\mu\frac{1}{2}(1-\gamma^5)\nu$$

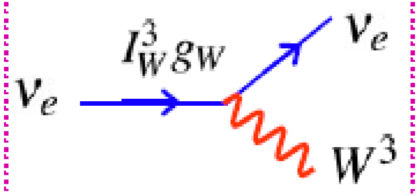
★ However have an additional interaction due to W³

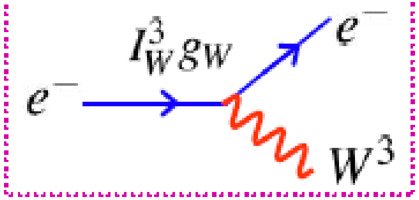
$$\boxed{j^\mu_3 = g_W\overline{\chi}_L\gamma^\mu\tfrac{1}{2}\sigma_3\chi_L}$$

expanding this:

$$\boxed{I^3_W = \pm\tfrac{1}{2}}$$

$$j^\mu_3 = g_W\frac{1}{2}(\overline{\nu}_L,\overline{e}_L)\gamma^\mu\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\nu\\e\end{pmatrix}_L = g_W\tfrac{1}{2}\overline{\nu}_L\gamma^\mu\nu_L - g_W\tfrac{1}{2}\overline{e}_L\gamma^\mu e_L$$





NEUTRAL CURRENT INTERACTIONS !

Electroweak unification

- ★Tempting to identify the W^3 as the Z
- ★However this is not the case, have two physical neutral spin-1 gauge bosons, γ, Z and the W^3 is a mixture of the two,
- ★ Equivalently write the photon and Z in terms of the W^3 and a new neutral spin-1 boson the B
- ★The physical bosons (the Z and photon field, A) are:

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

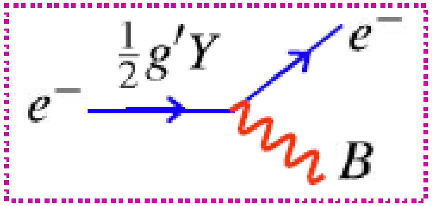
θ_W is the weak mixing angle

- ★The new boson is associated with a new gauge symmetry similar to that of electromagnetism : U(1)$_Y$
- ★The charge of this symmetry is called **WEAK HYPERCHARGE** Y

$$\boxed{Y = 2Q - 2I^3_W}$$

Q is the EM charge of a particle

I^3_W is the third comp. of weak isospin



- •By convention the coupling to the B_μ is $\frac{1}{2}g'Y$

e_L : $Y = 2(-1) - 2(-\frac{1}{2}) = -1$ $\quad$ ν_L : $Y = +1$

e_R : $Y = 2(-1) - 2(0) = -2$ $\quad$ ν_R : $Y = 0$

(this identification of hypercharge in terms of Q and I_3 makes all of the following work out)

★ For this to work the coupling constants of the W³, **B**, and photon must be related

e.g. consider contributions involving the neutral interactions of electrons:

γ $$j_\mu^{em} = e\overline{\psi}Q_e\gamma_\mu\psi = e\bar{e}_LQ_e\gamma_\mu e_L + e\bar{e}_RQ_e\gamma_\mu e_R$$

W³ $$j_\mu^{W^3} = -\frac{g_W}{2}\bar{e}_L\gamma_\mu e_L$$

B $$j_\mu^{Y} = \frac{g'}{2}\overline{\psi}Y_e\gamma_\mu\psi = \frac{g'}{2}\bar{e}_LY_{e_L}\gamma_\mu e_L + \frac{g'}{2}\bar{e}_RY_{e_R}\gamma_\mu e_R$$

★ The relation $A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W$ is equivalent to requiring

$$\boxed{j_\mu^{em} = j_\mu^{Y}\cos\theta_W + j_\mu^{W^3}\sin\theta_W}$$

- Writing this in full:

$$e\bar{e}_LQ_e\gamma_\mu e_L + e\bar{e}_RQ_e\gamma_\mu e_R = \tfrac{1}{2}g'\cos\theta_W[\bar{e}_LY_{e_L}\gamma_\mu e_L + \bar{e}_RY_{e_R}\gamma_\mu e_R] - \tfrac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$$

$$-e\bar{e}_L\gamma_\mu e_L - e\bar{e}_R\gamma_\mu e_R = \tfrac{1}{2}g'\cos\theta_W[-\bar{e}_L\gamma_\mu e_L - 2\bar{e}_R\gamma_\mu e_R] - \tfrac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$$

which works if: $\boxed{e = g_W\sin\theta_W = g'\cos\theta_W}$ (i.e. equate coefficients of L and R terms)

★ Couplings of electromagnetism, the weak interaction and the interaction of the $U(1)_Y$ symmetry are therefore related.

The Z boson

★ **In this model we can now derive the couplings of the Z Boson**

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W \qquad \boxed{I^3_W}$$

for the electron $I^3_W = \frac{1}{2}$

$$j^Z_\mu = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L Y_{e_L} \gamma_\mu e_L + \bar{e}_R Y_{e_R} \gamma_\mu e_R] - \tfrac{1}{2} g_W \cos\theta_W [e_L \gamma_\mu e_L]$$

- Writing this in terms of weak isospin and charge:

$$j^Z_\mu = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L \boxed{(2Q - 2I^3_W)} \gamma_\mu e_L + \bar{e}_R \underbrace{\boxed{(2Q)}}_{\text{For RH chiral states } I_3=0} \gamma_\mu e_R] + \boxed{I^3_W} g_W \cos\theta_W [e_L \gamma_\mu e_L]$$

- Gathering up the terms for LH and RH chiral states:

$$j^Z_\mu = [g' I^3_W \sin\theta_W - g' Q \sin\theta_W + g_W I^3_W \cos\theta_W]\, \bar{e}_L \gamma_\mu e_L - [g' Q \sin\theta_W]\, e_R \gamma_\mu e_R$$

- Using: $e = g_W \sin\theta_W = g' \cos\theta_W$ gives

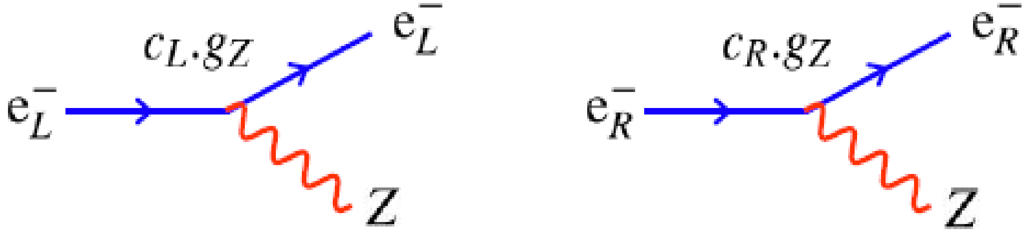
$$j^Z_\mu = \left[g' \frac{(I^3_W - Q\sin^2\theta_W)}{\sin\theta_W} \right] \bar{e}_L \gamma_\mu e_L - \left[g' \frac{Q \sin^2\theta_W}{\sin\theta_W} \right] e_R \gamma_\mu e_R$$

$$\boxed{j^Z_\mu = g_Z (I^3_W - Q\sin^2\theta_W)[\bar{e}_L \gamma_\mu e_L] - g_Z Q \sin^2\theta_W [e_R \gamma_\mu e_R]}$$

with $\boxed{e = g_Z \cos\theta_W \sin\theta_W}$ i.e. $\boxed{g_Z = \dfrac{g_W}{\cos\theta_W}}$

- Unlike for the Charged Current Weak interaction (W) the **Z** Boson couples to both LH and RH chiral components, but not equally…

$$\begin{aligned} j^Z_\mu &= g_Z(I^3_W - Q\sin^2\theta_W)[\bar{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W[e_R\gamma_\mu e_R] \\ &= g_Z c_L[\bar{e}_L\gamma_\mu e_L] + g_Z c_R[e_R\gamma_\mu e_R] \end{aligned}$$



$$\boxed{c_L = I^3_W - Q\sin^2\theta_W} \qquad \boxed{c_R = -Q\sin^2\theta_W}$$

W³ part of Z couples only to LH components (like W±)

B_μ part of Z couples equally to LH and RH components

- Use projection operators to obtain vector and axial vector couplings

$$\bar{u}_L\gamma_\mu u_L = \bar{u}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)u \qquad \bar{u}_R\gamma_\mu u_R = \bar{u}\gamma_\mu\tfrac{1}{2}(1+\gamma_5)u$$

$$j^Z_\mu = g_Z\bar{u}\gamma_\mu\left[c_L\tfrac{1}{2}(1-\gamma_5) + c_R\tfrac{1}{2}(1+\gamma_5))\right]u$$

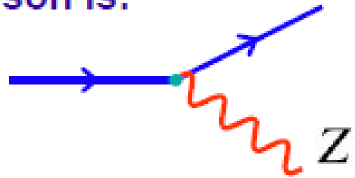
$$j_\mu^Z = \frac{g_Z}{2}\bar{u}\gamma_\mu\left[(c_L + c_R) + (c_R - c_L)\gamma_5\right]u$$

- ★ Which in terms of V and A components gives: $j_\mu^Z = \frac{g_Z}{2}\bar{u}\gamma_\mu\left[c_V - c_A\gamma_5\right]u$

with $\boxed{c_V = c_L + c_R = I_W^3 - 2Q\sin^2\theta_W}$ $\boxed{c_A = c_L - c_R = I_W^3}$

- ★ Hence the vertex factor for the **Z** boson is:

$$\boxed{-ig_Z\tfrac{1}{2}\gamma_\mu\left[c_V - c_A\gamma_5\right]}$$



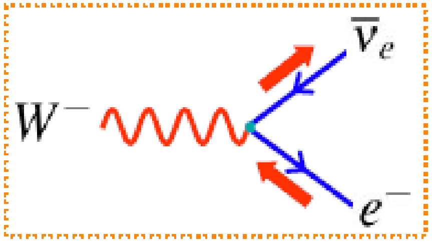
- ★ Using the experimentally determined value of the weak mixing angle:

$\boxed{\sin^2\theta_W \approx 0.23}$

Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

Z-boson decay: Γ_Z

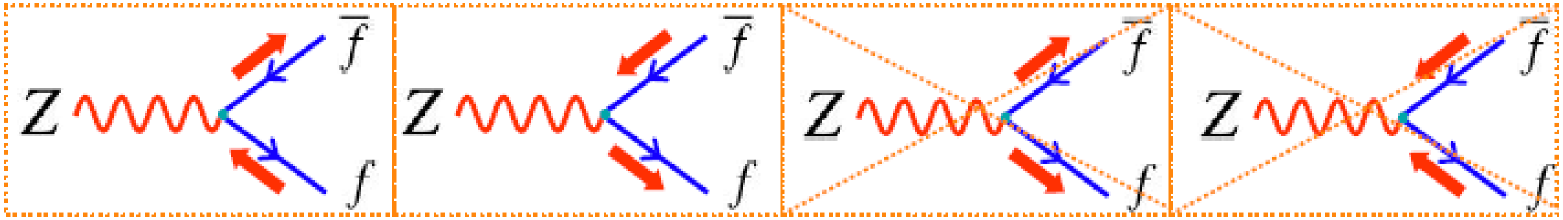
★ In W-boson decay only had to consider one helicity combination of (assuming we can neglect final state masses: helicity states = chiral states)



W-boson couples:
to LH particles
and RH anti-particles

★ But Z-boson couples to LH and RH particles (with different strengths)

★ Need to consider only two helicity (or more correctly chiral) combinations:

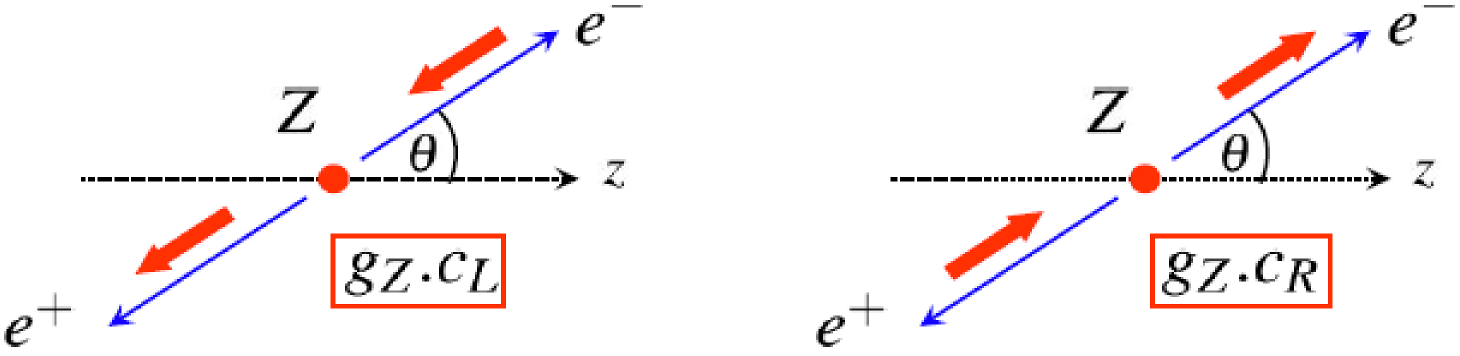


This can be seen by considering either of the combinations which give zero

e.g.
$$\overline{u}_R\gamma^\mu(c_V+c_A\gamma_5)v_R = u^\dagger\tfrac{1}{2}(1+\gamma^5)\gamma^0\gamma^\mu(c_V+c_A\gamma^5)\tfrac{1}{2}(1-\gamma^5)v$$
$$= \tfrac{1}{4}u^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)(c_V+c_A\gamma^5)v$$
$$= \tfrac{1}{4}\overline{u}\gamma^\mu(1+\gamma^5)(1-\gamma^5)(c_V+c_A\gamma_5)v = 0$$

Z-boson decay: Γ_Z

★ **In terms of left and right-handed combinations need to calculate:**



★ **For unpolarized Z bosons:**

$$\langle|M_{fi}|^2\rangle = \frac{1}{3}[2c_L^2 g_Z^2 m_Z^2 + 2c_R^2 g_Z^2 m_Z^2] = \frac{2}{3} g_Z^2 m_Z^2 (c_L^2 + c_R^2)$$

average over polarization

★ **Using** $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ **and** $\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$

⇨ $$\boxed{\Gamma(Z \to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)}$$

Z-boson branching ratios

★ (Neglecting fermion masses) obtain the same expression for the other decays

$$\Gamma(Z \to f\overline{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

- Using values for c_V and c_A obtain:

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \boxed{\approx 3.5\%}$$

$$Br(Z \to \nu_1\overline{\nu}_1) = Br(Z \to \nu_2\overline{\nu}_2) = Br(Z \to \nu_3\overline{\nu}_3) \boxed{\approx 6.9\%}$$

$$Br(Z \to d\overline{d}) = Br(Z \to s\overline{s}) = Br(Z \to b\overline{b}) \boxed{\approx 15\%}$$

$$Br(Z \to u\overline{u}) = Br(Z \to c\overline{c}) \boxed{\approx 12\%}$$

- The Z Boson therefore predominantly decays to hadrons

$$Br(Z \to \text{hadrons}) \approx 69\%$$

Mainly due to factor 3 from colour

- Also predict total decay rate (total width)

$$\Gamma_Z = \sum_i \Gamma_i = 2.5\,\text{GeV}$$

Experiment: $$\boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}}$$

Summary

★ The Standard Model interactions are mediated by spin-1 gauge bosons

★ The form of the interactions are completely specified by the assuming an underlying local phase transformation ➡ **GAUGE INVARIANCE**

$\boxed{U(1)_{em}} \Rightarrow$ **QED**

$\boxed{SU(2)_{L}} \Rightarrow$ **Charged Current Weak Interaction + W^3**

$\boxed{SU(3)_{col}} \Rightarrow$ **QCD**

★ In order to "unify" the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry : U(1) hypercharge

$\boxed{U(1)_{Y}} \Rightarrow \boxed{B_{\mu}}$

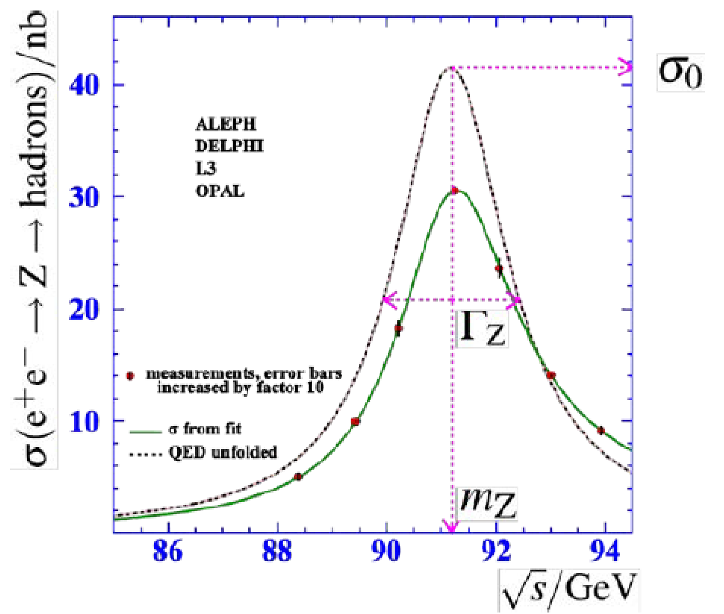
★ The physical **Z** boson and the photon are mixtures of the neutral W boson and **B** determined by the Weak Mixing angle

$$\boxed{\sin\theta_W \approx 0.23}$$

★ **Have we really unified the EM and Weak interactions ? Well not really…**

- Started with two independent theories with coupling constants g_W, e
- Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model θ_W
- Interactions not unified from any higher theoretical principle… but it works!

