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Rotation-invariant observables in parity-violating decays of vector particles to fermion pairs

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The di-fermion angular distribution observed in decays of inclusively produced vector particles is characterized by two frame-independent observables, reflecting the average spin-alignment of the produced particle and the magnitude of parity violation in the decay. The existence of these observables derives from the rotational properties of angular momentum eigenstates and is a completely general result, valid for any $J = 1$ state and independent of the production process. Rotation-invariant formulations of polarization and of the decay parity-asymmetry can provide more significant measurements than the commonly used frame-dependent definitions, also improving the quality of the comparisons between the measurements and the theoretical calculations.

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I. INTRODUCTION

Experimental studies of the decay angular distributions of vector particles represent a detailed way of testing fundamental theories. Quarkonium polarization measurements, in particular, are expected to provide key information for the understanding of quantum chromodynamics (QCD) [1], with competing production mechanisms leading to very different polarization predictions. Similarly, studies of the parity violating decays of heavy gauge bosons in a fermion-antifermion pair, such as $Z/\gamma^* \to \ell\ell$ and $W \to \ell\nu$, provide detailed tests of the electroweak theory, accurate determinations of the effective weak mixing angle, or lead to the discovery of new vector bosons coupling to fermion pairs [2]. The outcome of such studies can be limited — especially when the production mechanism of the particles under study is unknown — by the choice of specific polarization frames, which may result in misleading interpretations of polarization measurements, prevent model-independent physical conclusions, or lead to an effective reduction of the maximally observable parity-violation asymmetry.

In this paper we show how the rotation-covariance properties of angular momentum eigenstates imply the existence of a frame-invariant relation among the parameters characterizing the polar and azimuthal anisotropies of the di-fermion decay distribution of inclusively observed vector particles. This relation has been previously established in the case of parity-conserving decays [3] and its advantages in the studies of J/ψ and Υ polarizations have been specifically addressed in Refs. [4] and [5]. Here we show that the frame-invariant relation remains valid for any $J = 1$ particle decaying into a fermion-antifermion pair, even in the presence of parity-violating effects. Besides being useful in studies of the properties of W and Z bosons, this relation can be particularly relevant for the interpretation of new dilepton signals, where the production mechanisms and polarizations of the candidate bosons are, a priori, unknown. Besides improving the measurement of the spin alignment of the decaying particle, the adoption of a frame-independent perspective also provides more significant measurements of the parity asymmetry of the decay. Furthermore, with respect to a multidimensional analysis determining the frame-dependent angular coefficients and their correlated experimental errors, the rotation-invariant definitions of spin alignment and of parity asymmetry also provide an easier way of assessing the significance of the global anisotropy of the decay distributions. Finally, frame-independent results are less sensitive to specific choices of polarization frames or other experimental constraints.

II. DECAY ANGULAR DISTRIBUTION OF VECTOR PARTICLES IN FERMION PAIRS

We start by considering the most general angular distribution of the di-fermion decay, expressed keeping track of the average angular momentum composition of the decaying particle. First we address the case of a single production "subprocess", denoted by the index (i), in which the vector particle V is always formed as a specific superposition of the three J_z eigenstates, with eigenvalues $m = +1, -1, 0$, with respect to a chosen axis z,

$$|V^{(i)}\rangle = b_{+1}^{(i)}\,|+1\rangle + b_{-1}^{(i)}\,|-1\rangle + b_0^{(i)}\,|0\rangle\,. \tag{1}$$

The decay angular distribution is described in the V rest frame, where the common direction of the two leptons define the reference axis z'. We denote by ϑ and φ the (polar and azimuthal) angles formed by one of the two fermions with, respectively, the polarization axis z and the xz plane. For a sufficiently massive decaying particle, the fermions can be considered massless and helicity is conserved. The $|V^{(i)}\rangle$ decay angular distribution is, then,

$$W^{(i)}(\cos\vartheta,\varphi) \,\propto\, \frac{\mathcal{N}^{(i)}}{(3+\lambda_\vartheta^{(i)})}\,(1 + \lambda_\vartheta^{(i)}\cos^2\vartheta \, +$$

$$+\lambda_\varphi^{(i)}\sin^2\vartheta\cos 2\varphi+\lambda_{\vartheta\varphi}^{(i)}\sin 2\vartheta\cos\varphi \qquad (2)$$
$$+\lambda_\varphi^{\perp(i)}\sin^2\vartheta\sin 2\varphi+\lambda_{\vartheta\varphi}^{\perp(i)}\sin 2\vartheta\sin\varphi$$
$$+2A_\vartheta^{(i)}\cos\vartheta+2A_\varphi^{(i)}\sin\vartheta\cos\varphi+2A_\varphi^{\perp(i)}\sin\vartheta\sin\varphi)\,.$$

