Angular momentum and decay distributions in high energy physics: an introduction and use cases for the LHC

- Basics of dilepton decay distributions. Examples: quarkonium and vector bosons
- A general demonstration of an old and surprising “perturbative-QCD” relation, using only rotation invariance
- Model-independent spin characterization of the Higgs-like di-photon resonance

Pietro Faccioli, CERN, April 23th, 2013

Why should we study particle polarizations?

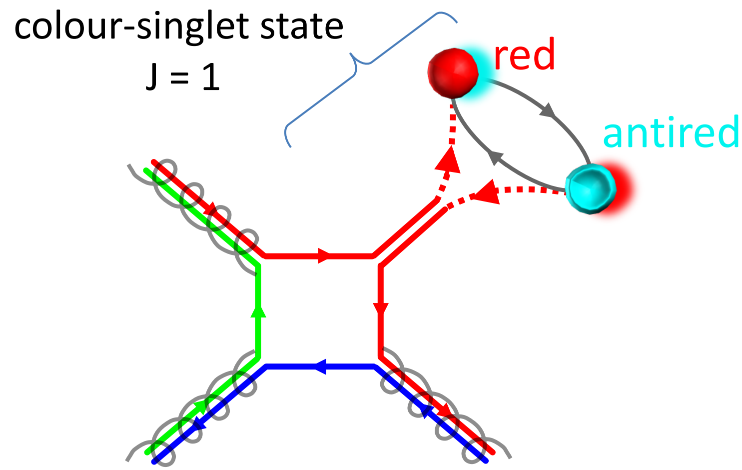
- test of perturbative QCD [***Z*** and ***W*** decay distributions]
- constrain universal quantities [**$\sin\theta_W$** and/or **proton PDFs** from $Z/W/\gamma^*$ decays]
- accelerate discovery of new particles or characterize them [**Higgs, *Z'*, anomalous *Z*+γ, graviton**, ...]
- understand the formation of hadrons (non-perturbative QCD)

Example: formation of ψ and ϒ

We want to know the relative contributions of the following processes, differing for how/when the observed *Q*-*Q*bar bound state acquires its quantum numbers

- **Colour-singlet processes**: quarkonia produced directly as observable ***colour-neutral*** *Q*-*Q*bar pairs

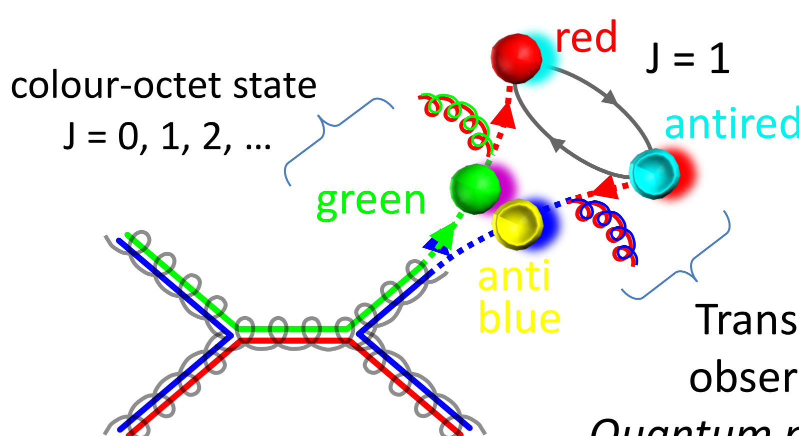
purely perturbative



+ analogous colour combinations

- **Colour-octet processes**: quarkonia are produced through ***coloured*** *Q*-*Q*bar pairs *of any possible quantum numbers*

perturbative
⊗
non-perturbative



Transition to the observable state.
Quantum numbers change!
*J can change! → **polarization**!*

Polarization of vector particles

$J = 1$ → three J_z eigenstates $|1, +1\rangle$, $|1, 0\rangle$, $|1, -1\rangle$ wrt a certain z

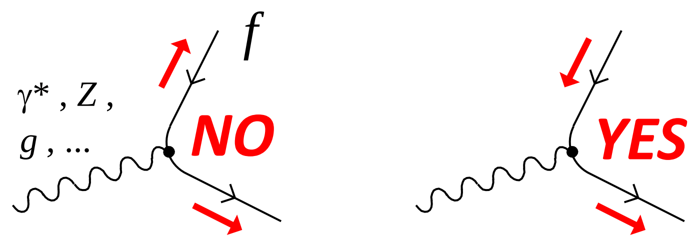
Measure polarization = measure (average) angular momentum composition

Method: study the **angular distribution of the particle decay** in its rest frame

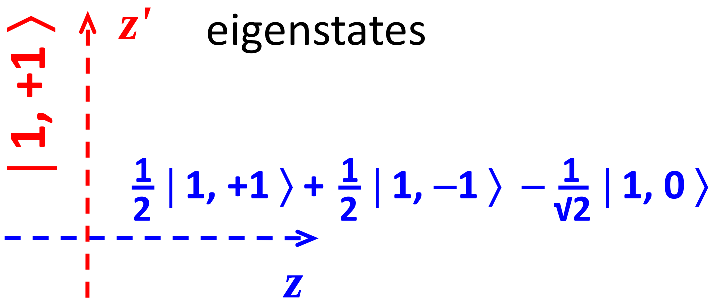
The decay **into a fermion-antifermion pair** is an especially clean case to be studied

The shape of the observable angular distribution is determined by a few basic principles:

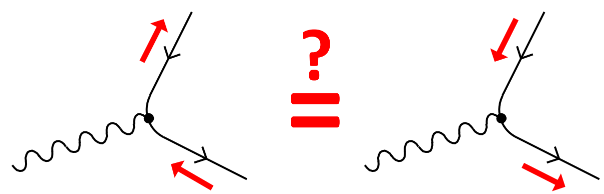
1) "helicity conservation"



2) rotational covariance of angular momentum eigenstates

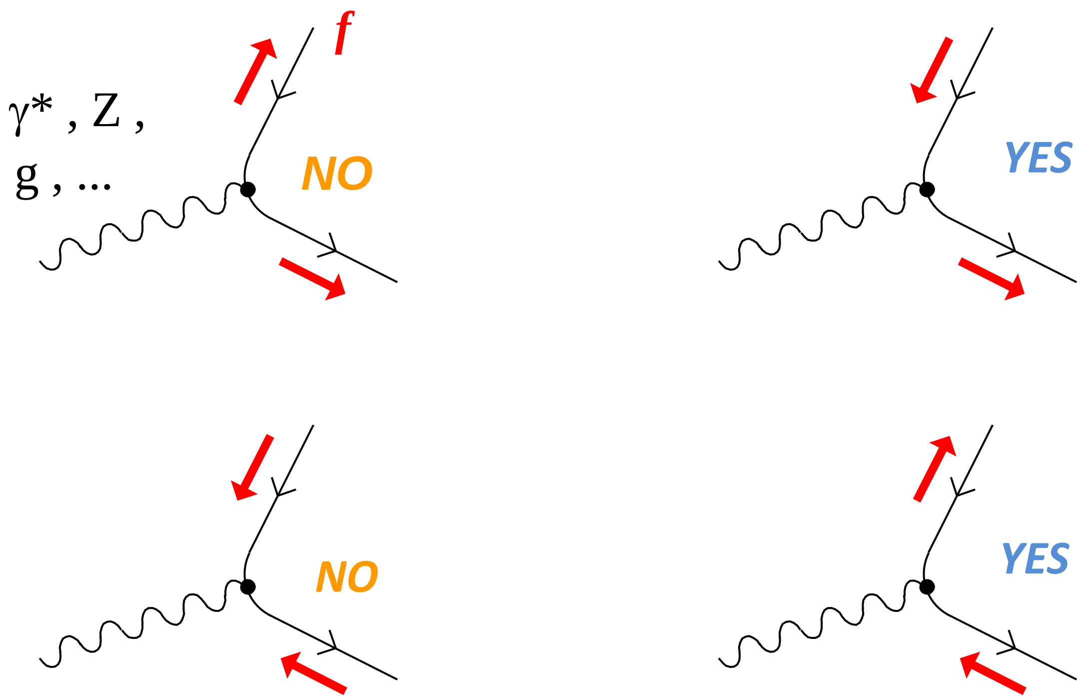


3) parity properties

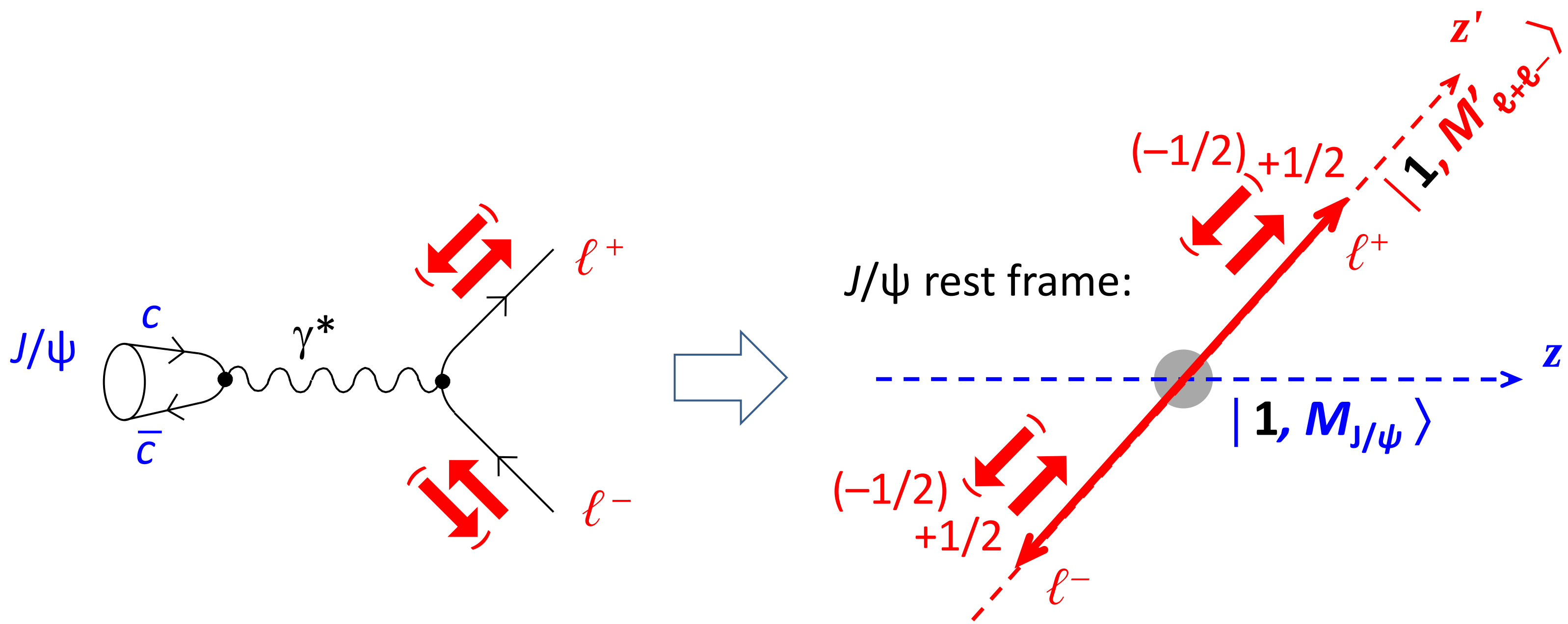


1: helicity conservation

EW and strong forces preserve the *chirality* (L/R) of fermions.
In the relativistic (massless) limit, *chirality* = *helicity* = spin-momentum alignment
→ the **fermion spin** never flips in the coupling to gauge bosons:



example: dilepton decay of J/ψ



J/ψ angular momentum component along the polarization axis ***z***:

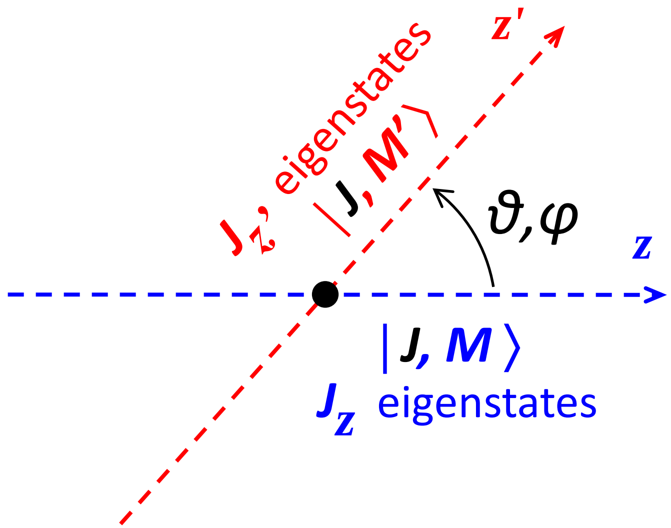
$$\boldsymbol{M_{J/\psi} = -1,\ 0,\ +1}$$ (determined by *production mechanism*)

The **two leptons** can only have total angular momentum component

$$\boldsymbol{M'_{\ell^+\ell^-} = +1 \text{ or } -1}$$ along their common direction ***z'***

0 is forbidden

2: rotation of angular momentum eigenstates



change of quantization frame:

