BALITSKI-KOVCHEGOV EQUATION I (8.04.2014)

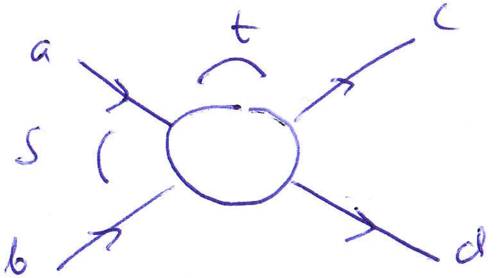
KRZYSZTOF GOLEC-BIERNAT, IFJ, UR

PLAN

1. HIGH ENERGY SCATTERING
2. REGGE LIMIT IN QCD
3. BFKL EQUATION
4. REGGE LIMIT IN DIS
5. DIPOLE PICTURE OF DIS
6. MUELLER DIPOLS
7. BK versus BFKL

HIGH ENERGY SCATTERING

Reaction: $a+b \to c+d$, $A(s,t)$ $\qquad s+t+u = \sum_i m_i^2 \equiv 0$



OPTICAL THEOREM

$$\boxed{\sigma_{TOT}(ab \to X) = \frac{1}{s}\,\mathrm{Im}\,A_{el}(s,0)}$$

Regge (high energy) limit: $s \gg |t|, m_i^2$, $\quad s \simeq |u|$

a) Regge pole models: $A_{el}(s,t) \sim s^{\alpha(t)}$

b) Regge trajectory: $\alpha(t) = \alpha_0 + \alpha' t$, α_0 = INTERCEPT, α' = SLOPE

c) Pomeron: dominant vaccum quantum number exchange with

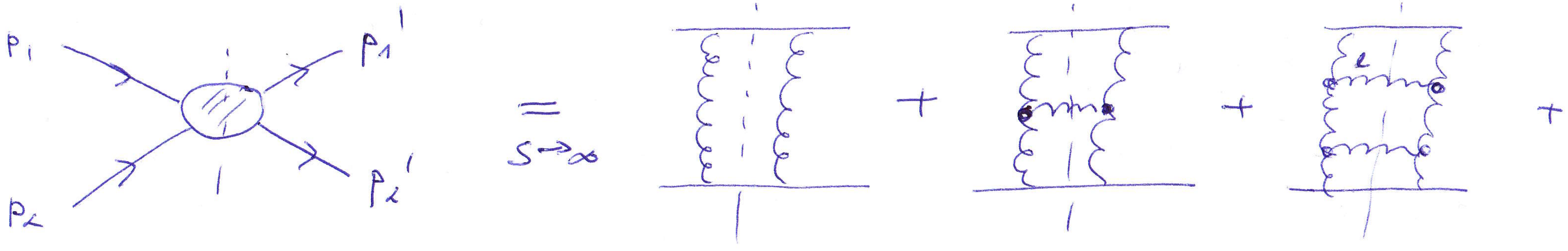
$\alpha_0 = 1+\varepsilon$, $\varepsilon > 0$ $\Rightarrow$ $\boxed{\sigma_{TOT} \sim s^{\varepsilon}}$ power-like behaviour

d) conflict with Froissard bound

$$\boxed{\sigma_{TOT} \leq \frac{\pi}{m_\pi^2} \ln^2\left(\frac{s}{s_0}\right)}$$

π = constant.