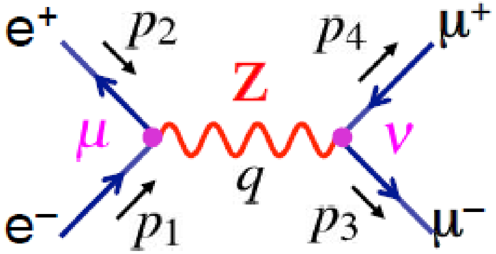
Precision tests of the Standard Model



The Z resonance

★ Want to calculate the cross-section for $e^+e^- \to Z \to \mu^+\mu^-$

- Feynman rules for the diagram below give:



e⁺e⁻ vertex: $\overline{v}(p_2) \cdot -ig_Z\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

μ⁺μ⁻ vertex: $\overline{u}(p_3) \cdot -ig_Z\gamma^\nu \frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot v(p_4)$

➡ $$-iM_{fi} = [\overline{v}(p_2) \cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}.[\overline{u}(p_3) \cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot v(p_4)]$$

➡ $$\boxed{M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot v(p_4)]}$$

★ Convenient to work in terms of helicity states by explicitly using the Z coupling to **LH and RH** chiral states (ultra-relativistic limit so helicity = chirality)

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\underbrace{\tfrac{1}{2}(1-\gamma^5)} + c_R\underbrace{\tfrac{1}{2}(1+\gamma^5)}$$

LH and RH projections operators

hence $c_V = (c_L + c_R), \;\; c_A = (c_L - c_R)$

and

$$\begin{aligned} \tfrac{1}{2}(c_V - c_A\gamma^5) &= \tfrac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) \\ &= c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5) \end{aligned}$$

with $c_L = \tfrac{1}{2}(c_V + c_A), \;\; c_R = \tfrac{1}{2}(c_V - c_A)$

- Rewriting the matrix element in terms of LH and RH couplings:

$$\begin{aligned} M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} &[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)] \\ &\times [c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)] \end{aligned}$$

- Apply projection operators remembering that in the ultra-relativistic limit

$$\tfrac{1}{2}(1-\gamma^5)u = u_\downarrow; \quad \tfrac{1}{2}(1+\gamma^5)u = u_\uparrow, \quad \tfrac{1}{2}(1-\gamma^5)v = v_\uparrow, \quad \tfrac{1}{2}(1+\gamma^5)v = v_\downarrow$$

⇨

$$\begin{aligned} M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu} &[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)] \\ &\times [c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)] \end{aligned}$$

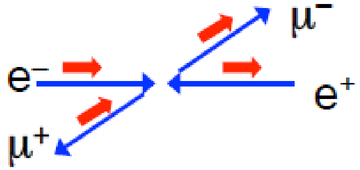
- For a combination of **V** and **A** currents, $\overline{u}_\uparrow \gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions

⇨

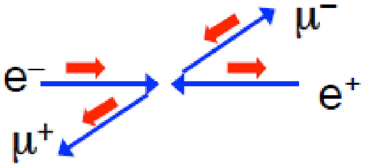
$$\boxed{\begin{aligned} -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} &[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)] \\ &\times [c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] \end{aligned}}$$

- Sum of 4 terms

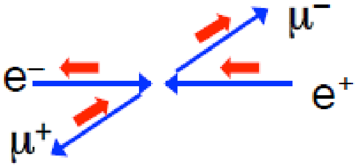
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



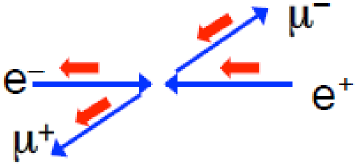
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state particles

- Fortunately we have calculated these terms before when considering $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ giving:

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1 + \cos\theta)$$ etc.

- ★ Applying the QED results to the Z exchange with

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

gives:

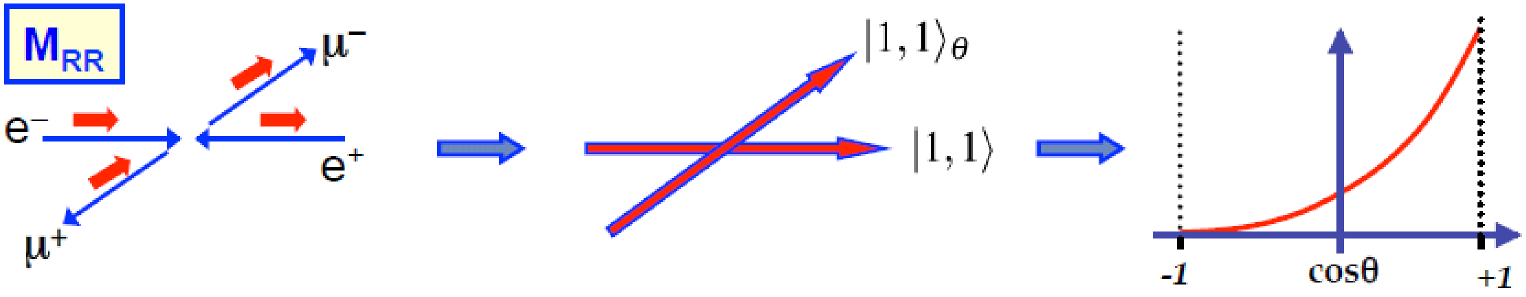
$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

- ★ As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.



The Breit-Wigner resonance

- Need to consider carefully the propagator term $1/(s-m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson
- To do this need to account for the fact that the Z boson is an unstable particle
  - For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

  - For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

  - Equivalent to making the replacement

$$m \longrightarrow m - i\Gamma/2$$

- In the Z boson propagator make the substitution:

$$m_Z \longrightarrow m_Z - i\Gamma_Z/2$$

- Which gives:

$$(s-m_Z^2) \longrightarrow [s-(m_Z - i\Gamma_Z/2)] = s - m_Z^2 + im_Z\Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

- Which gives

$$\left|\frac{1}{s-m_Z^2}\right|^2 \rightarrow \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}$$

- And the Matrix elements become

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2 \qquad \text{etc.}$$

- In the limit where initial and final state particle mass can be neglected:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

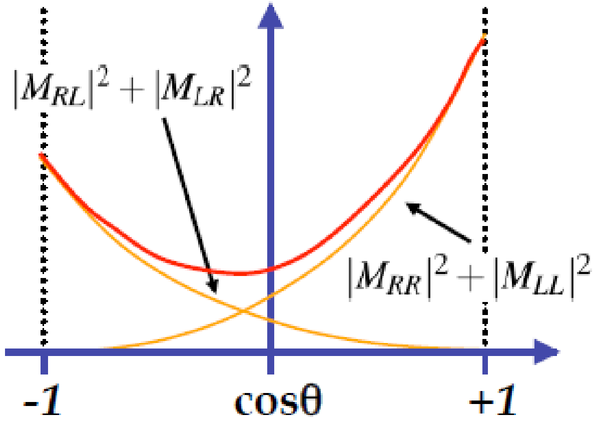
- Giving:

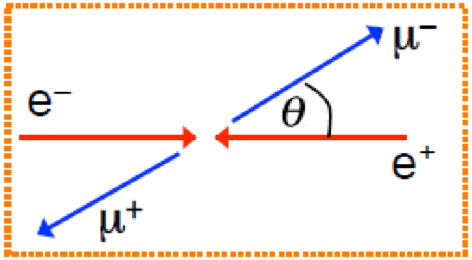
$$\frac{\mathrm{d}\sigma_{RR}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{LL}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{LR}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{RL}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$





- Because $|M_{LL}|^2+|M_{RR}|^2 \neq |M_{LR}|^2+|M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).

Cross-section with unpolarised beams

- ★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states**. Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\langle |M_{fi}|^2 \rangle = \overset{\searrow}{\frac{1}{2}} \cdot \overset{\swarrow}{\frac{1}{2}} . (|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \times \{[(c_R^e)^2 (c_R^\mu)^2 + (c_L^e)^2 (c_L^2)^2](1 + \cos\theta)^2 + [(c_L^e)^2 (c_R^\mu)^2 + (c_R^e)^2 (c_L^2)^2](1 - \cos\theta)^2\}$$

- ★ The part of the expression {…} can be rearranged:

$$\{\ldots\} = [(c_R^e)^2 + (c_L^e)^2][(c_R^\mu)^2 + (c_L^\mu)^2](1 + \cos^2\theta) + 2[(c_R^e)^2 - (c_L^e)^2][(c_R^\mu)^2 - (c_L^\mu)^2]\cos\theta \qquad (1)$$

and using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $c_V c_A = c_L^2 + c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2](1 + \cos^2\theta) + 2 c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta$$

★ Hence the complete expression for the unpolarized differential cross section is:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle \\ &= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times \\ & \left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\}\end{aligned}$$

★ Integrating over solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\,\mathrm{d}(\cos\theta) = 2\pi\mathrm{d}(\cos\theta)$

$\int_{-1}^{+1}(1+\cos^2\theta)\mathrm{d}(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \frac{8}{3}$ and $\int_{-1}^{+1}\cos\theta\,\mathrm{d}(\cos\theta) = 0$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]]}$$

★ Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions

$$(c_V^f)^2+(c_A^f)^2$$

Connection to Breit-Wigner formula

★ Can write the total cross section

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]]}$$

in terms of the Z boson decay rates (partial widths)

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^e)^2+(c_A^e)^2] \quad \text{and} \quad \Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^\mu)^2+(c_A^\mu)^2]$$

⇨ $$\sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

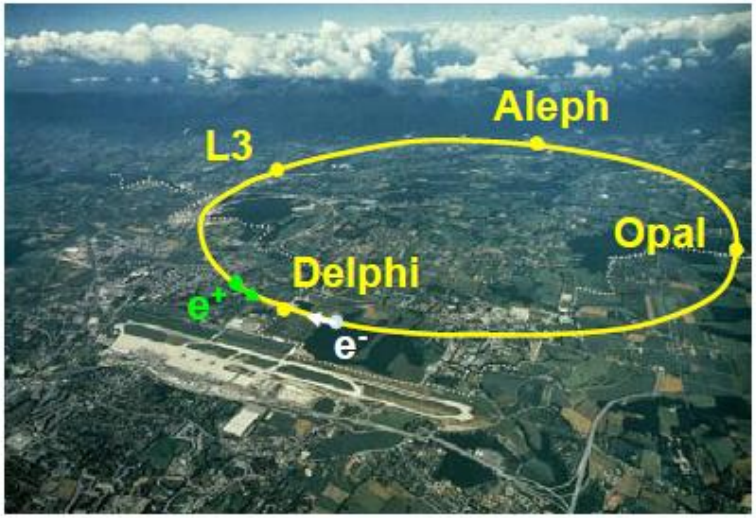
★ Writing the partial widths as $\Gamma_{ee} = \Gamma(Z\to e^+e^-)$ etc., the total cross section can be written

$$\boxed{\sigma(e^+e^-\to Z\to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff}} \qquad (2)$$

where f is the final state fermion flavour:

Electroweak measurements at LEP

★ The Large Electron Positron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.



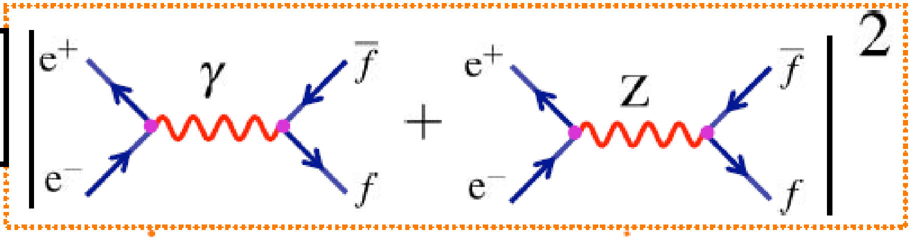
- 26 km circumference accelerator straddling French/Swiss boarder
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):
  - ALEPH,
  - DELPHI,
  - L3,
  - OPAL

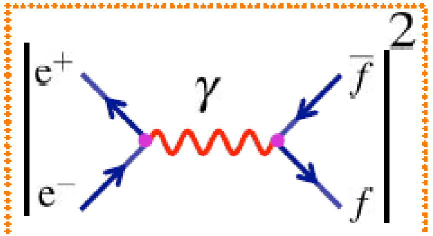
Basically a large Z and W factory:

- ★ 1989-1995: Electron-Positron collisions at $\sqrt{s}$ = 91.2 GeV
  - 17 Million Z bosons detected
- ★ 1996-2000: Electron-Positron collisions at $\sqrt{s}$ = 161-208 GeV
  - 30000 W^+W^- events detected

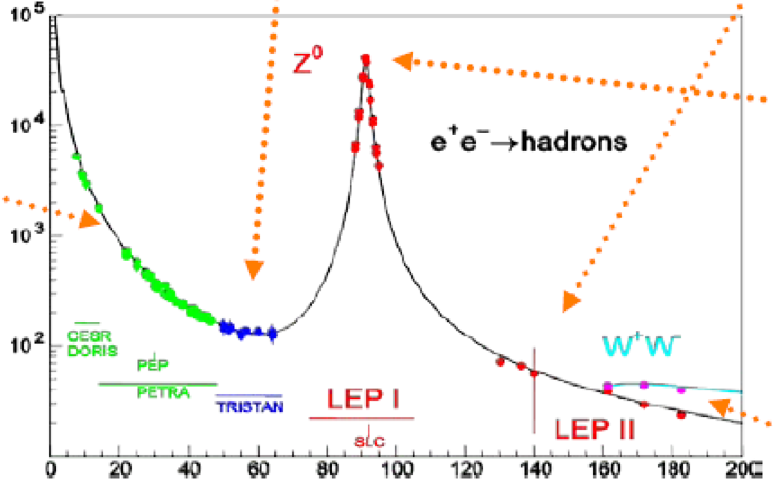
e+e- annihilation in Feynman diagrams

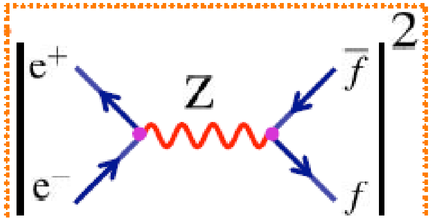
In general e^+e^- annihilation involves both photon and Z exchange : + interference





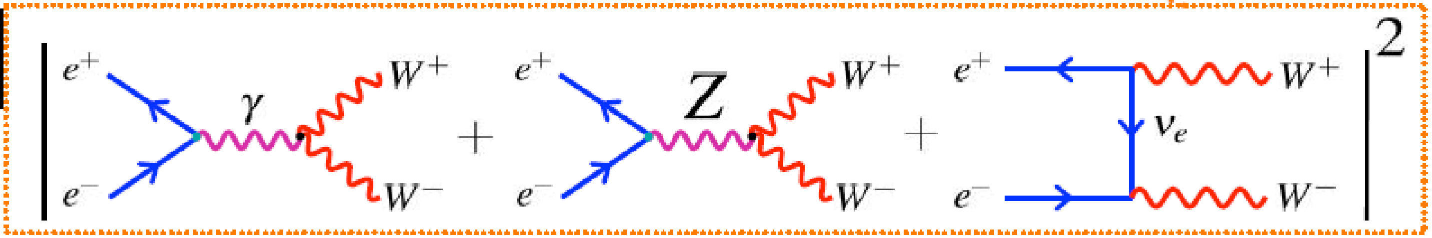
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

