# Advanced Statistical Physics

Lech Longa

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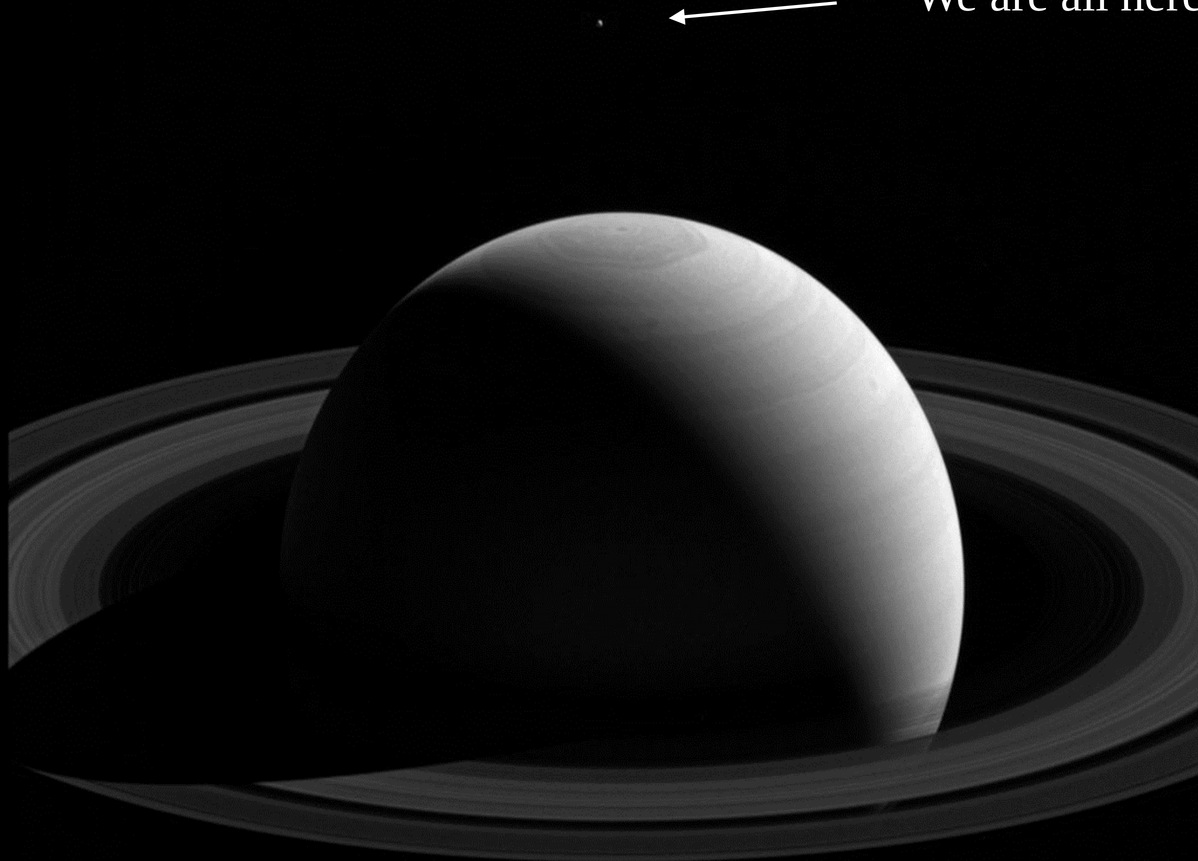
- *Dyżury: poniedziałek 12-14*

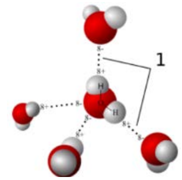
We are all here: good morning

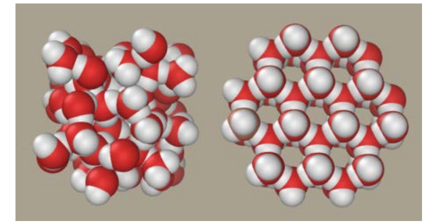


# LITERATURE

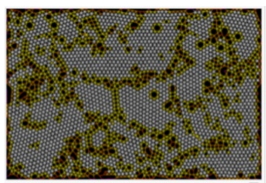
1. P.M.Chaikin and T. C. Lubensky ***Principles of condensed matter physics***
2. M. Plischke and B. Bergersen ***Equilibrium Statistical Physics, 3rd Edition***
3. Nigel Goldenfeld ***Lectures on phase transitions and the RG***
4. P.G. de Gennes and J. Prost ***The Physics of Liquid Crystals***
5. H. E. Stanley ***Introduction to Phase Transitions and Critical Phenomena***
6. S. R. de Groot and P. Mazur ***Non-equilibrium Thermodynamics***
7. J. Honerkamp, ***Statistical Physics, An advanced approach with applications***
8. L.E. Reichl, ***A Modern Course in Statistical Physics***

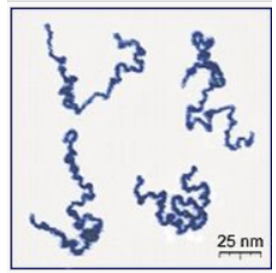
# Statistical Physics of **interacting systems:**





