BALITSKY-KOVCHEGOV EQ. II (20.05.2014)

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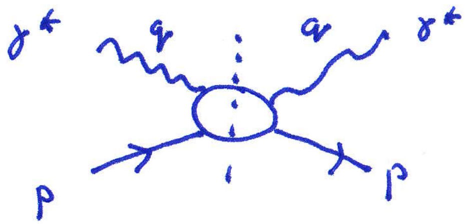
PLAN

1. SPACE-TIME PICTURE OF DIS FOR $x_B \to 0$
2. PHOTON WAVE FUNCTION
3. DIPOLE CROSS SECTION
4. EXAMPLE WITH TWO GLUON EXCHANGE
5. BK EQUATION

IN DIS PHYSICAL INFORMATION ABOUT $\gamma^* p$ INTERACTION IS CONTAINED IN HADRONIC TENSOR:



$$W_{\mu\nu}(P,q) = \frac{1}{2\pi m}\,\mathrm{Im}\left\{ i\int d^4x\, e^{iq\cdot x} \langle P|\, T(\underbrace{J_\mu(x)}_{\text{hadronic current}} J_\nu(0))\,|P\rangle \right\}$$

PROTON REST FRAME

$$P^\mu = (m, \vec{0}_\perp, 0) = (\overset{+}{m}, \overset{-}{m}, \vec{0}_\perp) \qquad p^\pm = p^0 \pm p^3 \qquad P\cdot q = \frac{1}{2}\left(p^+q^- + p^-q^+\right)$$

$$q^\mu = (q^0, \vec{0}_\perp, q^3) = (q^+, q^-, \vec{0}_\perp) \qquad q^\pm = q^0 \pm q^3$$

$$\left\{\begin{array}{l} q^2 = -Q^2 = q^+q^- \\ x_B = \dfrac{Q^2}{2P\cdot q} = \dfrac{Q^2}{m(q^+ + q^-)} \end{array}\right. \Rightarrow \left\{\begin{array}{l} q^+q^- = -Q^2 \\ q^+ + q^- = \dfrac{Q^2}{m x_B} \end{array}\right. \Rightarrow \left\{\begin{array}{l} q^+ \simeq \dfrac{Q^2}{m x_B} \longrightarrow \infty \\ q^- \simeq -m x_B \longrightarrow 0 \end{array}\right.$$

$$\boxed{x_B \to 0}$$

Integrand in Hadronic tensor is dominated by

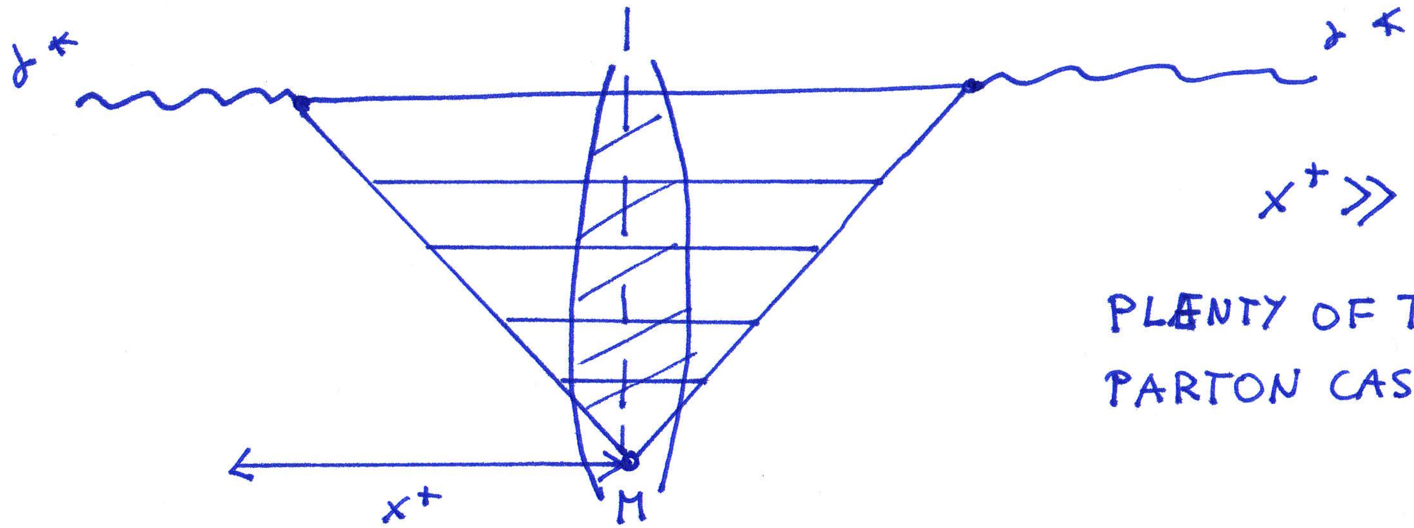
$$q\cdot x = \frac{1}{2}\left(q^+x^- + q^-x^+\right) \Rightarrow x^- \lesssim \frac{2}{q^+} = \frac{m}{Q^2}x_B \longrightarrow 0$$