$R(\vartheta,\varphi)$: $z \to z'$

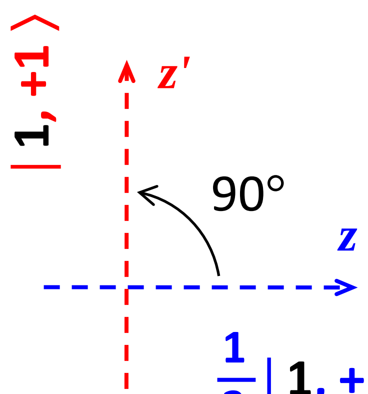
$y \to y'$

$x \to x'$

$$|J, M'\rangle = \sum_{M=-J}^{+J} D^{J}_{MM'}(\vartheta,\varphi)\ |J, M\rangle$$

Wigner D-matrices

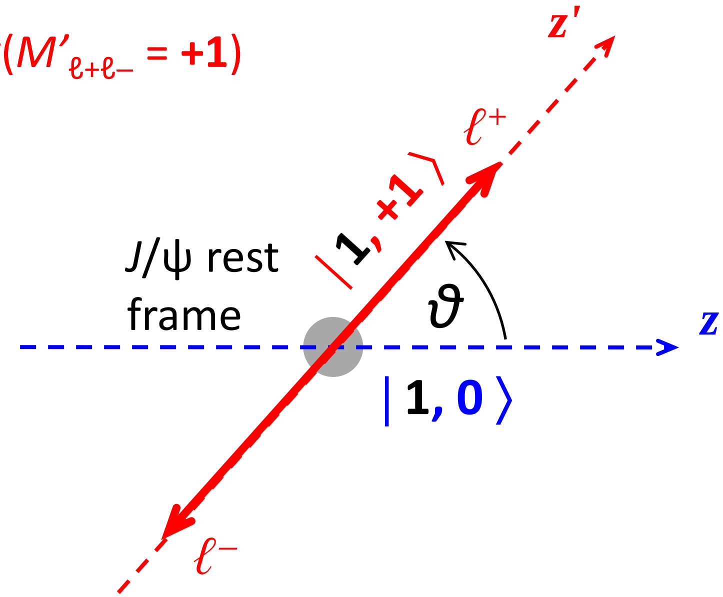
Example:



$$\frac{1}{2}|1,+1\rangle + \frac{1}{2}|1,-1\rangle - \frac{1}{\sqrt{2}}|1,0\rangle$$

example: $M = 0$

$J/\psi\ (M_{J/\psi} = 0) \rightarrow \ell^+\ell^-(M'_{\ell^+\ell^-} = +1)$



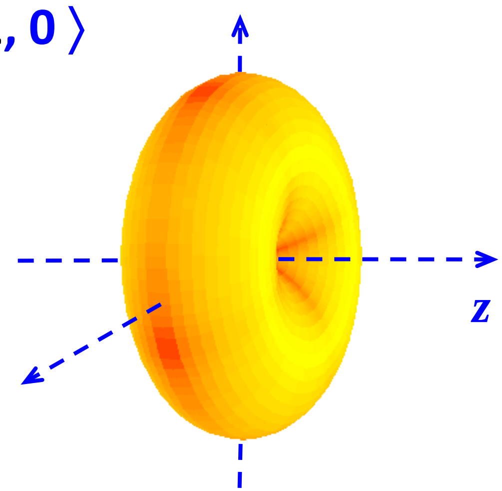
$$|1, +1\rangle = D^1_{-1,+1}(\vartheta,\varphi)\ |1, -1\rangle + D^1_{0,+1}(\vartheta,\varphi)\ |1, 0\rangle + D^1_{+1,+1}(\vartheta,\varphi)\ |1, +1\rangle$$

→ the $J_{z'}$ eigenstate $|1, +1\rangle$ "contains" the J_z eigenstate $|1, 0\rangle$ with component amplitude $D^1_{0,+1}(\vartheta,\varphi)$

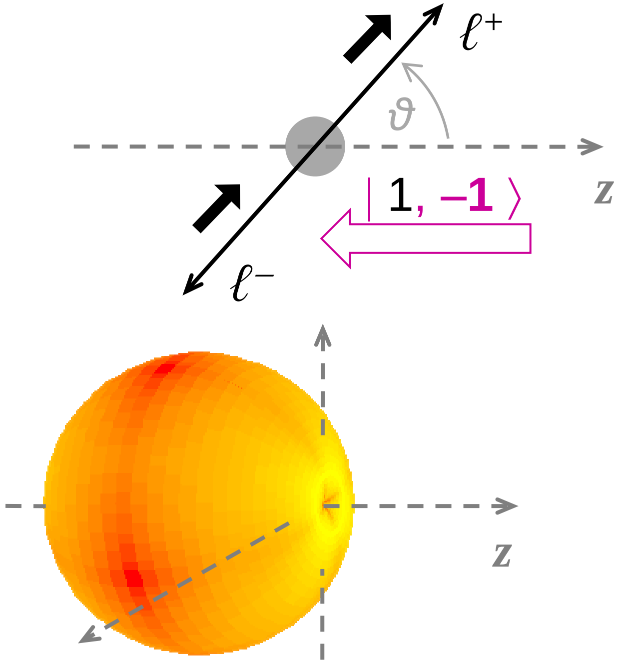
→ the decay distribution is

$$|\langle 1, +1\ |\ \mathcal{O}\ |\ 1, 0\rangle|^2 \propto |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 = \frac{1}{2}(1 - \cos^2\vartheta)$$

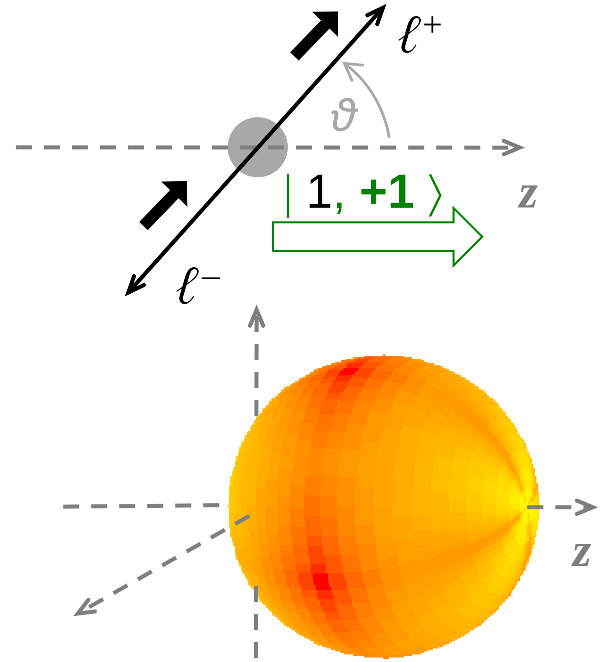
$\ell^+\ell^- \leftarrow J/\psi$



3: parity



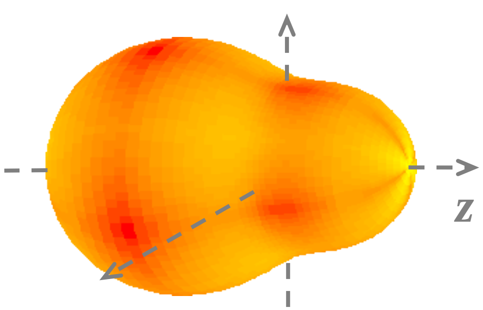
$|\,1, -1\,\rangle$ and $|\,1, +1\,\rangle$ distributions are mirror reflections of one another



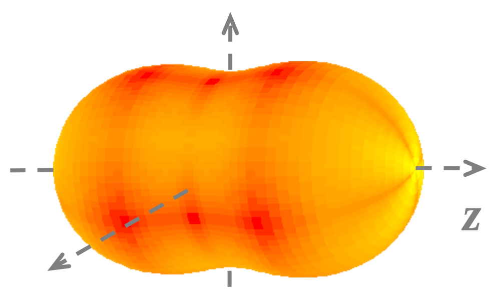
$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - 2\cos\vartheta$$

$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + 2\cos\vartheta$$

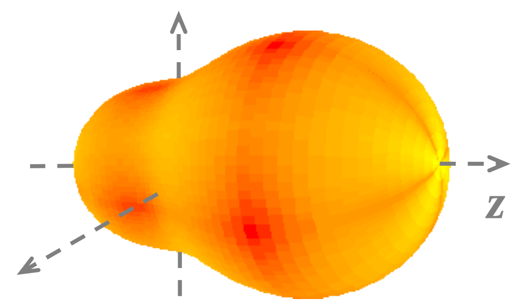
Are they equally probable?



$\mathcal{P}(-1) > \mathcal{P}(+1)$



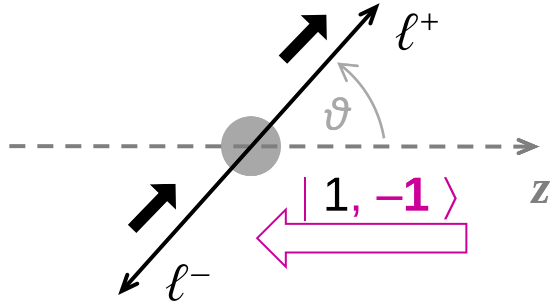
$\mathcal{P}(-1) = \mathcal{P}(+1)$

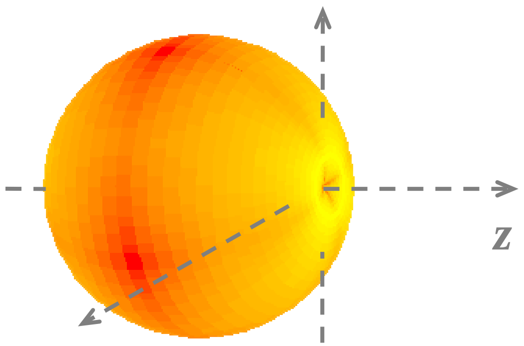


$\mathcal{P}(-1) < \mathcal{P}(+1)$

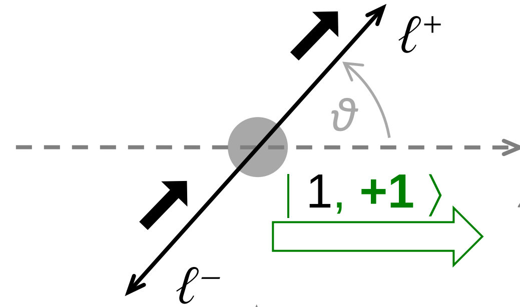
$$\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta + 2[\mathcal{P}(+1) - \mathcal{P}(-1)]\cos\vartheta$$

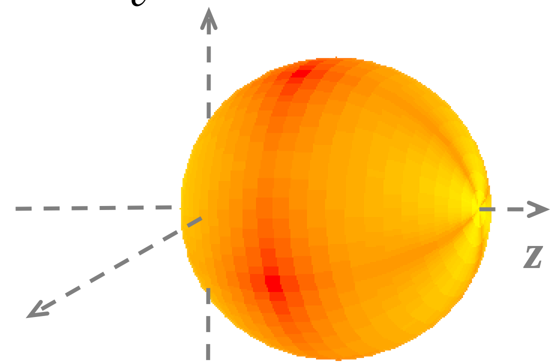
3: parity





$| 1, -1 \rangle$ and $| 1, +1 \rangle$ distributions are mirror reflections of one another

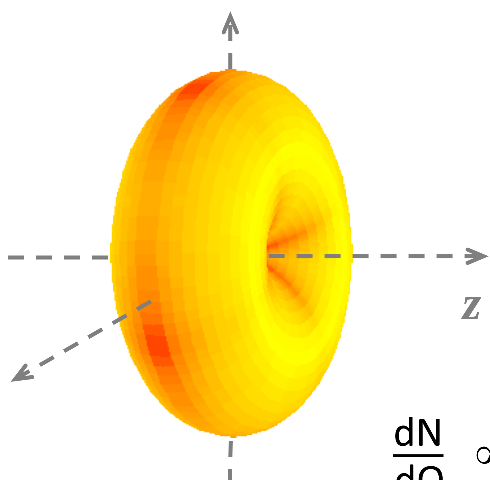




$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

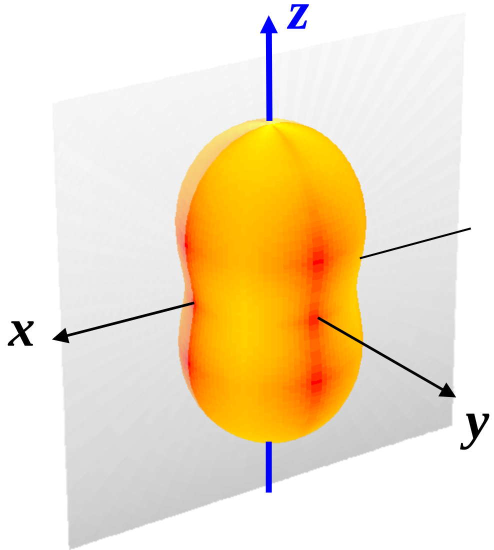
$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + \mathbf{2\cos\vartheta}$$

Decay distribution of $| 1, 0 \rangle$ state is always parity-symmetric:



$$\frac{dN}{d\Omega} \propto |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 \propto 1 - \cos^2\vartheta$$

"Transverse" and "longitudinal"

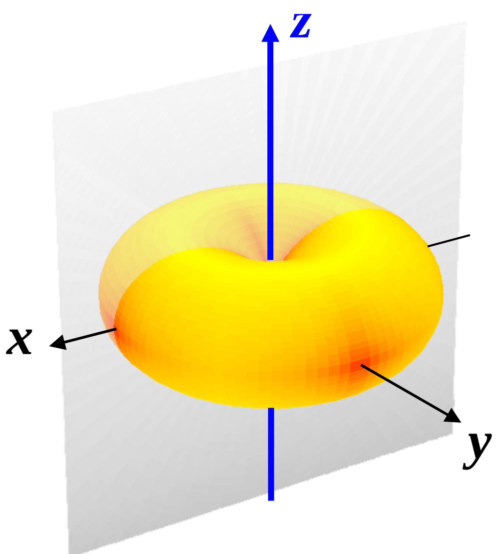


$$|J/\psi\rangle = |1, +1\rangle \text{ or } |1, -1\rangle$$

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta$$

(parity-conserving case)

"Transverse" polarization, like for *real photons*. The word refers to the alignment of the *field* vector, not to the *spin* alignment!