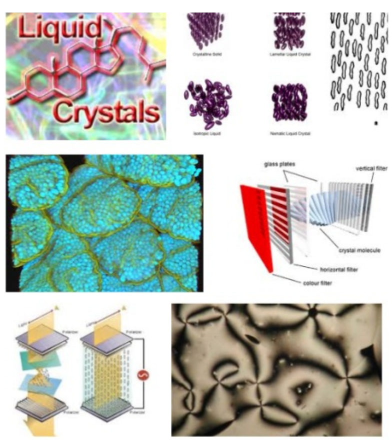

A crystal of amethyst


The connectivity of the crystals in the colloidal crystals



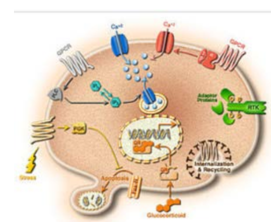
Appearance of real linear polymer chains as recorded using an atomic force microscope on surface under liquid medium.

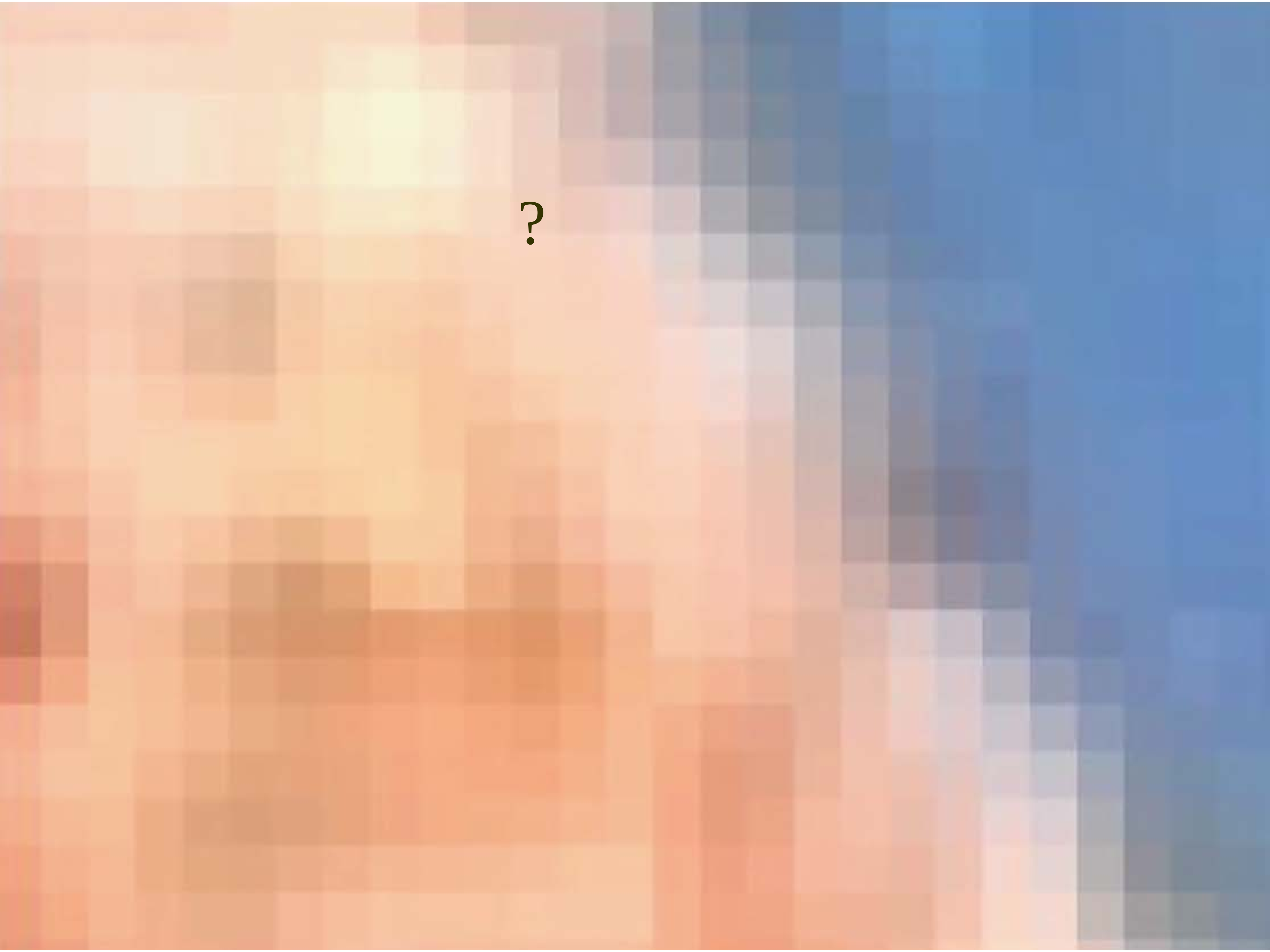

Roman Cage Cup from the 4th century A.D.