High energies: WW production



Cross-section measurements

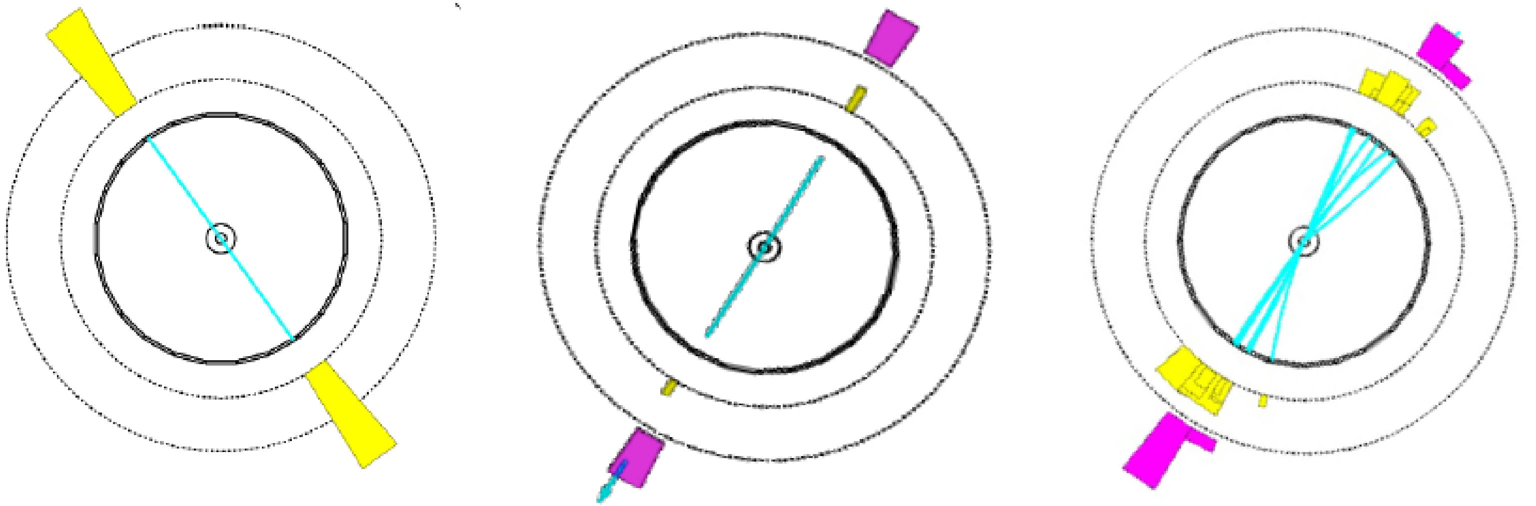
- At Z resonance mainly observe four types of event:

$$e^+e^- \rightarrow Z \rightarrow e^+e^- \qquad e^+e^- \rightarrow Z \rightarrow \mu^+\mu^- \qquad e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-$$

$$e^+e^- \rightarrow Z \rightarrow q\overline{q} \rightarrow \text{hadrons}$$

- Each has a distinct topology in the detectors, e.g.

$$e^+e^- \rightarrow Z \rightarrow e^+e^- \qquad e^+e^- \rightarrow Z \rightarrow \mu^+\mu^- \qquad e^+e^- \rightarrow Z \rightarrow \text{hadrons}$$



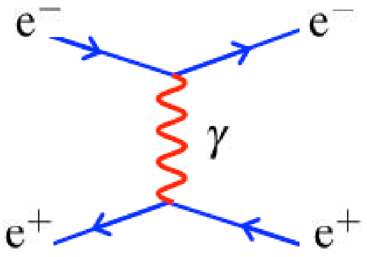
- To work out cross sections, first count events of each type
- Then need to know "integrated luminosity" of colliding beams, i.e. the relation between cross-section and expected number of interactions

$$N_{\text{events}} = \mathscr{L}\sigma$$

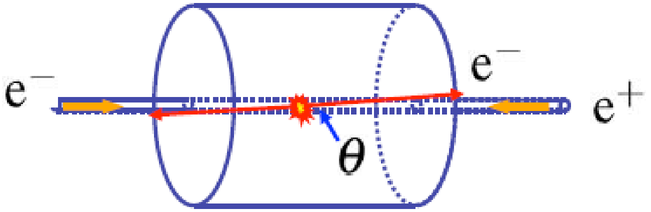
- ★ To calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile

  - very difficult to achieve with precision of better than 10%

- ★ Instead "normalise" using another type of event:



- Use the QED Bhabha scattering process
- QED, so cross section can be calculated very precisely
- Very large cross section – small statistical errors
- Reaction is very forward peaked – i.e. the electron tends not to get deflected much



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{\sin^4 \theta/2} \Rightarrow \boxed{\frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}}$$

Photon propagator

- Count events where the electron is scattered in the very forward direction

$$N_{\text{Bhabha}} = \mathscr{L}\sigma_{\text{Bhabha}} \Rightarrow \mathscr{L}$$

σ_{Bhabha} known from QED calc.

- ★ Hence all other cross sections can be expressed as

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}} \Rightarrow$$

Cross section measurements
Involve just event counting !

Measurements of the Z line-shape

- ★ Measurements of the Z resonance lineshape determine:
  - m_Z : peak of the resonance
  - Γ_Z : FWHM of resonance
  - Γ_f : Partial decay widths
  - N_ν : Number of light neutrino generations
- ★ Measure cross sections to different final states versus C.o.M. energy $\sqrt{s}$
- ★ Starting from

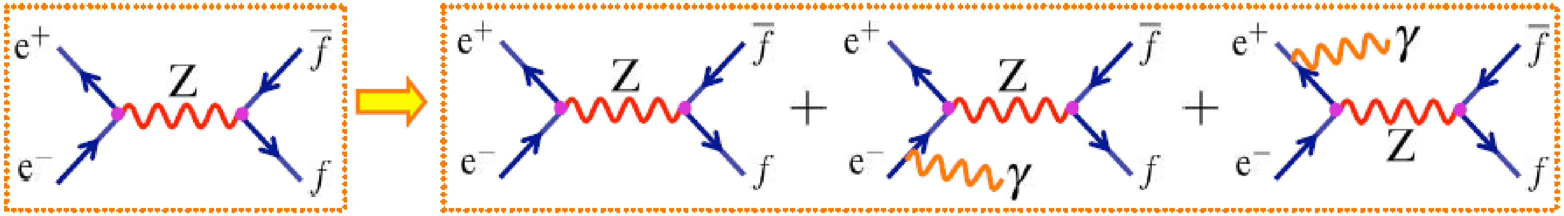
$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad (3)$$

maximum cross section occurs at $\sqrt{s} = m_Z$ with peak cross section equal to

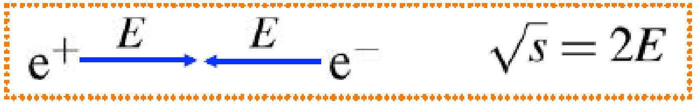
$$\boxed{\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}}$$

- ★ Cross section falls to half peak value at $\sqrt{s} \approx m_z \pm \frac{\Gamma_Z}{2}$ which can be seen immediately from eqn. (3)
- ★ Hence $\Gamma_Z = \frac{\hbar}{\tau_Z} = \text{FWHM of resonance}$

- In practise, it is not that simple, QED corrections distort the measured line-shape
- One particularly important correction: initial state radiation (**ISR**)

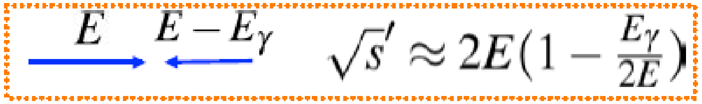


- Initial state radiation reduces the centre-of-mass energy of the e^+e^- collision



$$\sqrt{s} = 2E$$

becomes



$$\sqrt{s'} \approx 2E\left(1 - \frac{E_\gamma}{2E}\right)$$

- Measured cross section can be written:

$$\sigma_{\text{meas}}(E) = \int \sigma(E')f(E',E)\,\mathrm{d}E'$$

Probability of e+e- colliding with C.o.M. energy **E'** when C.o.M energy before radiation is **E**

- Fortunately can calculate $f(E',E)$ very precisely, just QED, and can then obtain Z line-shape from measured cross section

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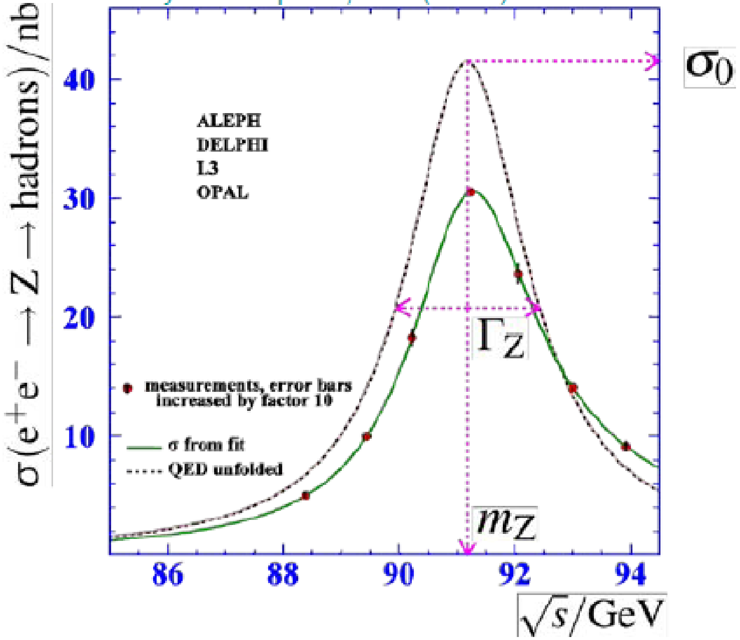


- ★ In principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM

$$\boxed{m_Z = 91.1875 \pm 0.0021\,\text{GeV}} \qquad \boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}}$$

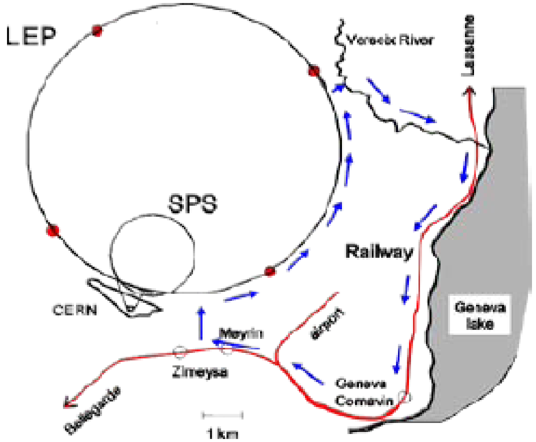
- ★ 0.002 % measurement of m_Z !
- ★ To achieve this level of precision – need to know energy of the colliding beams to better than 0.002 % : sensitive to unusual systematic effects…

Moon:

- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of **4.3 km** varies by **±0.15 mm**
- Changes beam energy by **~10 MeV** : need to correct for tidal effects !

Trains:

- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV



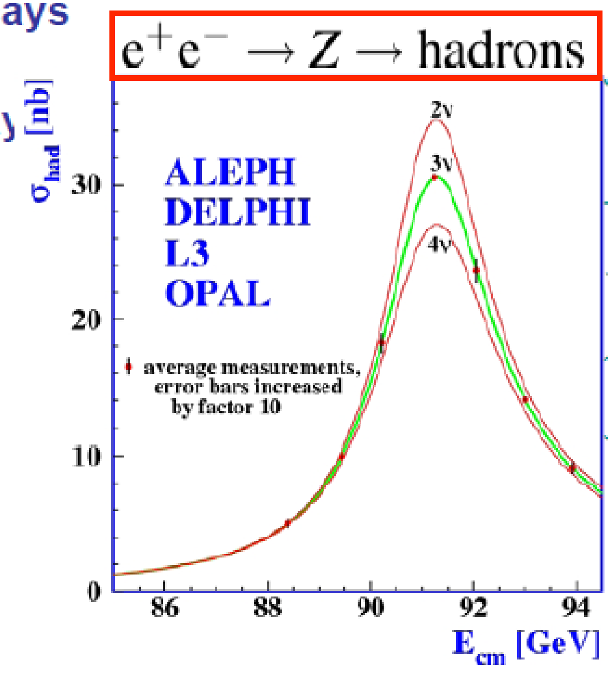
Number of generations

- ★ Total decay width measured from Z line-shape: $\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}$
- ★ If there were an additional 4th generation would expect $Z \to \nu_4\overline{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. > $m_Z/2$)
- ★ Total decay width is the sum of the partial widths:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

- ★ Although don't observe neutrinos, $Z \to \nu\overline{\nu}$ decays affect the Z resonance shape for all final states
- ★ For all other final states can determine partial decay widths from peak cross sections:

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$



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- ★ Assuming lepton universality:

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + \boxed{N_\nu}\Gamma_{\nu\nu}$$

measured from Z lineshape

measured from peak cross sections

calculated

$$\Rightarrow \boxed{N_\nu = 2.9840 \pm 0.0082}$$

- ★ ONLY 3 GENERATIONS (unless a new 4th generation neutrino has very large mass)

Forward-backward asymmetry

- ★ expression for the differential cross section:

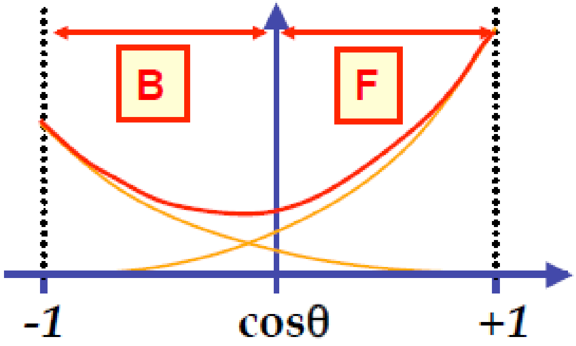
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

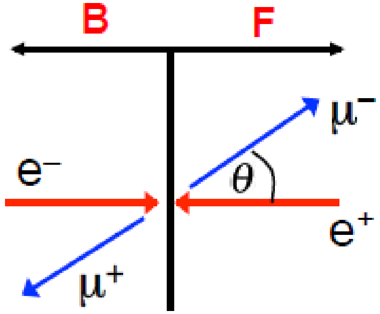
- ★ The differential cross sections is therefore of the form:

$$\frac{d\sigma}{d\Omega} = \kappa \times [A(1+\cos^2\theta)+B\cos\theta] \quad \begin{cases} A = [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B = [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{cases}$$

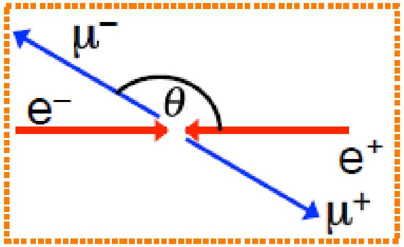
- ★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\boxed{\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta}d\cos\theta} \qquad \boxed{\sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta}d\cos\theta}$$



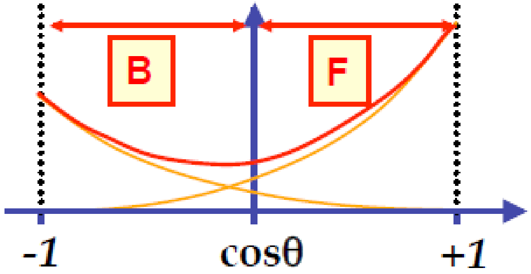


e.g. "backward hemisphere"



- The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



- Integrating equation (1):

$$\sigma_F = \kappa\int_0^1 [A(1+\cos^2\theta)+B\cos\theta]\mathrm{d}\cos\theta = \kappa\int_0^1 [A(1+x^2)+Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A+\frac{1}{2}B\right)$$

$$\sigma_B = \kappa\int_{-1}^0 [A(1+\cos^2\theta)+B\cos\theta]\mathrm{d}\cos\theta = \kappa\int_{-1}^0 [A(1+x^2)+Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A-\frac{1}{2}B\right)$$

- Which gives:

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2-(c_R^\mu)^2}{(c_L^\mu)^2+(c_R^\mu)^2}\right]$$

- This can be written as

$$\boxed{A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu}$$

with

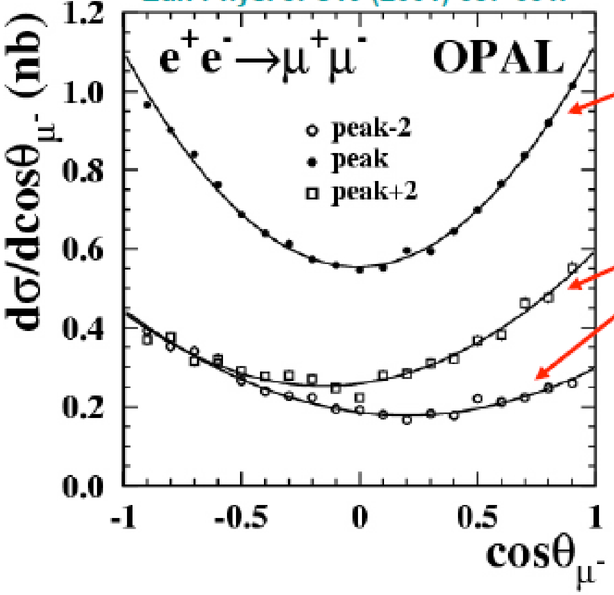
$$\boxed{A_f \equiv \frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2+(c_A^f)^2}} \qquad (4)$$

- Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

- ★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \to Z \to \mu^+\mu^-$

OPAL Collaboration, Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★ LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

- ★ To relate these measurements to the couplings uses $A_{FB} = \frac{3}{4}A_e A_\mu$
- ★ In all cases asymmetries depend on A_e
- ★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4}A_e^2$

Determination of the weak mixing angle

$$\left.\begin{array}{l}\bigstar\ \textbf{From LEP :}\ \ A_{FB}^{0,f} = \frac{3}{4} A_e A_f \\ \bigstar\ \textbf{From SLC :}\ \ A_{LR} = A_e \end{array}\right\}\quad A_e, A_\mu, A_\tau, \ldots$$

Putting everything together ⇨

$$\boxed{\begin{aligned} A_e &= 0.1514 \pm 0.0019 \\ A_\mu &= 0.1456 \pm 0.0091 \\ A_\tau &= 0.1449 \pm 0.0040 \end{aligned}}$$

includes results from other measurements

with $$A_f \equiv \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2\frac{c_V/c_A}{1+(c_V/c_A)^2}$$

