Introduction to particle physics: experimental part

Units, kinematics

Large fraction of those slides from M. Delmastro lectures at ESIPAP school

Prof. dr hab. Elżbieta Richter-Wąs

HEP, SI and „natural” units

Quantity	HEP units	SI units
length	1 fm	10^{-15} m
charge	e	$1.602 \cdot 10^{-19}$ C
energy	1 GeV	1.602×10^{-10} J
mass	1 GeV/c^2	1.78×10^{-27} kg
$\hbar = h/2$	6.588×10^{-25} GeV s	1.055×10^{-34} Js
c	2.988×10^{23} fm/s	2.988×10^{8} m/s
$\hbar c$	197 MeV fm	…

	“natural” units ($\hbar = c = 1$)
mass	1 GeV
length	1 GeV^{-1} = 0.1973 fm
time	1 GeV^{-1} = 6.59×10^{-25} s

Measuring particles

- Particles are characterized by
  - ✓ Mass [Unit: eV/c^2 or eV]
  - ✓ Charge [Unit: e]
  - ✓ Energy [Unit: eV]
  - ✓ Momentum [Unit: eV/c or eV]
  - ✓ (+ spin, lifetime, …)

Particle identification via measurement of:

e.g. (E, p, Q) or (p, β, Q)
(p, m, Q) ...

- … and move at relativistic speed

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\ell = \frac{\ell_0}{\gamma}$$ length contraption

$$t = t_0\gamma$$ time dilatation

$$E^2 = \vec{p}^2c^2 + m^2c^4$$

$$E = m\gamma c^2 = mc^2 + E_{\mathrm{kin}}$$

$$\vec{\beta} = \frac{\vec{p}c}{E} \qquad \vec{p} = m\gamma\vec{\beta}c$$

Relativistic kinematics in a nutschell

$$E^2 = \vec{p}^2 + m^2$$

$$\ell = \frac{\ell_0}{\gamma}$$

$$E = m\gamma$$

$$t = t_0\gamma$$

$$\vec{p} = m\gamma\vec{\beta}$$

$$\vec{\beta} = \frac{\vec{p}}{E}$$

Relativistic kinematics in a nutschell

Center of mass energy

- In the center of mass frame the total momentum is 0
- In laboratory frame center of mass energy can be computed as:

$$E_{\mathrm{cm}} = \sqrt{s} = \sqrt{\left(\sum E_i\right)^2 - \left(\sum \vec{p}_i\right)^2}$$

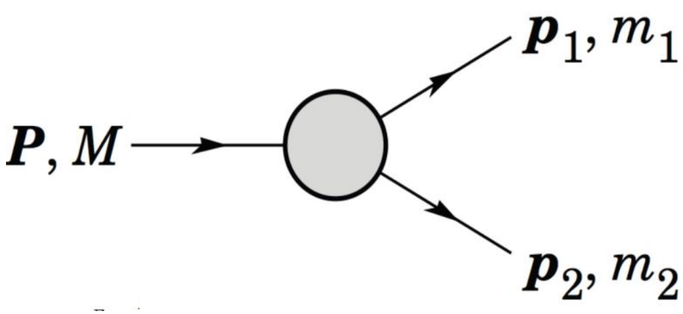
Hint: it can be computed as the "length" of the total four-momentum, that is invariant:

$$p = (E, \vec{p}) \qquad \sqrt{p \cdot p}$$

What is the "length" of a the four-momentum of a particle?

Kinematics

2-bodies decays



$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}$$

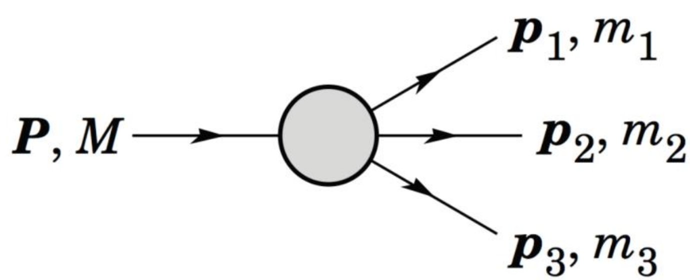
$$|\boldsymbol{p}_1| = |\boldsymbol{p}_2|$$

$$= \frac{\left[\left(M^2 - (m_1 + m_2)^2\right)\left(M^2 - (m_1 - m_2)^2\right)\right]^{1/2}}{2M}$$

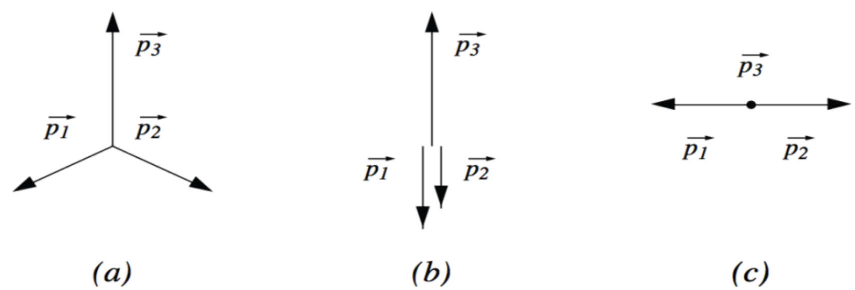
Invariant mass

$$M = \sqrt{\left(\sum E_i\right)^2 - \left(\sum \vec{p}_i\right)^2}$$

3-bodies decays



$$|\boldsymbol{p}_3| = \frac{\left[\left(M^2 - (m_{12} + m_3)^2\right)\left(M^2 - (m_{12} - m_3)^2\right)\right]^{1/2}}{2M}$$



$\max(|\vec{p}_3|)$ $(m_{12})_{min} = m_1 + m_2$

$\min(|\vec{p}_3|)$ $(m_{12})_{max} = M - m_3$

A real example: pion decays

pion decays at rest

$$|\mathbf{p}_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} c \simeq 30\,\mathrm{MeV/c}$$