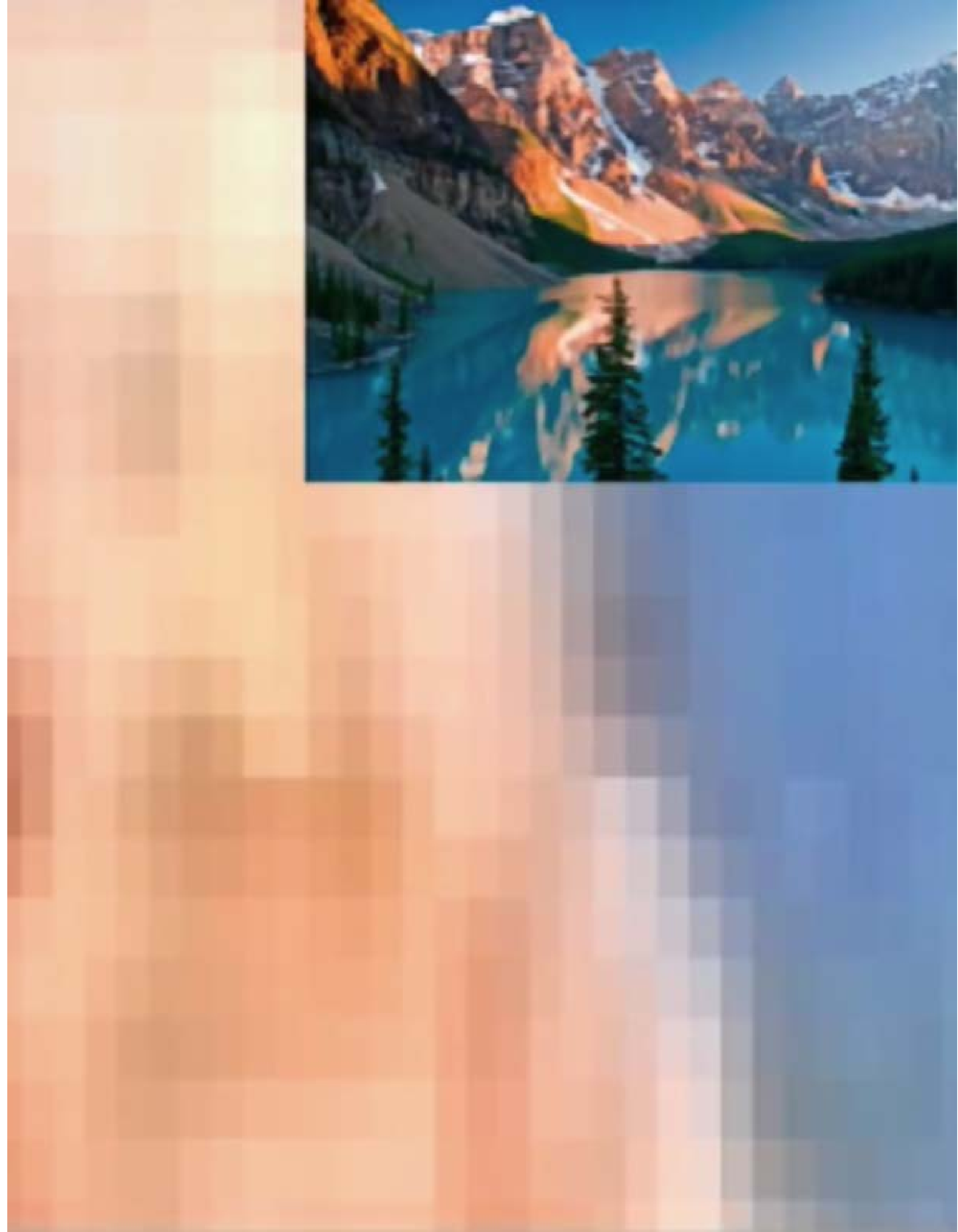
Microstructure of part of a DNA double helix **biopolymer**


Understanding cells in terms of their molecular components.