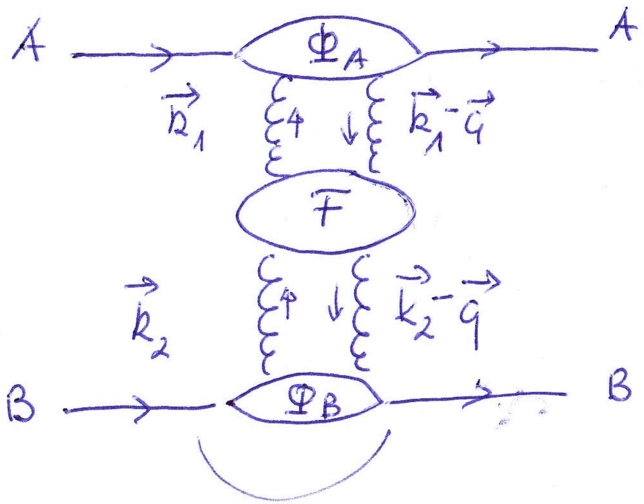
REGGE LIMIT IN QCD



a) QUASI FEYNMAN GRAPHS : reggeized gluons exchanged, non-local vertices

b) MULTI-REGGE KINEMATICS : $\hat{s}_i >> \hat{s}_{i-1} >> \hat{s}_{i-2}$, $\vec{\ell}_{i\perp}^{\,2} \sim s_0$

c) SUDAKOV DECOMPOSITION : $\ell = \alpha p + \beta \bar{p} + \vec{\ell}_\perp$, $p^2 = \bar{p}^2 = 0$, $2 p \cdot \bar{p} \equiv s$



$t = -\vec{q}^{\,2}$

$$\mathrm{Im}\, A_{el}(s,t) = \int \frac{d^2 k_1\, d^2 k_2}{\bar{k}_2^{\,2}\, (\bar{k}_1 - \bar{q})^2}\, \Phi_A(\bar{k}_1, \bar{q})\, \Phi_B(\bar{k}_2, \bar{q})$$

$$\times F(s, \bar{k}_1, \bar{k}_2, \bar{q})$$

↳ energy dependence

$\Phi_{A,B}$ = IMPACT FACTORS

F = 4-point off-shell GREEN FUNCTION OBTAINED BY SUMMING BFKL LADDERS

BFKL EQUATION

FORWARD LIMIT $\vec{q}=0$: $\sigma_{TOT} = 1/s \, \mathrm{Im}\, A(s,0)$

$$\boxed{\sigma_{TOT} = \int \frac{d^2k_1}{k_1^2}\frac{d^2k_2}{k_2^2}\, \Phi_A(\vec{k}_1,\vec{0})\, F(s,\vec{k}_1,\vec{k}_2,\vec{0})\, \Phi_B(\vec{k}_2,\vec{0})}$$

RAPIDITY : $y = \ln s/s_0$, $\bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}$

$$\boxed{\frac{\partial F(y,\vec{k}_1,\vec{k}_2)}{\partial y} = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'}{(\vec{k}_1-\vec{k}')^2}\Big[\underbrace{F(y,\vec{k}',\vec{k}_2)}_{\text{real emission}} - \underbrace{\frac{\vec{k}_1^{\,2}}{\vec{k}'^2+(\vec{k}_1-\vec{k}')^2}F(y,\vec{k}_1,\vec{k}_2)}_{\text{virtual corrections}}\Big]}$$

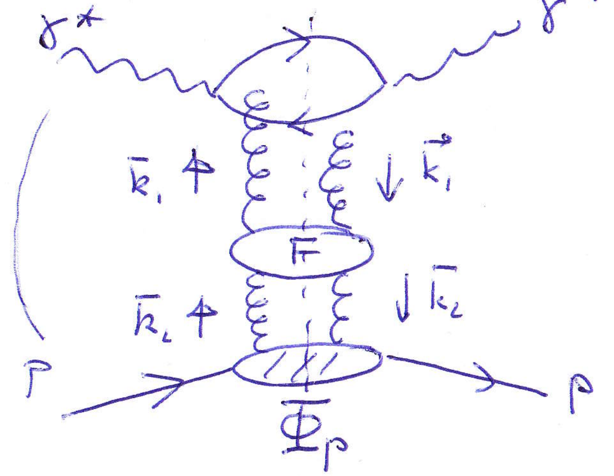
SOLUTION FOR $y\to\infty$ $(s\to\infty)$

$$F(y,\vec{k}_1,\vec{k}_2) = \frac{1}{\sqrt{k_1^2k_2^2}\,\pi}\; \underbrace{e^{\omega_0 y}}_{s^{\omega_0}}\; \underbrace{\frac{\exp\left\{\frac{-\ln^2(k_1^2/k_2^2)}{2\bar{\alpha}_s K''(\frac{1}{2})\,y}\right\}}{\sqrt{2\pi\bar{\alpha}_s K''(\frac{1}{2})\,y}}}_{\text{diffusion in transverse momenta}}$$