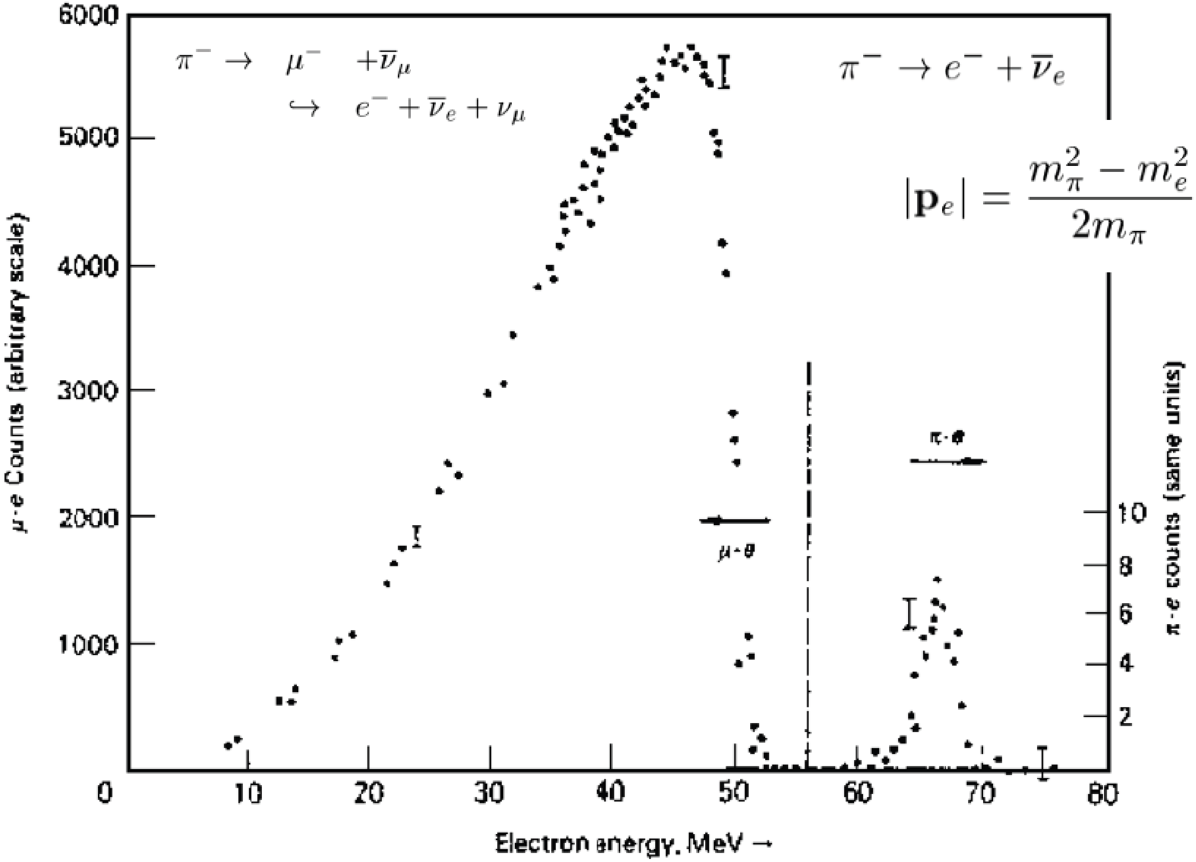
$$m_\nu = 0.$$

in most cases muon decays at rest

$$|\mathbf{p}_e|_{max} = \frac{m_\mu^2 - m_e^2}{2m_\mu} c \simeq 52\,\mathrm{MeV/c}$$