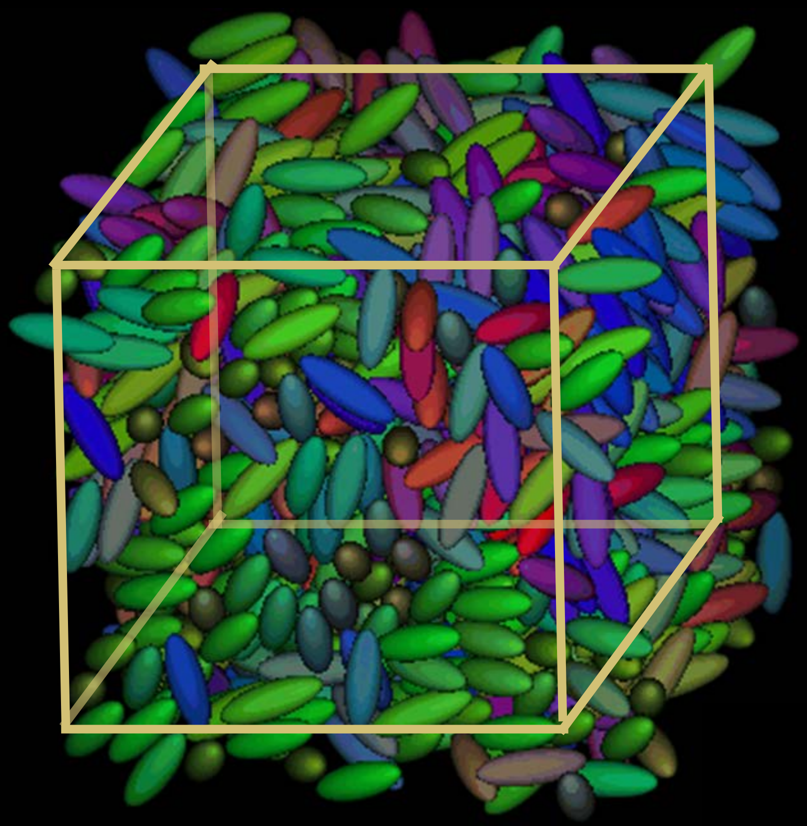
# Emergence

`We use concepts
and observe phenomena
at a mocroscopic scale,
which are derived from
a microscopic scale
where they have no
a priori meaning`



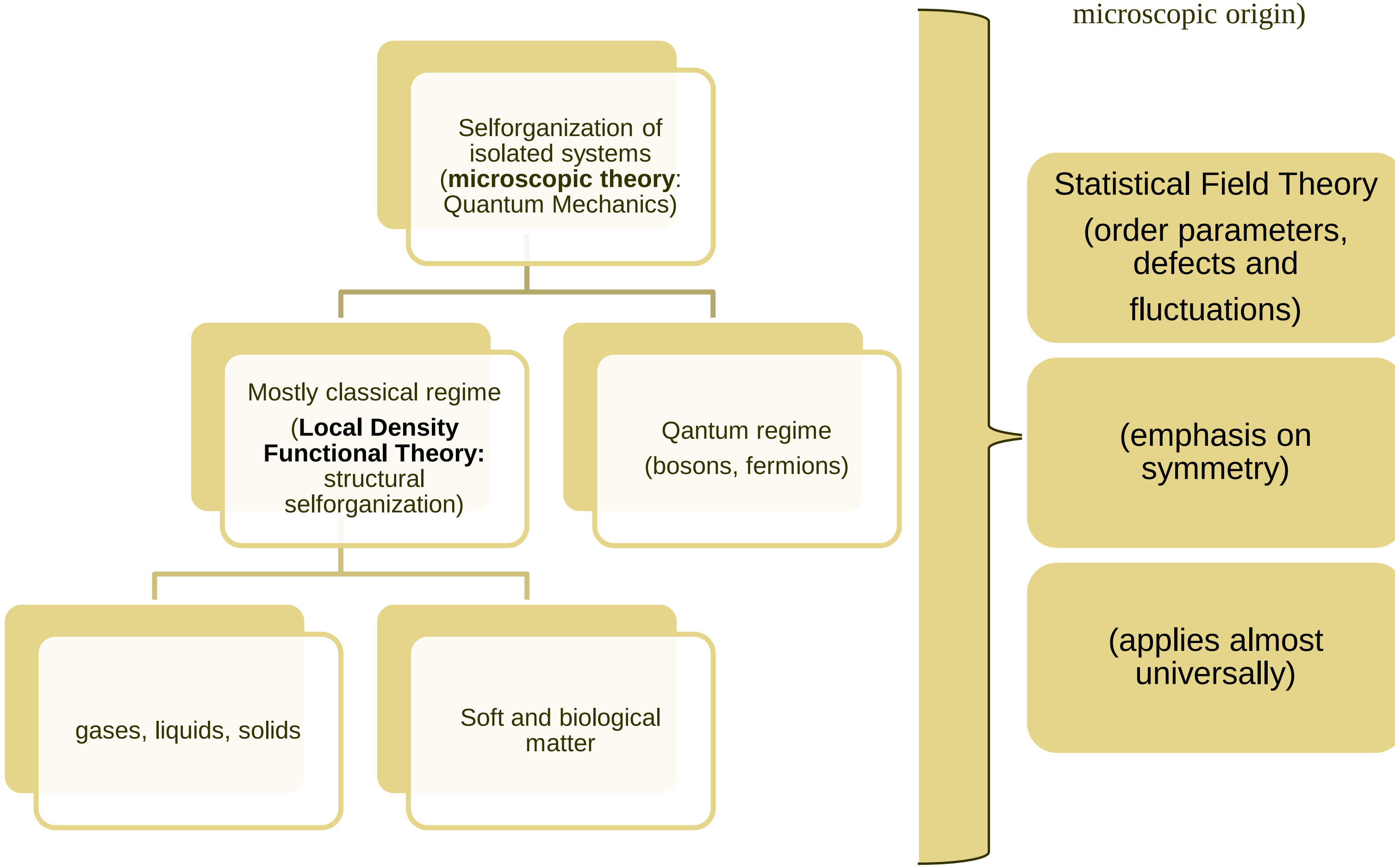
# *Emergence in Physics*

- Macroscopic properties are derived from microscopic quantities (e.g. by averaging)
- Temperature T equals change of  the average energy with entropy
- Entropy S measures the numer of microscopic possibilities and counts the amount of information needed to describe all the microscopic states
- Order parameters measure the amount of order of a given phase with respect to a reference phase



