Rotation-invariant relations
in vector meson decays into fermion pairs

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Abstract

The rotational properties of angular momentum eigenstates imply the existence of a frame-independent relation among the parameters of the decay distribution of vector mesons into fermions. This relation is a generalization of the Lam-Tung identity, a result specific to Drell-Yan production in perturbative QCD, here shown to be equivalent to the dynamical condition that the dilepton always originates from a transversely polarized photon.

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The partonic cross section for dilepton production in perturbative QCD obeys the so-called Lam-Tung identity [1], a relation between the helicity structure functions of the virtual photon or, equivalently, between the coefficients of the lepton angular distribution measured in the dilepton rest frame, $\lambda_\vartheta + 4\lambda_\varphi = 1$, with the λ_ϑ and λ_φ parameters introduced later in this Letter. The Lam-Tung relation (LTR) represents for lepton pair production the analog of the Callan-Gross relation in deep inelastic scattering, $F_1(x) - 2xF_2(x) = F_L(x) = 0$, where the Bjorken scaling functions F_1 and F_2 are reciprocally connected by the condition that the longitudinal helicity component F_L of the massive photon vanishes identically. The Callan-Gross relation, a consequence of the interaction between the photon probe and half-integer spin quarks, is not exact, being subject to substantial $O(\alpha_s)$ corrections due to gluon radiation.

The theoretical relevance of the LTR resides in the fact that, although the dilepton production cross section is substantially modified by QCD corrections, the *relation* between the different helicity contributions to this cross section remains unchanged up to $O(\alpha_s^2)$ [2] and is subject to relatively small corrections when the subsequent orders in α_s are taken into account [3]. In fact, the LTR is such a solid prediction of perturbative QCD that its violation is a strong signal of non-perturbative effects. Experimentally, the LTR has been shown to be violated in pion-nucleus collisions [5], raising speculations about the possible quantitative effects of intrinsic parton k_T [6] or of higher twist contributions [7]. Saturation effects are also expected to contribute to a violation of the LTR in proton-nucleus and deuteron-nucleus collisions at RHIC and at the LHC [4].

The distinctive feature of the LTR is that it is independent of the chosen orientation of the axes of the dilepton rest frame. In this Letter we show that this property is a completely general consequence of the rotational covariance of $J = 1$ angular momentum eigenstates. It does not depend, therefore, on the specific $J = 1$ state considered nor on the production process. This implies that the decay distribution of any vector state — including the case of non-perturbative effects in Drell-Yan production, as well as the case of quarkonium production — can be described in terms of a frame-independent relation analogous to the LTR.

We start by considering the case of a single production "subprocess", in which the vector meson V is always formed as a specific superposition of the three J_z eigenstates, with eigenvalues $m = +1, -1, 0$, with respect to a chosen polarization axis z:

$$|V^{(i)}\rangle = b_{+1}^{(i)}\,|+1\rangle + b_{-1}^{(i)}\,|-1\rangle + b_0^{(i)}\,|0\rangle\,. \tag{1}$$

Assuming helicity conservation at the di-fermion vertex, the angular distribution of the parity-conserving decay is

$$W^{(i)}(\cos\vartheta, \varphi) \propto \frac{\mathcal{N}^{(i)}}{(3 + \lambda_\vartheta^{(i)})}\,(1 + \lambda_\vartheta^{(i)}\cos^2\vartheta + \lambda_\varphi^{(i)}\sin^2\vartheta\cos 2\varphi \tag{2}$$

$$+\, \lambda_{\vartheta\varphi}^{(i)}\sin 2\vartheta\cos\varphi + \lambda_\varphi^{\perp(i)}\sin^2\vartheta\sin 2\varphi + \lambda_{\vartheta\varphi}^{\perp(i)}\sin 2\vartheta\sin\varphi)\,,$$

where ϑ and φ are the (polar and azimuthal) angles formed by the positive fermion

with, respectively, the polarization axis z and the xz plane, and

$$\mathcal{N}^{(i)} = |a_0^{(i)}|^2 + |a_{+1}^{(i)}|^2 + |a_{-1}^{(i)}|^2 \,,$$

$$\lambda_\vartheta^{(i)} = \frac{\mathcal{N}^{(i)} - 3|a_0^{(i)}|^2}{\mathcal{N}^{(i)} + |a_0^{(i)}|^2} \,, \quad \lambda_\varphi^{(i)} = \frac{2\,\mathrm{Re}(a_{+1}^{(i)*} a_{-1}^{(i)})}{\mathcal{N}^{(i)} + |a_0^{(i)}|^2} \,, \quad \lambda_{\vartheta\varphi}^{(i)} = \frac{\sqrt{2}\,\mathrm{Re}[a_0^{(i)*}(a_{+1}^{(i)} - a_{-1}^{(i)})]}{\mathcal{N}^{(i)} + |a_0^{(i)}|^2} \,,$$