$$|\mathbf{p}_e|_{min} = 0$$

$$|\mathbf{p}_e| = \frac{m_\pi^2 - m_e^2}{2m_\pi} c \simeq 70\,\mathrm{MeV/c}$$



3-bodies decay: Dalitz plot

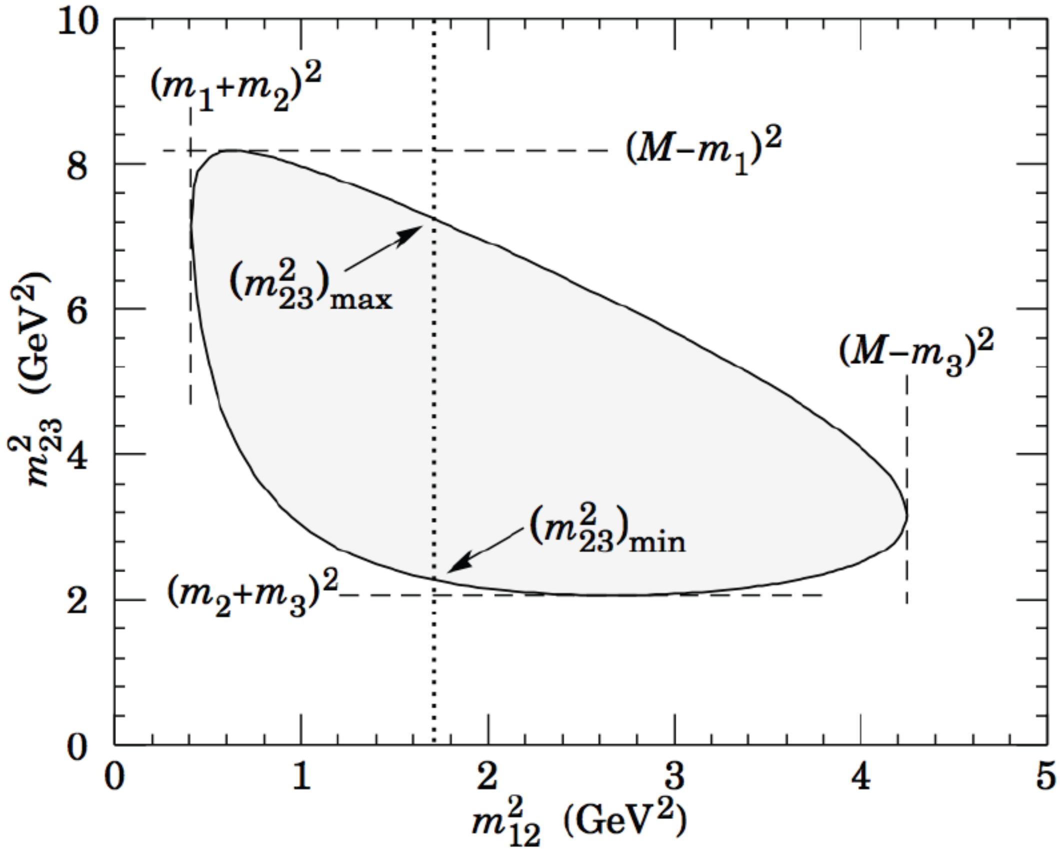
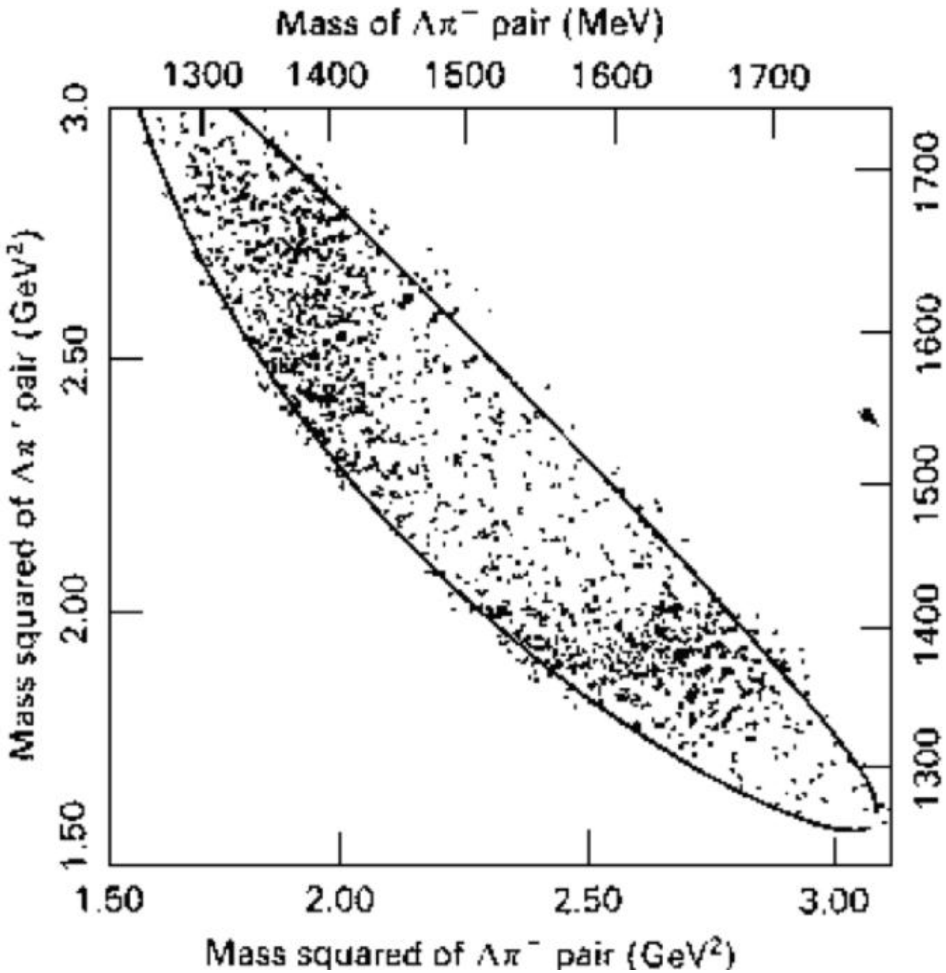


Figure 45.3: Dalitz plot for a three-body final state. In this example, the state is $\pi^+\overline{K}^0p$ at 3 GeV. Four-momentum conservation restricts events to the shaded region.

Multi-bodies decay

$$K^- + p \to \pi^+ + \pi^- + \Lambda$$



Reaction threshold

$$\sqrt{s} \geq \sum_i m_i c^2$$

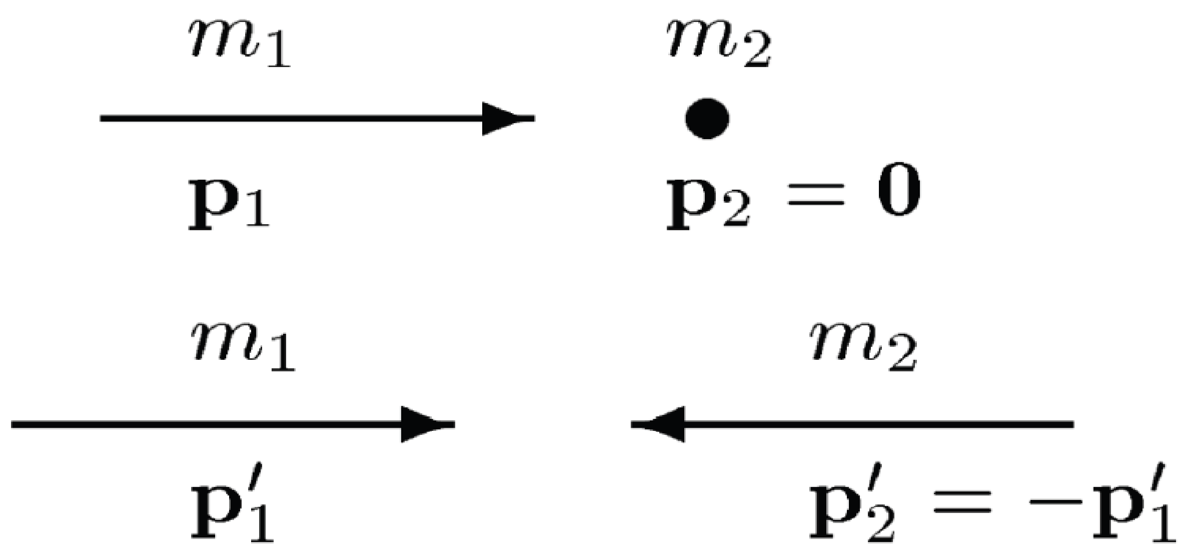
What energy should the pion have for this reaction to happen?

$$\pi + p \rightarrow \pi + \pi + \pi + p$$

$$\begin{aligned} s &= (p_\pi + p_p)^2 c^2 = (E_\pi + m_p c^2)^2 - |\mathbf{p}_\pi|^2 \\ &= (m_\pi c^2)^2 + (m_p c^2)^2 + 2E_\pi (m_p c^2) \end{aligned}$$

$$E_\pi \geq \frac{(\sum_i m_i c^2)^2 - (m_\pi c^2)^2 - (m_p c^2)^2}{2m_p c^2} \simeq 500\,\mathrm{MeV}$$