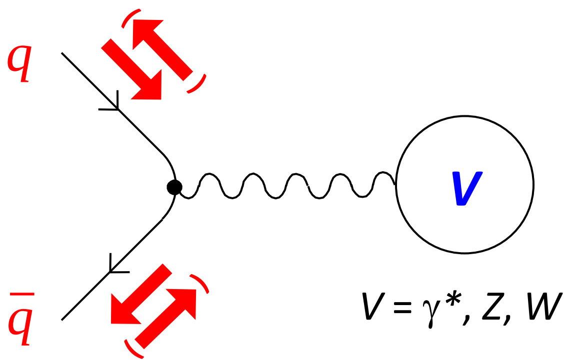
$$|J/\psi\rangle = |1, 0\rangle$$

$$\frac{dN}{d\Omega} \propto 1 - \cos^2\vartheta$$

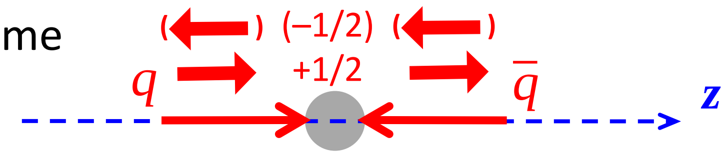
"Longitudinal" polarization

Why "photon-like" polarizations are common

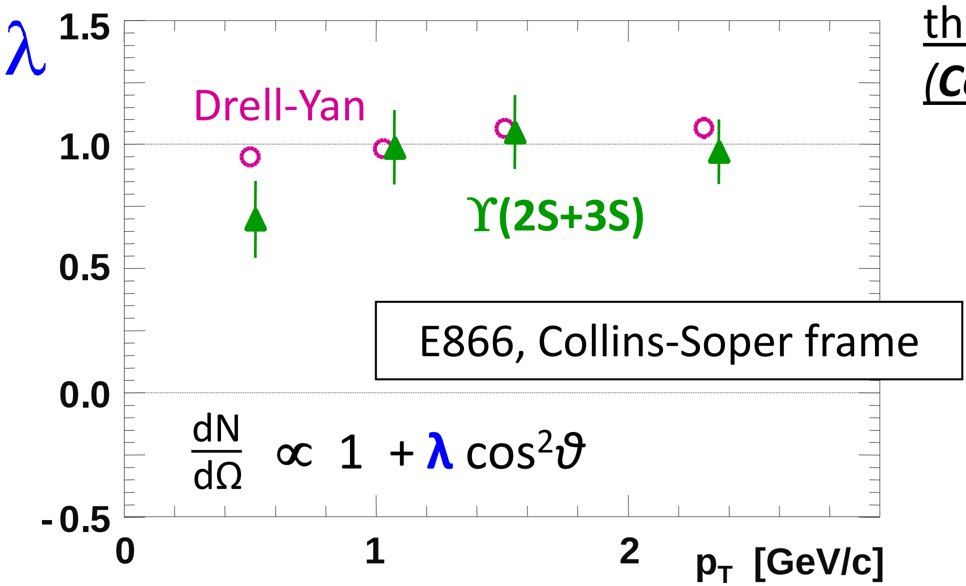
We can apply **helicity conservation at the *production* vertex** to predict that all *vector* states produced in ***fermion-antifermion annihilations* (q-$\bar{q}$ or e^+e^-)** at Born level have *transverse* polarization



q-$\bar{q}$ rest frame
= V rest frame



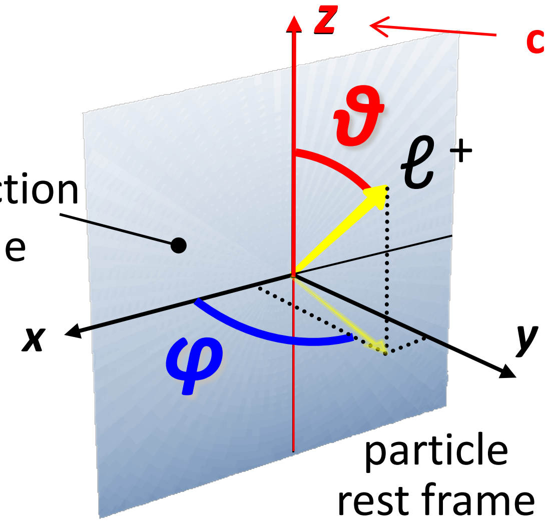
$|V\rangle = |1, +1\rangle$
$(|1, -1\rangle)$



The "natural" polarization axis in this case is the *relative direction of the colliding fermions* ***(Collins-Soper axis)***

Drell-Yan is a paradigmatic case
But not the only one

The most general distribution



average
polar anisotropy

average
azimuthal anisotropy

correlation
polar - azimuthal

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi$$

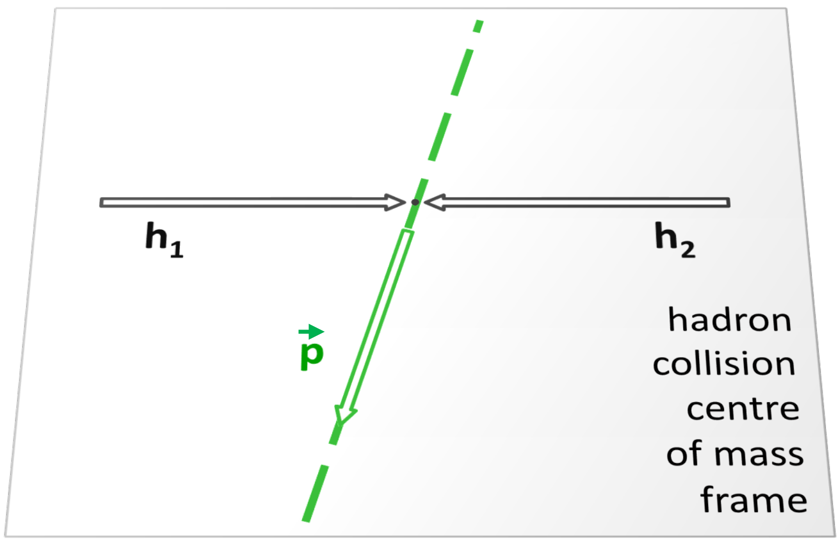
$$+ 2A_\theta \cos\theta + 2A_\varphi \sin\theta \cos\varphi$$

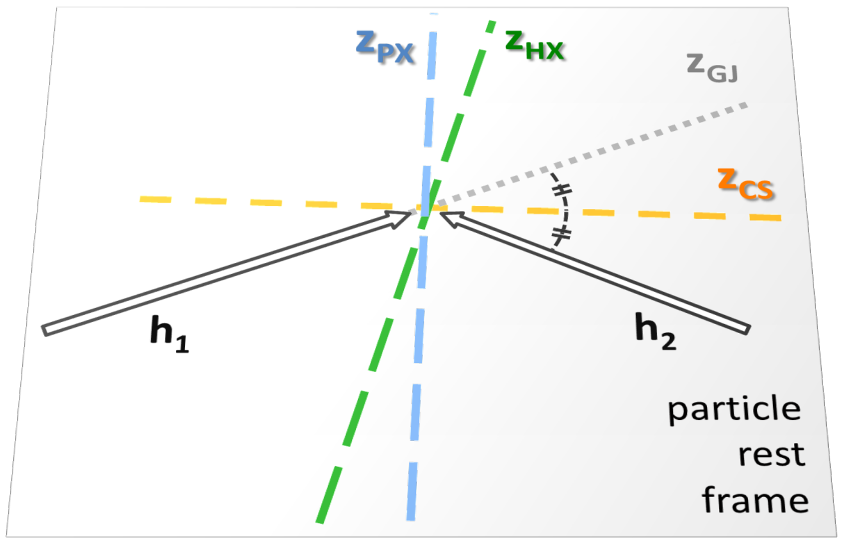
parity violating

Polarization frames

Helicity axis (HX): quarkonium momentum direction
Gottfried-Jackson axis (GJ): direction of one or the other beam
Collins-Soper axis (CS): average of the two beam directions
Perpendicular helicity axis (PX): perpendicular to CS

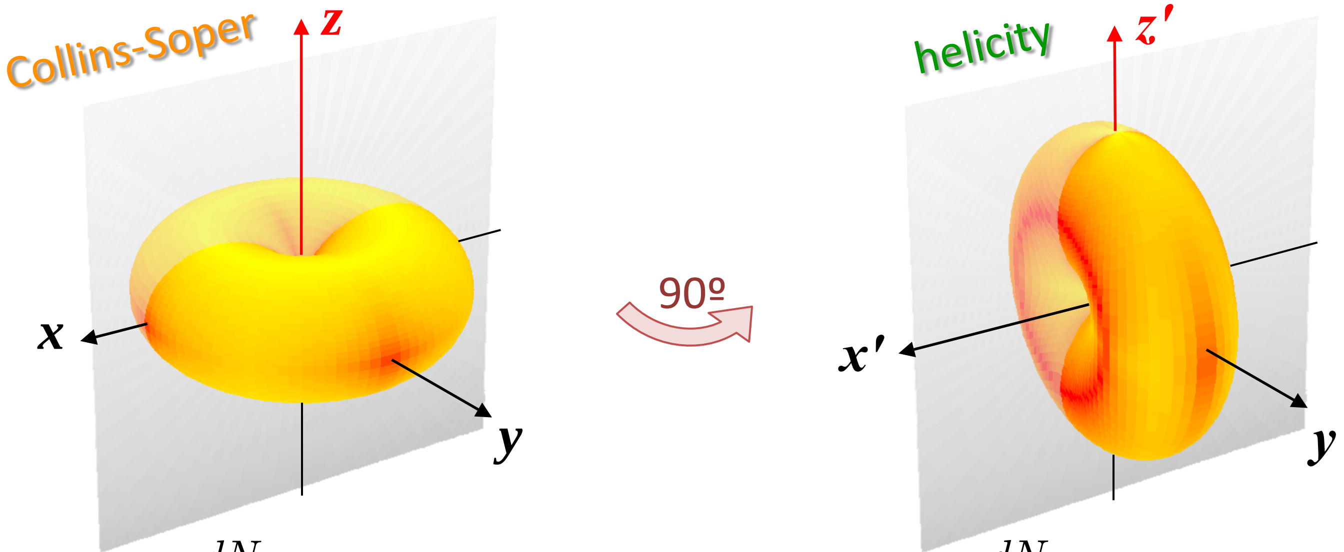
production plane





Frame dependence

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

longitudinal

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

"transverse"

$$|\psi\rangle = |0\rangle$$

(pure state)

$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

(mixed state)

All reference frames are equal… but some are more equal than others

What do different detectors measure with *arbitrary* frame choices?

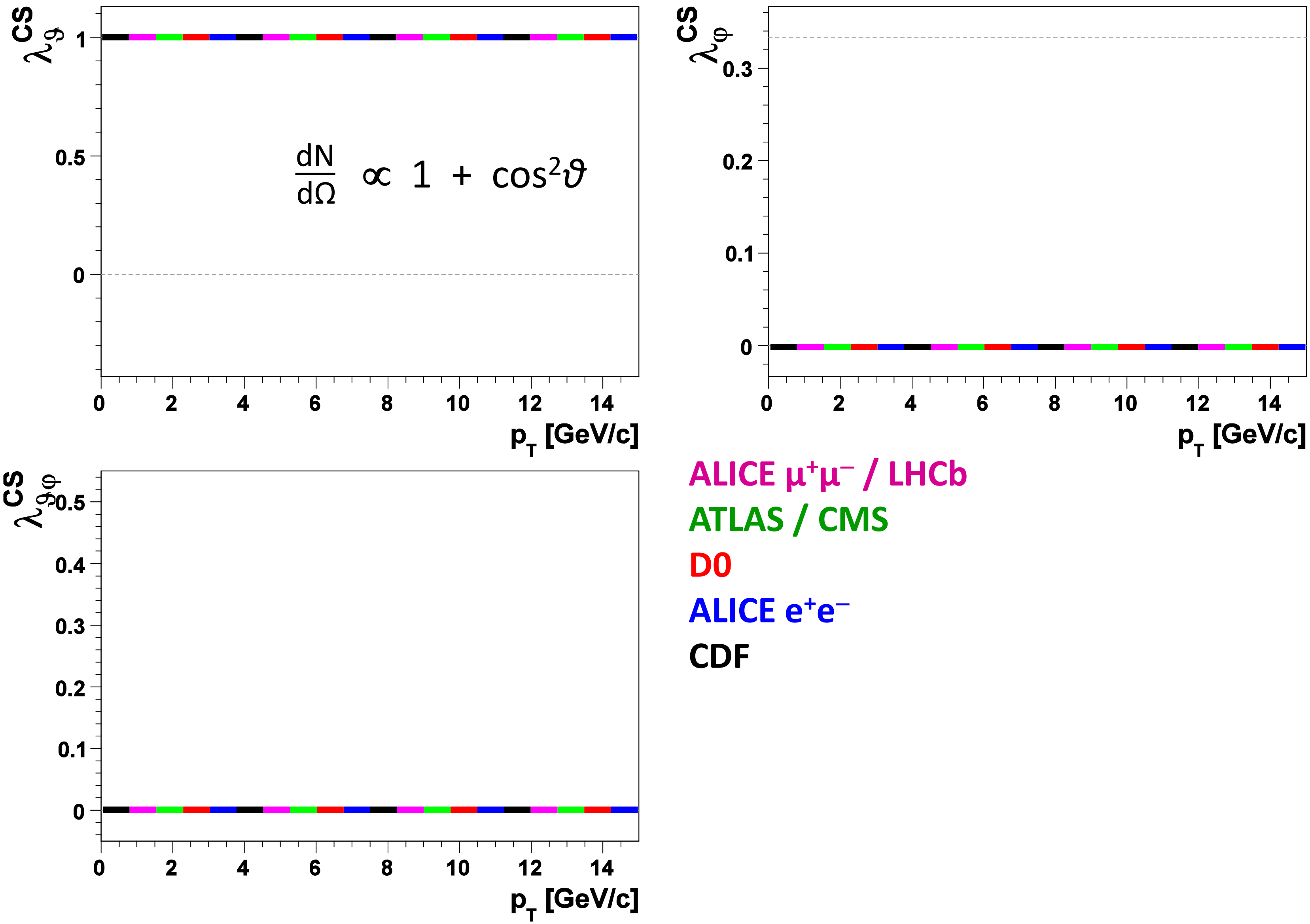
Gedankenscenario:

- **dileptons are fully transversely polarized in the CS frame**
- the decay distribution is measured at the $\Upsilon(1S)$ mass by 6 detectors with different **dilepton acceptances**:

CDF	$\|y\| < 0.6$
D0	$\|y\| < 1.8$
ATLAS & CMS	$\|y\| < 2.5$
ALICE e^+e^-	$\|y\| < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < y < 4$
LHCb	$2 < y < 4.5$

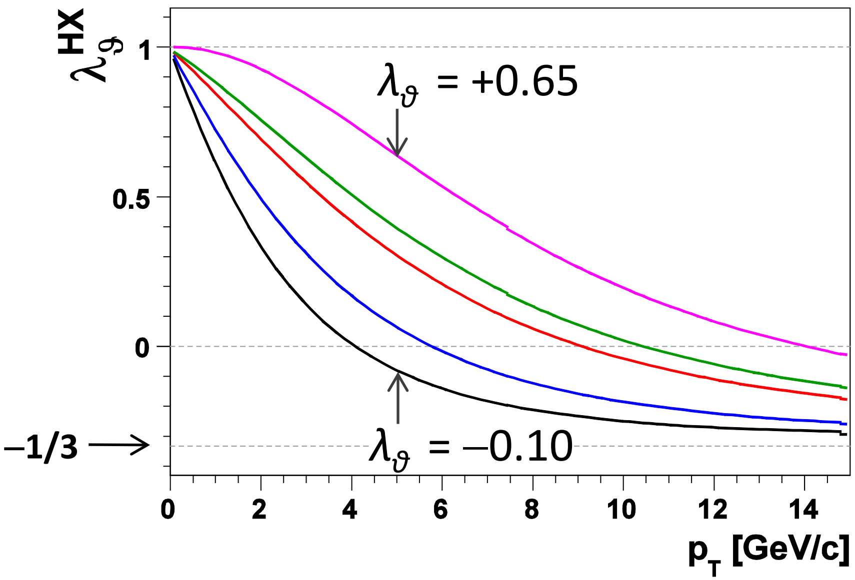
The lucky frame choice

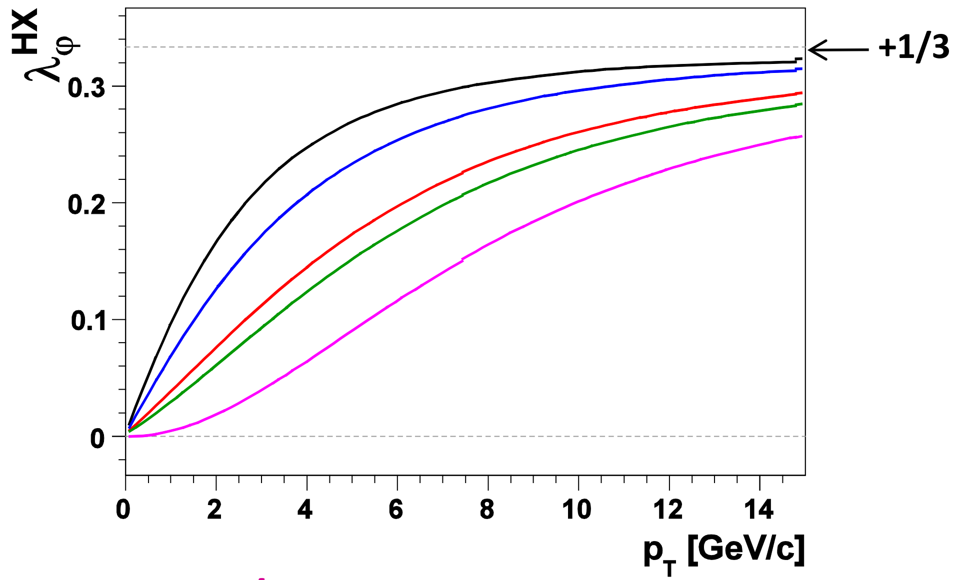
(CS in this case)

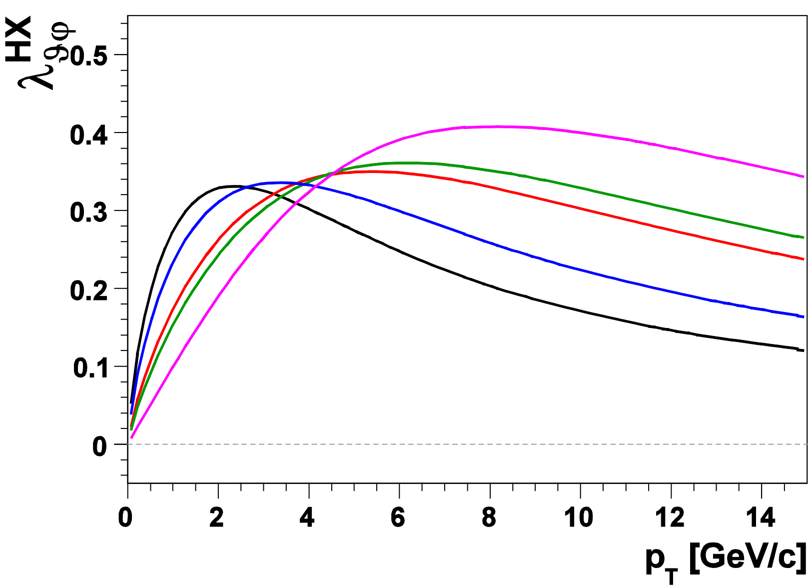


Less lucky choice

(HX in this case)







ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

D0

ALICE e^+e^-

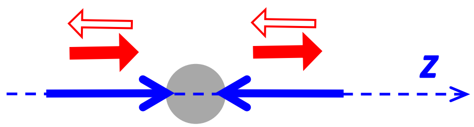
CDF

artificial (experiment-dependent!) kinematic behaviour
→ measure in more than one frame!

Frames for Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

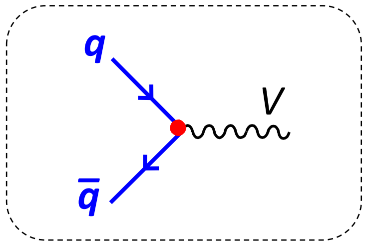
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

- ...but with respect to a ***subprocess-dependent quantization axis***

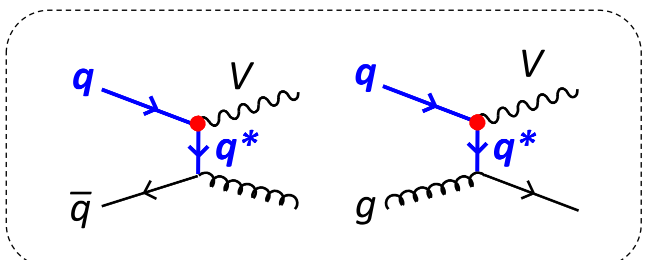
$O(\alpha_S^0)$



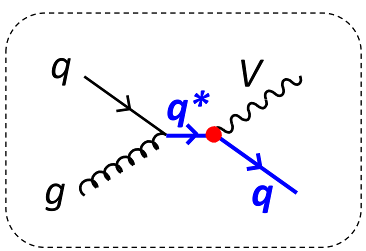
z = relative dir. of incoming *q* and *q*bar
(~ **Collins-Soper frame**)

important only up to $p_T = \mathcal{O}(\text{parton } k_T)$

$O(\alpha_S^1)$
QCD
corrections



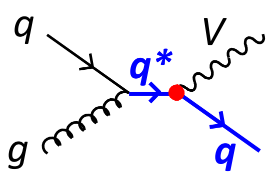
z = dir. of *one* incoming quark
(~ **Gottfried-Jackson frame**)



z = dir. of outgoing *q*
(= **parton-cms-helicity** $\approx$ **lab-cms-helicity**)

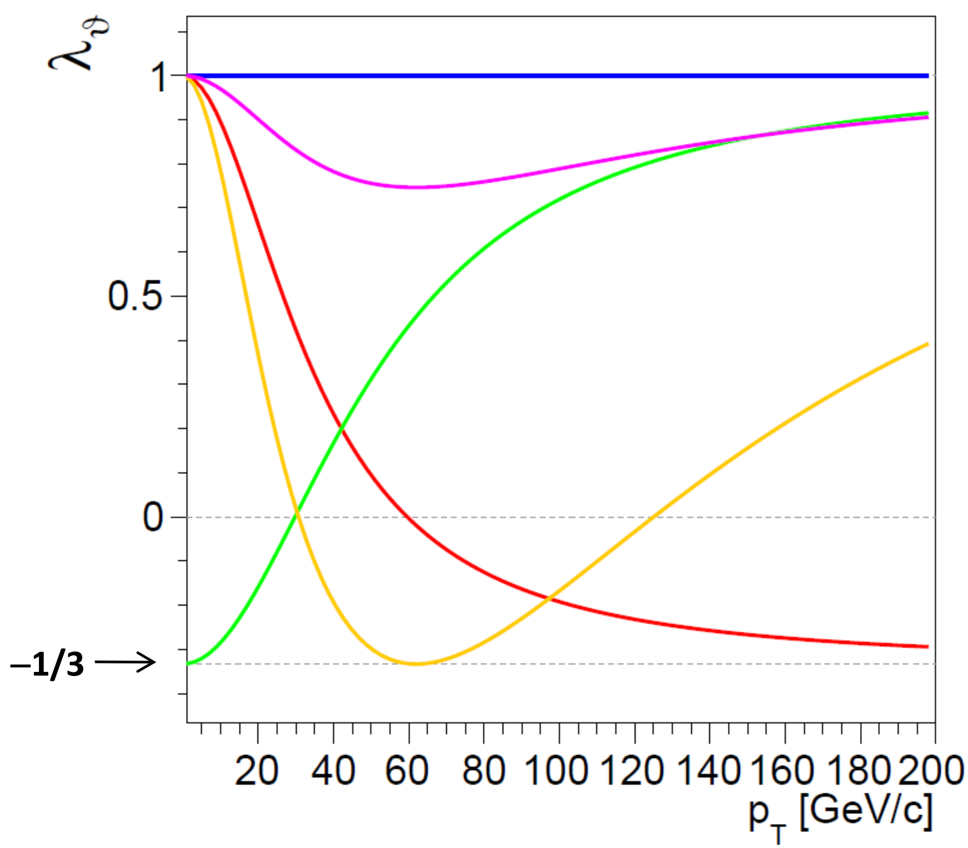
"Optimal" frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different "natural" quantization axes



For ***s*-channel processes** the natural axis is the direction of the outgoing quark (= direction of dilepton momentum)

→ optimal frame (= maximizing polar anisotropy): **HX** (neglecting parton-parton-cms vs proton-proton-cms difference!)