$$\lambda_\varphi^{\perp(i)} = \frac{-2\,\mathrm{Im}(a_{+1}^{(i)*} a_{-1}^{(i)})}{\mathcal{N}^{(i)} + |a_0^{(i)}|^2} \,, \quad \lambda_{\vartheta\varphi}^{\perp(i)} = \frac{-\sqrt{2}\,\mathrm{Im}[a_0^{(i)*}(a_{+1}^{(i)} + a_{-1}^{(i)})]}{\mathcal{N}^{(i)} + |a_0^{(i)}|^2} \,,$$

$$\tag{3}$$

with $a_0^{(i)}$, $a_{+1}^{(i)}$ and $a_{-1}^{(i)}$ being the partial decay amplitudes of the three J_z components of the vector state.

In this Letter we only consider inclusive production. Therefore, the only possible experimental definition of the xz plane coincides with the production plane, containing the directions of the colliding particles and of the decaying particle itself. The last two terms in Eq. 2 introduce an asymmetry of the distribution by reflection with respect to the production plane. Such asymmetry is not forbidden in individual parity-conserving events. In hadronic collisions, due to the intrinsic parton transverse momenta, for example, the "natural" polarization plane does not coincide event-by-event with the production plane. However, the symmetry by reflection must be a property of the observed *event distribution* when only parity-conserving processes contribute. Indeed, the terms in $\sin^2\vartheta\sin 2\varphi$ and $\sin 2\vartheta\sin\varphi$ are unobservable, because they vanish on average.

In the presence of n contributing production processes with weights $f^{(i)}$, the most general *observable* distribution can be written, therefore, as

$$W(\cos\vartheta, \varphi) = \sum_{i=1}^{n} f^{(i)} W^{(i)}(\cos\vartheta, \varphi)$$

$$\tag{4}$$

$$\propto \frac{1}{(3+\lambda_\vartheta)}\left(1 + \lambda_\vartheta\cos^2\vartheta + \lambda_\varphi\sin^2\vartheta\cos 2\varphi + \lambda_{\vartheta\varphi}\sin 2\vartheta\cos\varphi\right),$$

where

$$\lambda_\vartheta = \sum_{i=1}^{n} \frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}}\lambda_\vartheta^{(i)} \Bigg/ \sum_{i=1}^{n} \frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}} \,,$$

$$\lambda_\varphi = \sum_{i=1}^{n} \frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}}\lambda_\varphi^{(i)} \Bigg/ \sum_{i=1}^{n} \frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}} \,, \tag{5}$$

$$\lambda_{\vartheta\varphi} = \sum_{i=1}^{n} \frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}}\lambda_{\vartheta\varphi}^{(i)} \Bigg/ \sum_{i=1}^{n} \frac{f^{(i)}\mathcal{N}^{(i)}}{3+\lambda_\vartheta^{(i)}} \,.$$

Our considerations are based on two propositions concerning the rotational properties of the generic $J = 1$ state defined in Eq. 1.

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Prop. 1: Each amplitude combination $b_{+1}^{(i)} + b_{-1}^{(i)}$ is invariant by rotation around the y axis.

Prop. 2: For each subprocess there exists a quantization axis $z^{(i)*}$ with respect to which $b_0^{(i)*} = 0$; if $b_0^{(i)}$, $b_{+1}^{(i)}$ and $b_{-1}^{(i)}$ are real, $z^{(i)*}$ belongs to the xz plane.

Proposition 1 follows from the relations among rotation matrix elements $d_{+1,M}^1(\vartheta) + d_{-1,M}^1(\vartheta) = \delta_{|M|,1}$. When $|V^{(i)}\rangle$ is defined with a real $b_0^{(i)}$ (always possible), the frame defined in Prop. 2 is reached through successive rotations by the Euler angles φ^*, ϑ^* and $-\varphi^*$, defined as

$$\cos\vartheta^* = \frac{R_+ R_- + I_+ I_-}{\sqrt{2b_0^{(i)2}(R_+^2 + I_-^2) + (R_+ R_- + I_+ I_-)^2}},$$

$$\cos\varphi^* = \frac{R_+}{\sqrt{R_+^2 + I_-^2}}, \quad \sin\varphi^* = -\frac{I_-}{\sqrt{R_+^2 + I_-^2}}, \tag{6}$$

where $R_\pm = \mathrm{Re}(b_{+1}^{(i)} \pm b_{-1}^{(i)})$ and $I_\pm = \mathrm{Im}(b_{+1}^{(i)} \pm b_{-1}^{(i)})$. If all three amplitudes are real, then $\varphi^* = 0$ and the rotation is around the y axis.