BFKL Pomeron intercept $\boxed{\omega_0 = 2\bar{\alpha}_s \ln 2 \simeq 0.5 .}$ $\leftarrow$ Froissart bound ?

REGGE LIMIT IN DIS

$\boxed{\text{DIS}: \quad ep \to e'X}$



$A = \gamma^* =$ virtual photon : $Q^2 = -q^2 > 0$ $\qquad \Phi^{\gamma^*}_{T,L}(Q^2 \dots)$

$B = p =$ proton $\qquad \Phi_p(m_p \dots)$

$$\boxed{\sigma_{T,L}(\gamma^* p \to X) = \frac{F_{T,L}}{Q^2} \cdot 4\pi^2 \alpha_{em}}$$

F_T, F_L - structure functions ; $\quad F_2 = F_T + F_L$

$k_\perp$ - faktorization

$$\sigma^*_{T,L} = \int \frac{d^2k_1}{k_1^2} \Phi^{\gamma^*}_{T,L}(Q^2, \vec{k}_1) \underbrace{\int F(s, \vec{k}_1, \vec{k}_2) \Phi_p(m_p, \vec{k}_2) \frac{d^2k_2}{k_2^2}}$$

$$= \int \frac{d^2k_1}{\vec{k}_1^2} \Phi^{\gamma^*}_{T,L}(Q^2, \vec{k}_1) \quad \underbrace{f(x, \vec{k}_1)}$$

unintegrated gluon distribution obeys BFKL eq.

BJORKEN VARIABLE

$$x \doteq \frac{Q^2}{s} \quad \Rightarrow \quad y = \ln \frac{s}{Q^2} = \ln(1/x)$$

Regge limit : $\quad Y \to \infty \qquad \Longleftrightarrow \quad \boxed{x \to 0} \leftarrow$ small x limit

$\qquad\qquad\qquad s \to \infty$

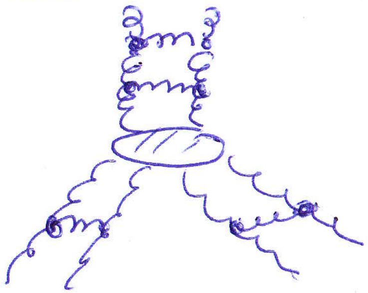
FROM BFKL

$$F_2(x, Q^2) \sim x^{-\omega_0} \quad , \quad \omega_0 = 4\bar{\alpha}_s \ln 2 \simeq 0.5$$

STRONG RISE DUE TO GLUON DISTRIBUTION $f(x, k)$

FROISSART BOUND $F_2 \leq c \ln^2(1/x)$ VIOLATED? $\boxed{\text{DOES IT EXIST?}}$!?

STH. IS DEFINITELY MISSING:

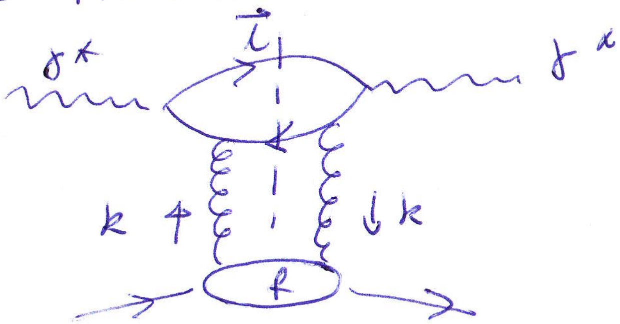


GLUON RECOMBINATION
POMERON INTERACTIONS – $3\mathbb{P}$ vertex
SATURATION
UNITARIZATION . . . .