- ★ **Measured asymmetries give ratio of vector to axial-vector Z coupings.**
- ★ **In SM these are related to the weak mixing angle**

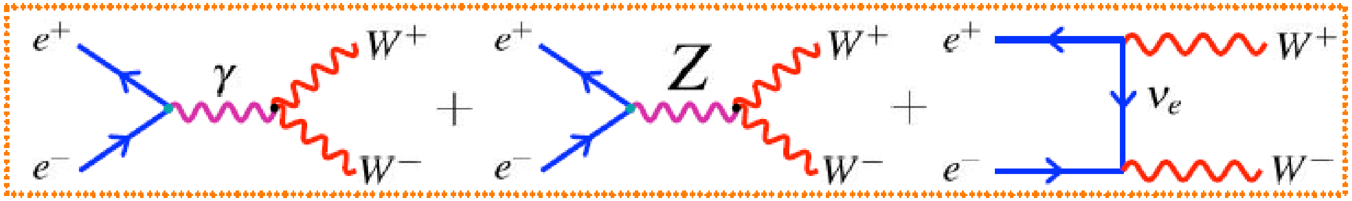
$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q\sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_3}\sin^2\theta_W = 1 - 4|Q|\sin^2\theta_W$$

- ★ **Asymmetry measurements give precise determination of** $\sin^2\theta_W$

$$\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$$

WW production

- From 1995-2000 LEP operated above the threshold for W-pair production
- Three diagrams “CC03” are involved

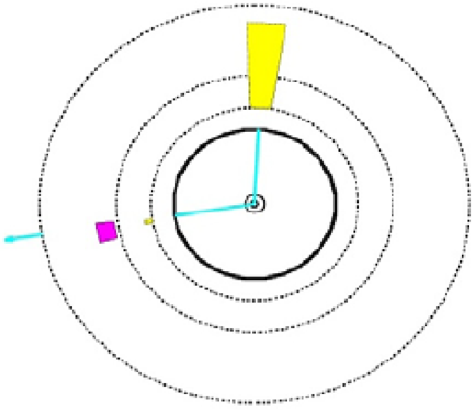


- W bosons decay (p.459) either to leptons or hadrons with branching fractions:

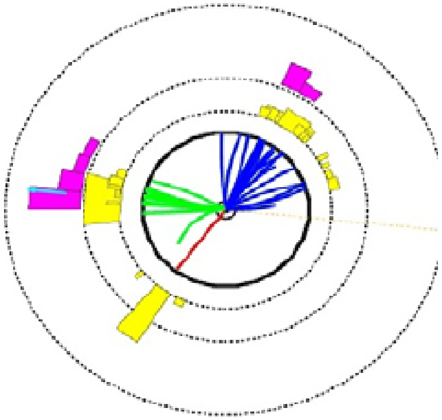
$$Br(W^- \rightarrow \text{hadrons}) \approx 0.67 \qquad Br(W^- \rightarrow e^-\overline{\nu}_e) \approx 0.11$$

$$Br(W^- \rightarrow \mu^-\overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \rightarrow \tau^-\overline{\nu}_\tau) \approx 0.11$$

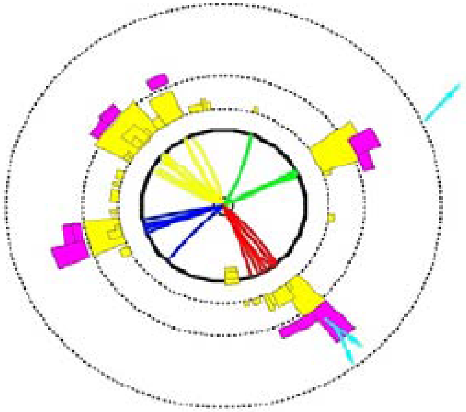
- Gives rise to three distinct topologies



$$\boxed{W^+W^- \rightarrow \ell^+\nu\ell^-\overline{\nu}}$$



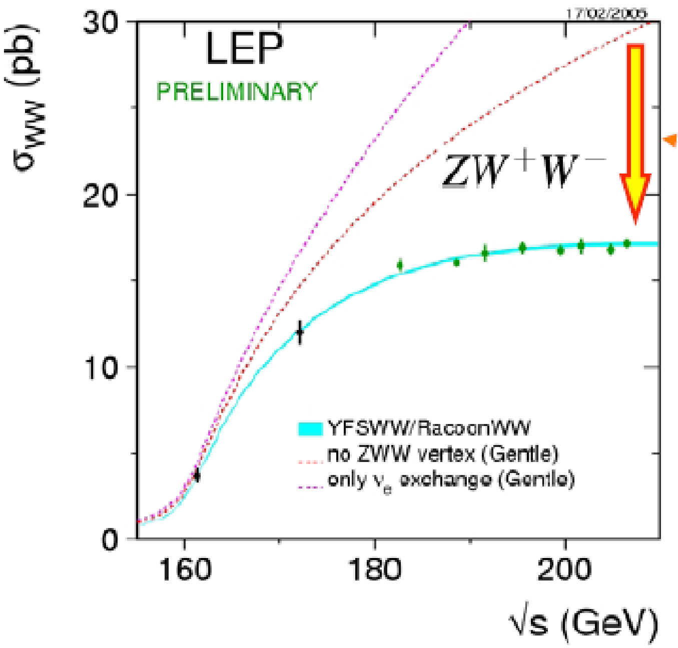
$$\boxed{W^+W^- \rightarrow q\overline{q}\ell\nu}$$



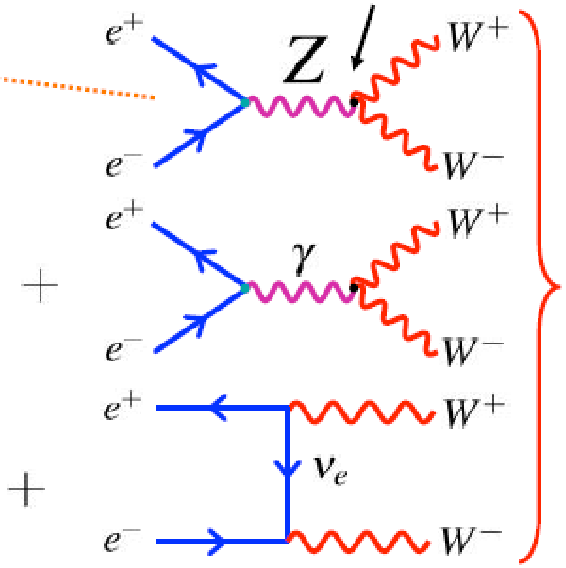
$$\boxed{W^+W^- \rightarrow q\overline{q}q\overline{q}}$$

e+e- -> WW cross-section

- ★ Measure cross sections by **counting events** and normalising to low angle Bhabha scattering events



- ★ Data consistent with SM expectation
- ★ Provides a direct test of ZW^+W^- vertex



- ★ Recall that without the Z diagram the cross section violates unitarity
- ★ Presence of Z fixes this problem

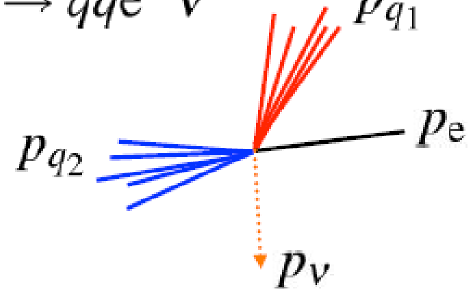
W-mass and W-width

★ Unlike $e^+e^- \rightarrow Z$, the process $e^+e^- \rightarrow W^+W^-$ is not a resonant process

⇨ Different method to measure W-boson Mass

•Measure energy and momenta of particles produced in the W boson decays, e.g.

$$W^+W^- \rightarrow q\bar{q}e^-\overline{\nu}$$



- Neutrino four-momentum from energy-momentum conservation !

$$p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$$

- Reconstruct masses of two W bosons

$$M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$$
$$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$

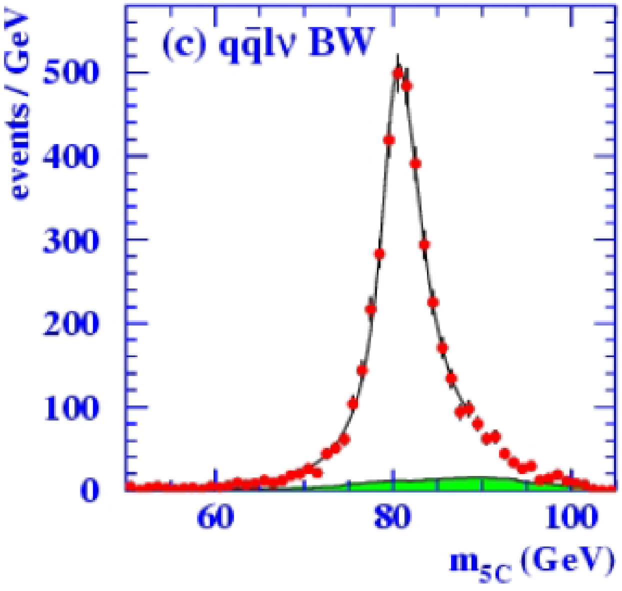
★ Peak of reconstructed mass distribution gives

$$\boxed{m_W = 80.376 \pm 0.033\,\text{GeV}}$$

★ Width of reconstructed mass distribution gives:

$$\boxed{\Gamma_W = 2.196 \pm 0.083\,\text{GeV}}$$

Does not include measurements from Tevatron at Fermilab



$$\boxed{\approx \frac{1}{2}(M_+ + M_-)}$$

The Higgs mechanism

- ★ Propose a scalar (spin 0) field with a non-zero vacuum expectation value (VEV)

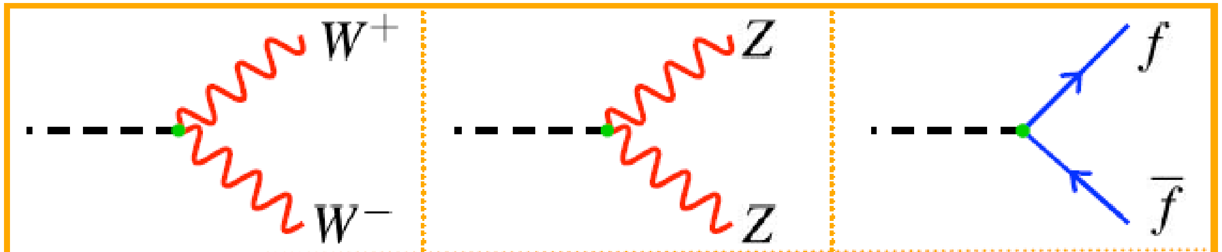
Massless Gauge Bosons propagating through the vacuum with a non-zero Higgs VEV correspond to massive particles.

- ★ The Higgs is electrically neutral but carries weak hypercharge of 1/2
- ★ The photon does not couple to the Higgs field and remains massless
- ★ The W bosons and the Z couple to weak hypercharge and become massive

More abous Higgs mechanism: next week lecture

- The Higgs mechanism results in absolute **predictions** for **masses of gauge bosons**
- In the SM, fermion masses are also ascribed to interactions with the Higgs field
  - however, here no prediction of the masses – **just put in by hand**

Feynman Vertex factors:



$ig_W m_W g^{\mu\nu}$	$ig_Z m_Z g^{\mu\nu}$	$-i\frac{g_W}{2m_W}m_f$

- Within the SM of Electroweak unification with the Higgs mechanism:

Relations between standard model parameters

$$m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W}$$

$$m_Z = \frac{m_W}{\cos\theta_W}$$

- Hence, if you know <u>any three</u> of : $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ predict the other two.

Precision tests of the Standard Model

★ From LEP and elsewhere have precise measurements – can test predictions of the Standard Model !

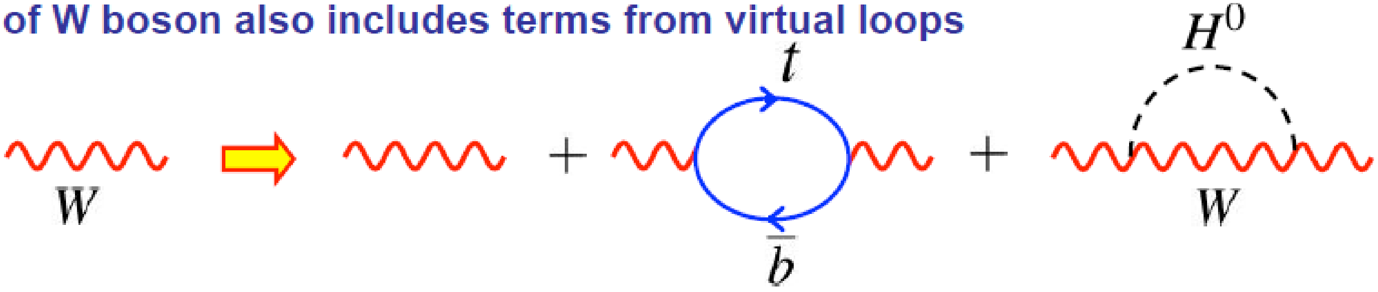
- e.g. predict: $\boxed{m_W = m_Z \cos\theta_W}$ measure $\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}}$ $\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$
- Therefore expect:

$\boxed{m_W = 79.946 \pm 0.008\,\mathrm{GeV}}$ but measure $\boxed{m_W = 80.376 \pm 0.033\,\mathrm{GeV}}$

Year 2011

★ Close, but not quite right – but have only considered lowest order diagrams

★ Mass of W boson also includes terms from virtual loops



$$\boxed{m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)}$$

★ Above "discrepancy" due to these virtual loops, i.e. by making very high precision measurements become sensitive to the masses of particles inside the virtual loops !

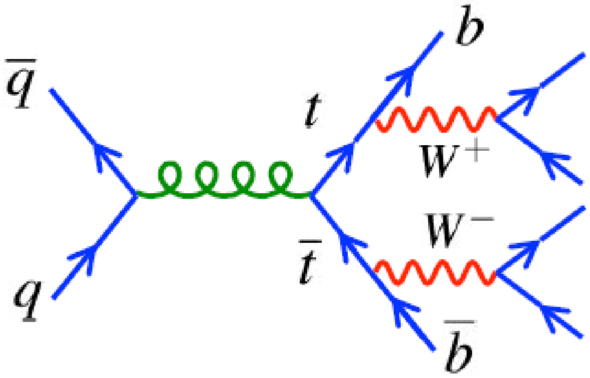
The top quark

- From virtual loop corrections and precise LEP data can predict the top quark mass:

$$\boxed{m_t^{\text{loop}} = 173 \pm 11\,\text{GeV}}$$

- In 1994 top quark observed at the Tevatron proton anti-proton collider at Fermilab

  – with the predicted mass !