$\boxed{\text{DIS IS LIGHT CONE DOMINATED}}$

$$\boxed{\begin{array}{c} x^2 = x^+x^- - \vec{x}_\perp^{\,2} \gtrsim 0 \\ |\vec{x}_\perp| \lesssim 1/Q \end{array}} \qquad \boxed{x^+ \lesssim \frac{2}{|q^-|} = \frac{1}{m x_B}} \longrightarrow \infty$$

LIGHT-CONE TIME
IOFFE TIME
COHERENCE TIME

SPACE-TIME PICTURE FOR $x_B \to 0$ ($\gamma^* p$ ENERGY $\to \infty$)

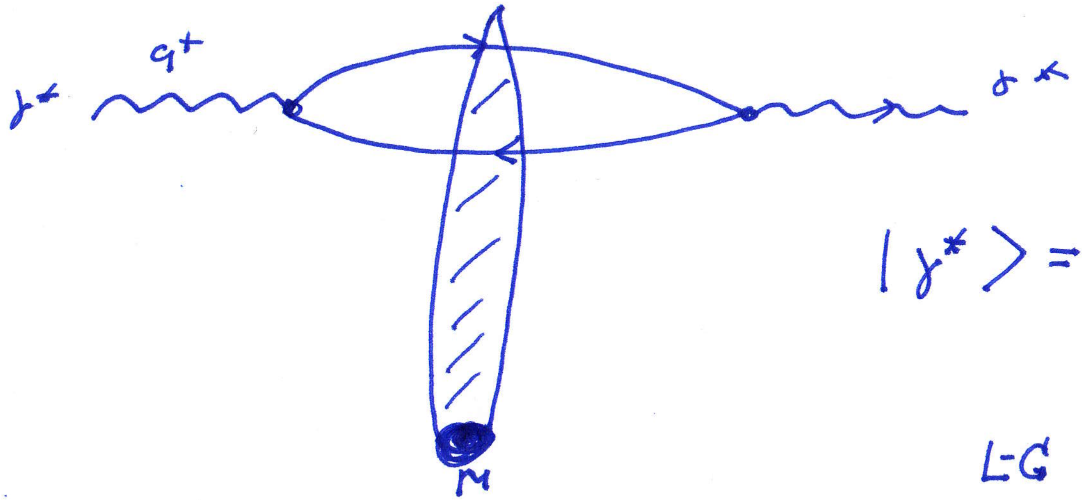


$x^+ \gg t_{DIS} \simeq \frac{1}{m}$

PLENTY OF TIME TO DEVELOPE PARTON CASCADE FROM γ^*

IN THE LOWEST QCD ORDER

$\gamma^* \longrightarrow q\bar{q}$



APPRORIATE TOOL TO GET PHYSICAL PICTURE LCPT

$$|\gamma^*\rangle = \Psi_{q\bar{q}}\,|q\bar{q}\rangle + \Psi_{q\bar{q}g}\,|q\bar{q}g\rangle + \dots$$

L-C WAVE FUNCTION OF γ^* (COMPUTABLE)