The subprocess coefficients depend on the partial amplitudes, $a_{m\kappa}^{(i)}$, corresponding to specific configurations of the decaying state and of the two-fermion system,

$$\begin{aligned}
\lambda_\vartheta^{(i)} &= 1/\mathcal{D}^{(i)}\,[\mathcal{N}^{(i)}-3(|a_{0,+1}^{(i)}|^2+|a_{0,-1}^{(i)}|^2)]\,,\\
\lambda_\varphi^{(i)} &= 2/\mathcal{D}^{(i)}\,\mathrm{Re}(a_{+1,+1}^{(i)*}a_{-1,+1}^{(i)}+a_{+1,-1}^{(i)*}a_{-1,-1}^{(i)})\,,\\
\lambda_{\vartheta\varphi}^{(i)} &= \sqrt{2}/\mathcal{D}^{(i)}\,\mathrm{Re}[a_{0,+1}^{(i)*}(a_{+1,+1}^{(i)}-a_{-1,+1}^{(i)})\\
&\qquad +a_{0,-1}^{(i)*}(a_{+1,-1}^{(i)}-a_{-1,-1}^{(i)})]\,,\\
\lambda_\varphi^{\perp(i)} &= 2/\mathcal{D}^{(i)}\,\mathrm{Im}(a_{+1,+1}^{(i)*}a_{-1,+1}^{(i)}+a_{+1,-1}^{(i)*}a_{-1,-1}^{(i)})\,,\\
\lambda_{\vartheta\varphi}^{\perp(i)} &= -\sqrt{2}/\mathcal{D}^{(i)}\,\mathrm{Im}[a_{0,+1}^{(i)*}(a_{+1,+1}^{(i)}+a_{-1,+1}^{(i)})\\
&\qquad +a_{0,-1}^{(i)*}(a_{+1,-1}^{(i)}+a_{-1,-1}^{(i)})]\,, \qquad (3)\\
A_\vartheta^{(i)} &= 1/\mathcal{D}^{(i)}\,(|a_{+1,+1}^{(i)}|^2+|a_{-1,-1}^{(i)}|^2\\
&\qquad -|a_{+1,-1}^{(i)}|^2-|a_{-1,+1}^{(i)}|^2)\,,\\
A_\varphi^{(i)} &= \sqrt{2}/(2\mathcal{D}^{(i)})\,\mathrm{Re}[a_{0,+1}^{(i)*}(a_{+1,+1}^{(i)}+a_{-1,+1}^{(i)})\\
&\qquad -a_{0,-1}^{(i)*}(a_{+1,-1}^{(i)}+a_{-1,-1}^{(i)})]\,,\\
A_\varphi^{\perp(i)} &= -\sqrt{2}/(2\mathcal{D}^{(i)})\,\mathrm{Im}[a_{0,+1}^{(i)*}(a_{+1,+1}^{(i)}-a_{-1,+1}^{(i)})\\
&\qquad -a_{0,-1}^{(i)*}(a_{+1,-1}^{(i)}-a_{-1,-1}^{(i)})]\,,
\end{aligned}$$

where m is the J_z component of V, $\kappa=\pm1$ is the total angular momentum projection of the two-fermion system along the fermion momentum direction in the V rest fame, $\mathcal{N}^{(i)}=\sum_{m,\kappa}|a_{m\kappa}^{(i)}|^2$, and $\mathcal{D}^{(i)}=\mathcal{N}^{(i)}+|a_{0,+1}^{(i)}|^2+|a_{0,-1}^{(i)}|^2$. In the special case of parity-conserving decays $a_{m,+1}^{(i)}=a_{m,-1}^{(i)}$ and, hence, $A_\vartheta^{(i)}=A_\varphi^{(i)}=A_\varphi^{\perp(i)}=0$.

In the presence of n contributing production processes with weights $f^{(i)}$, the resulting observable distribution has a general expression formally analogous to Eq. 2:

$$W(\cos\vartheta,\varphi)\;\propto\;\frac{1}{(3+\lambda_\vartheta)}\,(1+\lambda_\vartheta\cos^2\vartheta$$
$$+\lambda_\varphi\sin^2\vartheta\cos 2\varphi+\lambda_{\vartheta\varphi}\sin 2\vartheta\cos\varphi \qquad (4)$$
$$+\lambda_\varphi^\perp\sin^2\vartheta\sin 2\varphi+\lambda_{\vartheta\varphi}^\perp\sin 2\vartheta\sin\varphi$$
$$+2A_\vartheta\cos\vartheta+2A_\varphi\sin\vartheta\cos\varphi+2A_\varphi^\perp\sin\vartheta\sin\varphi)\,.$$

The observable coefficients, $X=\lambda_\vartheta$, λ_φ, $\lambda_{\vartheta\varphi}$, $\lambda_\varphi^\perp$, $\lambda_{\vartheta\varphi}^\perp$, A_ϑ, A_φ and $A_\varphi^\perp$, are weighted averages of the corresponding single-subprocess parameters, $X^{(i)}$,

$$X\;=\;\sum_{i=1}^{n}\frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}}\,X^{(i)}\Bigg/\sum_{i=1}^{n}\frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}}\;. \qquad (5)$$

While the most general form of the decay distribution is always given by Eq. 4 for any choice of polarization frame, the coefficients depend on this choice. In this paper we only consider inclusive production and, therefore, the only sensible experimental definition of the xz plane is the production plane, containing the directions of the colliding particles and of the decaying particle itself. A change of polarization frame is thus a rotation in the production plane, parametrized by one angle, δ. The corresponding transformations of the coefficients are