- The top quark almost exclusively decays to a bottom quark since

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

- Complicated final state topologies:

$$t\bar{t} \rightarrow b\bar{b}q\bar{q}q\bar{q} \rightarrow 6\text{ jets}$$

$$t\bar{t} \rightarrow b\bar{b}q\bar{q}\ell\nu \rightarrow 4\text{ jets} + \ell + \nu$$

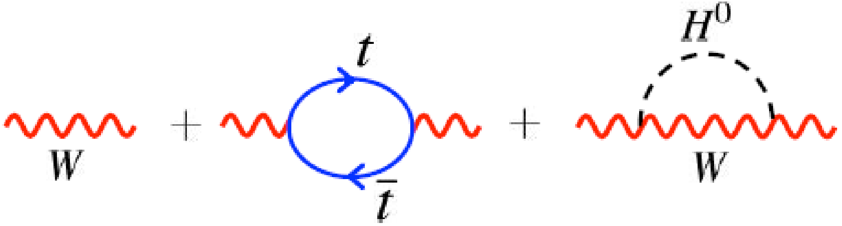
$$t\bar{t} \rightarrow b\bar{b}\ell\nu\ell\nu \rightarrow 2\text{ jets} + 2\ell + 2\nu$$

- Mass determined by direct reconstruction (see W boson mass)

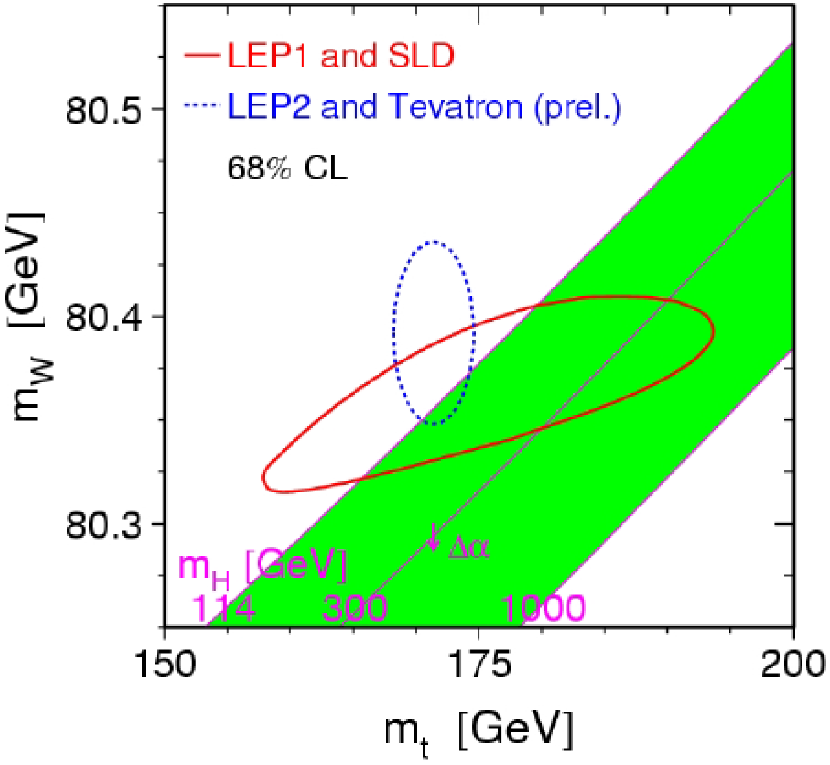
$$\boxed{m_t^{\text{meas}} = 174.2 \pm 3.3\,\text{GeV}}$$

Year 2011

- But the W mass also depends on the Higgs mass (albeit only logarithmically)



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



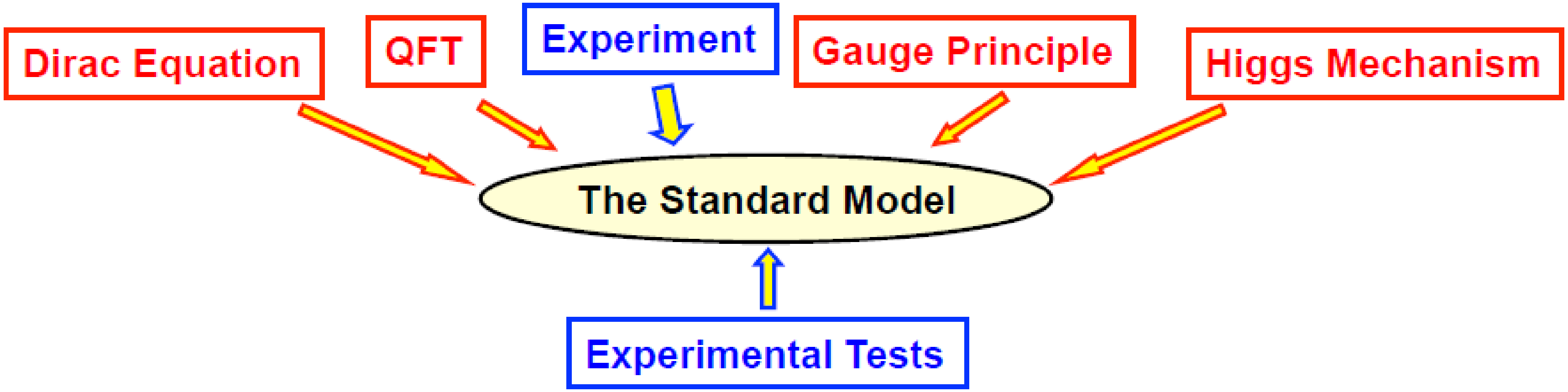
- Measurements are sufficiently precise to have some sensitivity to the Higgs mass
- Direct and indirect values of the top and W mass can be compared to prediction for different Higgs mass
  - **Direct:** W and top masses from direct reconstruction
  - **Indirect:** from SM interpretation of Z mass, θ_W etc. and
- Data favour a light Higgs:

$$\boxed{m_H < 200\,\mathrm{GeV}}$$

Year 2011

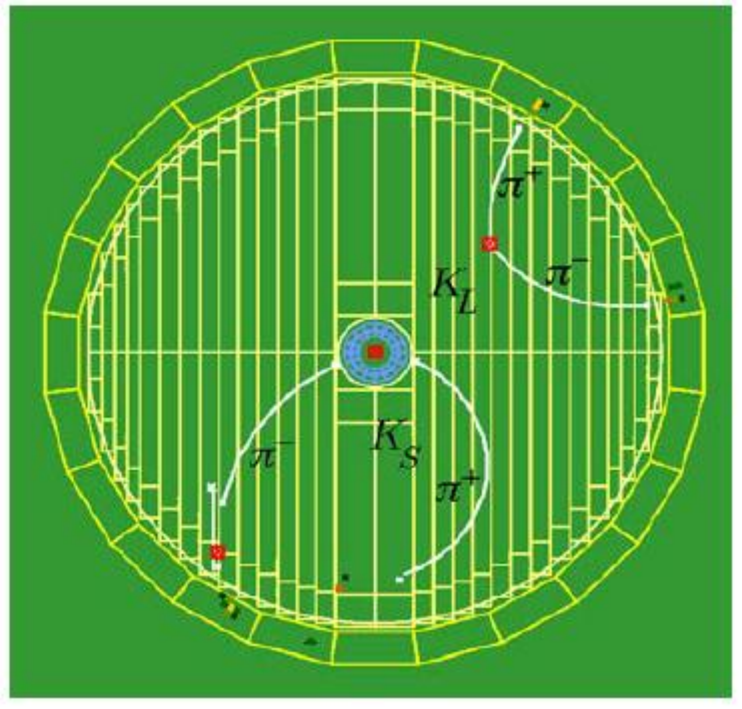
Summary

- ★ The Standard Model of Particle Physics is one of the great scientific triumphs of the late 20th century
- ★ Developed through close interplay of experiment and theory



- ★ Modern experimental particle physics provides many precise measurements. and the **Standard Model** successfully describes all current data !
- ★ Despite its great success, we should not forget that it is just a model; a collection of beautiful theoretical ideas cobbled together to fit with experimental data.
- ★ There are many issues / open questions…

The CKM matrix and CP violation



CP violation in the Early Universe

- Very early in the universe might expect equal numbers of baryons and anti-baryons
- However, today the universe is matter dominated (no evidence for anti-galaxies, etc.)
- From "Big Bang Nucleosynthesis" obtain the matter/anti-matter asymmetry

$$\xi = \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every baryon in the universe today there are 10^9 photons

- How did this happen?

★ Early in the universe need to create a very small asymmetry between baryons and anti-baryons

e.g. for every 10^9 anti-baryons there were 10^9+1 baryons

baryons/anti-baryons annihilate ⇨

1 baryon + ~10^9 photons + no anti-baryons

★ To generate this initial asymmetry three conditions must be met (Sakharov, 1967):

❶ "Baryon number violation", i.e. $n_B - n_{\overline{B}}$ is not constant

❷ "C and CP violation", if CP is conserved for a reaction which generates a net number of baryons over anti-baryons there would be a CP conjugate reaction generating a net number of anti-baryons

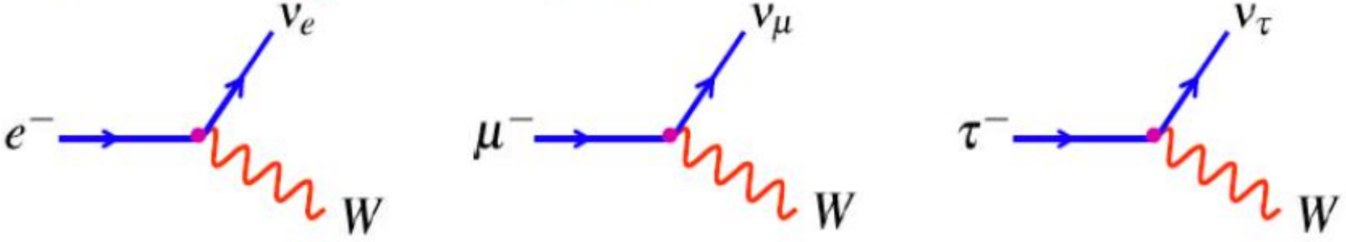
❸ "Departure from thermal equilibrium", in thermal equilibrium any baryon number violating process will be balanced by the inverse reaction

- CP Violation is an essential aspect of our understanding of the universe
- A natural question is whether the SM of particle physics can provide the necessary CP violation?
- There are two places in the SM where CP violation enters: the PMNS matrix and the CKM matrix
- To date CP violation has been observed only in the quark sector
- Because we are dealing with quarks, which are only observed as bound states, this is a fairly complicated subject. Here we will approach it in two steps:
  - i) Consider particle – anti-particle oscillations without CP violation
  - ii) Then discuss the effects of CP violation

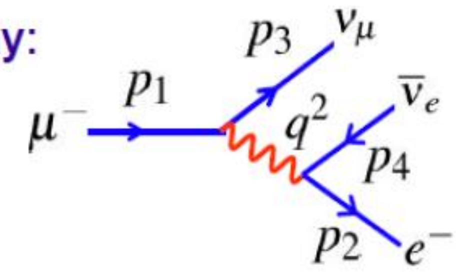
★ Many features in common with neutrino oscillations – except that we will be considering the oscillations of decaying particles (i.e. mesons) !

Muon decay and lepton universality

★The leptonic charged current (W±) interaction vertices are:



★Consider muon decay:



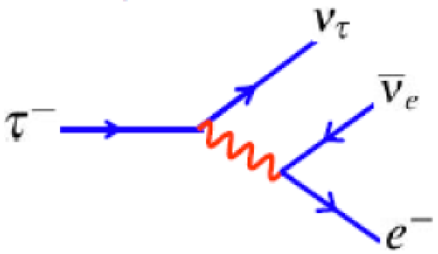
- It is straight-forward to write down the matrix element

$$M_{fi}=\frac{g_W^{(e)}g_W^{(\mu)}}{8m_W^2}[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)]g_{\mu\nu}[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)]$$

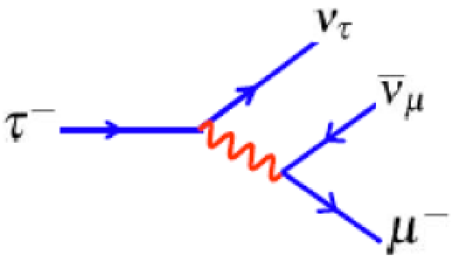
Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$
i.e. in limit of Fermi theory

- Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

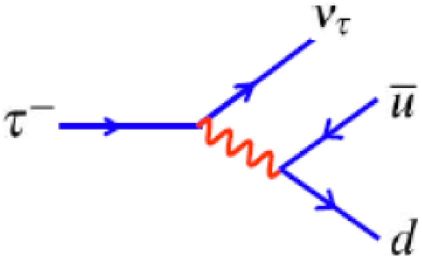
- The muon to electron rate $$\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu}$$ with $G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}$
- Similarly for tau to electron $$\Gamma(\tau \to e\nu\nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$$
- However, the tau can decay to a number of final states:



$Br(\tau \to e\nu\nu) = 0.1784(5)$



$Br(\tau \to \mu\nu\nu) = 0.1736(5)$



- Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

- Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

- Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$$

- All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94)\,\text{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\text{s}$$

$$m_\tau = 1.77699(28)\,\text{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\text{s}$$

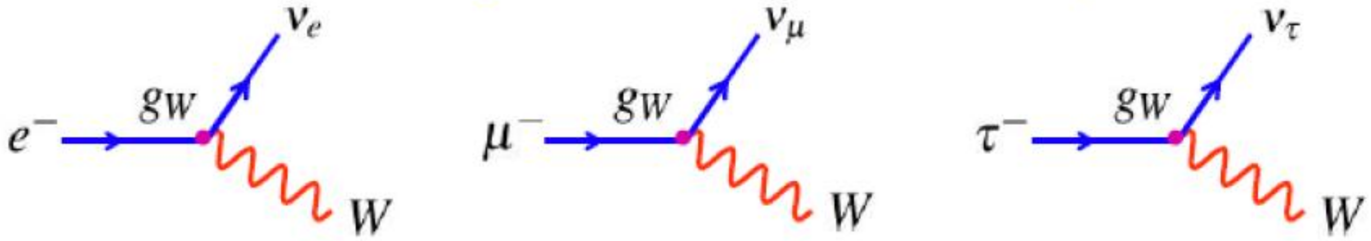
$$Br(\tau\to e\nu\nu) = 0.1784(5)$$

$$\Rightarrow \boxed{\frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5\tau_\mu}{m_\tau^5\tau_\tau}Br(\tau\to e\nu\nu) = 1.0024\pm 0.0033}$$

- Similarly by comparing $Br(\tau\to e\nu\nu)$ and $Br(\tau\to \mu\nu\nu)$

$$\boxed{\frac{G_F^e}{G_F^\mu} = 1.000\pm 0.004}$$

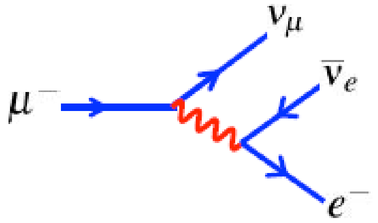
★ Demonstrates the weak charged current is the same for all leptonic vertices

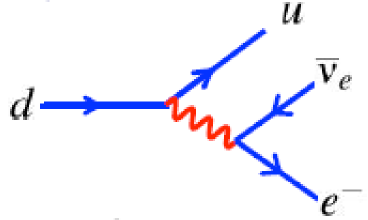


→ **Charged Current Lepton Universality**

The weak interaction of quarks

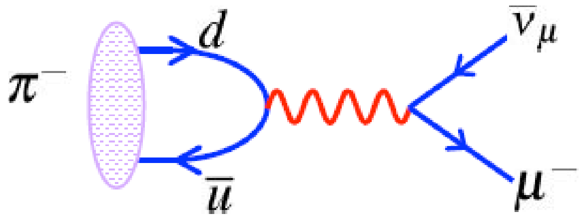
- ★ Slightly different values of G_F measured in μ decay and nuclear β decay:

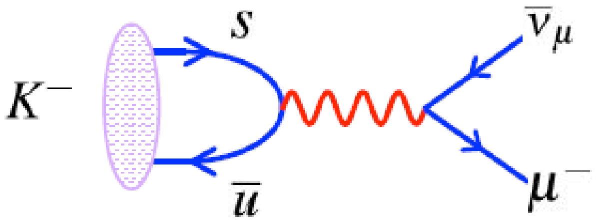




$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

- ★ In addition, certain hadronic decay modes are observed to be suppressed, e.g. compare $K^- \to \mu^- \overline{\nu}_\mu$ and $\pi^- \to \mu^- \overline{\nu}_\mu$. Kaon decay rate suppressed factor 20 compared to the expectation assuming a universal weak interaction for quarks.



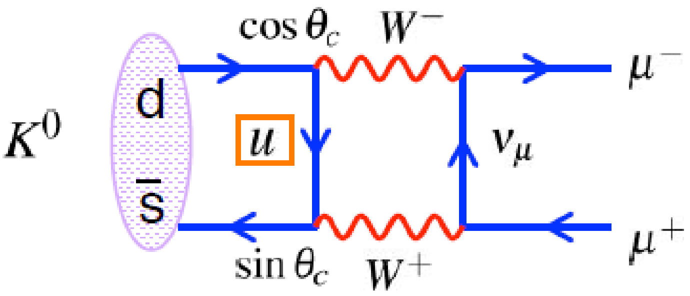


- Both observations explained by Cabibbo hypothesis (1963): weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

GIM mechanism

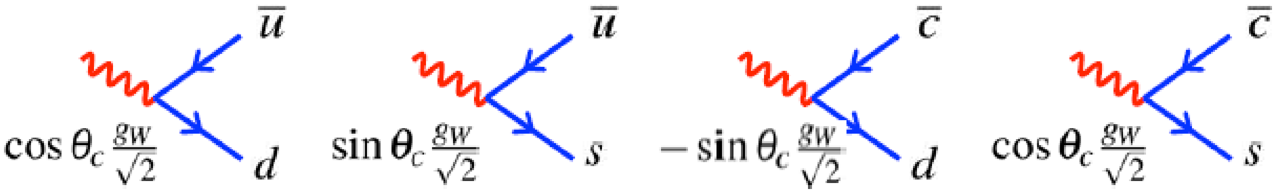
- ★ In the weak interaction have couplings between both ud and us which implies that neutral mesons can decay via box diagrams, e.g.