ASIDE: IN IMF PICTURE IS DIFFERENT

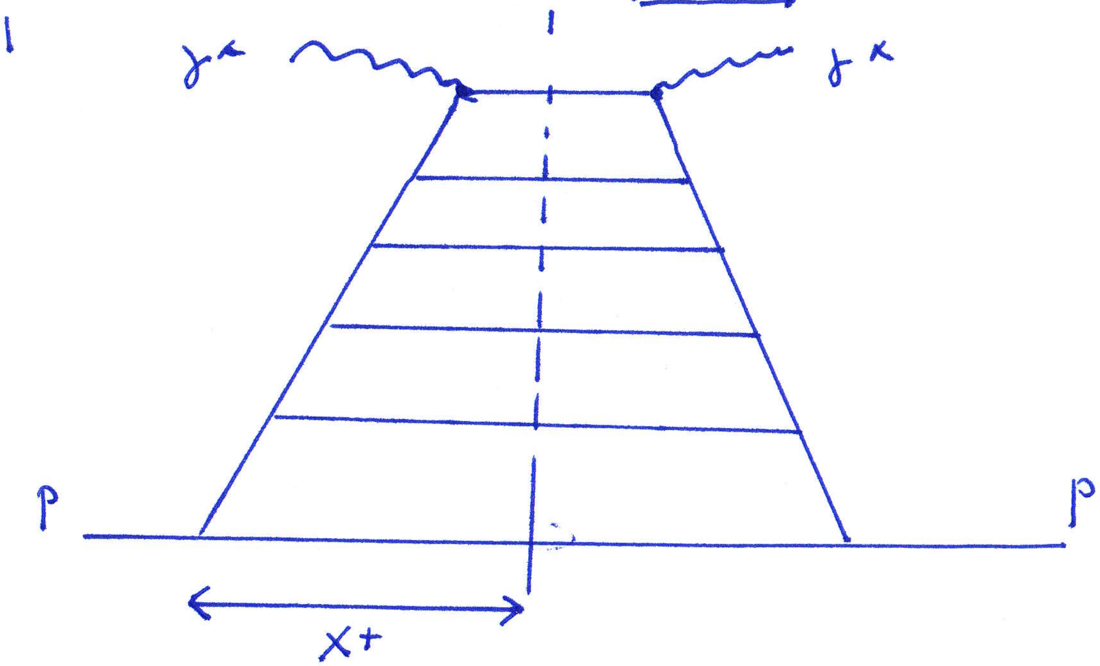
$$p^\mu = \left(P + \frac{m^2}{2P}, \vec{0}_\perp, -P\right) = \left(\overset{\oplus}{\frac{m^2}{2P}}, \overset{\ominus}{2P + \frac{m^2}{2P}}, \vec{0}_\perp\right) \quad \text{left mover}$$

$$q^\mu = \left(q^0, \vec{0}_\perp, q^3\right) = \left(q^+, q^-, \vec{0}_\perp\right) \quad \text{right mover}$$

NOW

$$\begin{cases} q^- \simeq -\dfrac{Q^2}{2Px_B} \longrightarrow 0 \\ q^+ \simeq 2Px_B \xrightarrow[\boxed{P\to\infty}]{} \infty \end{cases}$$

BUT $\boxed{q^+ \ll P^-}$ FOR $x_B \ll 1$



$$x^+ \simeq \frac{2}{|q^-|} \longrightarrow \infty$$

BUT PARTON CASCADE FROM PROTON

$$|p\rangle = \sum_{\text{partons}} \Psi_{\text{partons}} \, |\text{partons}\rangle$$

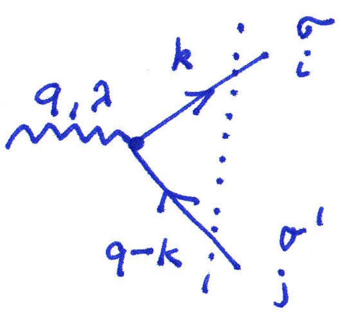
$\downarrow$

LC WAVE FUNCTION OF PROTON

(NOT EASY TO COMPUTE)

FROM NOW ON – PROTON REST FRAME

$$\boxed{|\gamma^*\rangle = \Psi^{\gamma^*\to q\bar q}_{T,L}\,|q\bar q\rangle + \Psi^{\gamma^*\to q\bar q g}_{T,L}\,|q\bar q g\rangle + \ldots}$$