Microscopic Models

Effective models (weakly dependent of microscopic origin)



# Summary of the course:

1. *Introduction to phase transitions:* general mechanisms, examples (magnets, nematics, smectics A, superconductors, TGB phases);

2. Model interaction potentials; quantum corrections in classical regime.

3. *Statistical, microscopic description of condensed matter*: calculation of averages; distribution functions;

4. *Classical systems (microscopic)*:

    - Gases and simple liquids (Virial expansion)
    - Local density functional (LDF) theory of more complex systems with spontaneous symmetry breaking (crystals, soft matter, concept of order parameters, examples)

5. *Near critical point (microscopic):* scaling, critical exponents, real space RG

6. Beyond microscopic models of phases and phase transitions:
    - Role of symmetry and Landau theory
    - Role of residual symmetry (defects)
    - Role of fluctuations (statistical field theory); examples; RG in momentum space

7. Elements of nonequilibrium statistical mechanics:

This series of lectures aims to address three issues that anyone interested in the study of principles macroscopic world works should be familiar with:

- Microscopic description of condensed matter (gases, liquids, solids, soft matter, …)
- Spontaneous Symmetry Breaking and its consequences
- Statistical (phenomenological) approach to time-dependent processes

## Phase transitions (general)

- *Introduction: general mechanisms, examples (magnets, nematics, smectics A, superconductors, TGB phases):*

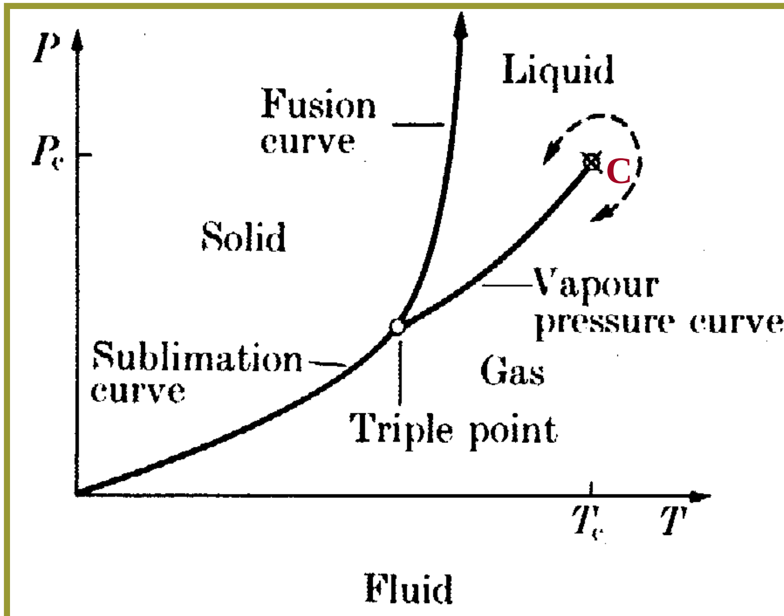
1. discrete symmetry breaking and domain wals
2. continuous symmetry breaking and Goldstone modes
3. theories with local gauge symmetry: Higgs mechanism