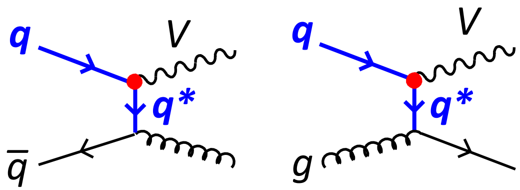


HX
CS
PX
GJ1 (negative beam)
GJ2 (positive beam)

example: Z
$y = +0.5$

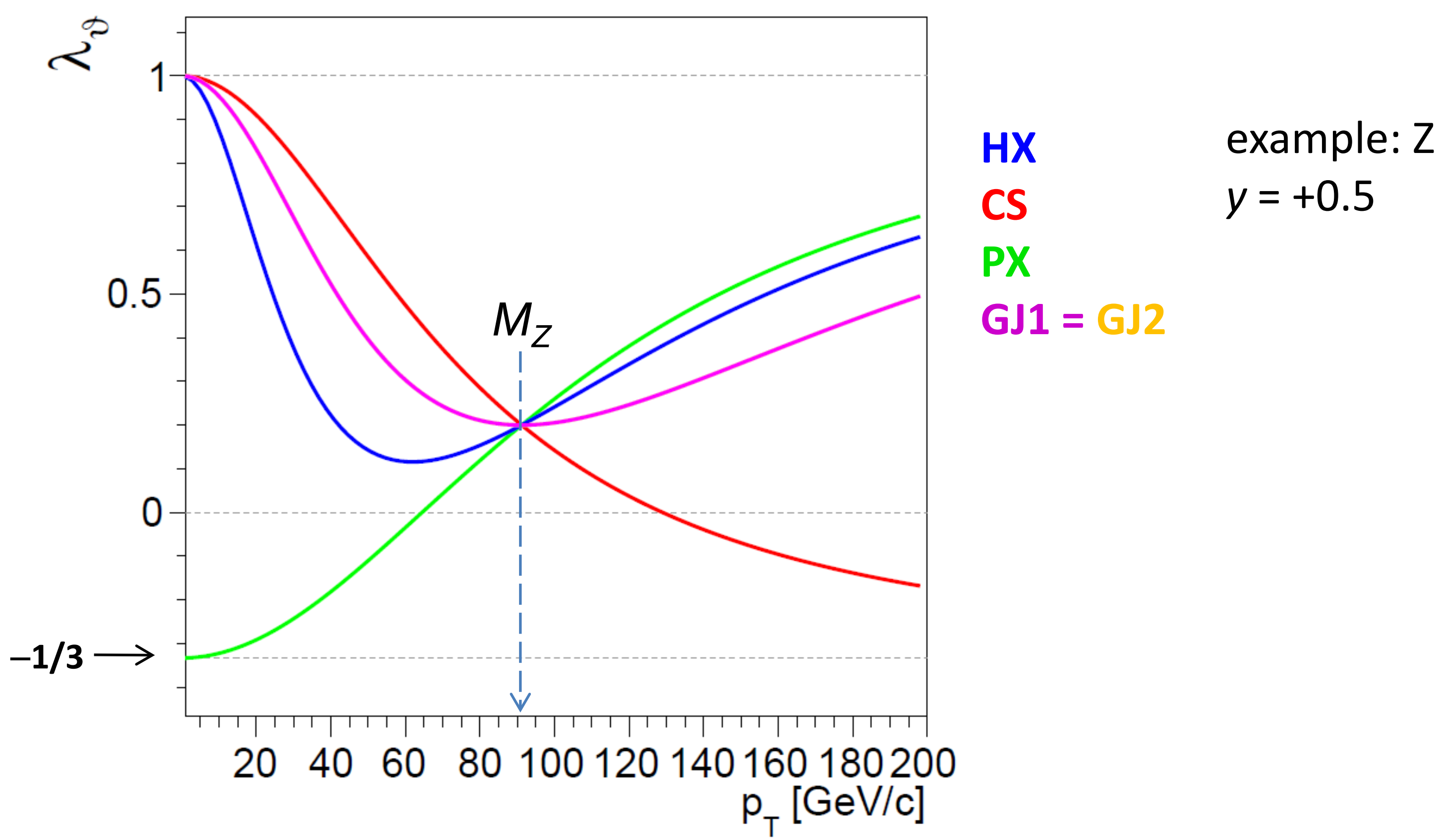
"Optimal" frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different "natural" quantization axes



For ***t*- and *u*-channel processes** the natural axis is the direction of either one or the other incoming parton (~ "Gottfried-Jackson" axes)

→ optimal frame: geometrical average of GJ1 and GJ2 axes = **CS ($p_T < M$)** and **PX ($p_T > M$)**

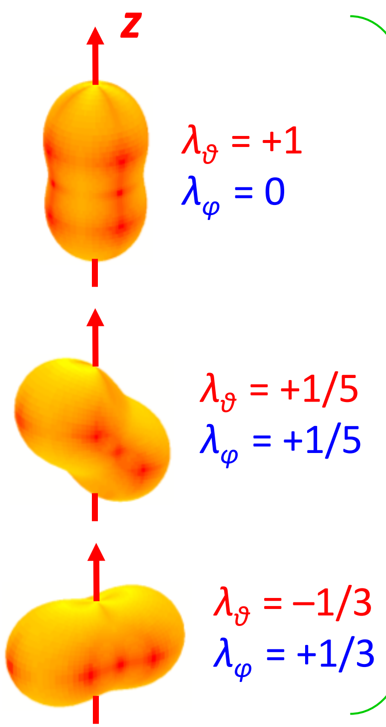


A complementary approach: frame-independent polarization

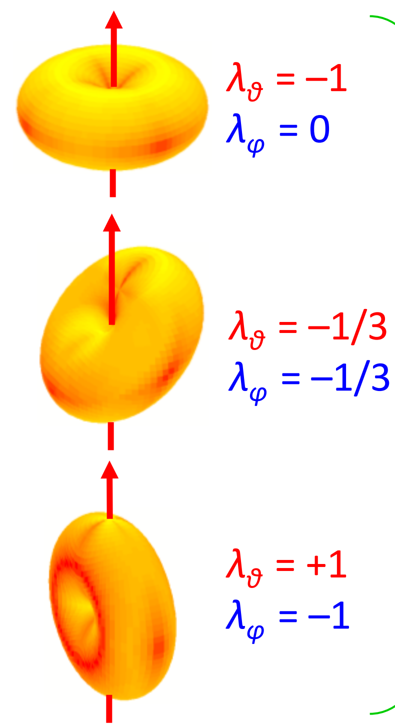
The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation)

→ it can be characterized by frame-independent parameters:

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} \qquad \lambda^* = \frac{\lambda_\vartheta - 3\Lambda^*}{1 + \Lambda^*} \quad \Lambda^* = \frac{1}{4}\left\{\lambda_\vartheta - \lambda_\varphi \pm \sqrt{\left(\lambda_\vartheta - \lambda_\varphi\right)^2 + 4\lambda_{\vartheta\varphi}^2}\right\} \qquad \tilde{\mathcal{A}} = \frac{\sqrt{A_\vartheta^2 + A_\varphi^2}}{3 + \lambda_\vartheta}$$



$$\tilde{\lambda} = +1$$



$$\tilde{\lambda} = -1$$

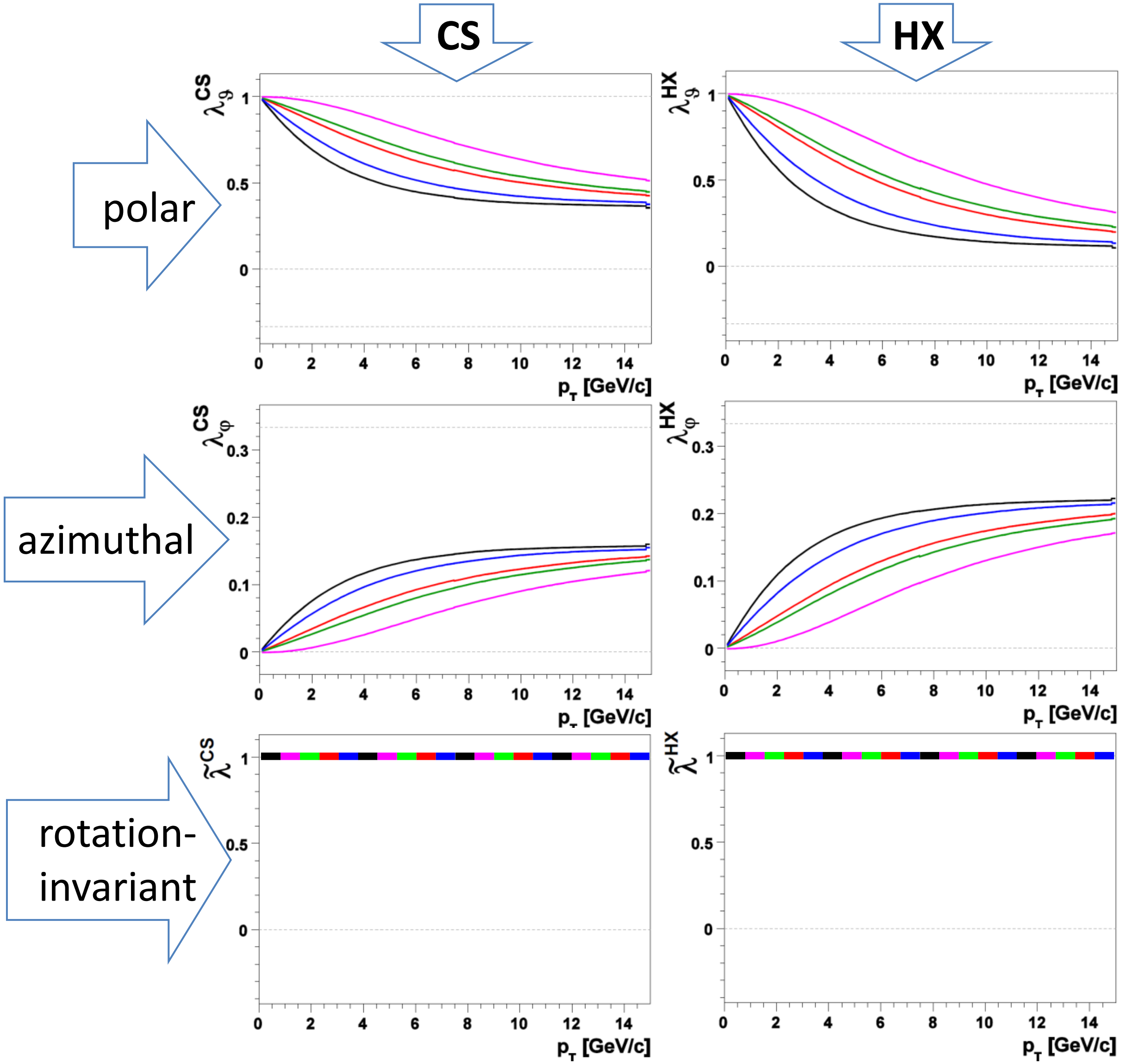
rotations *in the production plane*

Reduces acceptance dependence

Gedankenscenario: vector state produced in this subprocess admixture:

- **60%** processes with natural **transverse** polarization in the **CS** frame
- **40%** processes with natural **transverse** polarization in the **HX** frame

assumed indep. of kinematics, for simplicity



M = 10 GeV/c^2

CDF	**|y| < 0.6**
D0	**|y| < 1.8**
ATLAS/CMS	**|y| < 2.5**
ALICE e^+e^-	**|y| < 0.9**
ALICE $\mu^+\mu^-$	**2.5 < y < 4**
LHCb	**2 < y < 4.5**

- Immune to "extrinsic" kinematic dependencies

→ *less acceptance-dependent*

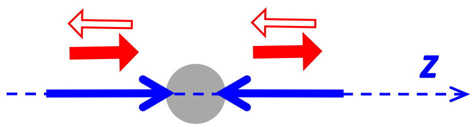
→ *facilitates comparisons*

- *useful as closure test*

Physical meaning: Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

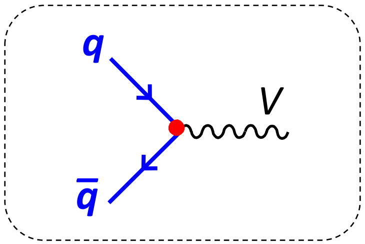
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

- ...but with respect to a ***subprocess-dependent quantization axis***

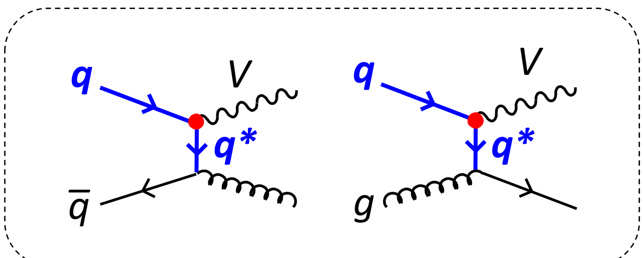
$O(\alpha_S^0)$



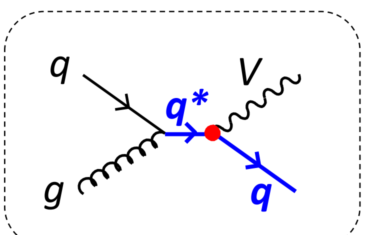
"natural" z = relative dir. of *q* and *q*bar
$\rightarrow \lambda_\vartheta(\text{"CS"}) = +1$

wrt ***any*** axis: $\tilde{\lambda} = +1$

$O(\alpha_S^1)$
(LO) QCD corrections



z = dir. of *one* incoming quark
$\rightarrow \lambda_\vartheta(\text{"GJ"}) = +1$
$\tilde{\lambda} = +1$



z = dir. of outgoing *q*
$\rightarrow \lambda_\vartheta(\text{"HX"}) = +1$
$\tilde{\lambda} = +1$

N.B.: $\tilde{\lambda} = +1$ in both pp-HX and qg-HX frames!