$$\begin{aligned}
\lambda_\vartheta^\prime &= \frac{\lambda_\vartheta-3\Lambda}{1+\Lambda}\,,\quad \lambda_\varphi^\prime=\frac{\lambda_\varphi+\Lambda}{1+\Lambda}\,,\\
\lambda_{\vartheta\varphi}^\prime &= \frac{\lambda_{\vartheta\varphi}\cos 2\delta-\frac{1}{2}\,(\lambda_\vartheta-\lambda_\varphi)\sin 2\delta}{1+\Lambda}\,,\\
\lambda_\varphi^{\perp\prime} &= \frac{\lambda_\varphi^\perp\cos\delta-\lambda_{\vartheta\varphi}^\perp\sin\delta}{1+\Lambda}\,,\\
\lambda_{\vartheta\varphi}^{\perp\prime} &= \frac{\lambda_\varphi^\perp\sin\delta+\lambda_{\vartheta\varphi}^\perp\cos\delta}{1+\Lambda}\,, \qquad (6)\\
A_\vartheta^\prime &= \frac{A_\vartheta\cos\delta+A_\varphi\sin\delta}{1+\Lambda}\,,\\
A_\varphi^\prime &= \frac{-A_\vartheta\sin\delta+A_\varphi\cos\delta}{1+\Lambda}\,,\quad A_\varphi^{\perp\prime}=\frac{A_\varphi^\perp}{1+\Lambda}\,,\\
\text{with}\quad \Lambda &= \frac{1}{2}\,(\lambda_\vartheta-\lambda_\varphi)\sin^2\delta-\frac{1}{2}\,\lambda_{\vartheta\varphi}\sin 2\delta\,.
\end{aligned}$$

In this paper we focus on quantities which are invariant under this transformation. In fact, from Eq. 6 it is immediate to derive the following covariance relations:

$$\begin{aligned}
3+\lambda_\vartheta^\prime &= \frac{3+\lambda_\vartheta}{1+\Lambda}\,,\\
1-\lambda_\varphi^\prime &= \frac{1-\lambda_\varphi}{1+\Lambda}\,,\\
\sqrt{\lambda_\varphi^{\perp\prime 2}+\lambda_{\vartheta\varphi}^{\perp\prime 2}} &= \frac{\sqrt{\lambda_\varphi^{\perp 2}+\lambda_{\vartheta\varphi}^{\perp 2}}}{1+\Lambda}\,, \qquad (7)\\
\sqrt{A_\vartheta^{\prime 2}+A_\varphi^{\prime 2}+A_\varphi^{\perp\prime 2}} &= \frac{\sqrt{A_\vartheta^2+A_\varphi^2+A_\varphi^{\perp 2}}}{1+\Lambda}\,,\\
\sqrt{A_\vartheta^{\prime 2}+A_\varphi^{\prime 2}} &= \frac{\sqrt{A_\vartheta^2+A_\varphi^2}}{1+\Lambda}\,.
\end{aligned}$$

Naturally, relations linearly depending on these are also covariant. The ratio of any two of the covariant quantities defined by these relations is independent of the chosen polarization frame. Certain specific frame-independent quantities are discussed in the following sections.

In the absence of parity-violating processes, the coefficients A_ϑ, A_φ and $A_\varphi^\perp$ are zero. Besides, the observed *event distribution* must be symmetric by reflection with respect to the production plane, even if the "natural" polarization plane does not coincide *event-by-event* with the production plane. Therefore, also $\lambda_\varphi^\perp$ and $\lambda_{\vartheta\varphi}^\perp$, multiplying terms that are asymmetric by the reflection transformation $\varphi\to\pi+\varphi$, are unobservable, because they vanish on average. In the more general case when parity is not conserved, as in $Z/\gamma^*\to\ell\ell$ and $W\to\ell\nu$, event-by-event topological asymmetries with respect to the experimental production plane can result in non-zero values of the observed $\lambda_\varphi^\perp$ and $\lambda_{\vartheta\varphi}^\perp$, even if these coefficients, as well as $A_\varphi^\perp$, are expected to remain negligible because of the kinematic averaging. In summary, in inclusive-production studies the observable di-fermion distribution is essentially described by five coefficients: the three "spin-alignment" parameters λ_ϑ, λ_φ and $\lambda_{\vartheta\varphi}$, containing information about the average angular momentum composition of the decaying particle, and the two "parity-asymmetry" parameters A_ϑ and A_φ, expressing the parity properties of the decay.

The parity-conserving part of the distribution can be written as a quadratic form in the momentum coordinates p_x, p_y, p_z of the lepton,

$$1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi \propto \begin{pmatrix} p_x & p_y & p_z \end{pmatrix} \begin{pmatrix} 1+\lambda_\varphi & 0 & \lambda_{\vartheta\varphi} \\ 0 & 1-\lambda_\varphi & 0 \\ \lambda_{\vartheta\varphi} & 0 & 1+\lambda_\vartheta \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}, \quad (8)$$

showing that the $\lambda_{\vartheta\varphi}$ term is non-zero when the axes of the reference frame do not coincide with the principal axes of symmetry of the parity-conserving distribution. There always exists a polarization frame where $\lambda_{\vartheta\varphi} = 0$ and the parity-conserving part of the decay distribution is "diagonal". In this frame, the shape of the decay distribution is univocally described by the parameters λ_ϑ, λ_φ, A_ϑ and A_φ. These parameters are always bound, in any frame, between -1 and $+1$, while $|\lambda_{\vartheta\varphi}| \leq \sqrt{2}/2$.