USING THE RULES OF LCPT, $z = \dfrac{k^+}{q^+}$

$$\Psi^{\gamma^*\to q\bar q}_{T,L}(\vec k_\perp, z) = e\, Z_f\, \frac{z(1-z)\,\delta_{ij}}{\vec k_\perp^{\,2} + m_f^2 + Q^2 z(1-z)}\; \underbrace{\bar u_\sigma(k)\,\gamma\cdot\epsilon^\lambda_{T,L}(q)\, v_{\sigma'}(q-k)}_{\text{need to be worked out}}$$

FOURIER TRANSFORM $\vec k_\perp \Rightarrow \vec x_\perp \equiv \vec r$

$$\Psi^{\gamma^*\to q\bar q}_{T,L}(\vec r, z) = \int \frac{d^2 \vec k_\perp}{(2\pi)^2}\, e^{i\vec k_\perp\cdot\vec r}\, \Psi^{\gamma^*\to q\bar q}_{T,L}(\vec k_\perp, z)$$

In cross section

$$\boxed{\sigma^{\gamma^* N}_{T,L}(x, Q^2) = \int d^2\vec r \int_0^1 dz\, \left|\Psi^{\gamma^*\to q\bar q}_{T,L}(\vec r, z)\right|^2 \underbrace{\sigma^{q\bar q N}_{TOT}(\vec r, x)}}\; ; \quad Y = \ln 1/x$$

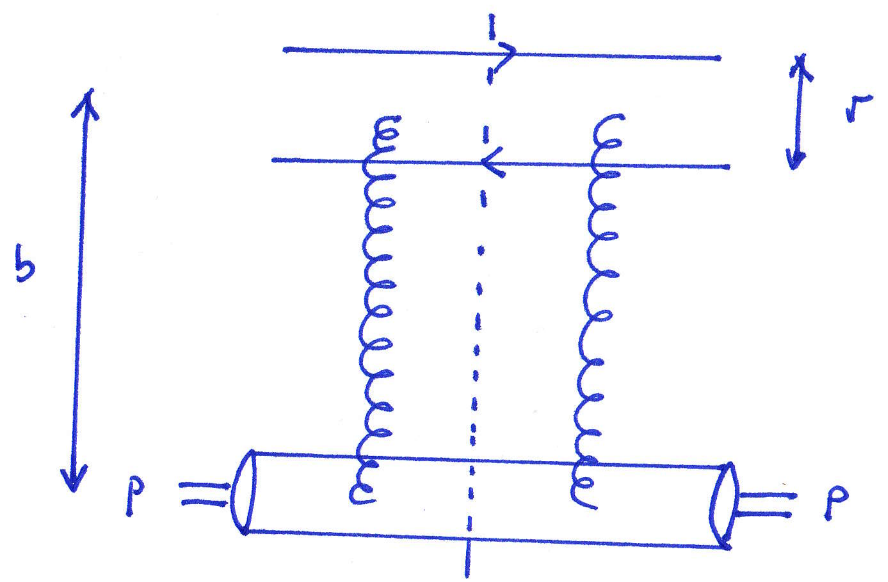
interaction of $q\bar q$ dipole with N

DIPOLE CROSS SECTION AND DIPOLE SCATTERING AMPLITUDE

$$\sigma^{q\bar{q}N}_{tot} = 2 \int d^2b \, \underbrace{N(\vec{r}, \vec{b}, Y)}$$

$$Y = \ln 1/x = \ln \frac{\hat{s}}{s_0}$$

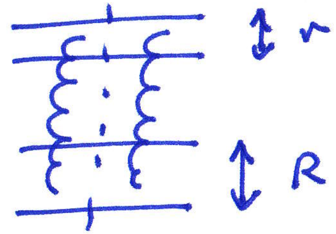
Imaginary part of the scattering amplitude (forward) for a dipole of transverse size $\vec{r}$ interacting with a target at impact parameter $\vec{b}$ with rapidity Y.