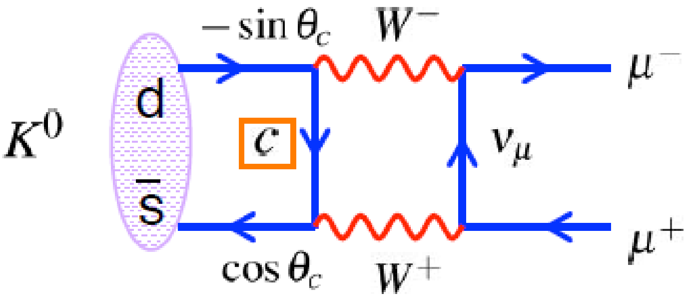
$$\boxed{M_1 \propto g_W^4 \cos\theta_c \sin\theta_c}$$

- Historically, the observed branching was **much smaller** than predicted

- ★ Led **Glashow, Illiopoulos** and **Maiani** to postulate existence of an extra quark - **before** discovery of charm quark in **1974**. Weak interaction couplings become



- ★ Gives another box diagram for $K^0 \rightarrow \mu^+\mu^-$



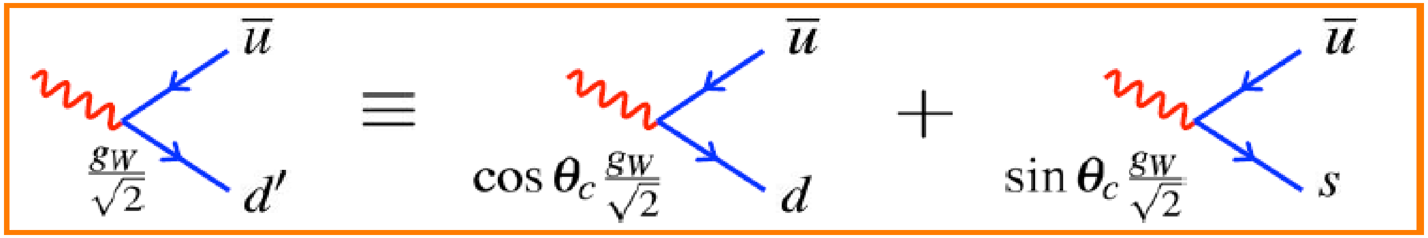
$$\boxed{M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c}$$

- Same final state so sum amplitudes

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

- Cancellation not exact because $m_u \neq m_c$

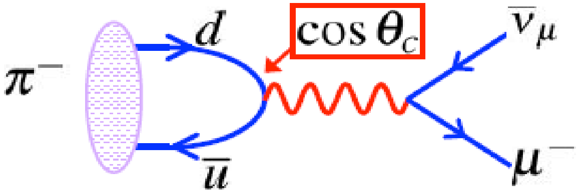
i.e. **weak interaction couples different generations of quarks**

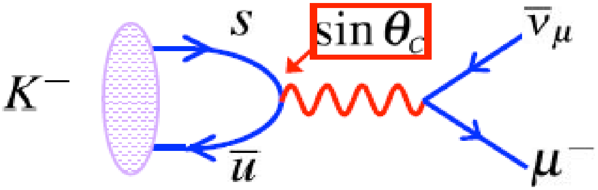


(The same is true for leptons e.g. $e^-\nu_1$, $e^-\nu_2$, $e^-\nu_3$ couplings – connect different generations)

★ Can explain the observations on the previous pages with $\theta_c = 13.1^\circ$

- Kaon decay suppressed by a factor of $\tan^2\theta_c \approx 0.05$ relative to pion decay

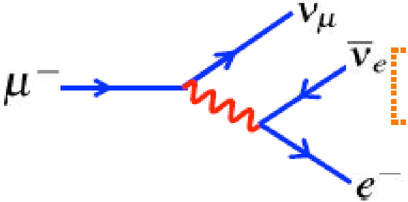




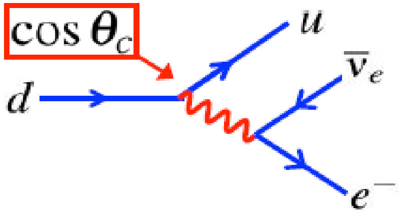
$$\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c$$

$$\Gamma(K^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$$

- Hence expect $G_F^\beta = G_F^\mu \cos\theta_c$



$\propto G_F.G_F$



$\propto G_F.(G_F\cos^2\theta_c)$

CKM matrix

- ★ Extend ideas to three quark flavours (analogue of three flavour neutrino treatment)

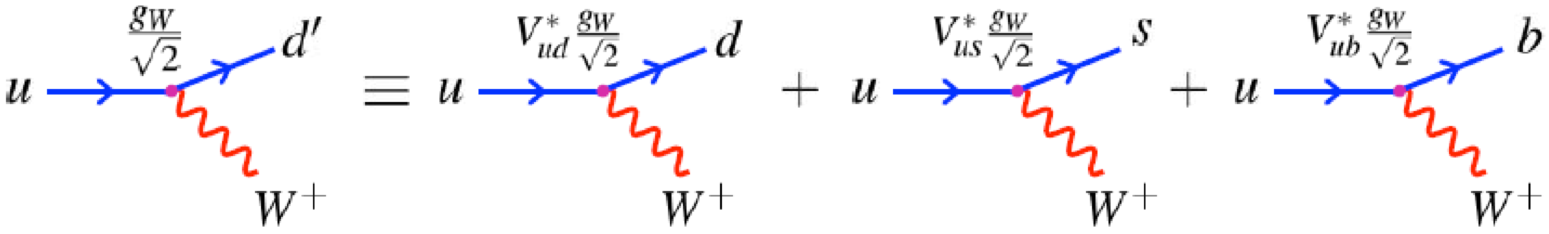
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$

Weak eigenstates **CKM Matrix** **Mass Eigenstates**

(**Cabibbo, Kobayashi, Maskawa**)

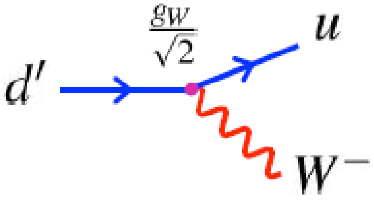
- ★ e.g. Weak eigenstate d' is produced in weak decay of an up quark:



- The CKM matrix elements V_{ij} are complex constants
- The CKM matrix is unitary
- The V_{ij} are not predicted by the SM – have to determined from experiment

Feynman rules

- Depending on the order of the interaction, $u \to d$ or $d \to u$, the CKM matrix enters as either V_{ud} or V_{ud}^*
- Writing the interaction in terms of the WEAK eigenstates



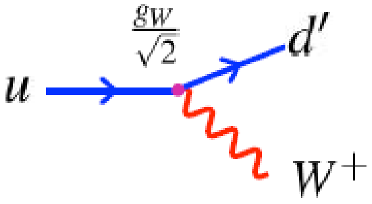
$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] d'$$

NOTE: $\bar{u}$ is the adjoint spinor not the anti-up quark

- Giving the $\boxed{d \to u}$ weak current:

$$\boxed{j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] V_{ud} d}$$

- For $u \to d'$ the weak current is:



$$j_{ud'} = \bar{d}'\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u$$

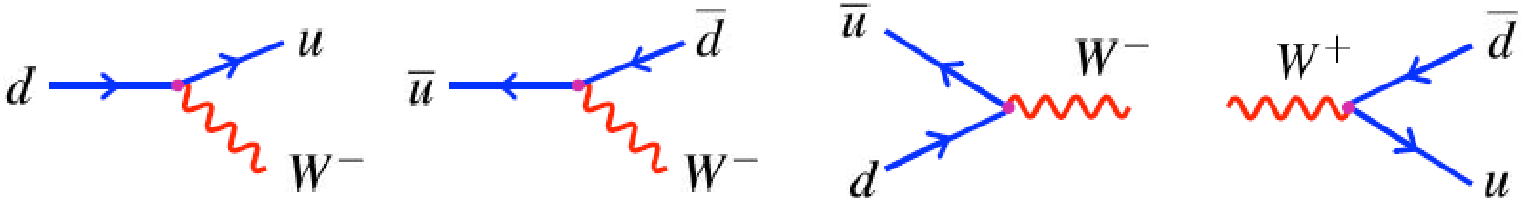
- In terms of the mass eigenstates $\bar{d}' = d'^\dagger\gamma^0 \to (V_{ud}d)^\dagger\gamma^0 = V_{ud}^* d^\dagger\gamma^0 = V_{ud}^*\bar{d}$

- Giving the $\boxed{u \to d}$ weak current:

$$\boxed{j_{ud} = \bar{d}V_{ud}^*\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u}$$

- Hence, when the charge $-\frac{1}{3}$ quark enters as the adjoint spinor, the complex conjugate of the CKM matrix is used

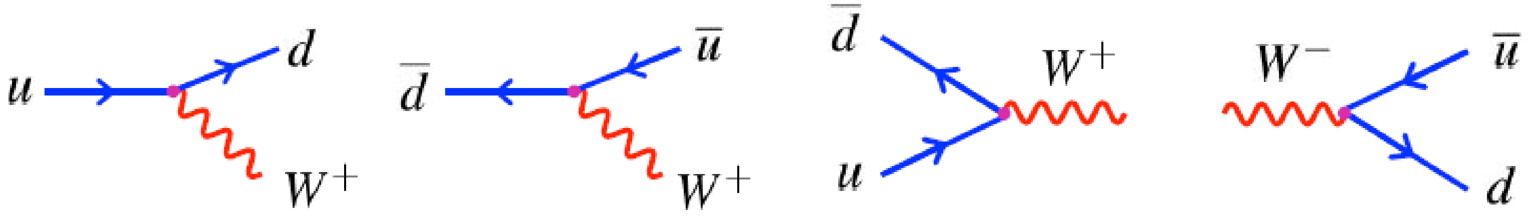
★ The vertex factor the following diagrams:



is

$$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^\mu\,\tfrac{1}{2}(1-\gamma^5)}$$

★ Whereas, the vertex factor for:



is

$$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}^*\gamma^\mu\,\tfrac{1}{2}(1-\gamma^5)}$$

- ★ Experimentally determine

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

- ★ Currently little direct experimental information on V_{td}, V_{ts}, V_{tb}
- ★ Assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$ gives:

Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

Near diagonal – very different from PMNS

- ★ **NOTE: within the SM,** the charged current, $W^{\pm}$, weak interaction:
  ① Provides the only way to change flavour !
  ② only way to change from one generation of quarks or leptons to another !

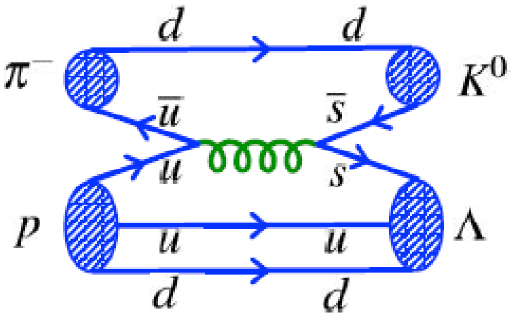
- ★ **However,** the off-diagonal elements of the CKM matrix are relatively small.
  - Weak interaction largest between quarks of the same generation.
  - Coupling between first and third generation quarks is very small !
- ★ **Just as for the PMNS matrix – the CKM matrix allows CP violation in the SM**

The neutral Kaon system

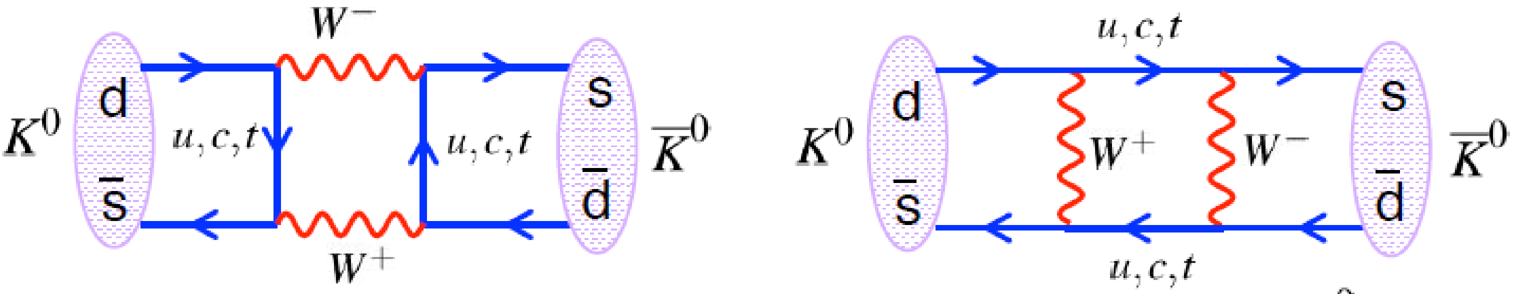
- Neutral Kaons are produced copiously in strong interactions, e.g.

$$\pi^-(d\bar{u}) + p(uud) \rightarrow \Lambda(uds) + K^0(d\bar{s})$$

$$\pi^+(u\bar{d}) + p(uud) \rightarrow K^+(u\bar{s}) + \overline{K}^0(s\bar{d}) + p(uud)$$



- Neutral Kaons decay via the weak interaction
- The Weak Interaction also allows mixing of neutral kaons via “box diagrams”



- This allows transitions between the strong eigenstates states $K^0, \overline{K}^0$
- Consequently, the neutral kaons propagate as eigenstates of the overall strong + weak interaction ; i.e. as linear combinations of $K^0, \overline{K}^0$

- These neutral kaon states are called the “K-short” K_S and the “K-long” K_L

- These states have approximately the same mass $m(K_S) \approx m(K_L) \approx 498\,\text{MeV}$

- But very different lifetimes: $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\text{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\text{s}}$

CP eigenstates

★The K_S and K_L are closely related to eigenstates of the combined charge conjugation and parity operators: CP

- The strong eigenstates $K^0(d\bar{s})$ and $\overline{K}^0(s\bar{d})$ have $J^P = 0^-$

with $$\hat{P}|K^0\rangle = -|K^0\rangle, \quad \hat{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

- The charge conjugation operator changes particle into anti-particle and *vice versa*

$$\hat{C}|K^0\rangle = \hat{C}|d\bar{s}\rangle = +|s\bar{d}\rangle = |\overline{K}^0\rangle$$

similarly $$\hat{C}|\overline{K}^0\rangle = |K^0\rangle$$

The + sign is purely conventional, could have used a - with no physical consequences

- Consequently

$$\hat{C}\hat{P}|K^0\rangle = -|\overline{K}^0\rangle \qquad \hat{C}\hat{P}|\overline{K}^0\rangle = -|K^0\rangle$$

i.e. neither K^0 or $\overline{K}^0$ are eigenstates of CP

- Form CP eigenstates from linear combinations:

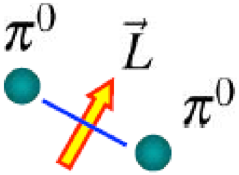
$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

$$\hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$$
$$\hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$$

Decays of CP eigenstates

- Neutral kaons often decay to pions (the lightest hadrons)
- The kaon masses are approximately **498 MeV** and the pion masses are approximately **140 MeV**. Hence neutral kaons can decay to either 2 or 3 pions

Decays to Two Pions:



☆ $K^0 \rightarrow \pi^0\pi^0 \qquad J^P: \quad 0^- \rightarrow 0^- + 0^-$

- Conservation of angular momentum → $\vec{L} = 0$

$$\Rightarrow \quad \hat{P}(\pi^0\pi^0) = -1.-1.(-1)^L = +1$$

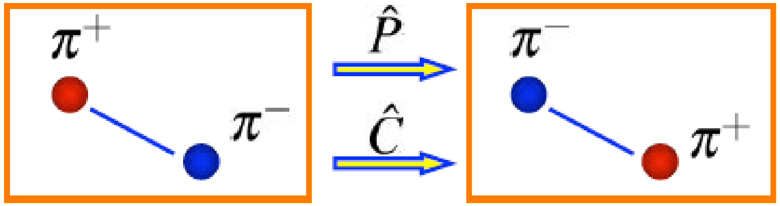
- The $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ is an eigenstate of $\hat{C}$

$$C(\pi^0\pi^0) = C\pi^0.C\pi^0 = +1.+1 = +1$$

$$\Rightarrow \quad \boxed{CP(\pi^0\pi^0) = +1}$$

☆ $K^0 \rightarrow \pi^+\pi^-$ as before $\hat{P}(\pi^+\pi^-) = +1$

★ Here the **C** and **P** operations have the identical effect



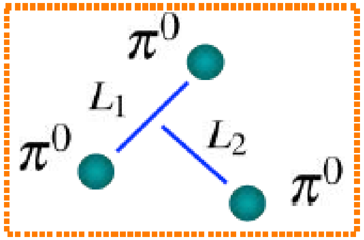
Hence the combined effect of $\hat{C}\hat{P}$ is to leave the system unchanged

$$\boxed{\hat{C}\hat{P}(\pi^+\pi^-) = +1}$$

Neutral kaon decays to two pions occur in CP even (i.e. +1) eigenstates

Decays to Three Pions:

- $K^0 \to \pi^0\pi^0\pi^0$ $J^P: \quad 0^- \to 0^- + 0^- + 0^-$



Remember L is magnitude of angular momentum vector

- Conservation of angular momentum:

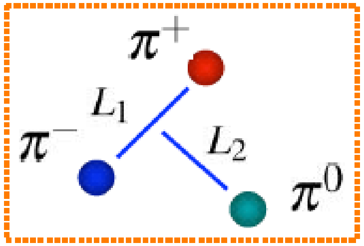
$$L_1 \oplus L_2 = 0 \quad \Rightarrow \quad L_1 = L_2$$

$$P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0\pi^0\pi^0) = +1.+1.+1$$

$$\Rightarrow \quad \boxed{CP(\pi^0\pi^0\pi^0) = -1}$$

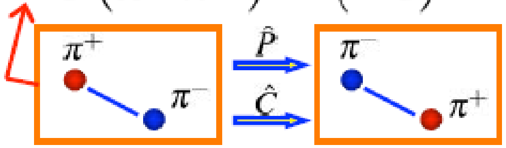
- $K^0 \to \pi^+\pi^-\pi^0$



- Again $L_1 = L_2$

$$P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^{L_1}$$



Hence: $\boxed{CP(\pi^+\pi^-\pi^0) = -1.(-1)^{L_1}}$

- The small amount of energy available in the decay, $m(K) - 3m(\pi) \approx 70\,\mathrm{MeV}$ means that the L>0 decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)