III. ROTATION-INVARIANT POLARIZATION OBSERVABLE

The rotation-covariance properties of the generic $J = 1$ state defined in Eq. 1 imply that each amplitude combination $b_{+1}^{(i)} + b_{-1}^{(i)}$ is invariant by rotation about the y axis [6]. In terms of decay amplitudes, this means that the combinations $a_{+1,\kappa}^{(i)} + a_{-1,\kappa}^{(i)}$ (with $\kappa = \pm 1$) are invariant by rotation about y. The "normalization" $\sum_{m,\kappa} |a_{m\kappa}^{(i)}|^2$ is obviously invariant under any rotation. In inclusive production studies, all experimentally definable polarization axes belong to the production plane, implying that a change of polarization frame means a rotation about y. Hence, for each subprocess, the quantity

$$\mathcal{F}^{(i)} = \frac{|a_{+1,+1}^{(i)} + a_{-1,+1}^{(i)}|^2 + |a_{+1,-1}^{(i)} + a_{-1,-1}^{(i)}|^2}{2 \sum_{m,\kappa} |a_{m\kappa}^{(i)}|^2} \quad (9)$$

(bound between 0 and 1) is independent of the chosen polarization frame. Using also Eqs. 3 and 5, we find that the combination of observable parameters

$$\mathcal{F} = \frac{\sum_{i=1}^n f^{(i)} \mathcal{N}^{(i)} \mathcal{F}^{(i)}}{\sum_{i=1}^n f^{(i)} \mathcal{N}^{(i)}} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta} \quad (10)$$

is frame-invariant. This observable invariant polarization parameter, $\mathcal{F}$, is the average of the corresponding sub-process invariants, calculated irrespectively of the directions of the n polarization axes.

For example, $\mathcal{F} = 0$ when all sub-process distributions are of the kind $1 - \cos^2\vartheta^{(i)}$, with respect to n specific polarization axes $z^{(i)}$ (belonging to the production plane). When the individual distributions have the shapes $1 + \cos^2\vartheta^{(i)}$, $\mathcal{F} = 1/2$. When the observed distribution is isotropic, $\mathcal{F} = 1/3$. We note that $\mathcal{F}$ does not depend on the spin-alignment parameter $\lambda_{\vartheta\varphi}$. In fact, while $\lambda_{\vartheta\varphi}$ can be interpreted as a measure of the "tilt" of the chosen system of axes with respect to the principal axes of symmetry of the distribution, $\mathcal{F}$ represents an intrinsic (rotation-independent) characteristic of the *shape* of the angular distribution.

The existence of a frame-independent polarization observable is a completely general result, valid for the decay of any vector particle into fermion pairs, irrespectively of its production mechanism.

As a special case, it is easy to recognize that Eq. 10 resembles the "Lam-Tung relation" [6], $\lambda_\vartheta + 4\,\lambda_\varphi = 1$, a result valid for Drell-Yan production in perturbative QCD. This relation is trivially derivable when the invariants $\mathcal{F}^{(i)}$ (and, thus, $\mathcal{F}$) are equal to 1/2. The Lam-Tung relation can therefore be reinterpreted as a consequence of rotational invariance and of the fact that all contributing processes produce transversely polarized lepton pairs. The latter ingredient can in turn be inferred from helicity conservation and, thus, the Lam-Tung relation can be extended to the description of direct Z/γ^* and W production. Figure 1 shows the processes representing the $O(\alpha_s^0)$ and $O(\alpha_s^1)$ contributions to direct Z/γ^* and W production. The assumption of helicity conservation at each diquark-boson vertex leads in each case to the prediction of transverse polarizations, even if with respect to different axes: Collins–Soper (CS) [7], approximation (neglecting the parton intrinsic transverse momentum) of the direction of the colliding quark and antiquark (Fig. 1 a), Gottfried–Jackson (GJ) [8], approximation of the direction of the single quark or antiquark coupling to the vector boson (Fig. 1 b,c,d), and helicity, direction of the vector boson momentum (Fig. 1 e).

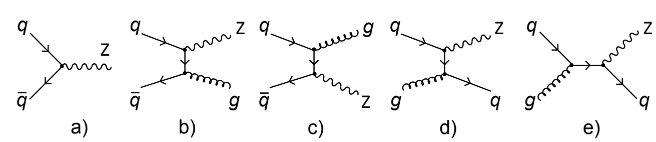

FIG. 1. $O(\alpha_s^0)$ and $O(\alpha_s^1)$ processes for Z/γ^* and W production, giving rise to transverse dilepton polarizations along different quantization axes: Collins–Soper (a), Gottfried–Jackson (b, c, d) and helicity (e).

It can be convenient to consider the following alternative frame-independent observable:

$$\tilde{\lambda} \equiv \frac{3\mathcal{F} - 1}{1 - \mathcal{F}} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}. \quad (11)$$

In the special case when the observed distribution is the superposition of n "elementary" distributions of the kind $1+\lambda_\vartheta^{(i)}\cos^2\vartheta$ with respect to n different polarization axes, $\tilde{\lambda}$ represents a weighted average of the n polarizations, made irrespectively of the orientations of the corresponding axes,

$$\tilde{\lambda} \;=\; \sum_{i=1}^{n} \frac{f^{(i)}}{3+\lambda_\vartheta^{(i)}}\,\lambda_\vartheta^{(i)} \Bigg/ \sum_{i=1}^{n} \frac{f^{(i)}}{3+\lambda_\vartheta^{(i)}}\,, \tag{12}$$

with $\tilde{\lambda} = +1$ when all contributing processes have natural transverse polarizations and $\tilde{\lambda} = -1$ when they are all longitudinal.