ONIUM

BK equation is written for $N(\vec{r}, \vec{b}, Y)$: $\frac{\partial}{\partial Y} N = \dots$

Example – onium-onium scattering with two gluon exchange



$q\bar{q}$ = small dipole r

N = large dipole $R = \frac{1}{\Lambda}$, $r \ll R$

$$\sigma_{tot}^{q\bar{q}N} = \frac{2\pi \alpha_s^2 C_F}{N_c} r^2 \left\{ \ln\left(\frac{R^2}{r^2}\right) + 1 \right\} = \frac{2\pi \alpha_s^2 C_F}{N_c} r^2 \ln\left(\frac{1}{r^2 \Lambda^2}\right)$$

CONNECTION TO GLUON DISTRIBUTION

– UNITEGRATED GLUE :

$$\varphi(x, k_\perp^2) = \frac{\alpha_s C_F}{\pi} \int d^2R \int_0^1 dz \, |\Psi(R, z)|^2 \left(\frac{2 - e^{-i\vec{k}_\perp \cdot \vec{R}} - e^{i\vec{k}_\perp \cdot \vec{R}}}{k_\perp^2} \right)$$

$$= \frac{\alpha_s C_F}{\pi} \frac{2}{k_\perp^2} \, , \quad k_\perp \gg \Lambda$$

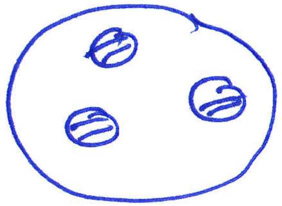
– INTEGRATED GLUE

$$x G_N(x, Q^2) = \int_{\Lambda^2}^{Q^2} dk_\perp^2 \, \varphi(x, k_\perp^2) = \frac{\alpha_s C_F}{\pi} 2 \ln \frac{Q^2}{\Lambda^2}$$

NOW

$$\boxed{\sigma_{tot}^{q\bar{q}N} \approx \frac{\alpha_s \pi^2}{N_c} r^2 \, x G_N\left(x, \frac{1}{r^2}\right)}$$

Introducing impact parameter b ≡ nucleons (large dipoles) in nucleus A



A

$$N(\vec{r},\vec{b},Y) = \frac{\alpha_s \pi^2}{2N_c}\, r^2 \underbrace{T(b)}_{\text{nuclear profile function}} \times G_N\left(x, \frac{1}{r^2}\right)$$

ONLY ONE NUCLEON IN NUCLEUS INTERACTS WITH SMALL DIPOLE $\Rightarrow \underbrace{N > 0}_{\text{Violation of unitarity}}$

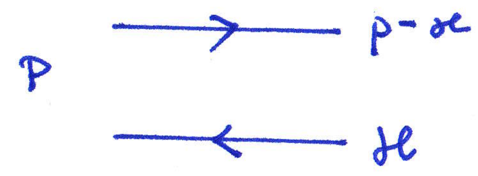
GRIBOV - GLAUBER - MUELLER RESCATTERING

$$N_{GGM}(\vec{r},\vec{b},\overset{\text{no energy dependence}}{Y=0}) = 1 - \exp\left\{-\frac{\alpha_s \pi^2}{2N_c}\, r^2\, T(b) \times \underbrace{G_N(x}_{\text{fake dependence}}, \frac{1}{r^2})\right\}$$

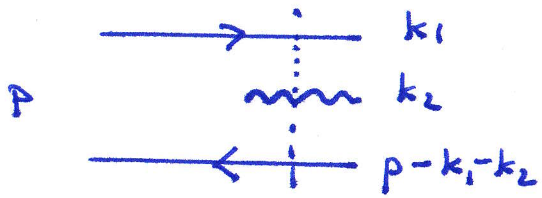
UNITARITY RESTORED

$$0 < N \leq 1$$

MUELLER CASCADE – SOFT PART OF ONIUM WAVE FUNCTION



$$|\psi(-\infty)\rangle = \int d^3\varkappa\ \psi^{(0)}(\varkappa)\,|p-\varkappa,\varkappa\rangle \qquad d^3\varkappa = d\varkappa^+ d^2\varkappa$$



$$|\psi(0)\rangle = |\psi(-\infty)\rangle + \int d^3k_1\, d^3k_2\ \psi^{(1)}(k_1,k_2)\,|p-k_1-k_2, k_1, k_2\rangle$$