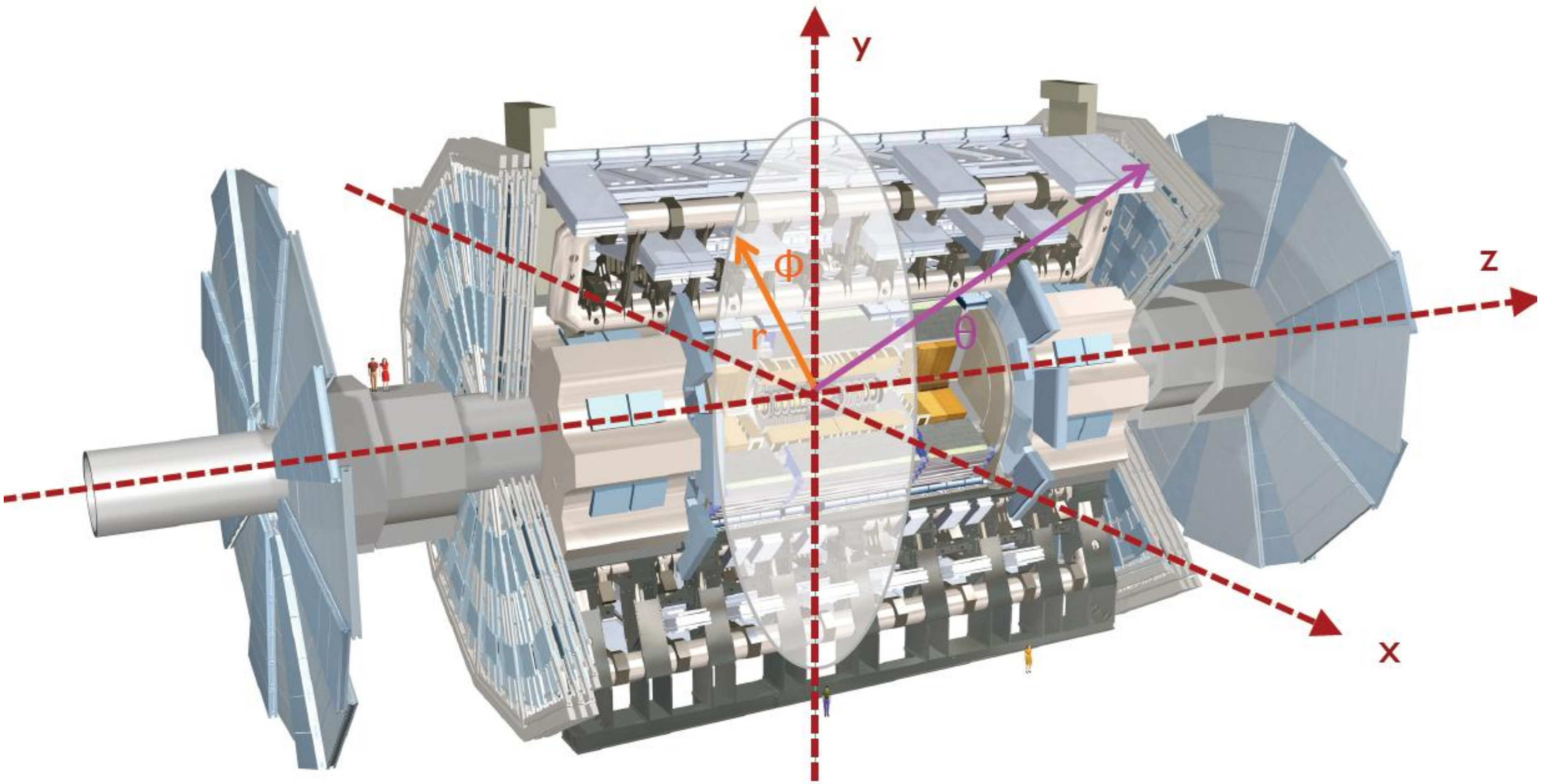
Fixed target vs collider



How much energy should a fixed target experiment have to equal the center of mass energy of two colliding beam?

$$E_{\text{fix}} = 2\frac{E_{\text{col}}^2}{m} - m$$

Collider experiment coordinates



Rapidity

Lorentz factor $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\varphi$ Hyperbolic cosine of "rapidity"

$$E = m\cosh\varphi$$
$$|\vec{p}| = m\sinh\varphi$$

$$\varphi = \tanh^{-1}\frac{E}{|\vec{p}|} = \frac{1}{2}\ln\frac{E+|\vec{p}|}{E-|\vec{p}|}$$

- Particle physicists prefer to use modified rapidity along beam axis

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$$

Pseudorapidity

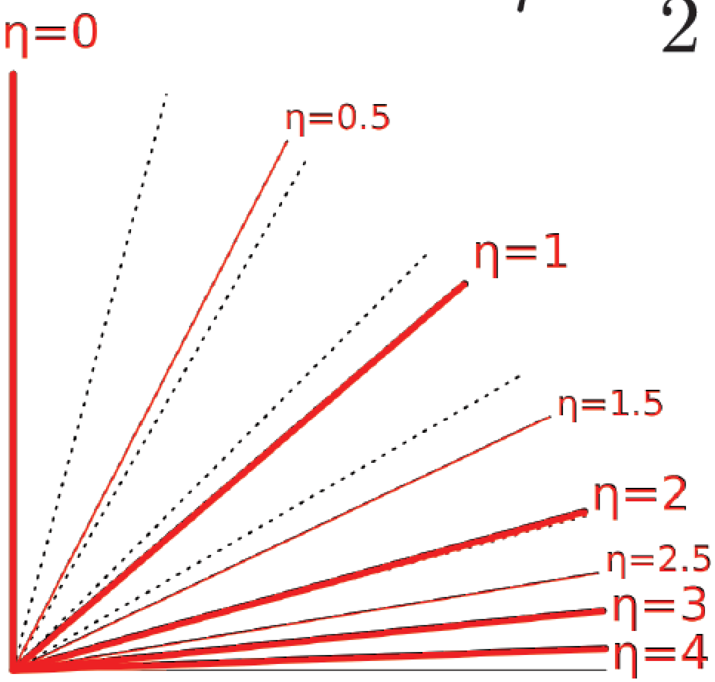
$$\eta = \frac{1}{2}\ln\frac{|\vec{p}| + p_z}{|\vec{p}| - p_z}$$