# Introduction

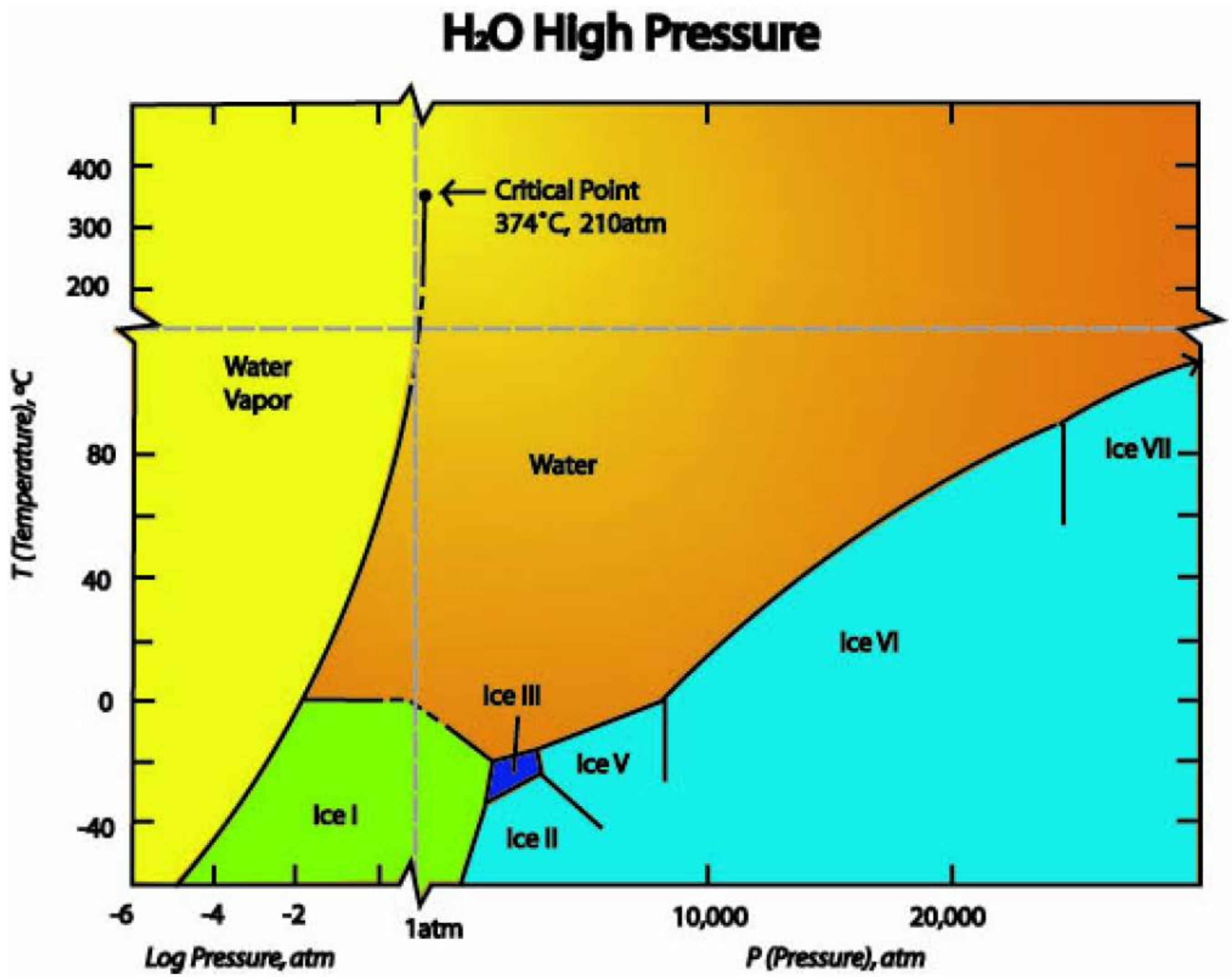
**One way in which matter can change its <u>qualitative</u> properties are phase transitions**