Proposition 1 and the obvious rotation invariance of $|a_0^{(i)}|^2 + |a_{+1}^{(i)}|^2 + |a_{-1}^{(i)}|^2$ imply that the quantities

$$\mathcal{F}^{(i)} = \frac{1}{2}\frac{|a_{+1}^{(i)} + a_{-1}^{(i)}|^2}{|a_0^{(i)}|^2 + |a_{+1}^{(i)}|^2 + |a_{-1}^{(i)}|^2} \tag{7}$$

(bound between 0 and 1) are independent of the chosen frame. Using also Eqs. 3 and 5, we find that the following combination of observable parameters is frame-independent:

$$\mathcal{F} = \frac{\sum_{i=1}^n f^{(i)}\mathcal{N}^{(i)}\mathcal{F}^{(i)}}{\sum_{i=1}^n f^{(i)}\mathcal{N}^{(i)}} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta}. \tag{8}$$

Equation 8 can be written as

$$(1 - \mathcal{F})(3 + \lambda_\vartheta) = 2(1 - \lambda_\varphi), \tag{9}$$

an expression formally analogous to the LTR [1], which, as mentioned above, accounts for Drell-Yan production up to first-order QCD modifications, neglecting parton transverse momenta.

At this level of description, the topology of each contributing subprocess (quark-antiquark annihilation without or with single gluon emission, Compton-like quark-gluon scattering, etc.) is characterized by one reaction plane, coinciding with the experimental production plane. Therefore, for each single subprocess $\lambda_\varphi^{\perp(i)} = \lambda_{\vartheta\varphi}^{\perp(i)} = 0$, implying (Eq. 3) that the three partial decay amplitudes (and, thus, the three components of the produced angular momentum state) can be chosen to be real. Proposition 2, together with Eq. 3, implies, then, that the observed dilepton distribution is a convolution of sub-distributions of the kind

$$\lambda_\vartheta^{(i)*} = +1, \quad \lambda_\varphi^{(i)*} = 2\mathcal{F}^{(i)} - 1, \lambda_\varphi^{\perp(i)*} = \lambda_{\vartheta\varphi}^{(i)*} = \lambda_{\vartheta\varphi}^{\perp(i)*} = 0, \tag{10}$$

each one referred to a specific polarization axis $z^{(i)*}$ belonging to the production plane.

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The LTR is obtained from Eq. 9 in the special case when the invariants $\mathcal{F}^{(i)}$ (and, thus, $\mathcal{F}$) are equal to $1/2$. This means, according to Eq. 10, that all competing subprocesses lead to the same kind of *fully transverse, purely polar* decay anisotropy, *with respect to possibly different natural axes.* In other words, the *frame independence* of the LTR is the *kinematic* consequence of the rotational properties of the $J = 1$ angular momentum eigenstates, while its specific *form* ($\mathcal{F} = 1/2$) derives from the *dynamical* input that all contributing subprocesses produce the dilepton via one transversely polarized photon.

More generally, the decay of a vector state into fermion pairs in a given kinematic condition is always described in frame-independent terms by a specific form of Eq. 9. The advantages of this kind of representation, complementary to the determination of the full angular distribution, are described in Ref. [8] for the specific case of quarkonium polarization studies.

In summary, we have shown that rotational invariance imposes frame-invariant constraints on the polar and azimuthal anisotropy parameters of the di-fermion decay distribution of vector mesons. In particular, for any mixture of production mechanisms in a given kinematic condition there exists a frame-invariant relation among the angular coefficients, depending on one calculable parameter, $\mathcal{F}$. The Lam-Tung relation corresponds to the special case when all processes produce the dilepton via one transverse photon and their respective natural axes belong to the production plane. Any violation of this relation will continue to be described by a suitably modified *frame-invariant* relation. The frame-invariant formalism can be extended to the study of the spin alignment of quarkonium and other vector particles.

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