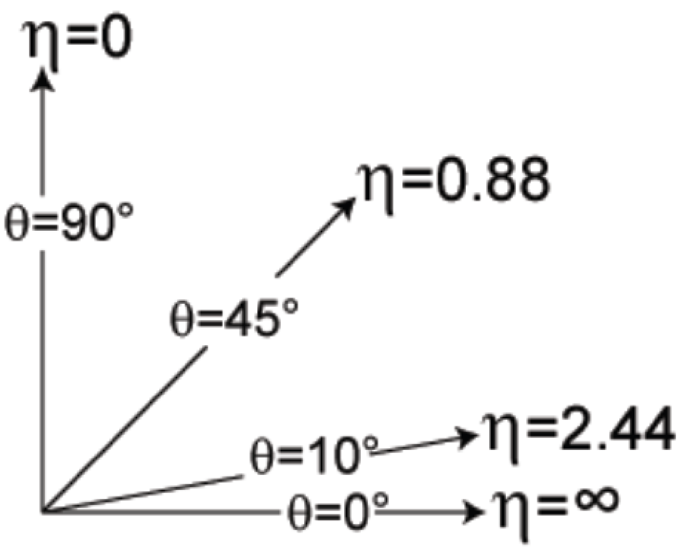
$$\eta = \frac{1}{2}\ln\left(\tan\frac{\theta}{2}\right)$$

$$y = \frac{1}{2}\ln\frac{E + p_z}{E - p_z}$$

$$\eta \simeq y$$

if E >> m





Transverse variables

- At hadron colliders, a significant and unknown fraction of the beam energy in each event escapes down the beam pipe.
- Net momentum can only be constrained in the plane transverse to the beam z-axis!

$$p_T = \sqrt{p_x^2 + p_y^2}$$

$$p_x = p_T \cos\phi$$
$$p_y = p_T \sin\phi$$
$$p_z = p_T \sinh\eta$$

$$|p| = p_T \cosh\eta$$

$$E_T = \frac{E}{\cosh\eta}$$

$$\sum p_X(i) = 0 \qquad \sum p_Y(i) = 0$$

Missing transverse energy and transverse mass

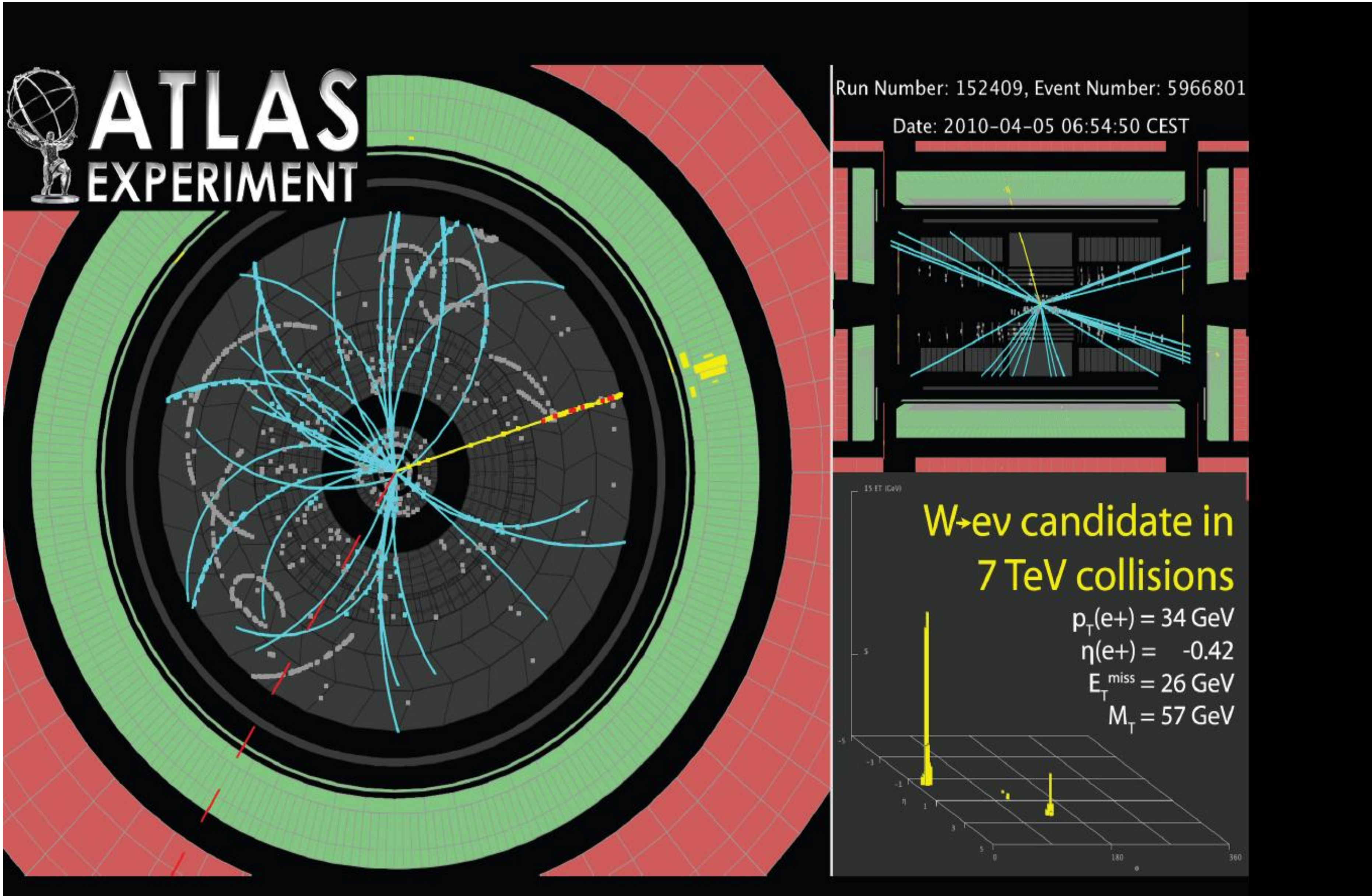
- If invisible particle are created, only their transverse momentum can be constrained: missing transverse energy