In the case of direct Z/γ^* or W production, for example, the polar anisotropy observed in the Collins–Soper frame is maximal only at $p_\mathrm{T} = 0$ (when all frames are degenerate). With increasing p_T, the polarization axes become increasingly different from each other and the processes represented in Fig. 1 will add up to a significantly reduced observable polarization. Figure 2 shows how this reduction of $\lambda_\vartheta^\mathrm{CS}$, for high p_T W production, is affected by the relative contribution of the sub-processes transversely polarized in the GJ and helicity frames. The measurement of $\tilde{\lambda}$ always yields, instead, the value $+1$, directly reflecting the intrinsic, fully transverse polarization characteristic of the produced W.

On the other hand, the measurement of the frame-dependent parameters can provide further insight into the production processes. For example, from the comparison of the top and bottom panels of Fig. 2 we deduce that accurate rapidity-dependent measurements of $\lambda_\vartheta^\mathrm{CS}$, made at not too high p_T, can reveal if the QCD corrections are dominated by quark-antiquark or by quark-gluon diagrams, given that only in the latter case $\lambda_\vartheta^\mathrm{CS}$ would depend on rapidity.

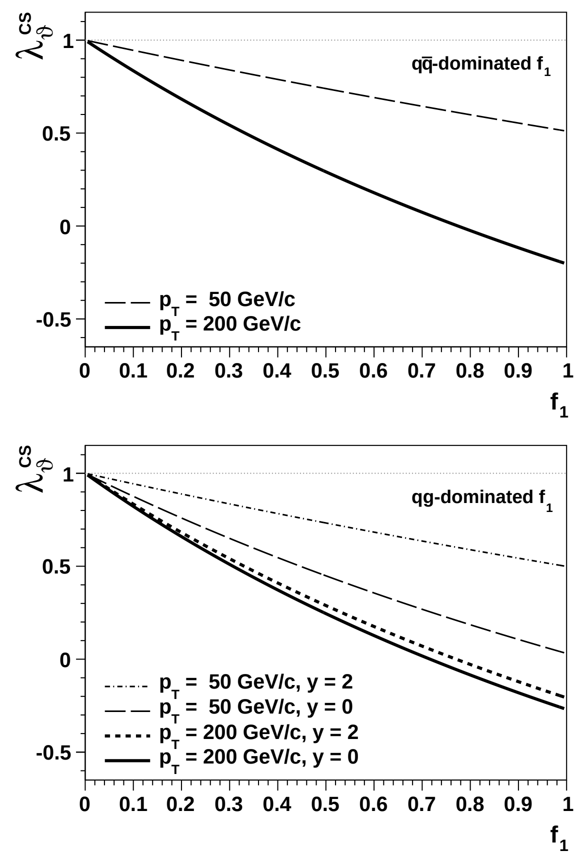

FIG. 2. Polar anisotropy of W decay, observable in the Collins–Soper frame, as a function of the relative contribution of $O(\alpha_s^1)$ processes, f_1, for different p_T and rapidity values. The $O(\alpha_s^1)$ contributions are assumed to be dominated by the quark-antiquark diagrams (Fig. 1 b-c) in the top panel and by the quark-gluon diagrams (Fig. 1 d-e) in the bottom panel. In the top panel only two curves are drawn because $\lambda_\vartheta^\mathrm{CS}$ is rapidity-independent in the case of quark-antiquark dominance.

IV. ROTATION-INVARIANT PARITY-VIOLATING ASYMMETRY

We will now study the rotational properties of the parity-violating coefficients A_ϑ, A_φ and $A_\varphi^\perp$. We start by noticing that, for each subprocess, the combination

$$\tilde{\mathcal{A}}^{(i)} \;=\; \frac{4}{3+\lambda_\vartheta^{(i)}}\sqrt{A_\vartheta^{(i)2} + A_\varphi^{(i)2} + A_\varphi^{\perp(i)2}}\,, \tag{13}$$

taking values between 0 and 1, is invariant under *any* rotation. Furthermore, the rotational properties of the angular momentum eigenstate $|V^{(i)}\rangle$ imply that there exists a quantization axis $z^{(i)\star}$ with respect to which $a_{0,+1}^{(i)\star} = a_{0,-1}^{(i)\star} = 0$ [6]. Along this axis (from Eq. 3)

$$\lambda_\vartheta^{(i)\star} = +1\,, \quad A_\varphi^{(i)\star} = 0\,, \quad A_\varphi^{\perp(i)\star} = 0 \tag{14}$$

and, moreover, $\lambda_{\vartheta\varphi}^{(i)\star} = \lambda_{\vartheta\varphi}^{\perp(i)\star} = 0$, while $A_\vartheta^{(i)\star}$, $\lambda_\varphi^{(i)\star}$ and $\lambda_\varphi^{\perp(i)\star}$ can be different from zero (in fact, $\lambda_\varphi^{(i)\star} = 2\,\mathcal{F}^{(i)} - 1$, from Eq. 10). This means that each single subprocess is characterized by its own "natural" axis, along which the polarization is fully transverse and the entire magnitude of the parity-violating effect is expressed by the coefficient $A_\vartheta^{(i)\star}$,

$$A_\vartheta^{(i)\star} = \frac{|a_{+1,+1}^{(i)\star}|^2 + |a_{-1,-1}^{(i)\star}|^2 - |a_{+1,-1}^{(i)\star}|^2 - |a_{-1,+1}^{(i)\star}|^2}{|a_{+1,+1}^{(i)\star}|^2 + |a_{-1,-1}^{(i)\star}|^2 + |a_{+1,-1}^{(i)\star}|^2 + |a_{-1,+1}^{(i)\star}|^2}\,. \tag{15}$$

Therefore, the axis-independent parity asymmetry $\tilde{\mathcal{A}}^{(i)}$ represents the magnitude of the *maximum* parity-violating asymmetry, as would be measured if it were possible to isolate the i-th single subprocess and choose $z^{(i)\star}$ as polarization axis, $\tilde{\mathcal{A}}^{(i)} = |A_\vartheta^{(i)\star}|$.