HOW TO TAME POWER-LIKE GROWTH AS $x \to 0$ $(s \to \infty)$?

DIPOLE PICTURE FROM FEYNMAN DIAGRAMS

COMPUTE PHOTON IMPACT FACTOR:



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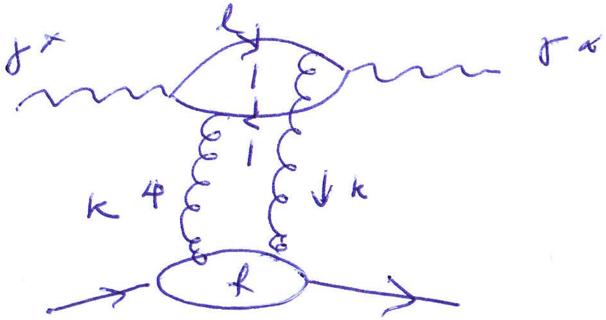


RESULT FOR σ_T:

$$\sigma_T \sim \int \frac{d^2k}{\vec{k}^2} \Big\{ \underbrace{\int_0^1 dz \int d^2l \left[z^2 + (1-z)^2\right] \left\{ \frac{\vec{l}}{D(\vec{l})} - \frac{\vec{k}+\vec{l}}{D(\vec{k}+\vec{l})} \right\}^2}_{\Phi_T^{\gamma^*}(Q^2, \vec{k})} \Big\} \times f(x, k)$$

PROPAGATORS: $D(\vec{l}) = \vec{l}^{\,2} + \bar{Q}^2 = \vec{l}^{\,2} + z(1-z)Q^2$

USING FOURIER TRADE $\vec{l} \Longleftrightarrow \vec{r}$ = transverse vector

$$\boxed{\frac{\vec{l}}{D(\vec{l})} = i\bar{Q} \int \frac{d^2r}{2\pi}\, e^{-i\vec{l}\cdot\vec{r}}\, \frac{\vec{r}}{r} K_1(\bar{Q}r)}$$

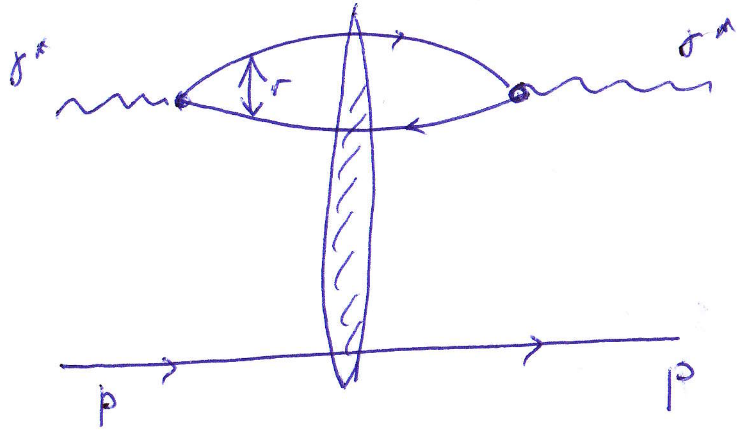
RESULT

$$\sigma_T \sim \int d^2r \int_0^1 dz \left\{ \underbrace{\left[z^2+(1-z)^2\right]\bar{Q}^2 K_1^2(\bar{Q}r)} \right\} \times \left\{ \underbrace{\int \frac{d^2k}{k^2} f(x,\vec{k})\left(1-e^{-i\vec{k}\cdot\vec{r}}\right)\left(1-e^{i\vec{k}\cdot\vec{r}}\right)} \right\}$$

$$\sigma_T \sim \int d^2r \int_0^1 dz \quad \left|\Psi_T^{\gamma^*}(z,\vec{r},Q^2)\right|^2 \quad \times \quad \hat{\sigma}(x,r)$$