$$E_T^{\mathrm{miss}} = \sum p_T(i)$$

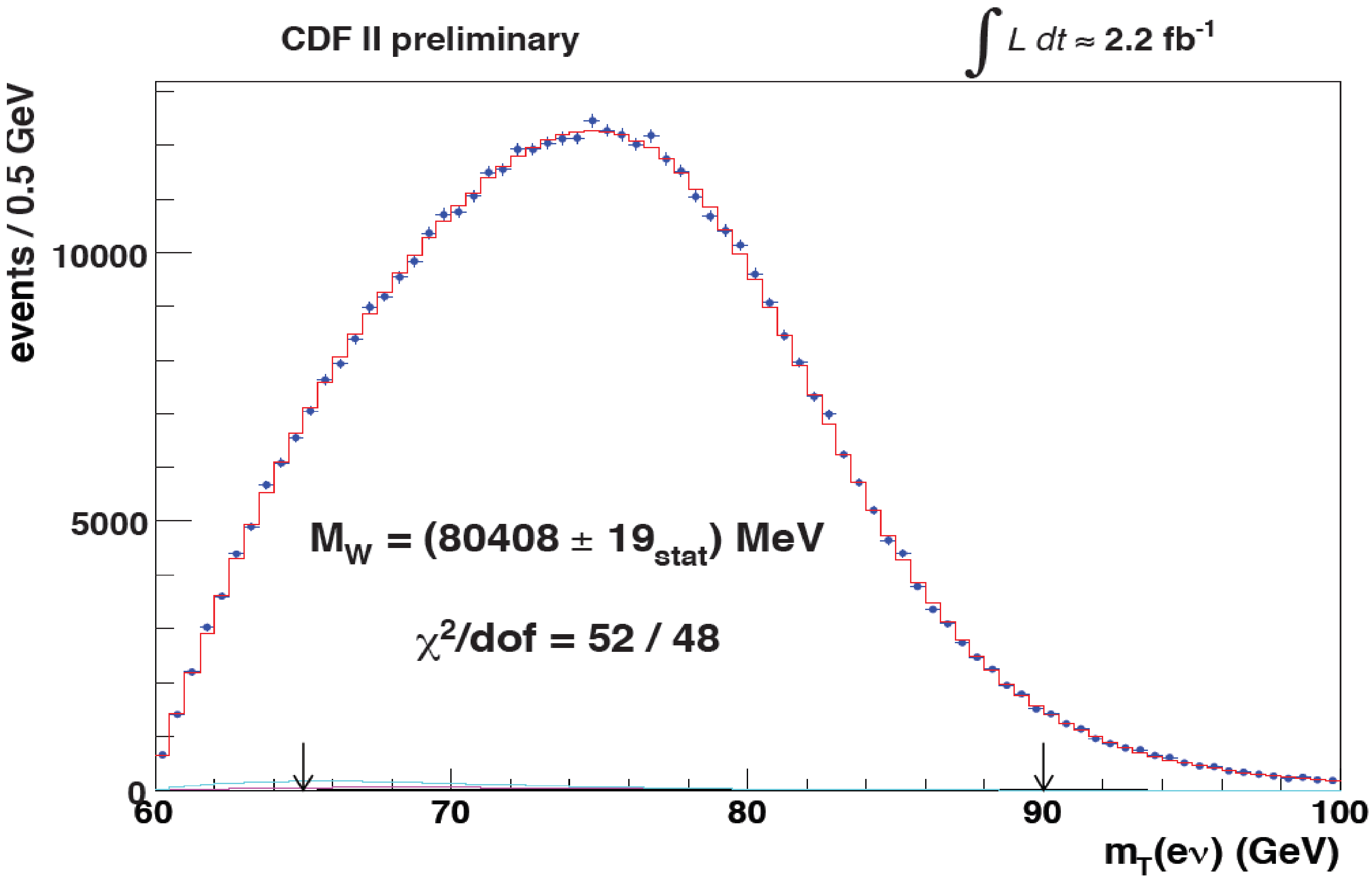
- If a heavy particle is produced and decays in two particles one of which is invisible, the mass of the parent particle can constrained with the transverse mass quantity

$$\begin{aligned} M_T^2 &\equiv [E_T(1) + E_T(2)]^2 - [\boldsymbol{p}_T(1) + \boldsymbol{p}_T(2)]^2 \\ &= m_1^2 + m_2^2 + 2[E_T(1)E_T(2) - \boldsymbol{p}_T(1) \cdot \boldsymbol{p}_T(2)] \end{aligned}$$

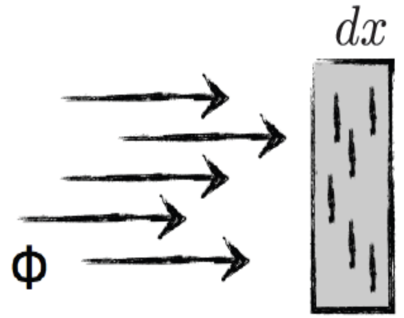
if $m_1 = m_2 = 0$ $\qquad M_T^2 = 2|\boldsymbol{p}_T(1)||\boldsymbol{p}_T(2)|(1 - \cos\phi_{12})$


ATLAS
EXPERIMENT
Run Number: 152409, Event Number: 5966801
Date: 2010-04-05 06:54:50 CEST
W→eν candidate in
7 TeV collisions
$p_T(e+)$ = 34 GeV
η(e+) = -0.42
E_T^{miss} = 26 GeV
M_T = 57 GeV

Mass of the W boson



Interaction cross-section



Flux $$\Phi = \frac{1}{S}\frac{dN_i}{dt}$$ [L^{-2} t^{-1}]

area obscured by target particle

Reactions per unit of time $$\frac{dN_{\mathrm{reac}}}{dt} = \Phi\sigma N_{\mathrm{target}}dx$$ [t^{-1}]

[L^{-2} t^{-1}] [?] [L^{-1}] [L]

Reaction rate per target particle $$W_{if} = \Phi\sigma$$ [t^{-1}]

Cross section per target particle $$\sigma = \frac{W_{if}}{\Phi}$$ [L^2] = reaction rate per unit of flux

1b = 10^{-28} m^2 (roughly the area of a nucleus with A = 100)