While the polarization information contained in the observable parameter $\mathcal{F}$ is not smeared in the superposition of subprocesses with different natural axes, smearing effects are unavoidable in the observable parity asymmetry, as a consequence of the intrinsic directionality induced by parity asymmetry. To quantify these effects, we define the asymmetry "vectors"

$$\vec{\mathcal{A}}^{(i)} \;=\; \frac{4}{3+\lambda_\vartheta^{(i)}}\,(A_\vartheta^{(i)}, A_\varphi^{(i)}, A_\varphi^{\perp(i)})\,, \tag{16}$$

each one directed along the corresponding natural axis $z^{(i)\star}$ and equal to $\tilde{\mathcal{A}}^{(i)}$ in magnitude. The observable rotation-invariant quantity

$$\tilde{\mathcal{A}} \;=\; \frac{4}{3+\lambda_\vartheta}\,\sqrt{A_\vartheta^2 + A_\varphi^2 + A_\varphi^{\perp 2}}\,, \tag{17}$$

bound between 0 and 1, is equal to the following combination of the elementary single-process asymmetries:

$$\tilde{\mathcal{A}} \;=\; \frac{\sqrt{\sum_{i,j=1}^{n} f^{(i)}\mathcal{N}^{(i)}\vec{\mathcal{A}}^{(i)}\cdot f^{(j)}\mathcal{N}^{(j)}\vec{\mathcal{A}}^{(j)}}}{\sum_{i=1}^{n} f^{(i)}\mathcal{N}^{(i)}}. \tag{18}$$

The inequality

$$\tilde{\mathcal{A}} \;\leq\; \frac{\sum_{i=1}^{n} f^{(i)}\mathcal{N}^{(i)}\tilde{\mathcal{A}}^{(i)}}{\sum_{i=1}^{n} f^{(i)}\mathcal{N}^{(i)}} \tag{19}$$

expresses the fundamental observation that superimposed processes characterized by different polarization axes partially cancel each other, leading to a less significant measurable asymmetry, whatever the observation frame. With this in mind, Eq. 17 provides the magnitude of the *maximum observable* parity asymmetry, i.e. of the net asymmetry as it can be measured along the polarization axis that maximizes it.

It may be experimentally convenient, when the detector itself does not induce "parity-violating" acceptance effects on the two decay fermions, to measure the following asymmetries:

$$\begin{aligned}
\mathcal{A}_{\cos\vartheta} \;&=\; \frac{N(\cos\vartheta>0)-N(\cos\vartheta<0)}{N(\cos\vartheta>0)+N(\cos\vartheta<0)} \;=\; \frac{3A_\vartheta}{3+\lambda_\vartheta}\,,\\
\mathcal{A}_{\cos\varphi} \;&=\; \frac{N(\cos\varphi>0)-N(\cos\varphi<0)}{N(\cos\varphi>0)+N(\cos\varphi<0)} \;=\; \frac{3A_\varphi}{3+\lambda_\vartheta}\,,\\
\mathcal{A}_{\sin\varphi} \;&=\; \frac{N(\sin\varphi>0)-N(\sin\varphi<0)}{N(\sin\varphi>0)+N(\sin\varphi<0)} \;=\; \frac{3A_\varphi^\perp}{3+\lambda_\vartheta}\,.
\end{aligned} \tag{20}$$

The first one is the so-called forward-backward asymmetry, $\mathcal{A}_{FB}$, studied, for example, in experimental analyses of direct Z/γ^* and W production, and usually defined in the CS frame. However, at high p_T, where processes beyond leading order contribute significantly, the CS frame no longer closely reflects the topology of the decay process. As discussed in the previous section, the polarization parameter undergoes a strong reduction from its leading-order expectation for direct production $\lambda_\vartheta^\mathrm{CS} = +1$. In a similar way, the polar "projection" of the asymmetry becomes smaller than the maximum observable asymmetry. For example, among the processes of Fig. 1 for direct Z/γ^* and W production, the quark-gluon diagrams (d) and (e) are naturally polarized along the GJ and helicity axes, respectively, and can lead, therefore, to observable azimuthal components of the parity asymmetry in the CS frame. Instead, the diagrams (b) and (c) do not affect A_φ because the vector boson is emitted either by the quark or by the antiquark and the two cases cancel exactly their contribution. The term A_φ is especially important when the observed process has significant contributions from sub-processes having a natural polarization along the helicity axis.