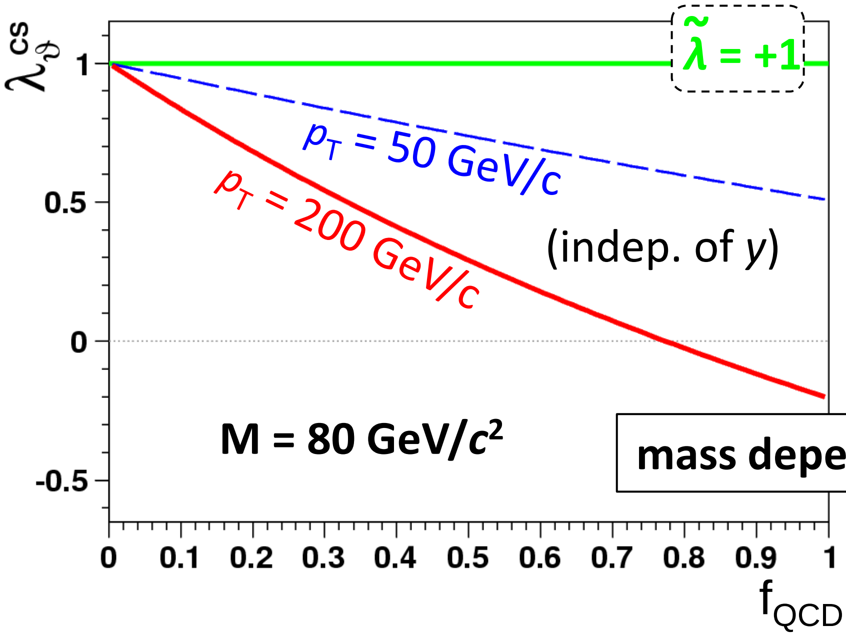
$\tilde{\lambda} = +1$
any frame

In all these cases the *q*-*q*-*V* lines are in the production plane (planar processes);
The CS, GJ, pp-HX and qg-HX axes only differ by a rotation in the production plane

λ_ϑ vs $\tilde{\lambda}$

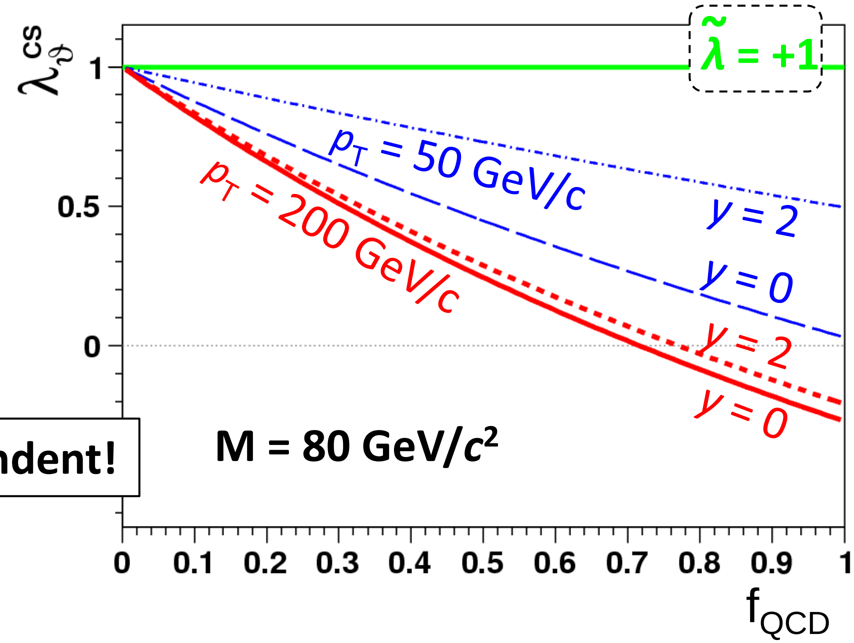
Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

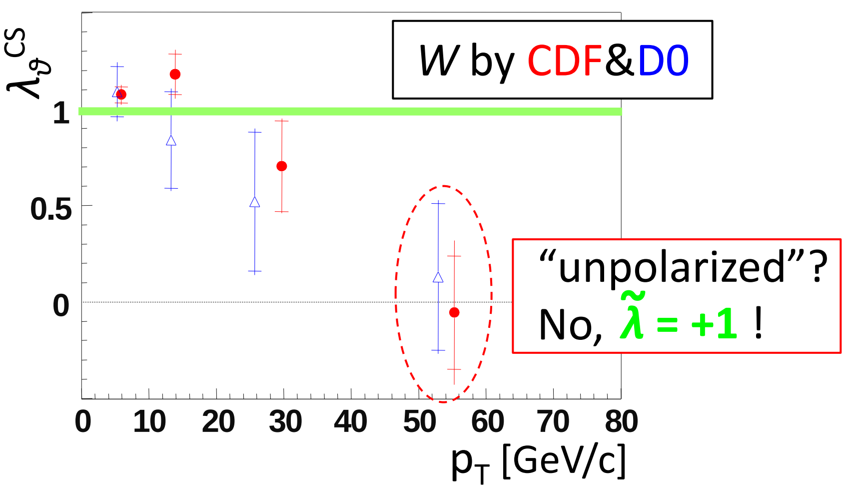
Case 1: dominating ***q-qbar*** QCD corrections



mass dependent!

Case 2: dominating ***q-g*** QCD corrections





λ_ϑ

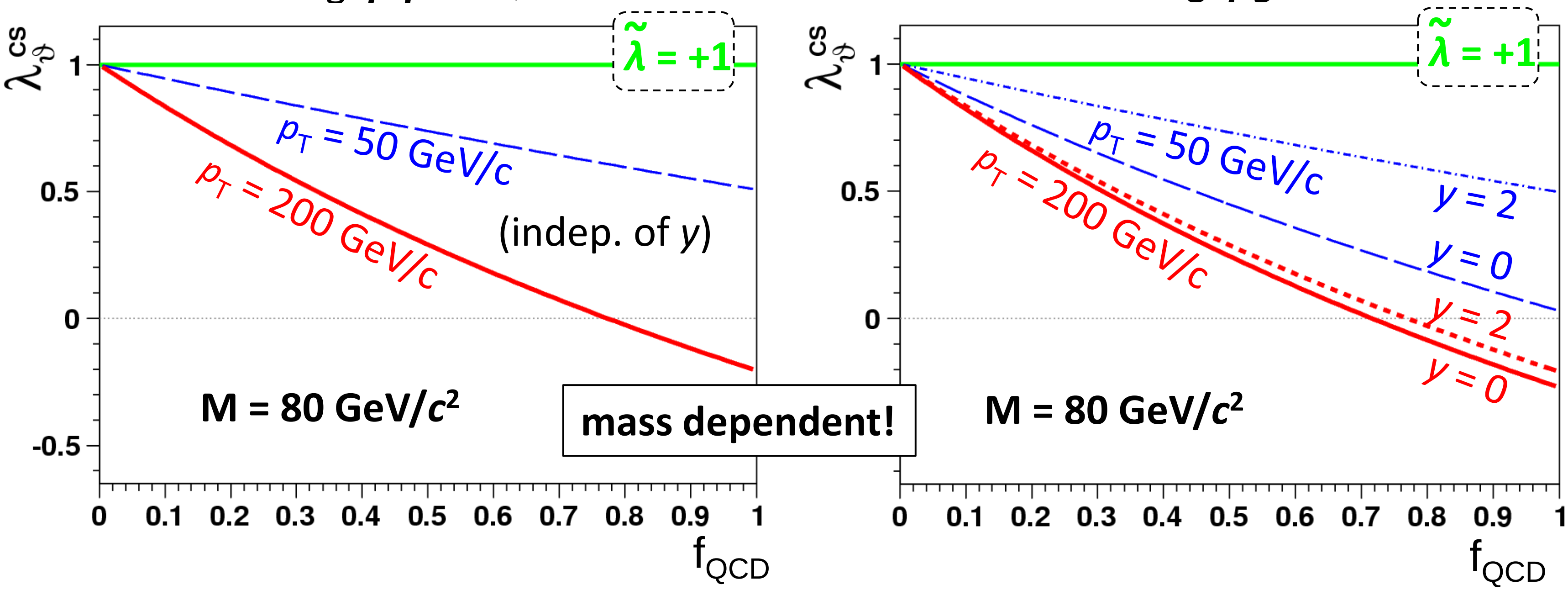
- depends on p_T , y and mass
  → by integrating we lose significance
- is far from being maximal
- depends on process admixture
  → need pQCD and PDFs

$\tilde{\lambda}$ is constant, maximal and independent of process admixture

λ_ϑ vs $\tilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating ***q-qbar*** QCD corrections   Case 2: dominating ***q-g*** QCD corrections



On the other hand, $\tilde{\lambda}$ forgets about the direction of the quantization axis.
This information is crucial if we want to **disentangle the *qg* contribution**,
the only one resulting in a ***rapidity-dependent*** $\boldsymbol{\lambda_\vartheta}$

Measuring λ_ϑ(CS) *as a function of rapidity* gives information on the gluon content of the proton

The Lam-Tung relation

A fundamental result of the theory of vector-boson polarizations (Drell-Yan, directly produced *Z* and *W*) is that, at leading order in **perturbative QCD**,

$$\lambda_\vartheta + 4\lambda_\varphi = 1$$ independently of the polarization frame

Lam-Tung relation, PRD 18, 2447 (1978)

This identity was considered as a surprising result of cancellations in the calculations

Today we know that it is only a *special* case of general frame-independent polarization relations, corresponding to a *transverse* intrinsic polarization:

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} = +1 \quad \Rightarrow \lambda_\vartheta + 4\lambda_\varphi = 1$$

It is, therefore, simply a consequence of

1) rotational invariance

2) properties of the quark-photon/Z/W coupling

Experimental tests of the LT relation **are not tests of QCD**!

Beyond the Lam-Tung relation

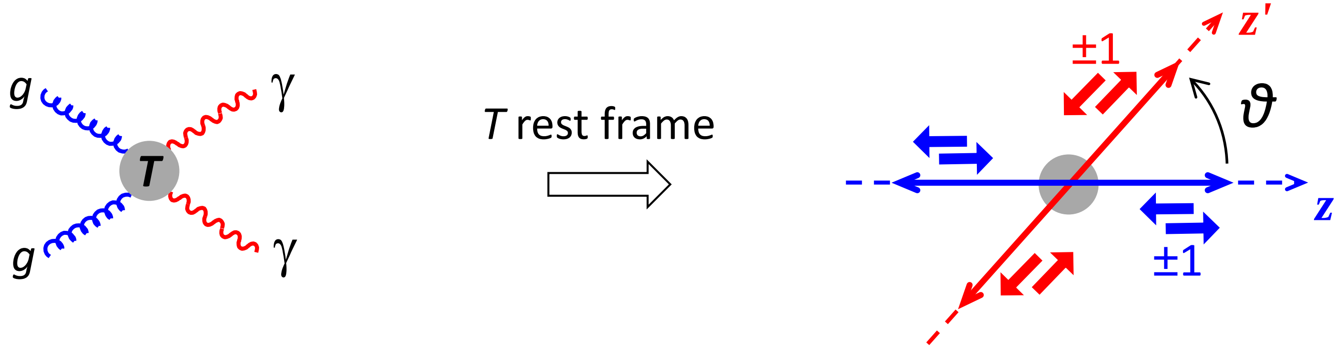
Even when the Lam-Tung relation is violated,
$\tilde{\lambda}$ can always be defined and is always frame-independent

$\tilde{\lambda} = +1$ → Lam-Tung. New interpretation: only ***vector boson – quark – quark*** couplings (in planar processes) $\rightarrow$ automatically verified in DY at QED & LO QCD levels and in several higher-order QCD contributions

$\tilde{\lambda} = +1 - \mathcal{O}(0.1)$ $\rightarrow +1 \text{ for } p_T \rightarrow 0$ → vector-boson – quark – quark couplings in ***non-planar* processes** (higher-order contributions)

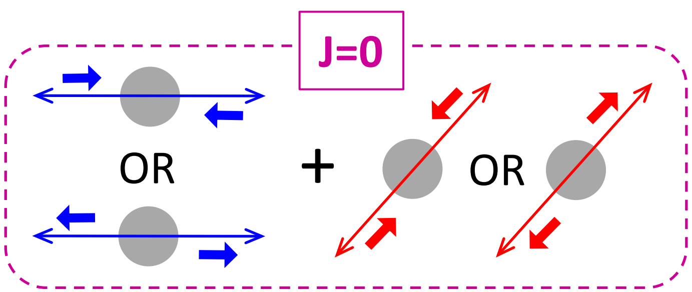
$\left.\begin{array}{l}\tilde{\lambda} \ll +1 \\ \tilde{\lambda} > +1\end{array}\right\}$ → contribution of ***different/new couplings or processes***
(e.g.: *Z* from Higgs, *W* from top, triple *ZZ*γ coupling,
higher-twist effects in DY production, etc…)

Spin characterization of the Higgs-like di-photon resonance



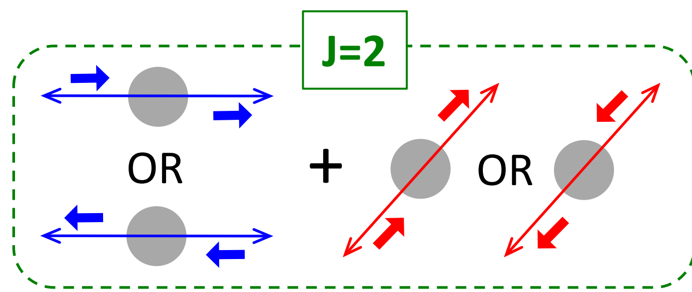
Usual approach to "determine" the J of *T*:
comparison between **J=0 hypothesis** and **ONE alternative hypothesis**. Example:

SM Higgs boson



$M_{gg} = 0$ $M_{\gamma\gamma} = 0$

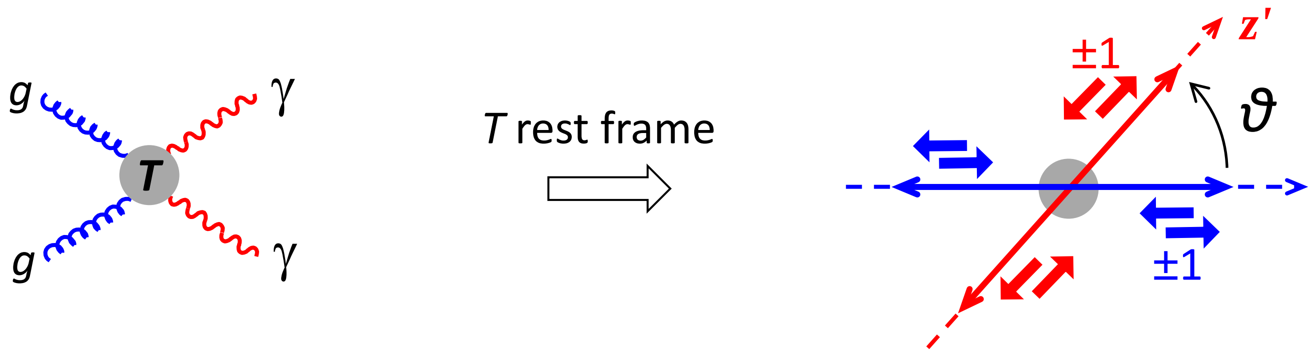
graviton with minimal-couplings to SM bosons (~ "boson helicity conservation")