Neutral kaon decays to three pions occur in CP odd (i.e. -1) eigenstates

★ If CP were conserved in the Weak decays of neutral kaons, would expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1 , K_2)

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \qquad K_1 \rightarrow \pi\pi$$

CP EVEN

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \qquad K_2 \rightarrow \pi\pi\pi$$

CP ODD

★ Expect lifetimes of CP eigenstates to be very different

- For two pion decay energy available: $m_K - 2m_\pi \approx 220\,\mathrm{MeV}$
- For three pion decay energy available: $m_K - 3m_\pi \approx 80\,\mathrm{MeV}$

★ Expect decays to two pions to be more rapid than decays to three pions due to increased phase space

★ This is exactly what is observed: a short-lived state "K-short" which decays to (mainly) to two pions and a long-lived state "K-long" which decays to three pions

★ In the absence of CP violation we can identify

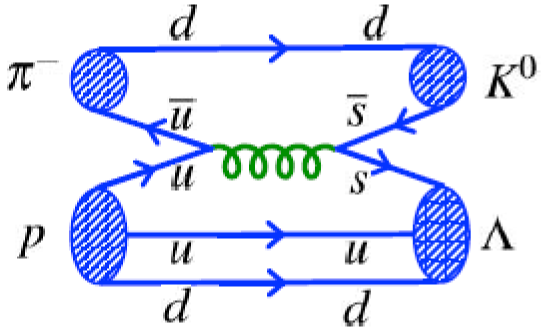
$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

with decays: $K_S \rightarrow \pi\pi$

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

with decays: $K_L \rightarrow \pi\pi\pi$

Neutral Kaon decays to pions



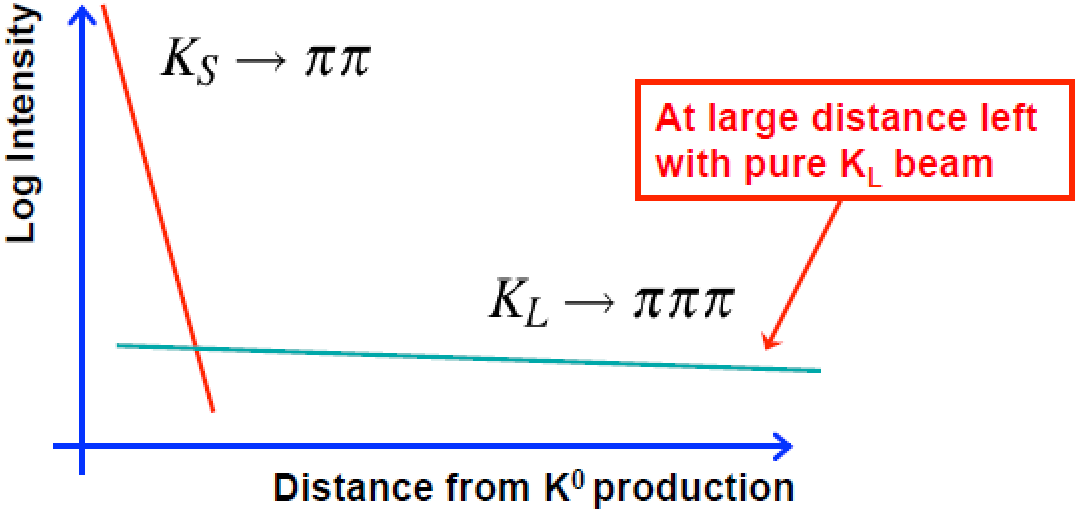
- Consider the decays of a beam of K^0
- The decays to pions occur in states of definite CP
- If CP is conserved in the decay, need to express K^0 in terms of K_S and K_L

$$|K_0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

- Hence from the point of view of decays to pions, a K^0 beam is a linear combination of CP eigenstates:

  a rapidly decaying CP-even component and a long-lived CP-odd component

- Therefore, expect to see predominantly two-pion decays near start of beam and predominantly three pion decays further downstream



At large distance left with pure K_L beam

★To see how this works algebraically:

- Suppose at time t=0 make a beam of pure K^0

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

- Put in the time dependence of wave-function

$$|K_S(t)\rangle = |K_S\rangle e^{-im_St-\Gamma_St/2}$$

K$_S$ mass: m_S
K$_S$ decay rate: $\Gamma_S = 1/\tau_S$

NOTE the term $e^{-\Gamma_St/2}$ ensures the K$_S$ probability density decays exponentially

i.e. $|\psi_S|^2 = \langle K_S(t)|K_S(t)\rangle = e^{-\Gamma_St} = e^{-t/\tau_S}$

- Hence wave-function evolves as

$$|\psi(t)\rangle \quad = \quad \frac{1}{\sqrt{2}}\left[|K_S\rangle e^{-(im_S+\frac{\Gamma_S}{2})t} + |K_L\rangle e^{-(im_L+\frac{\Gamma_L}{2})t}\right]$$

- Writing $\theta_S(t) = e^{-(im_S+\frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L+\frac{\Gamma_L}{2})t}$

$$|\psi(t)\rangle \quad = \quad \frac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

- **The decay rate to two pions** for a state which was produced as K^0:

$$\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_S|\psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_St} = e^{-t/\tau_S}$$

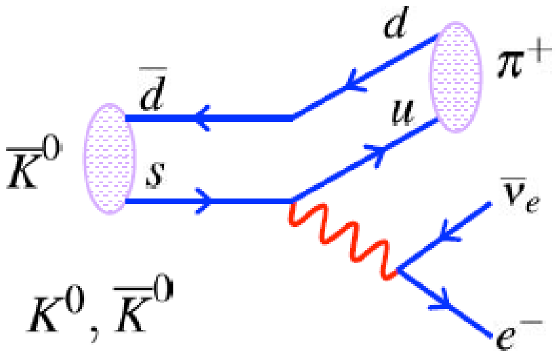
which is as anticipated, i.e. decays of the short lifetime component **K$_S$**

Neutral Kaon decays to leptons

- Neutral kaons can also decay to leptons

$$\overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}_e \qquad \overline{K}^0 \rightarrow \pi^+ \mu^- \overline{\nu}_\mu$$

$$K^0 \rightarrow \pi^- e^+ \nu_e \qquad K^0 \rightarrow \pi^- \mu^+ \nu_\mu$$



- **Note:** the final states are not CP eigenstates which is why we express these decays in terms of $K^0, \overline{K}^0$
- Neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L . The main decay modes/branching fractions are:

K_S	$\rightarrow$	$\pi^+\pi^-$	$BR = 69.2\%$
	$\rightarrow$	$\pi^0\pi^0$	$BR = 30.7\%$
	$\rightarrow$	$\pi^- e^+ \nu_e$	$BR = 0.03\%$
	$\rightarrow$	$\pi^+ e^- \overline{\nu}_e$	$BR = 0.03\%$
	$\rightarrow$	$\pi^- \mu^+ \nu_\mu$	$BR = 0.02\%$
	$\rightarrow$	$\pi^+ \mu^- \overline{\nu}_\mu$	$BR = 0.02\%$

K_L	$\rightarrow$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$
	$\rightarrow$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$
	$\rightarrow$	$\pi^- e^+ \nu_e$	$BR = 20.2\%$
	$\rightarrow$	$\pi^+ e^- \overline{\nu}_e$	$BR = 20.2\%$
	$\rightarrow$	$\pi^- \mu^+ \nu_\mu$	$BR = 13.5\%$
	$\rightarrow$	$\pi^+ \mu^- \overline{\nu}_\mu$	$BR = 13.5\%$

- Leptonic decays are more likely for the K-long because the three pion decay modes have a lower decay rate than the two pion modes of the K-short

Strangeness Oscillations (neglecting CP violation)

- The "semi-leptonic" decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state. Hence to calculate the expected decay rate, need to know the K^0 component of the wave-function. For example, for a beam which was initially K^0 we have (1)

$$|\psi(t)\rangle \quad = \quad \tfrac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

- Writing K_S, K_L in terms of $K^0, \overline{K}^0$

$$\begin{aligned}|\psi(t)\rangle \quad &= \quad \tfrac{1}{2}\left[\theta_S(t)(|K^0\rangle - |\overline{K}^0\rangle) + \theta_L(t)(|K^0\rangle + |\overline{K}^0\rangle)\right] \\ &= \quad \tfrac{1}{2}(\theta_S + \theta_L)|K^0\rangle + \tfrac{1}{2}(\theta_L - \theta_S)|\overline{K}^0\rangle\end{aligned}$$

- Because $\theta_S(t) \neq \theta_L(t)$ a state that was initially a K^0 evolves with time into a mixture of K^0 and $\overline{K}^0$ - "strangeness oscillations"

- The K^0 intensity (i.e. K^0 fraction):

$$\Gamma(K^0_{t=0} \rightarrow K^0) = |\langle K^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S + \theta_L|^2 \qquad (2)$$

- Similarly

$$\Gamma(K^0_{t=0} \rightarrow \overline{K}^0) = |\langle \overline{K}^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S - \theta_L|^2 \qquad (3)$$

- Using the identity $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$

$$\begin{aligned}
|\theta_S \pm \theta_L|^2 &= |e^{-(im_S+\frac{1}{2}\Gamma_S)t} \pm e^{-(im_L+\frac{1}{2}\Gamma_L)t}|^2 \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\Re\{e^{-im_S t} e^{-\frac{1}{2}\Gamma_S t} . e^{+im_L t} e^{-\frac{1}{2}\Gamma_L t}\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t} \Re\{e^{-i(m_S-m_L)t}\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t} \cos(m_S - m_L)t \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t} \cos\Delta m t
\end{aligned}$$

- Oscillations between neutral kaon states with frequency given by the mass splitting $\Delta m = m(K_L) - m(K_S)$
- Reminiscent of neutrino oscillations ! Only this time we have decaying states.
- Using equations (2) and (3):

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2} \cos\Delta m t\right] \qquad (4)$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2} \cos\Delta m t\right] \qquad (5)$$

- Experimentally we find: $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\mathrm{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\mathrm{s}}$

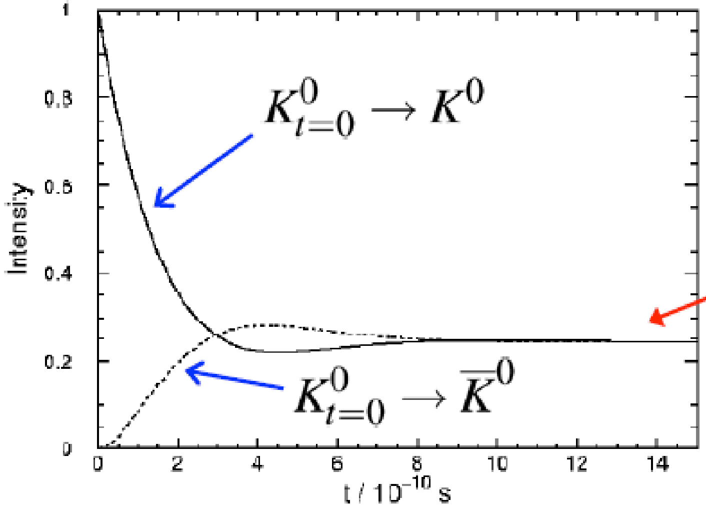
  and $\boxed{\Delta m = (3.506 \pm 0.006) \times 10^{-15}\,\mathrm{GeV}}$

  i.e. the K-long mass is greater than the K-short by 1 part in 10^{16}

- The mass difference corresponds to an oscillation period of

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\,\mathrm{s}$$

- The oscillation period is relatively long compared to the K_S lifetime and consequently, do not observe very pronounced oscillations

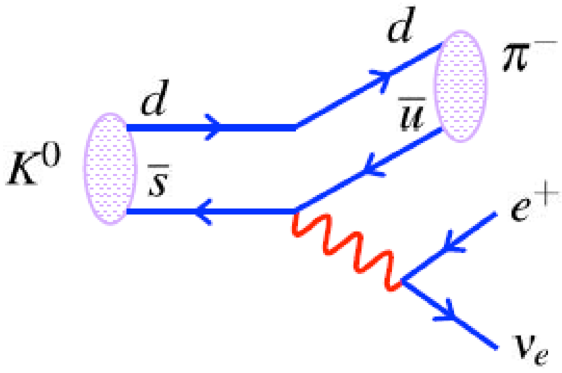


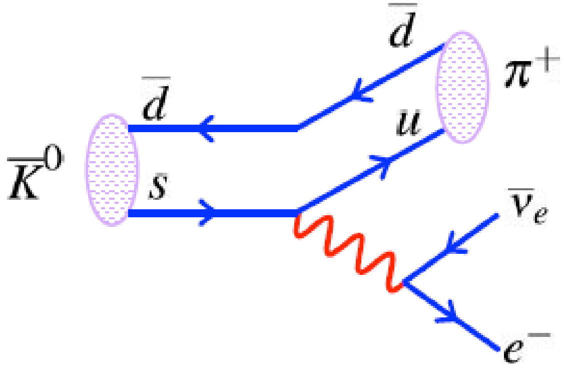
$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

- Strangeness oscillations can be studied by looking at semi-leptonic decays





- The charge of the observed pion (or lepton) tags the decay as from either a $\overline{K}^0$ or K^0 because

$$K^0 \to \pi^- e^+ \nu_e$$
$$\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$$

but

$$\left.\begin{array}{l}\overline{K}^0 \not\to \pi^- e^+ \nu_e \\ K^0 \not\to \pi^+ e^- \overline{\nu}_e\end{array}\right\}$$ **NOT ALLOWED**

- So for an initial K^0 beam, observe the decays to both charge combinations:

$$K^0_{t=0} \to K^0 \qquad\qquad K^0_{t=0} \to \overline{K}^0$$
$$\hookrightarrow \pi^- e^+ \nu_e \qquad\qquad \hookrightarrow \pi^+ e^- \overline{\nu}_e$$

which provides a way of measuring strangeness oscillations

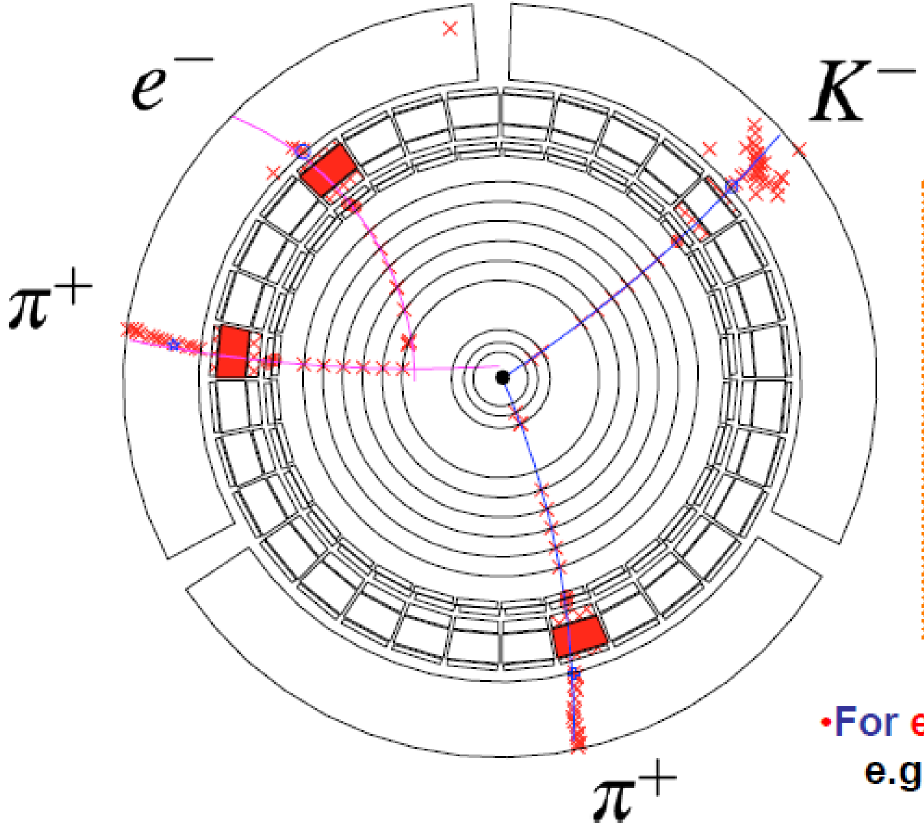
The CPLEAR experiement

- CERN : 1990-1996
- Used a low energy anti-proton beam
- Neutral kaons produced in reactions

$$\overline{p}p \rightarrow K^-\pi^+K^0$$
$$\overline{p}p \rightarrow K^+\pi^-\overline{K}^0$$

- Low energy, so particles produced almost at rest
- Observe production process and decay in the same detector
- Charge of $K^{\pm}\pi^{\mp}$ in the production process tags the initial neutral kaon as either K^0 or $\overline{K}^0$
- Charge of decay products tags the decay as either as being either K^0 or $\overline{K}^0$
- Provides a direct probe of strangeness oscillations

An example of a CPLEAR event



$$\boxed{\begin{array}{c} K^-(s\bar{u}) \\ K^0(d\bar{s}) \\ \overline{K}^0(s\bar{d}) \end{array}}$$