**gas ⟷ liquid**

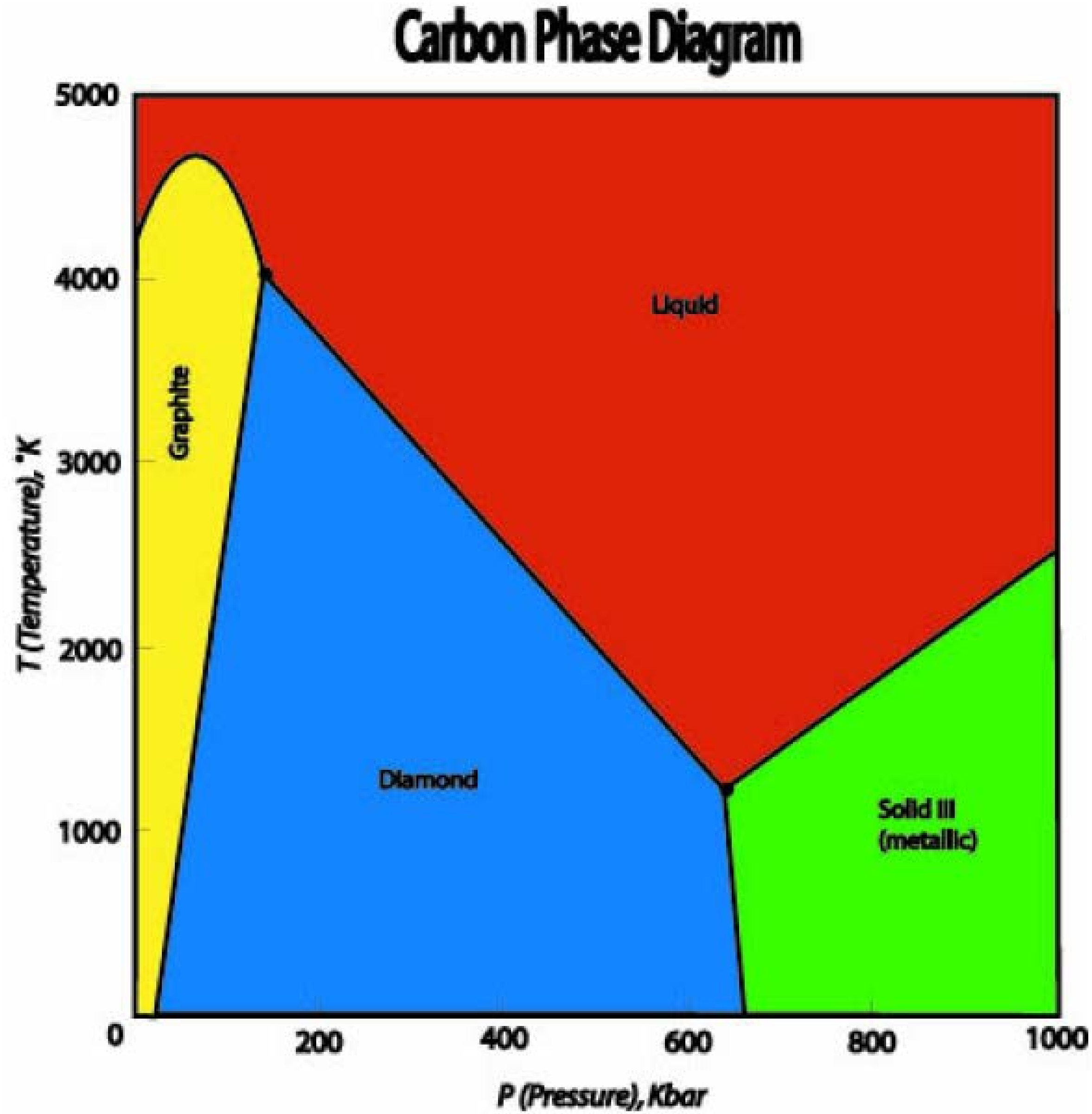
**liquid ⟷ crystalline phase**




**Phase diagram of a fluid.**
**All the phase transitions are first-order except at the critical point C.**
**Beyond C it is possible to move continuously from a liquid to a gas.**
**The boundary between the solid and liquid phases is thought to be always first-order and not to terminate in a critical point.**




$T$-$P$ phase diagram for pure water showing several different solid phases.

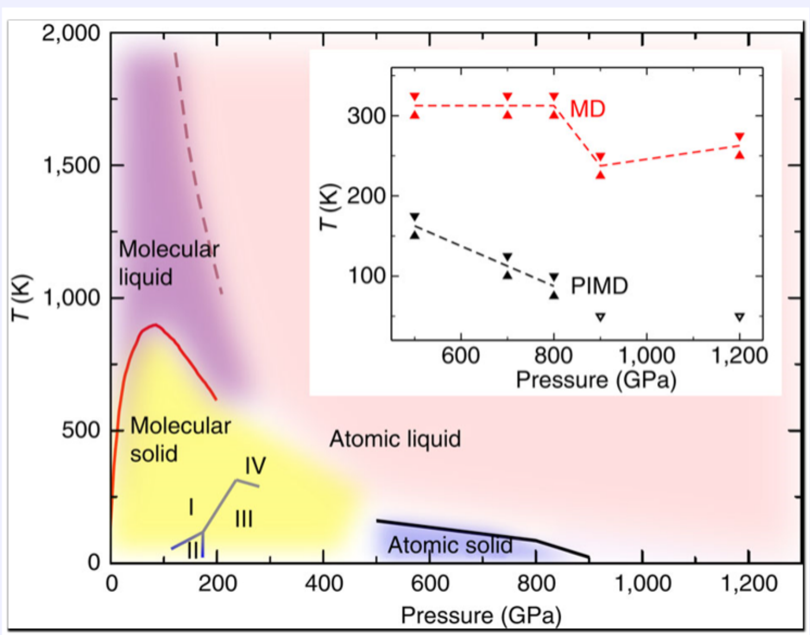



$T$-$P$ phase diagram for carbon—proving that diamonds are not forever.