For indirectly produced vector bosons (for example, W from top quark decay) the CS axis loses completely its role of optimal spin quantization axis. In such situations the significance of the measured parity-violating effect can be improved, independently of the choice of the polarization frame, by using a rotation-invariant combination of the *three* asymmetries, which coincides with the already defined $\tilde{\mathcal{A}}$:

$$\tilde{\mathcal{A}} \;=\; \frac{4}{3}\,\sqrt{\mathcal{A}_{\cos\vartheta}^2 + \mathcal{A}_{\cos\varphi}^2 + \mathcal{A}_{\sin\varphi}^2}\,. \tag{21}$$

The parameter $A_\varphi^\perp$ and the corresponding asymmetry $\mathcal{A}_{\sin\varphi}$ should be small, as a consequence of the approximate symmetry with respect to the production plane expected for the decay distribution of inclusively produced vector bosons. In any case, these parameters can be neglected without affecting the exactness of the frame-independent formalism. In fact, the "reduced" invariant asymmetry

$$\tilde{\mathcal{A}}_\mathrm{R} \;=\; \frac{4}{3+\lambda_\vartheta}\,\sqrt{A_\vartheta^2 + A_\varphi^2} \;=\; \frac{4}{3}\,\sqrt{\mathcal{A}_{\cos\vartheta}^2 + \mathcal{A}_{\cos\varphi}^2} \tag{22}$$

is invariant under rotation about the y axis (while $\tilde{\mathcal{A}}$ is invariant under *any* rotation) and is, therefore, exactly independent of the choice of a polarization axis belonging to the production plane.

V. EXPERIMENTAL ADVANTAGES OF THE ROTATION-INVARIANT FORMALISM

The rotation-invariant parameters $\mathcal{F}$ and $\tilde{\mathcal{A}}$ can be determined directly from a fit of the angular distribution (taking into account acceptance and efficiencies) with *any* choice of polarization frame, through a suitable substitution of parameters in Eq. 4. For example:

$$\begin{aligned}
\lambda_\varphi \;&\rightarrow\; 1 - 1/2(1-\mathcal{F})(3+\lambda_\vartheta)\,,\\
A_\vartheta \;&\rightarrow\; \frac{(3+\lambda_\vartheta)\tilde{\mathcal{A}}}{4}\cos\xi\,,\\
A_\varphi \;&\rightarrow\; \frac{(3+\lambda_\vartheta)\tilde{\mathcal{A}}}{4}\sin\xi\cos\zeta\,,\\
A_\varphi^\perp \;&\rightarrow\; \frac{(3+\lambda_\vartheta)\tilde{\mathcal{A}}}{4}\sin\xi\sin\zeta\,,
\end{aligned} \tag{23}$$

where, $A_\varphi^\perp$ being negligible, ζ may be set to zero.

The determination of $\mathcal{F}$ and $\tilde{\mathcal{A}}$ has the advantage that these parameters directly estimate the significance of the global anisotropy of the distribution, while to derive the same information from the frame-dependent coefficients we need to study the correlation between λ_ϑ and λ_φ, and also between A_ϑ and A_φ. This is especially important if the polarization, or parity asymmetry, is not large, or if the frame-dependent parameters are poorly determined because of insufficient statistics.

The kinematic dependence of the frame-dependent coefficients has, in general, a "spurious" component reflecting the choice of observation frame rather than the intrinsic characteristics of the production processes, or how their mixture changes with kinematics. This can be understood from Eq. 6, δ explicitly depending on the momentum of the produced particle [5]. For instance, a subprocess with a "natural" parity-asymmetry axis coinciding with the helicity axis would give the contributions

$$\begin{aligned} A_\vartheta^{(i)} &= \frac{m\, p_\mathrm{L}\, p\, m_\mathrm{T}}{p^2\, m_\mathrm{T}^2 + \frac{1}{2}\, p_\mathrm{T}^2\, E^2\, (1 - \lambda_\varphi^{(i)\star})}\, A_\vartheta^{(i)\star}\,, \\ A_\varphi^{(i)} &= -\frac{E\, p_\mathrm{T}\, p\, m_\mathrm{T}}{p^2\, m_\mathrm{T}^2 + \frac{1}{2}\, p_\mathrm{T}^2\, E^2\, (1 - \lambda_\varphi^{(i)\star})}\, A_\vartheta^{(i)\star} \end{aligned} \tag{24}$$

to the parameters A_ϑ and A_φ observed in the CS frame, with m, m_T, E, p, p_T and p_L being, respectively, the mass, the transverse mass, the energy, and the total, transverse and longitudinal momenta of the meson in the center-of-mass of the collision. At sufficiently high p_T, $A_\vartheta^{(i)}$ vanishes and $A_\varphi^{(i)}$ acquires the same significance of $A_\vartheta^{(i)\star}$. Moreover, the strong, explicit dependence of $A_\vartheta^{(i)}$ and $A_\varphi^{(i)}$ on the momentum of the particle implies a significant dependence of the observed asymmetries on the experimental acceptance. In general, such misleading effects cannot be reduced with a suitable frame choice, being impossible to eliminate in the presence of a mixture of processes characterized by different natural axes. By definition, the frame-independent polarization and asymmetry parameters are less sensitive (or even immune) to these effects. Their use improves the representation of the results and facilitates the comparison between different experiments, and with theoretical calculations [4].

We will now motivate the use of a frame-independent evaluation of the parity asymmetry by considering a concrete (albeit arbitrary) scenario, where a heavy vector boson would be produced as a superposition of two processes, one transversely polarized in the CS frame and the other transversely polarized in the helicity frame. This is illustrated in Fig. 3. For simplicity of illustration, we assume that the relative weight of these processes is independent of kinematics. As long as the process naturally polarized in the CS frame dominates, the use of the standard forward-backward asymmetry, $\mathcal{A}_{FB}$, provides a relatively good approximation of the maximally observable asymmetry, $\tilde{\mathcal{A}}$, apart from the trivial scale factor 4/3. If, instead, a large fraction of the events are transversely polarized in the helicity frame, the two approaches give very different results.