M_{gg} = ~~0~~, ~~±1~~, ±2 $M_{\gamma\gamma}$ = ~~0~~, ~~±1~~, ±2

Decay distribution calculated case-by-case

Spin characterization of the Higgs-like di-photon resonance



Usual approach to "determine" the J of *T*:
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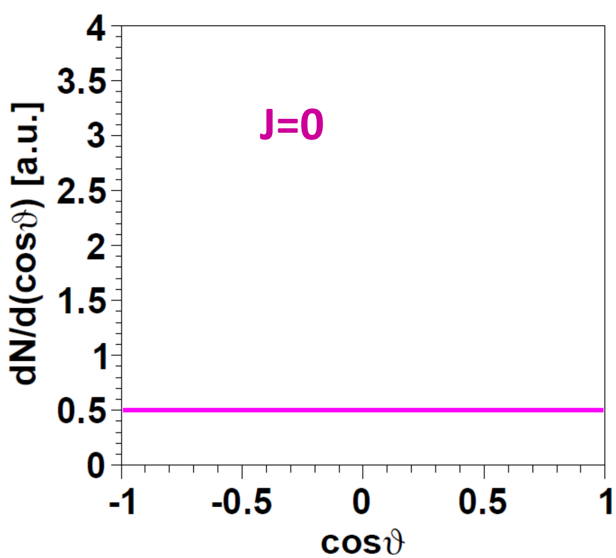
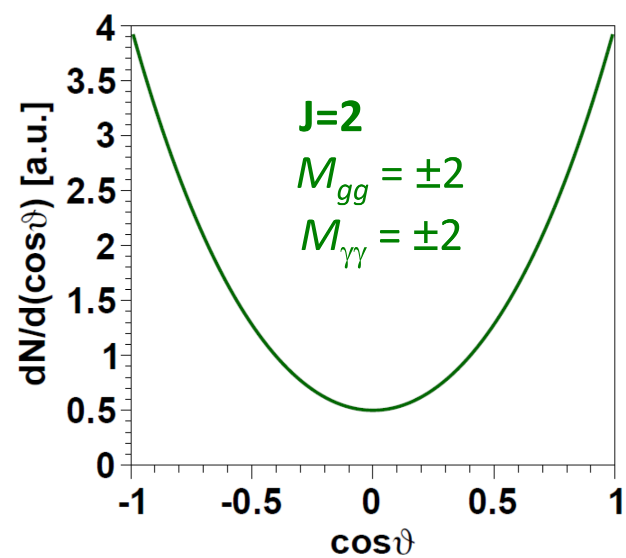


graviton with minimal-couplings to SM bosons (~ "boson helicity conservation")



Likelihood Ratio Approach

- **Method**:
  - measure distribution of the likelihood ratio between **hypothesis A** and **hypothesis B**

    $\mathscr{L}$[**B**] / $\mathscr{L}$[**A**] $\mathscr{L} \propto$ decay angular distribution
  - here **A = SM Higgs (J_A = 0)**, **B = a new-physics hypothesis (J_B)**

- **Ingredients** (for each set of A and B hypotheses):
  - the angular momentum quantum numbers J_A and J_B
  - the coupling properties of A and B to initial and final particles (gluons and photons)
  - calculations of the helicity amplitudes for the production and decay processes

- **Question** addressed:
  - is the observed resonance more likely to be particle A or particle B?
- The **answer**
  - may be given unhesitatingly, i.e. $\mathscr{L}$[**A**] >> $\mathscr{L}$[**B**], even when neither A nor B coincide with the correct hypothesis
  - is never conclusive until the whole set of possible models for A and B is explored.
    Do we know this set of *models* in a totally *model-independent* way?
    As a matter of fact, a very restricted set of "B" models is currently considered

MPC approach

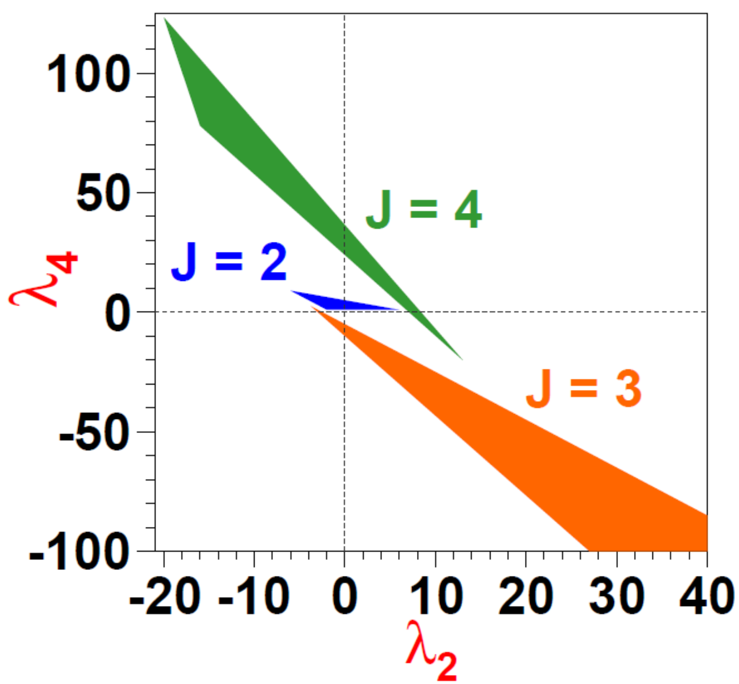
- **Method**:

MPC = Minimal Physical Constraints

  - measure the angular distribution

$$\frac{dN}{d\Omega} \propto 1 + \lambda_2 \cos^2\vartheta + \lambda_4 \cos^4\vartheta + \lambda_6 \cos^6\vartheta + \ldots + \lambda_N \cos^N\vartheta$$

- **Ingredients**:
  - angular momentum conservation
  - initial gluons and final photons are transversely polarized
  - no hypothesis on J nor on couplings, no explicit calculations of helicity amplitudes



[J=1 hypothesis forbidden by Landau-Yang theorem]

The general physical parameter domains of the J=2, 3 and 4 cases are mutually exclusive!

MPC approach

- **Method**: MPC = Minimal Physical Constraints
  - measure the angular distribution

$$\frac{dN}{d\Omega} \propto 1 + \lambda_2 \cos^2\vartheta + \lambda_4 \cos^4\vartheta + \lambda_6 \cos^6\vartheta + \ldots + \lambda_N \cos^N\vartheta$$

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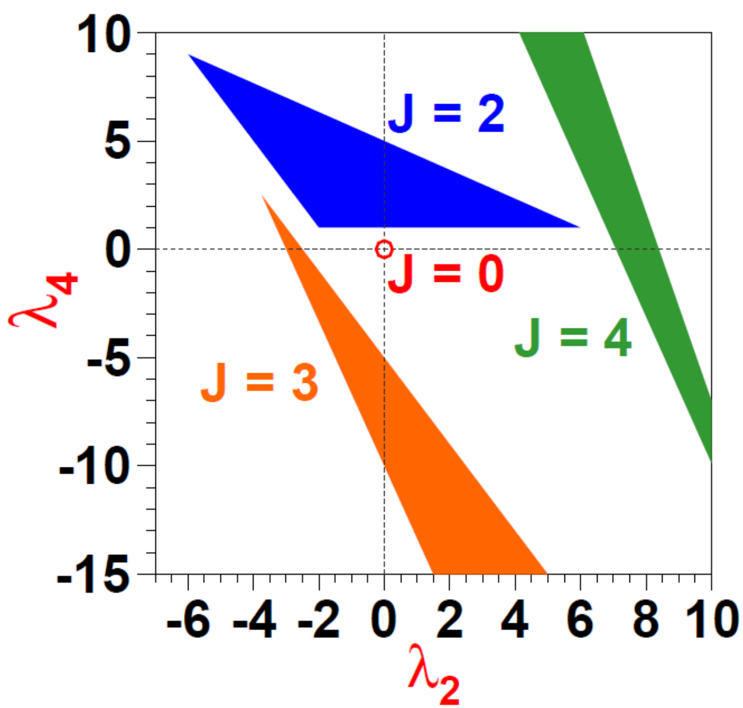


[J=1 hypothesis forbidden by Landau-Yang theorem]

The general physical parameter domains of the J=2, 3 and 4 cases are mutually exclusive!

And do not include the origin (J=0)!

MPC approach

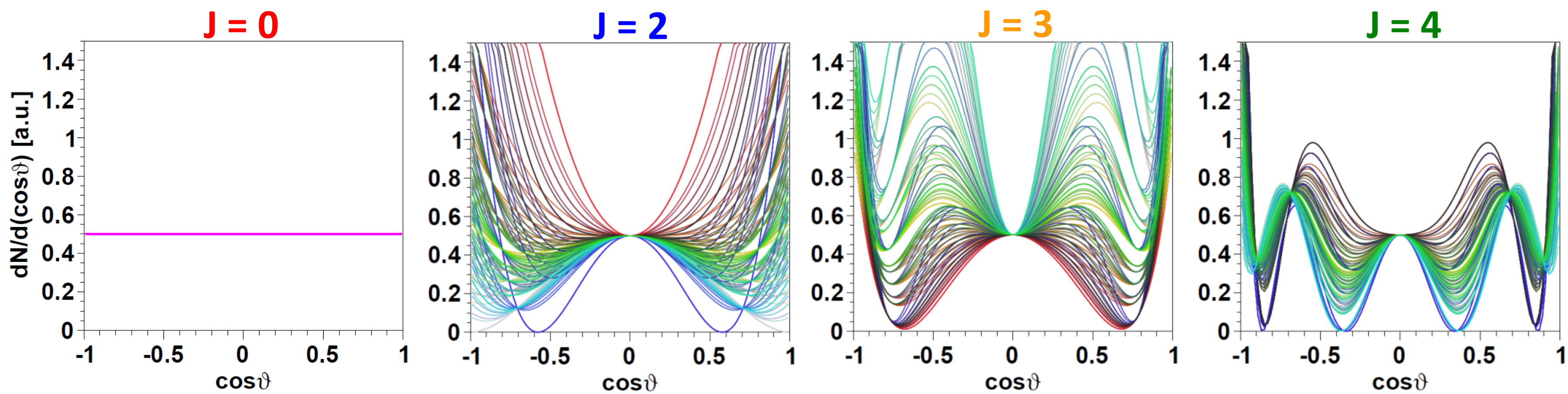
MPC = Minimal Physical Constraints

- **Method**:
  - measure the angular distribution

$$\frac{dN}{d\Omega} \propto 1 + \lambda_2 \cos^2\vartheta + \lambda_4 \cos^4\vartheta + \lambda_6 \cos^6\vartheta + \ldots + \lambda_N \cos^N\vartheta$$

- **Ingredients**:
  - angular momentum conservation
  - Initial gluons and final photons are transversely polarized
  - no hypothesis on J nor on couplings, no explicit calculations of helicity amplitudes

The cosϑ distribution discriminates the spin univocally:



MPC approach

MPC = Minimal Physical Constraints

- **Method**:
  - measure the angular distribution

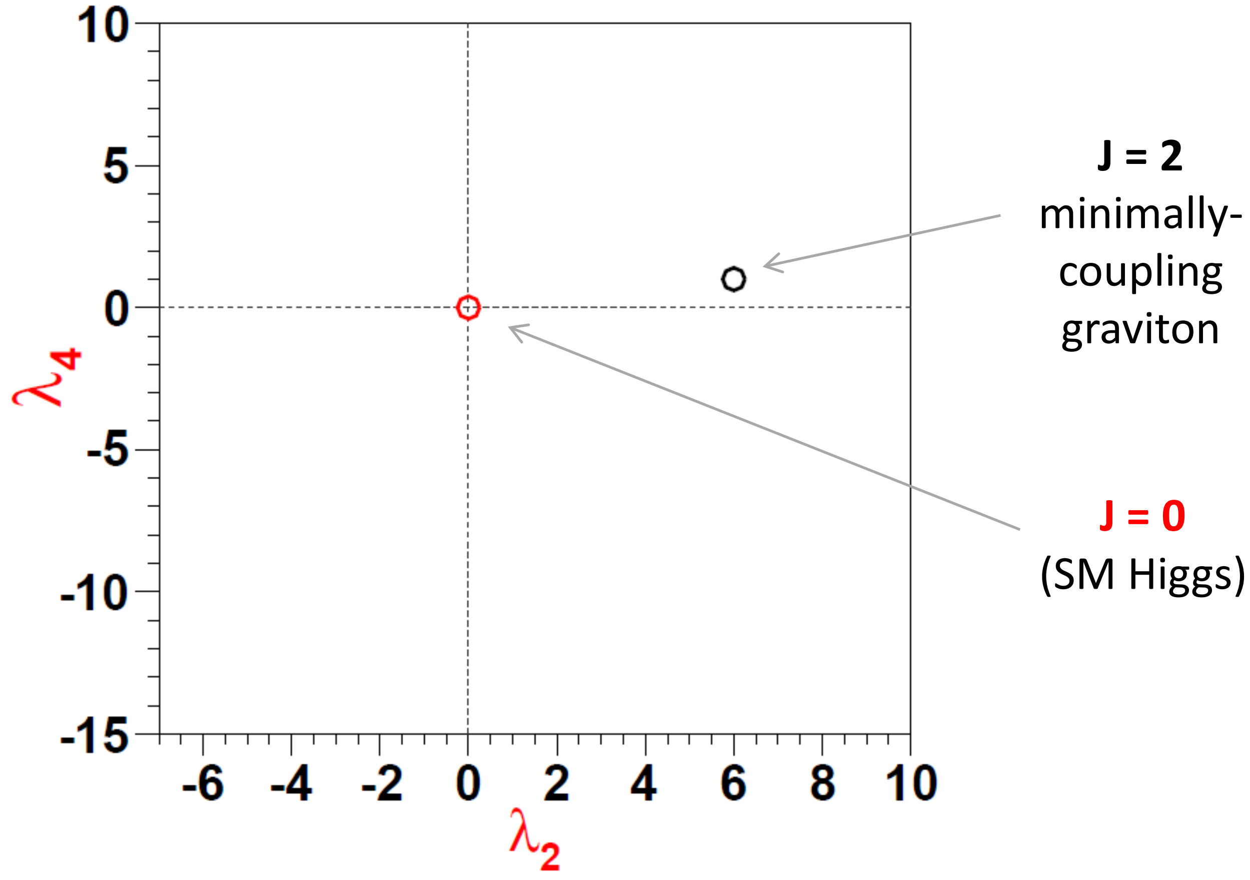
$$\frac{dN}{d\Omega} \propto 1 + \lambda_2 \cos^2\vartheta + \lambda_4 \cos^4\vartheta + \lambda_6 \cos^6\vartheta + \ldots + \lambda_N \cos^N\vartheta$$