Production:

$$\bar{p}p \rightarrow \boxed{K^-\pi^+}K^0$$

Mixing

Decay:

$$\overline{K}^0 \rightarrow \boxed{\pi^+e^-}\bar{\nu}_e$$

- For each event know initial wave-function, e.g. here: $|\psi(t=0)\rangle = |K^0\rangle$

- Can measure decay rates as a function of time for all combinations:

e.g. $R^{+} = \Gamma(K^0_{t=0} \to \pi^- e^+ \overline{\nu}_e) \propto \Gamma(K^0_{t=0} \to K^0)$

- From equations (4), (5) and similar relations:

$$R_+ \equiv \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu}\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$R_- \equiv \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) = N_{\pi e \nu}\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\overline{R}_- \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) = N_{\pi e \nu}\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\overline{R}_+ \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu}\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

where $N_{\pi e \nu}$ is some overall normalisation factor

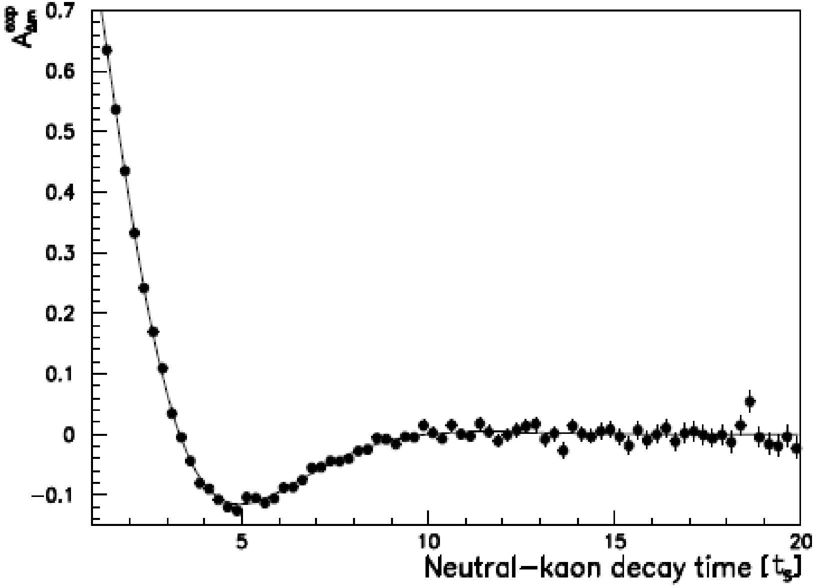
- Express measurements as an “asymmetry” to remove dependence on $N_{\pi e \nu}$

$$A_{\Delta m} = \frac{(R_+ + \overline{R}_-) - (R_- + \overline{R}_+)}{(R_+ + \overline{R}_-) + (R_- + \overline{R}_+)}$$

- Using the above expressions for R_+ etc., obtain

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta mt}{e^{-\Gamma_S t}+e^{-\Gamma_L t}}$$

A. Angelopoulos et al., Eur. Phys. J. C22 (2001) 55



- ★ Points show the data
- ★ The line shows the theoretical prediction for the value of Δm most consistent with the **CPLEAR** data:

$$\boxed{\Delta m = 3.485\times 10^{-15}\,\mathrm{GeV}}$$

- The sign of Δm is not determined here but is known from other experiments
- When the CPLEAR results are combined with experiments at FermiLab obtain:

$$\boxed{\Delta m = m(K_L)-m(K_S) = (3.506\pm 0.006)\times 10^{-15}\,\mathrm{GeV}}$$

CP violation in the Kaon system

- ★ So far we have ignored CP violation in the neutral kaon system
- ★ Identified the K-short as the CP-even state and the K-long as the CP-odd state

$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$ with decays: $K_S \rightarrow \pi\pi$ $\boxed{\text{CP = +1}}$

$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$ with decays: $K_L \rightarrow \pi\pi\pi$ $\boxed{\text{CP = -1}}$

- ★ At a long distance from the production point a beam of neutral kaons will be 100% K-long (the K-short component will have decayed away). Hence, if CP is conserved, would expect to see only three-pion decays.
- ★ In 1964 Fitch & Cronin (joint Nobel prize) observed 45 $K_L \rightarrow \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point

⇨ **Weak interactions violate CP**

- CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000

K$_L$ to pion BRs:

K_L	$\rightarrow$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$	$CP = -1$
	$\rightarrow$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$	$CP = -1$
	$\rightarrow$	$\pi^+\pi^-$	$BR = 0.20\%$	$CP = +1$
	$\rightarrow$	$\pi^0\pi^0$	$BR = 0.08\%$	$CP = +1$

★ Two possible explanations of CP violation in the kaon system:

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$\boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right]} \quad \boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]}$$

with $|\varepsilon| \sim 2\times 10^{-3}$

- In this case the observation of $K_L \to \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

$$\begin{tikzcd} |K_2\rangle \arrow[d] & \varepsilon|K_1\rangle \arrow[d] \\ \pi\pi\pi \ \boxed{\text{CP = -1}} & \pi\pi \ \boxed{\text{CP = +1}} \end{tikzcd}$$

ii) and/or CP is violated in the decay

$$|K_L\rangle = |K_2\rangle \quad \boxed{\text{CP = -1}}$$

$$\begin{tikzcd} |K_2\rangle \arrow[r] \arrow[dr] & \pi\pi\pi \ \boxed{\text{CP = -1}} \\ & \pi\pi \ \boxed{\text{CP = +1}} \end{tikzcd}$$

Parameterised by ε'

★ Experimentally both known to contribute to the mechanism for CP violation in the kaon system but i) dominates: $\varepsilon'/\varepsilon = (1.7 \pm 0.3)\times 10^{-3}$ $\left\{ \begin{array}{l} \text{NA48 (CERN)} \\ \text{KTeV (FermiLab)} \end{array} \right.$

CP violation in semileptonic decays

- ★ If observe a neutral kaon beam a long time after production (i.e. a large distances) it will consist of a pure K_L component

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\underset{\llcorner\!\!\longrightarrow \pi^+e^-\overline{\nu}_e}{|K_0\rangle} + (1-\varepsilon)\underset{\llcorner\!\!\longrightarrow \pi^-e^+\nu_e}{|\overline{K}^0\rangle}\right]$$

- ★ Decays to $\pi^-e^+\nu_e$ must come from the $\overline{K}^0$ component, and decays to $\pi^+e^-\overline{\nu}_e$ must come from the K^0 component

$$\Gamma(K_L \to \pi^+e^-\overline{\nu}_e) \propto |\langle\overline{K}^0|K_L\rangle|^2 \propto |1-\varepsilon|^2 \approx 1-2\Re\{\varepsilon\}$$

$$\Gamma(K_L \to \pi^-e^+\nu_e) \propto |\langle K^0|K_L\rangle|^2 \propto |1+\varepsilon|^2 \approx 1+2\Re\{\varepsilon\}$$

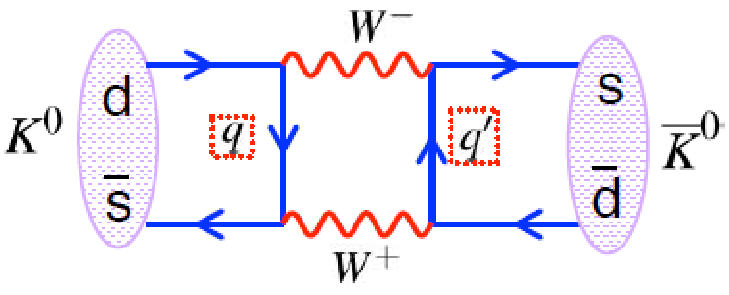
- ★ Results in a small difference in decay rates: the decay to $\pi^-e^+\nu_e$ is 0.7 % more likely than the decay to $\pi^+e^-\overline{\nu}_e$
  - This difference has been observed and thus provides the first direct evidence for an absolute difference between matter and anti-matter.

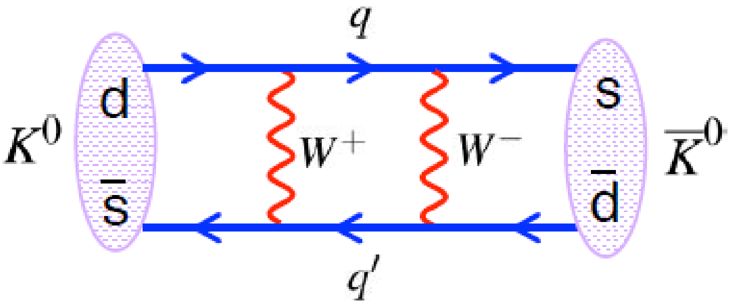
- ★ It also provides an unambiguous definition of matter which could, for example, be transmitted to aliens in a distant galaxy

"The electrons in our atoms have the same charge as those emitted least often in the decays of the long-lived neutral kaon"

CP violation and the CKM matrix

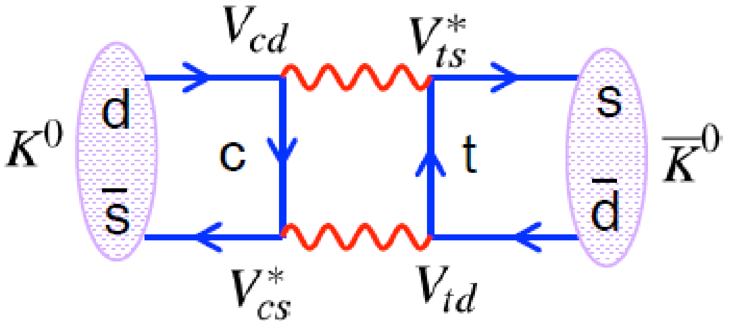
- How can we explain $\Gamma(\overline{K}^0_{t=0} \rightarrow K^0) \neq \Gamma(K^0_{t=0} \rightarrow \overline{K}^0)$ in terms of the CKM matrix ?
  - Consider the box diagrams responsible for mixing, i.e.





where $q = \{u, c, t\}, \quad q' = \{u, c, t\}$

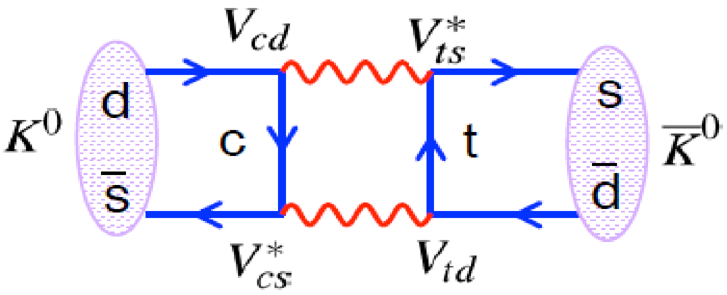
- Have to sum over all possible quark exchanges in the box. For simplicity consider just one diagram

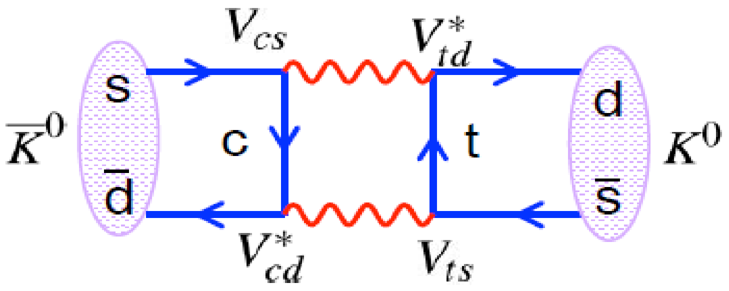


$$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$$

A constant related to integrating over virtual momenta

- Compare the equivalent box diagrams for $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$





$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^* \qquad M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

- Therefore difference in rates

$$\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M_{fi}^* = 2\Im\{M_{fi}\}$$

- Hence the rates can only be different if the CKM matrix has imaginary component

$$|\varepsilon| \propto \Im\{M_{fi}\}$$

- In the kaon system we can show

$$|\varepsilon| \propto A_{ut}.\Im\{V_{ud}V_{us}^*V_{td}V_{ts}^*\} + A_{ct}.\Im\{V_{cd}V_{cs}^*V_{td}V_{ts}^*\} + A_{tt}.\Im\{V_{td}V_{ts}^*V_{td}V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

Summary

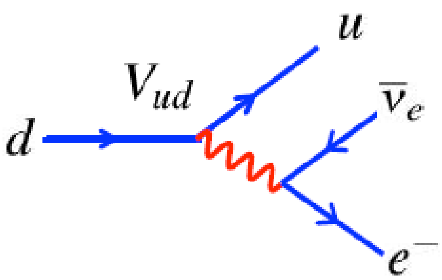
- ★ The weak interactions of quarks are described by the **CKM** matrix
- ★ Similar structure to the lepton sector, although unlike the **PMNS** matrix, the **CKM** matrix is nearly diagonal
- ★ CP violation enters through via a complex phase in the **CKM** matrix
- ★ A great deal of experimental evidence for CP violation in the weak interactions of quarks
- ★ CP violation is needed to explain matter – anti-matter asymmetry in the Universe
- ★ **HOWEVER**, CP violation in the SM is **not sufficient** to explain the matter – anti-matter asymmetry. There is probably another mechanism.

Appendix: determination of CKM matrix

- The experimental determination of the **CKM matrix** elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision
- Contrast this with the measurements of the **PMNS matrix**, where there are few theoretical uncertainties and the experimental difficulties in dealing with neutrinos limits the precision.

❶ **$|V_{ud}|$** **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



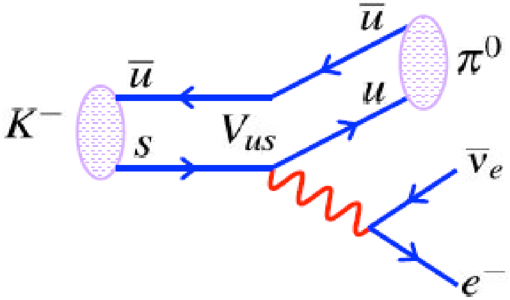
Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$\boxed{|V_{ud}| = 0.97377 \pm 0.00027} \qquad (\approx \cos\theta_c)$$

❷ **$|V_{us}|$** from semi-leptonic kaon decays

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



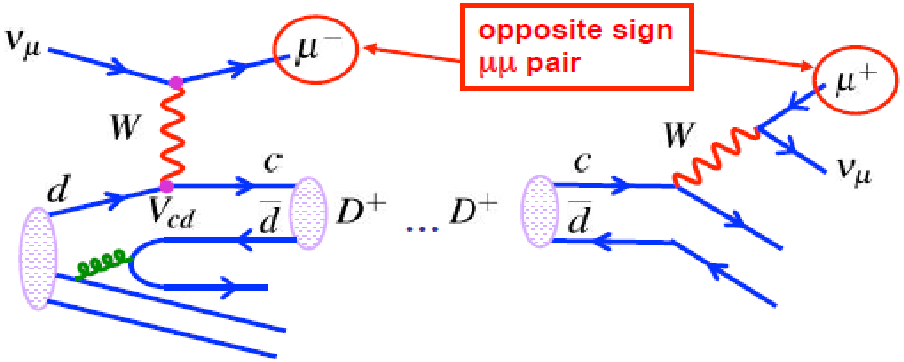
$$\Gamma \propto |V_{us}|^2$$

$$\boxed{|V_{us}| = 0.2257 \pm 0.0021} \qquad (\approx \sin\theta_c)$$

❸ **$|V_{cd}|$** from neutrino scattering $\nu_\mu + N \rightarrow \mu^+ \mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\overline{d})$ meson



opposite sign μμ pair

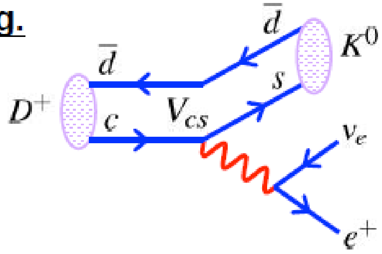
$$\text{Rate} \propto |V_{cd}|^2 \underbrace{\text{Br}(D^+ \rightarrow X\mu^+\nu_\mu)}_{\text{Measured in various collider experiments}}$$

⇨ $\boxed{|V_{cd}| = 0.230 \pm 0.011}$

❹ **$|V_{cs}|$** from semi-leptonic charmed meson decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cs}|^2$$

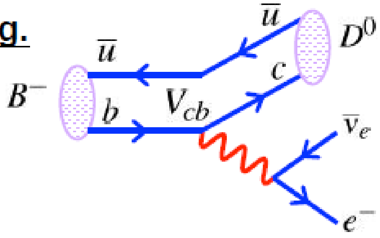
- Precision limited by theoretical uncertainties

$$\boxed{|V_{cs}| = 0.957 \pm \underbrace{0.017}_{\text{experimental error}} \pm \underbrace{0.093}_{\text{theory uncertainty}}}$$

❺ **$|V_{cb}|$** from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



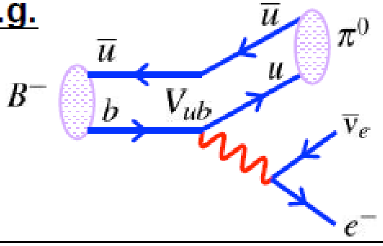
$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

❻ **$|V_{ub}|$** from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$$