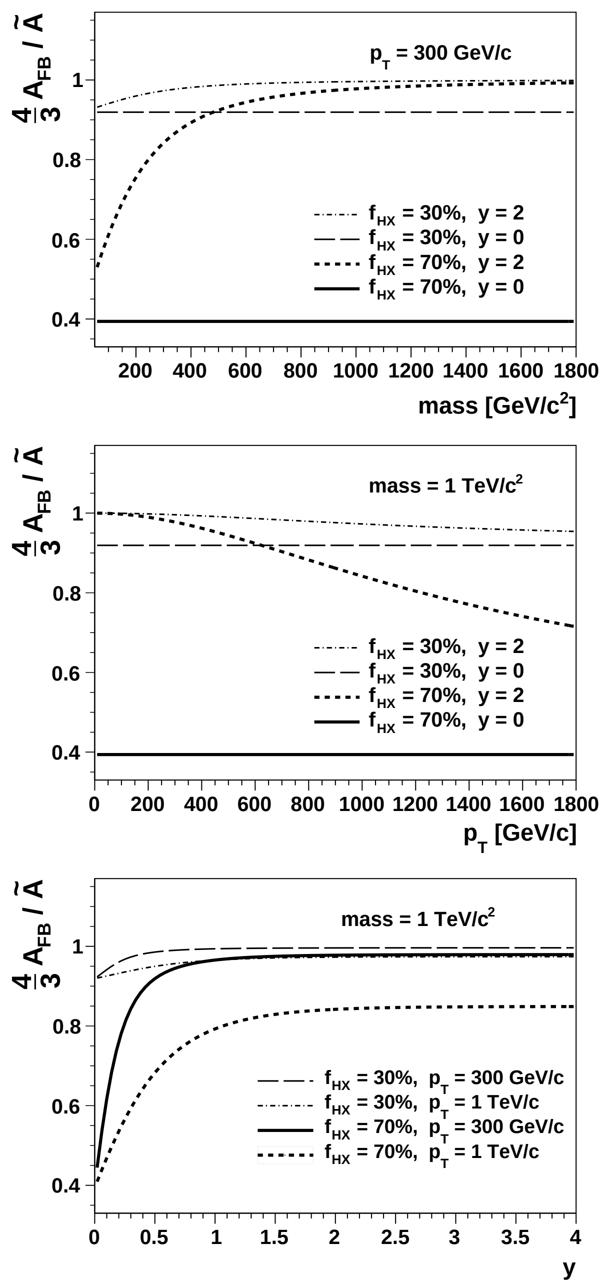

FIG. 3. Ratio between the forward-backward asymmetry (scaled up for consistency) and the maximally observable parity asymmetry, as a function of mass, p_T and rapidity of the vector boson, when a fraction $f_\mathrm{HX} = 30\%$ or 70% of the cross section comes from processes naturally polarized in the helicity frame, the others being polarized in the CS frame.

Finally, the explicit verification of the frame-independence of the invariant quantities represents a non-trivial check of the correctness of the experimental analysis. In fact, acceptance limitations due to the geometry of the detector and/or to its limited sensitivity to the particle momenta act as strong kinematic cuts in the reconstructed angular spectra and induce a fake polarization, which must be accurately understood for a correct interpretation of the measurements. Such polarizing effects do not follow, in general, the physical transformation rules of the decay distribution in the particle rest frame, i.e., they do not obey the rotational-covariance properties of the angular parameters. Comparing frame-independent results obtained in two significantly distinct frames can, thus, reveal unaccounted systematic effects and lead to an improved evaluation of systematic uncertainties.

VI. SUMMARY

The spin alignment of a vector state decaying into fermion-antifermion (or lepton-neutrino) pairs and inclusively observed in a given kinematic condition is always described by a specific form of Eq. 10. This relation is invariant with respect to rotations about the axis perpendicular to the production plane and provides, therefore, a frame-independent definition of spin-alignment. Its existence derives from basic rotational covariance properties of $J = 1$ angular momentum eigenstates and is a general result valid for parity-conserving and parity-violating difermion decays.

The parity-violating terms of the decay distribution can be combined in a second frame-independent relation, representing the magnitude of the maximum observable parity asymmetry, as could be measured with the best possible choice of quantization axis.

The adoption of the frame-invariant formalism for the measurement of polarization and parity asymmetry presents several advantages in studies of the decays of high-p_T bosons. The rotation-invariant spin-alignment and parity-asymmetry parameters always provide a representation of the maximum observable polarization and parity-asymmetry effects. They are independent of assumptions on the production mechanisms or on the optimal spin-quantization axis, and are immune to the effect of a possible "unlucky choice" of the polarization frame. For example, with respect to the usually adopted definition of the forward-backward asymmetry for Z/γ^* decays, referred to one chosen polarization axis (typically the Collins–Soper axis), the rotation-invariant formulation always gives a more significant observable parity asymmetry.

By eliminating spurious kinematic dependencies induced by the momentum-dependence of the angle between different physical quantization axes, the use of frame-independent quantities also provides a more robust comparison between different experiments, as well as between measurements and calculations. Additionally, the results can be cross-checked in two orthogonal frames, probing the existence of unaccounted biases induced by experimental limitations in the kinematic acceptance of the decay fermions.

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