LCPT

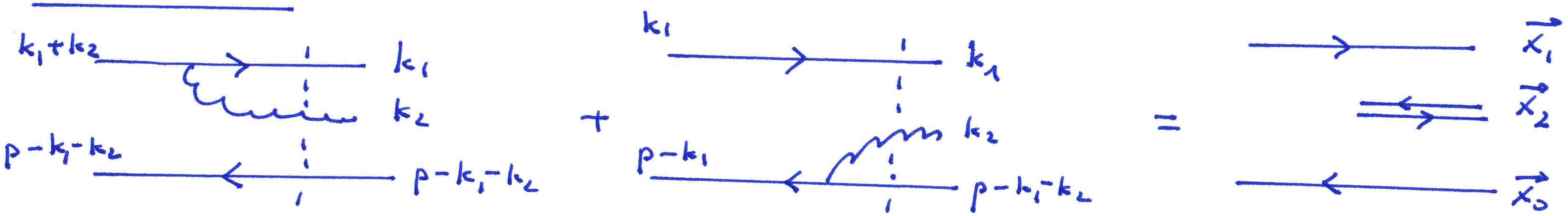
$$\psi^{(1)}(k_1,k_2) = \frac{\langle p-k_1-k_2, k_1, k_2|H_I|\psi(-\infty)\rangle}{p^- - k_1^- - \underbrace{k_2^-}_{\text{dominates}} - (p-k_1-k_2)^-}$$

SOFT GLUON EMISSION

$$z_1 = \frac{k_1^+}{p^+}, \quad z_2 = \frac{k_2^+}{p^+} \quad ; \qquad z_2 \ll z_1,\ (1-z_1)$$

$$k_2^- = -\frac{\vec{k}_2^{\,2}}{2k_2^+} = -\frac{\vec{k}_2^{\,2}}{2p^+}\frac{1}{z_2} \gg p^-,\ k_1^-,\ (p-k-k_2)^- \Rightarrow \overset{k_2^-}{\text{dominates}} \text{ propagator}$$

NUMERATOR

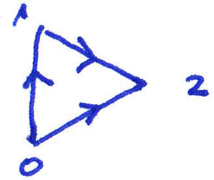


$$\psi^{(1)}(k_1,k_2) = 2 g_s T^a \frac{\vec{\epsilon}_2\cdot\vec{k}_2}{\vec{k}_2^{\,2}} \left\{ \psi^{(0)}(z_1,\vec{k}_1+\vec{k}_2) - \psi^{(1)}(z_1,\vec{k}_1) \right\}$$

FOURIER TRANSFORM

$$\tilde{\psi}^{(1)}(\vec{x}_{01},\vec{x}_{02},z_1,z_2) = \int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\, e^{i(\vec{k}_1\cdot\vec{x}_{01}+\vec{k}_2\cdot\vec{x}_{02})}\,\psi^{(1)}(k_1,k_2)$$

$$= \frac{i g_s T^a}{\pi}\,\vec{\epsilon}_2\cdot\left\{ \frac{\vec{x}_{12}}{\vec{x}_{12}^{\,2}} - \frac{\vec{x}_{02}}{\vec{x}_{02}^{\,2}} \right\}\psi^{(0)}$$



SQUARE

$$\boxed{|\tilde{\psi}^{(1)}|^2 = 4 C_F \frac{\alpha_s}{\pi}\frac{\vec{x}_{01}^{\,2}}{\vec{x}_{02}^{\,2}\,\vec{x}_{12}^{\,2}}\,|\tilde{\psi}^{(0)}|^2}$$

PROBABILITY TO FIND ONE EXTRA GLUON IN ONIUM WAVE FUNCTION

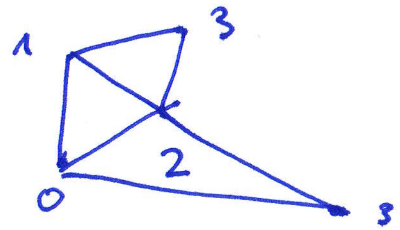
$$\underbrace{\int_{z_0}^{z_1}\frac{dz_2}{z_2}\int\frac{d^2\vec{x}_2}{4\pi}\,|\tilde{\psi}^{(1)}|^2}_{\text{probability}} = \int_{z_0}^{z_1}\frac{dz_2}{z_2}\int d^2x_2\,\frac{\alpha_s C_F}{\pi^2}\,\frac{\vec{x}_{01}^{\,2}}{\vec{x}_{02}^{\,2}\,\vec{x}_{12}^{\,2}}\,|\tilde{\Psi}^{(0)}|^2$$