↳ photon wave function ↳ dipole cross section

Interpretation

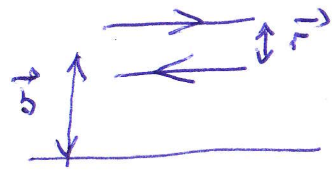


$\Psi_T^{\gamma^*}$ – DESCRIBES SPLITTING $\gamma^* \to q\bar{q}$ DIPOLE

$\hat{\sigma}$ – FORWARD SCATTERING AMPLITUDE OF DIPOLE

$$\boxed{\text{IN HIGH ENERGY LIMIT DIPOLE SCATTERS WITH FROZEN } z \text{ AND DIPOLE SIZE } \vec{r}}$$

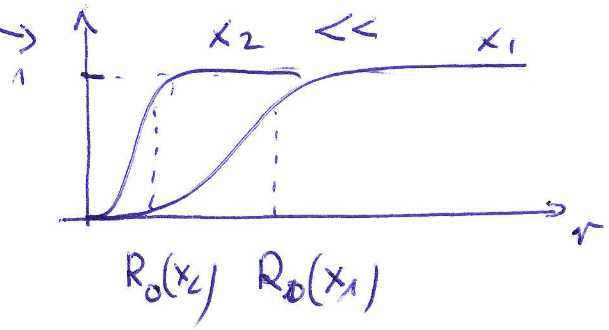
$$\boxed{\hat{\sigma}(x,r) = \int d^2b \; N(x,\vec{r},\vec{b})}$$



FORWARD DIPOLE SCATT. AMPL ← BASIC OBJECT

1 GBW MODEL OF $N(x,\vec{r},b)$ = dipole scattering amplitude

$$N(x,\bar{r},\bar{b}) = \left\{1 - e^{-r^2/R_0^2(x)}\right\}\Theta(b_0-b) \qquad \boxed{R_0(x) \sim x^{0.3}}$$



(a) x^{λ} growth tamed, $N \leq 1$

(b) for $x \to 0$ proton blackens

(c) SCALING $N\left(\frac{r}{R_0(x)}\right) \longrightarrow \sigma_T\left(\frac{Q^2}{Q_s^2(x)}\right)$

2 KOVCHEGOV EQUATION

- DERIVED IN DIPOLE PICTURE OF DIS.
- CAPTURES THE GBW MODEL FEATURES
- EQ. FOR DIPOLE SCATTERING AMPLITUDE

$$\boxed{\frac{\partial N_0}{\partial Y}(Y,\vec{r},\vec{b}) = \int K_{BFKL} \otimes \left\{ \underbrace{N_1 + N_2 - N_0}_{\text{linear}} - \underbrace{N_1 N_2}_{\text{nonlinear}} \right\}}$$

like in BFKL

MUELLER (NIKOLAEV, ZAKHAROV) 1993

LIGHT-CONE QUANTIZATION $\quad x^+ = x^0 + x^3 = \text{time} \quad P^- = P^0 - P^3 = \text{energy} = H.$

LIGHT-CONE PERTURBATION THEORY

PHYSICAL PICTURE OF BFKL GROWTH

$$|\gamma^*\rangle = \sim|q\bar{q}\rangle + \sim|q\bar{q}g\rangle + \ldots \sim|q\bar{q}\underbrace{gg..g}_{\text{soft gluons}}\rangle$$

a) GROWTH IN NUMBER OF SOFT GLUONS IN PHOTON WAVE FUNCTION

b) SIMPLE TWO GLUON EXCHANGE BETWEEN SOFT GLUONS AND PROTON

c) IN PROTON REST FRAME GLUON CASCADE DEVELOPS FAR BEFORE INTERACTION POINT.