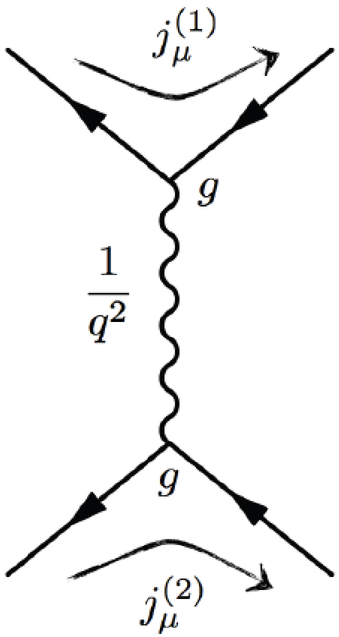
Fermi Golden rule

From non-relativistic perturbation theory…

transition probability — *matrix element* — *energy density of final states*

$$W_{if} = \frac{2\pi}{\hbar}|M_{if}|^2 \frac{dN}{dE_f}$$

[t⁻¹] [E] [E⁻¹]



$$M_{if} = -i\int j_\mu^{(1)}\left(\frac{1}{q^2}\right)j_\mu^{(2)}d^4x$$

$$\sigma \sim |M_{if}|^2 \sim g^4\left(\frac{1}{q^4}\right)$$

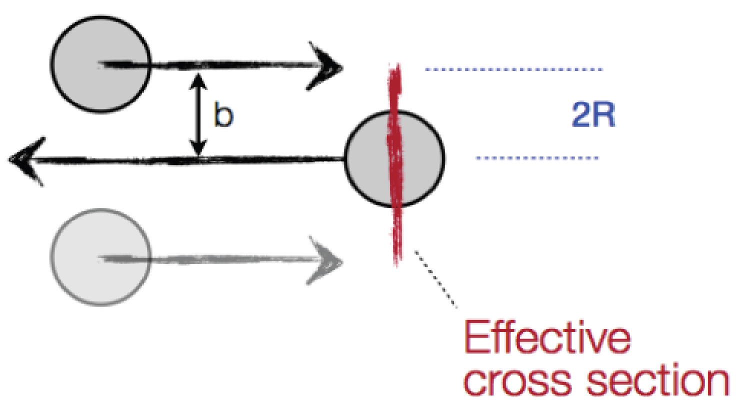
Cross-section: magnitude and units

Standard
cross section unit: $[\sigma]$ = mb with 1 mb = 10^{-27} cm^2

or in
natural units: $[\sigma] = \text{GeV}^{-2}$ with 1 GeV^{-2} = 0.389 mb
1 mb = 2.57 GeV^{-2}

using: $\hbar c$ = 0.1973 GeV fm
$(\hbar c)^2$ = 0.389 GeV^2 mb

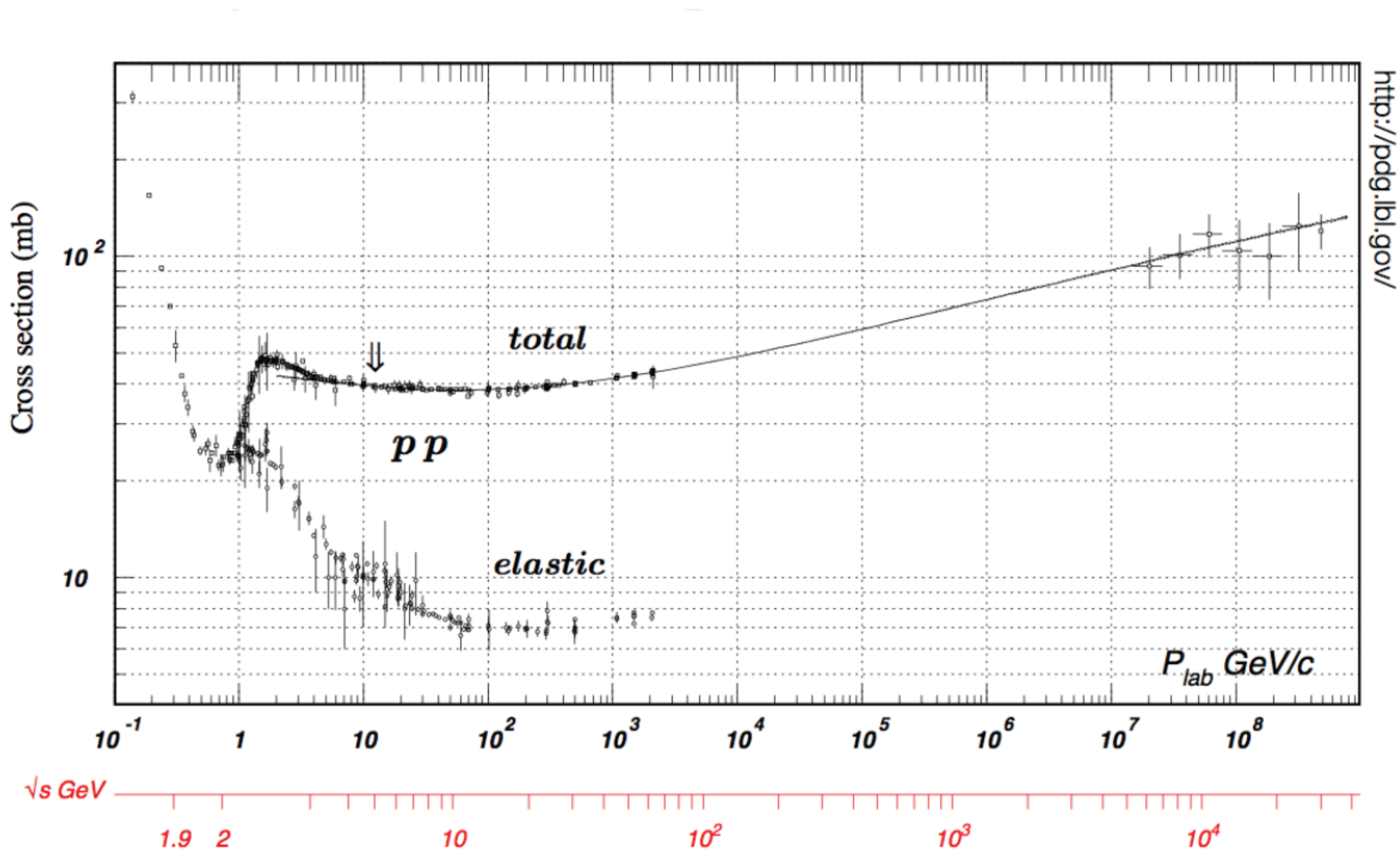
Estimating the
proton-proton cross section:



Proton radius: R = 0.8 fm
Strong interactions happens up to b = 2R

$$\sigma = \pi(2R)^2 = \pi \cdot 1.6^2 \text{ fm}^2$$
$$= \pi \cdot 1.6^2 \; 10^{-26} \text{ cm}^2$$
$$= \pi \cdot 1.6^2 \; 10 \text{ mb}$$
$$= 80 \text{ mb}$$

Proton-proton scattering cross-section



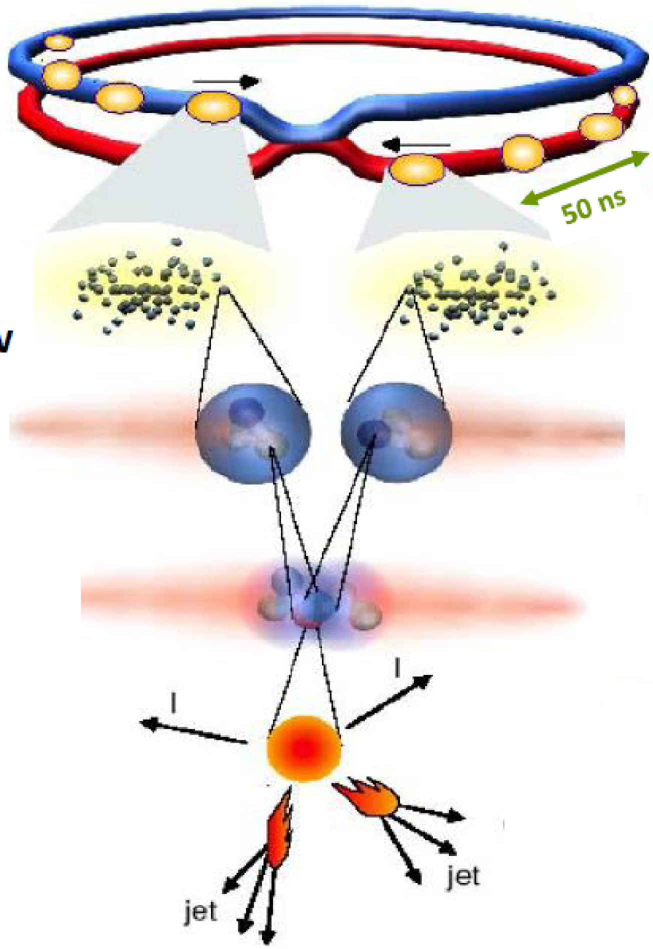
Proton-proton collisions at LHC

Paczka protonów

Proton

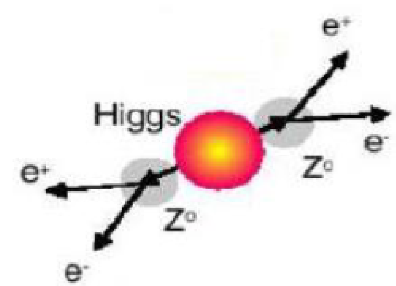
Składniki protonu

Produkty zderzeń



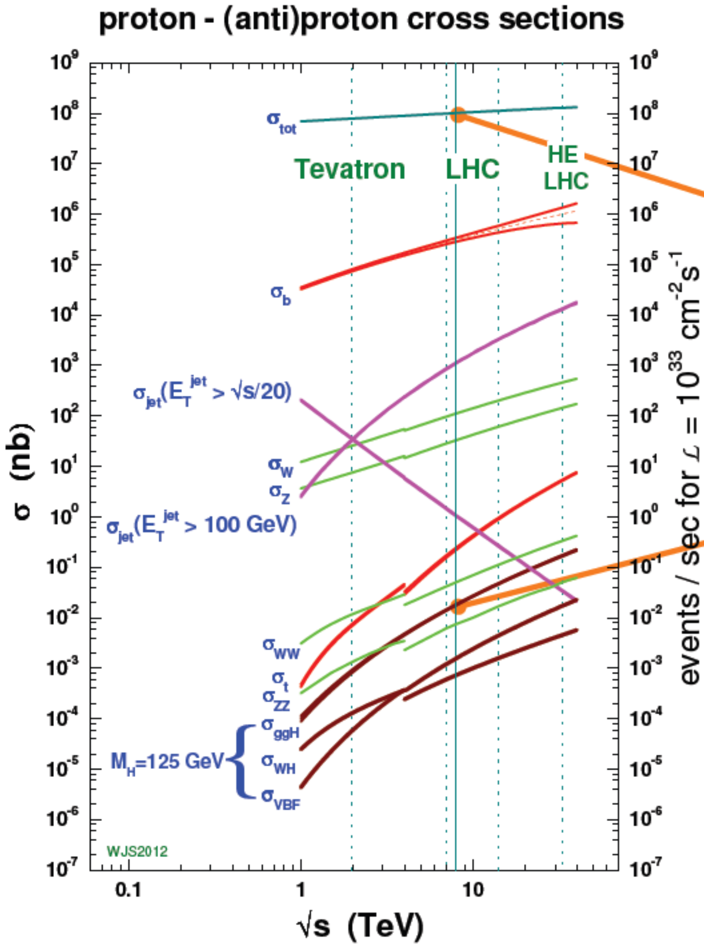
Proton-Proton 1380 paczek/wiązkę
Protonów/paczka 1.7 10^{11}
Energia wiązki 4 TeV

Każdy proton porusza się z prędkością bliską prędkości światła i niesie kinetyczną energię muchy w locie, okrąża pierścień akceleratora 1100 razy na sekundę.
Rozmiar poprzeczny wiązki: 16μm (4 razy mniejszy niż grubość ludzkiego włosa).
Każda z wiązek niesie energię pociągu TGV o dł. 200 m i jadącego z prędkością 155km/godz (360M Jula).



Takie zdarzenie pojawia się raz na 10 bilionów zderzeń

Cross-sections at LHC



10^8 events/s

$\sim 10^{10}$

10^{-2} events/s ~ 10 events/min

[m_H ~ 125 GeV]

0.2% H → γγ
1.5% H → ZZ