EMISSION OF SECOND GLUON – PROBA. TO FIND TWO GLUONS

$$\underbrace{\int_{z_0}^{z_1}\frac{dz_2}{z_2}\int_{z_0}^{z_2}\frac{dz_3}{z_3}\int d^2x_2\,d^2x_3\left(\frac{\alpha_s C_F}{\pi^2}\right)^2}_{\alpha_s^2\,\ln^2(z_1/z_0)}\;\frac{\vec{x}_{01}^{\,2}}{\vec{x}_{02}^{\,2}(\vec{x}_{01}^{\,2})}\left(\frac{\vec{x}_{12}^{\,2}}{\vec{x}_{13}^{\,2}\,\vec{x}_{23}^{\,2}}+\frac{\vec{x}_{02}^{\,2}}{\vec{x}_{03}^{\,2}\,\vec{x}_{23}^{\,2}}\right)|\tilde{\Psi}^{(0)}|^2$$

if $z_1 \gg z_2 \gg z_3$

SUBSEQUENT GLUON EMISSIONS SOFTER



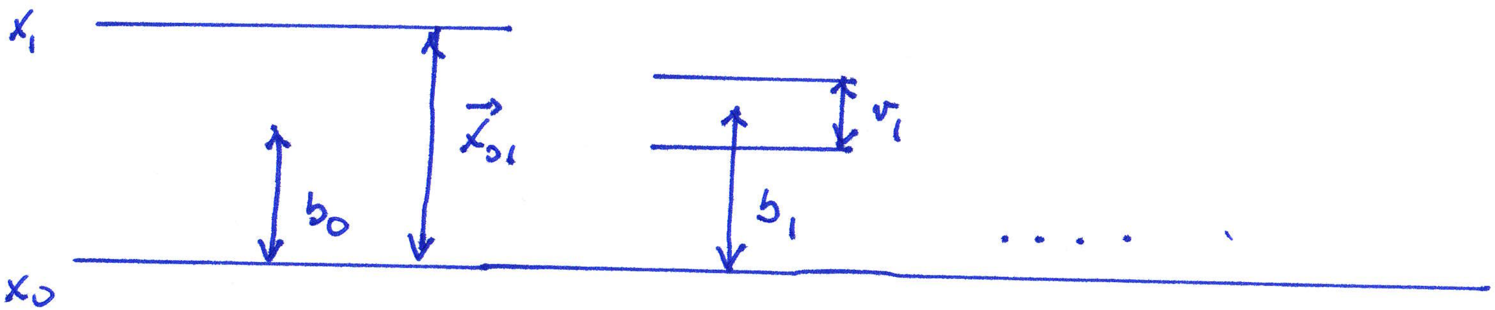
GENERATING FUNCTIONAL TO INCLUDE CORRECTIONS TO ALL ORDERS IN $\alpha_s \ln\left(\frac{z_1}{z_0}\right)$ $\left(N_c \to \infty,\ C_F = \frac{N_c}{2}\right)$

$$Z(\vec{x}_{01}, \vec{b}_0, Y, u)\ |\Psi^{(0)}|^2 = \int d^2 r_1\, d^2 b_1\, |\tilde{\Psi}^{1}|^2\, u(r_1, b_1)$$

$$+ \frac{1}{2!} \int d^2 r_1\, d^2 b_1\, d^2 r_2\, d^2 b_2\, |\tilde{\Psi}^{(2)}|^2\, u(r_1, b_1)\, u(r_2, b_2) + \ldots$$

WHERE

$\tilde{\Psi}^{(n)}(r_1, b_1, r_2, b_2 \ldots r_n, b_n, Y)$ = ONIUM STATE WITH n DIPOLES



$$\boxed{|\Psi^{(n)}|^2 = |\Psi^{(0)}|^2 \left.\frac{\delta^2 Z(\ldots Y, u)}{\delta u(r_1, b_1) \ldots \delta u(r_n, b_n)}\right|_{u=0}}$$