- **Ingredients**:
  - angular momentum conservation
  - Initial gluons and final photons are transversely polarized
  - no hypothesis on J nor on couplings, no explicit calculations of helicity amplitudes

- This method directly addresses the **question**:
  - how much is J?
- The **answer**
  - is model-independent and can be compared to any theory
  - is always conclusive, if the measurement is sufficiently precise

LR vs MPC

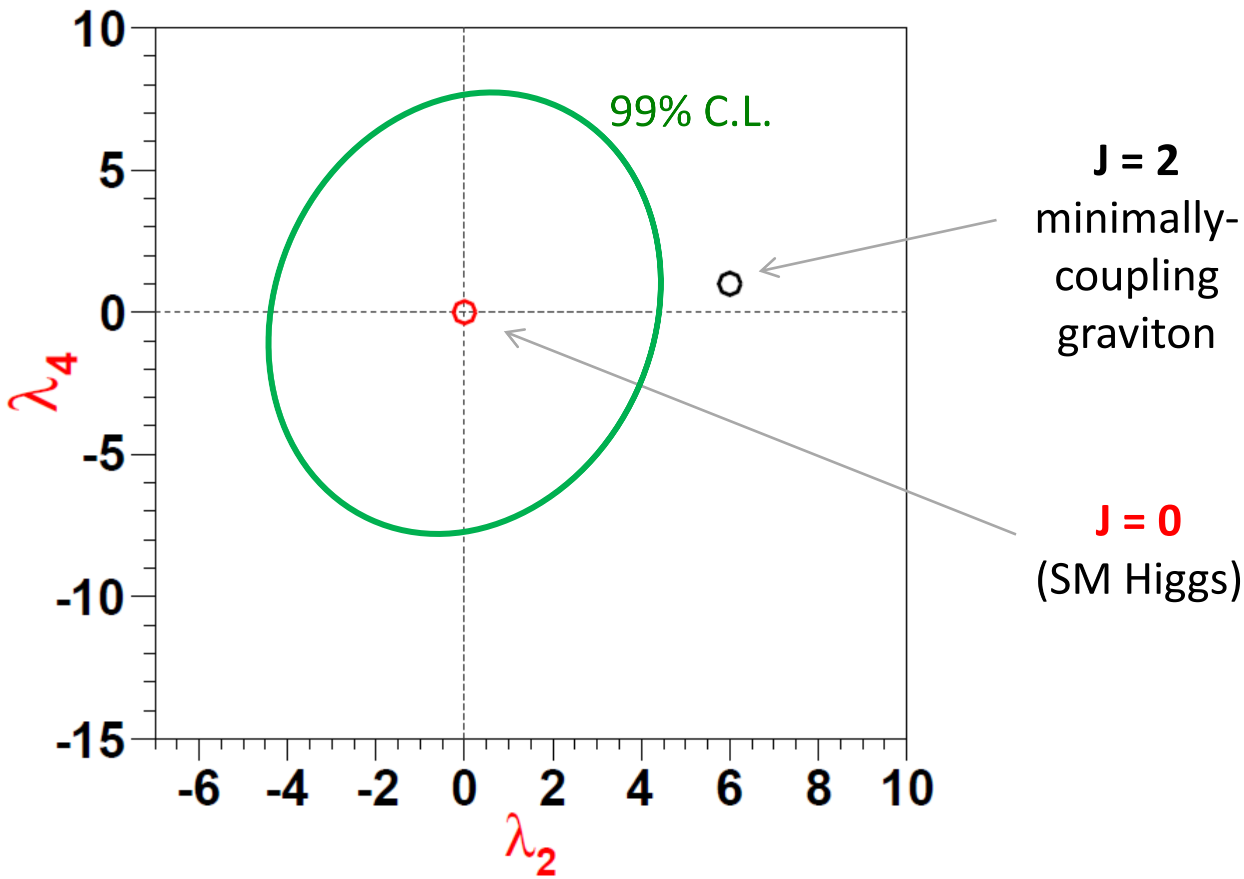
The binary strategy of the LR approach aims at discriminating between two hypotheses:



LR vs MPC

The binary strategy of the LR approach aims at discriminating between two hypotheses:

From this point of view, **this measurement** would correspond to a J=0 characterization

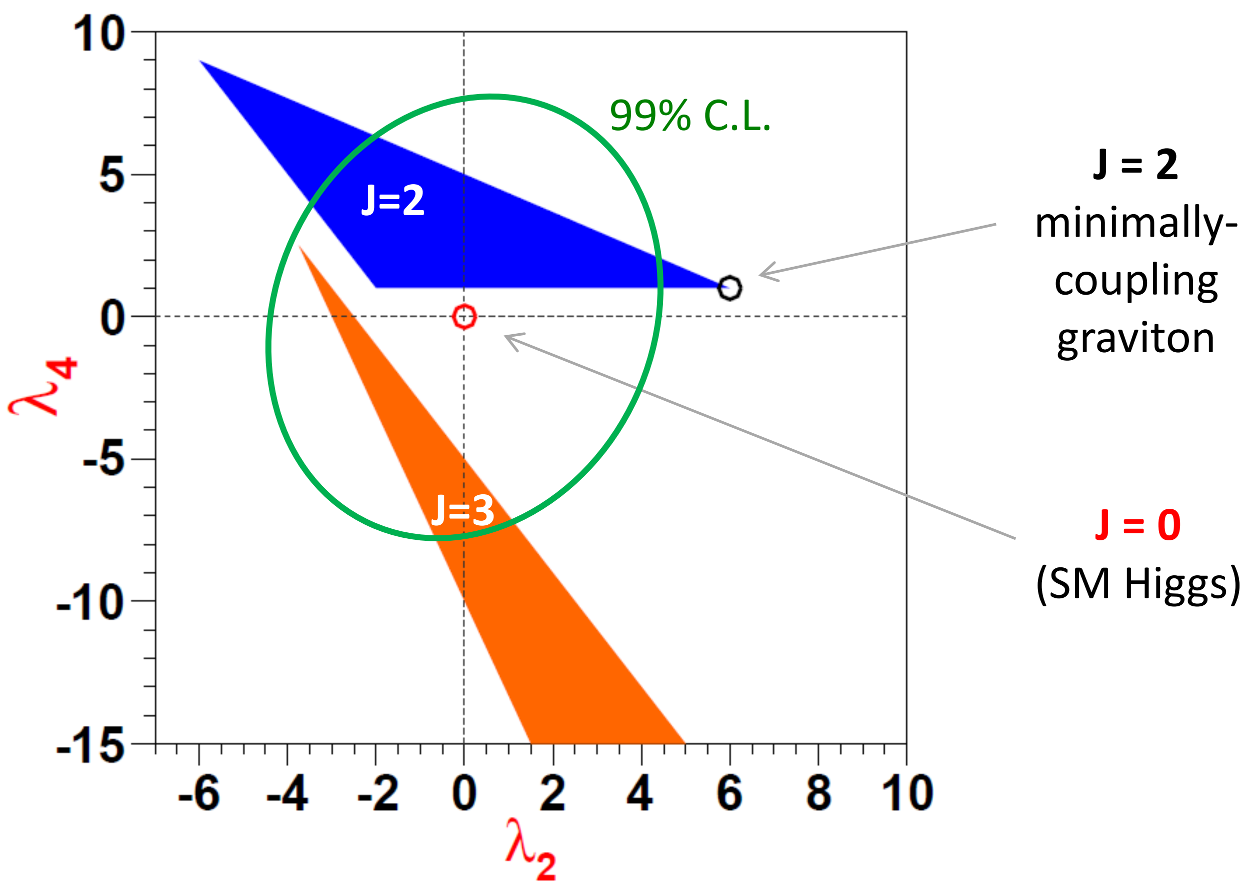


LR vs MPC

The binary strategy of the LR approach aims at discriminating between two hypotheses:

From this point of view, **this measurement** would correspond to a J=0 characterization

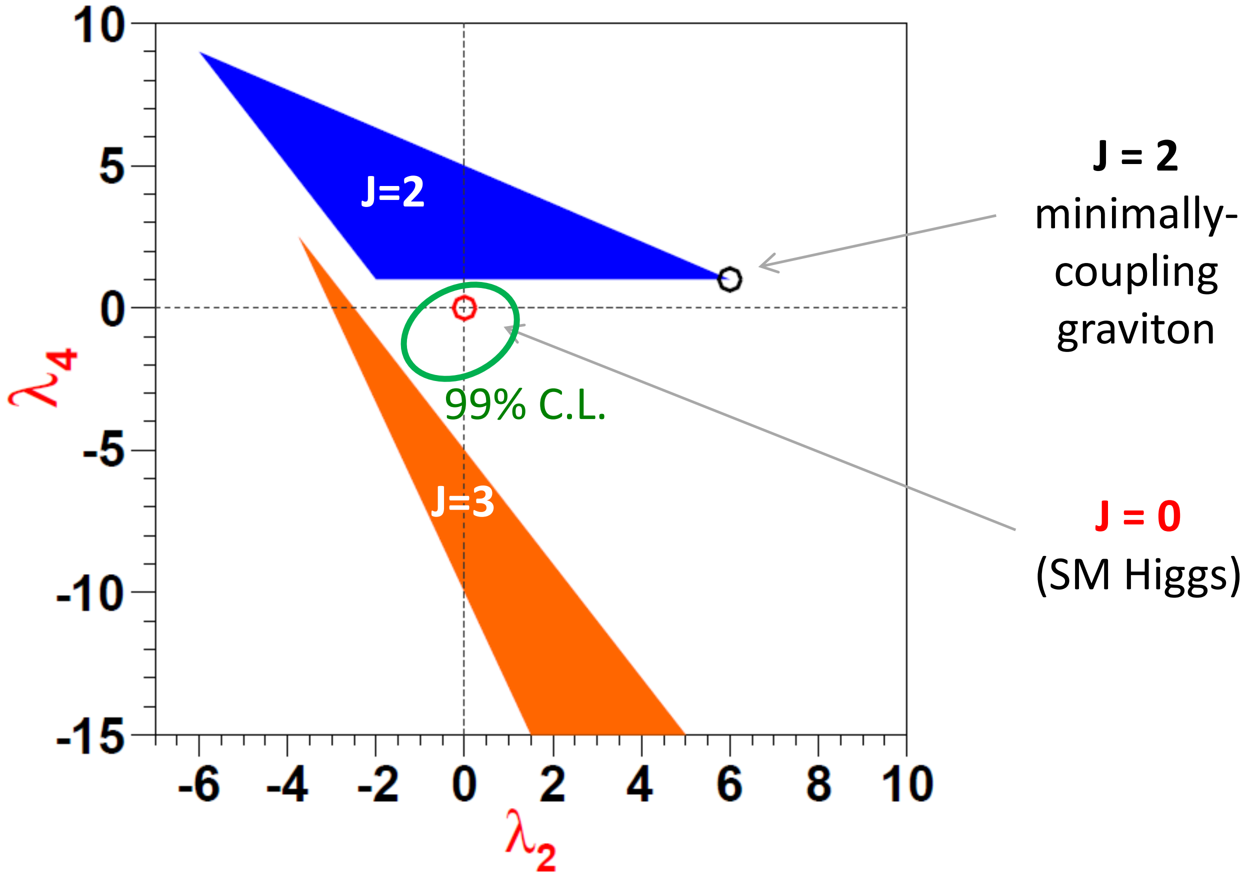
In the MPC approach it would exclude all models lying outside the ellipse, but it would not exclude **J=2**, nor **J=3**!



LR vs MPC

The binary strategy of the LR approach aims at discriminating between two hypotheses:

In the MPC approach **this measurement** would represent an *unequivocal* spin-0 characterization



Further reading

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