MUELLER EQUATION $\left(N_c \to \infty , \; C_F = \frac{N_c}{2} \right)$

$$\frac{\partial}{\partial Y} Z(\vec{x}_0, \vec{x}_1, Y, u) = \frac{\alpha_s N_c}{(2\pi)^2} \int d^2 x_2 \frac{\vec{x}_{01}^{\,2}}{\vec{x}_{02}^{\,2} \, \vec{x}_{12}^{\,2}} \left\{ Z(\vec{x}_1, \vec{x}_2, Y, u) \, Z(\vec{x}_0, \vec{x}_2, Y, u) - \underbrace{Z(\vec{x}_0, \vec{x}_1, Y, u)}_{\text{virtual correction}} \right\}$$

VIRTUAL CORRECTION~~S~~ REGULARIZE ∞ FOR $\vec{x}_2 = \vec{x}_1$ or $\vec{x}_2 = \vec{x}_0$.

KOVCHEGOV (1999)

SCATTERING MATRIX

$$S(\vec{x}_{01}, \vec{b}_0, Y) = 1 - N(\vec{x}_{01}, \vec{b}_0, Y)$$

$$S(\vec{x}_{01}, \vec{b}_0, Y) = \sum_{k=1}^{\infty} \frac{1}{k!} \int d^2 r_1 \, d^2 b_1 \ldots d^2 r_k \, d^2 b_k$$

$$\times \underbrace{\frac{\delta^k Z(\vec{x}_{01}, \vec{b}_0, Y, u)}{\delta u(r_1, b_1) \ldots \delta u(r_n, b_n)}}_{\text{proba. to find } k\text{-dipoles in onium wave funktion}} \; \underbrace{\gamma_0(r_1 \, b_1) \ldots \gamma_0(r_k, b_k)}_{\text{multiple scattering interaction of 1 dipole}}$$

AND

$$\gamma_0 = e^{-\frac{\vec{r}^2 Q_s^2(b^2)}{4}\ln\left(\frac{1}{rA}\right)} = S(r, b, Y=0)$$

WE SEE THAT

multiple interaction of one dipole

$$\boxed{S(\vec{x}_{01}, \vec{b}_0, Y) = Z(\vec{x}_{01}, \vec{b}_0, Y, u = \gamma_0)}$$

THUS EVOLUTION EQUATION IS THE SAME $\longrightarrow$ $\boxed{\text{BK. EQ.}}$

$$\frac{\partial}{\partial Y} S(\vec{x}_{01}, \vec{b}_0, Y) = \frac{\alpha_s N_c}{2\pi^2}\int d^2x_2 \frac{\vec{x}_{01}^{\,2}}{\vec{x}_{02}^{\,2}\,\vec{x}_{12}^{\,2}}\left\{S(\vec{x}_1, \vec{x}_2, Y)\, S(\vec{x}_0, \vec{x}_2, Y) - S(\vec{x}_0, \vec{x}_1, Y\right\}$$

FOR FORWARD SCATTERING AMPLITUDE : $N = 1 - S$

$$\frac{\partial}{\partial Y} N(\vec{x}_0, \vec{x}_1, Y) = \frac{\alpha_s N_c}{2\pi^2}\int d^2x_2 \frac{\vec{x}_{01}^{\,2}}{\vec{x}_{12}^{\,2}\,\vec{x}_{02}^{\,2}}\left\{N(\vec{x}_1, \vec{x}_2, Y) + N(\vec{x}_0, \vec{x}_2, Y) - N(\vec{x}_0, \vec{x}_1, Y)\right.$$

$$\left. - N(\vec{x}_1, \vec{x}_2, Y)\, N(\vec{x}_0, \vec{x}_2, Y)\right\}$$

WITHOUT QUADRATIC TERMS : BFKL EQUATON

PROPERTIES

1. LOCAL UNITARITY $N \leq 1$

2. SATURATON SCALE : $Q_s(x) \sim x^{\omega_0}$ $\omega_0 = 4\bar{\alpha}_s \ln 2$

3. GEOMETRIC SCALING $N \sim N(r\,Q_s(x))$ TRAVELING WAVE

4. SUPPRESION OF INFRARED : (LARGE DIPOLES)

PROBLEMS

1. RUNNIG α_s
2. NLL CORRECTIONS
3. BETTER KINEMATICS
4. IMPACT PARAMETER DEPENDENCE - LARGE TAILS IN b.