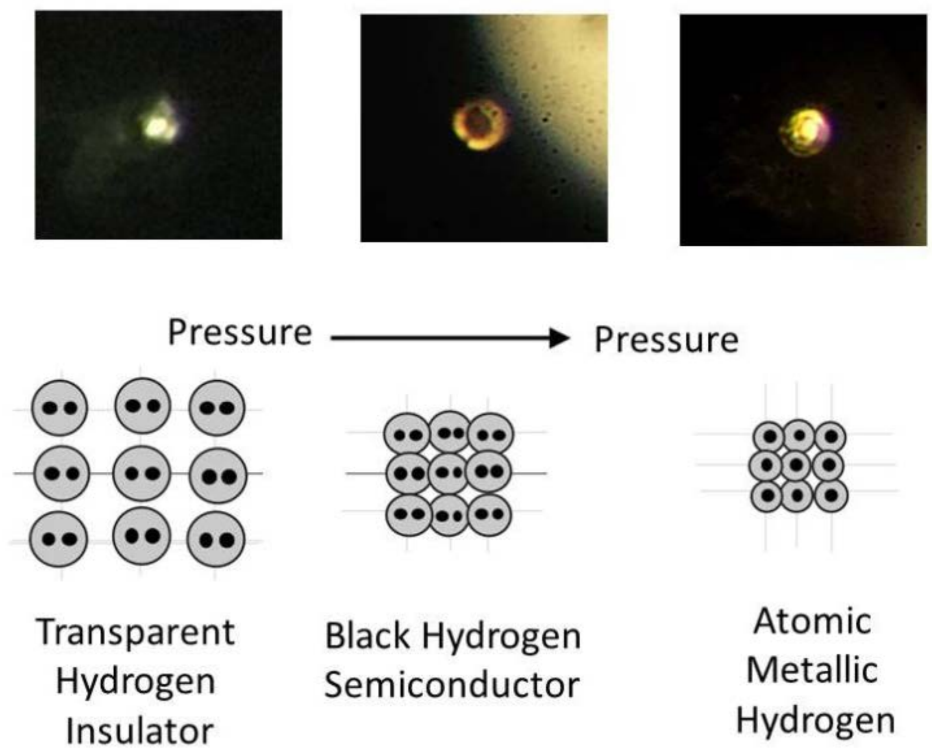
# (Metallic) Hydrogen


**It's Real: Metallic Hydrogen Has Been Created for the First Time**

"This is the holy grail of high-pressure physics," says lead researcher Isaac F. Silvera from Harvard University. "It's the first-ever sample of metallic hydrogen on Earth, so when you're looking at it, you're looking at something that's never existed before.„
To create the sample, the team trapped hydrogen gas inside a tiny diamond casket, chilled it to 5.5 Kelvin and put it under incredibly high pressure, between 465 and 495 GPa - nearly 20 times higher than initially predicted.
1 GPa equals 1 million kilopascals (KPa), and the average pressure at sea level on Earth is 101.325 KPa




**Phase diagram of hydrogen. : Quantum simulation of low-temperature ...**



# Spontaneous Symmetry Breaking

- **First we shall explain what *spontaneous symmetry-breaking* is and then**

- **consider the *mechanism* of spontaneous breaking of the *global* (discrete and continuous) invariance emphasizing their specific features.**

- **Discuss the role of gauge symmetry (superconductors, smectics, TGB phases)**

- **Finally, we shall dwell on the *residual* symmetry.**

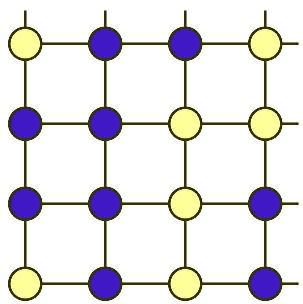
# Example 1:

## Ising ferromagnet (breaking of global discrete symmetry: domain walls)

# Example 1

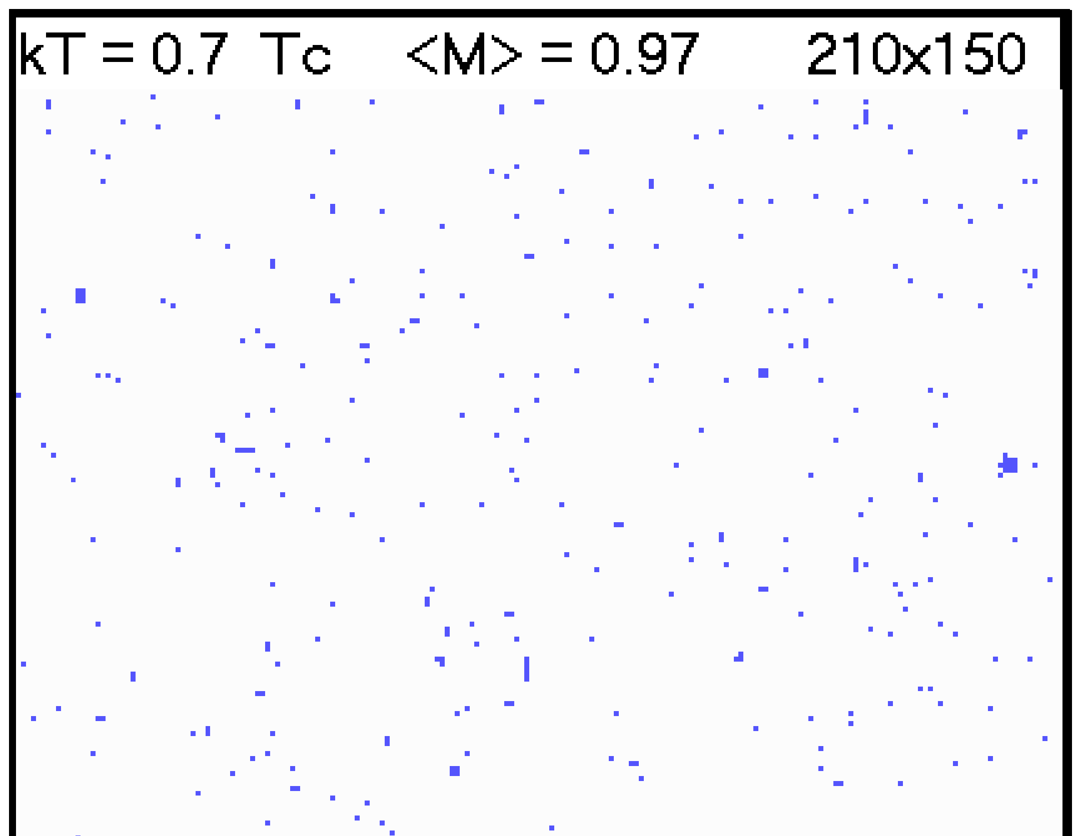
**Ising model: (2d)**

$$\mathcal{H} = -J \sum_{<i,j>} s_i s_j; \qquad s = \pm 1$$



$\downarrow$ (s = +1)

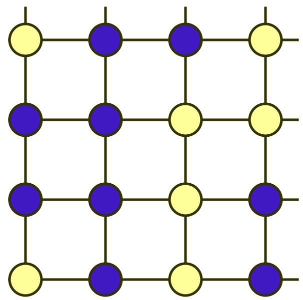
$\uparrow$ (s = - 1)

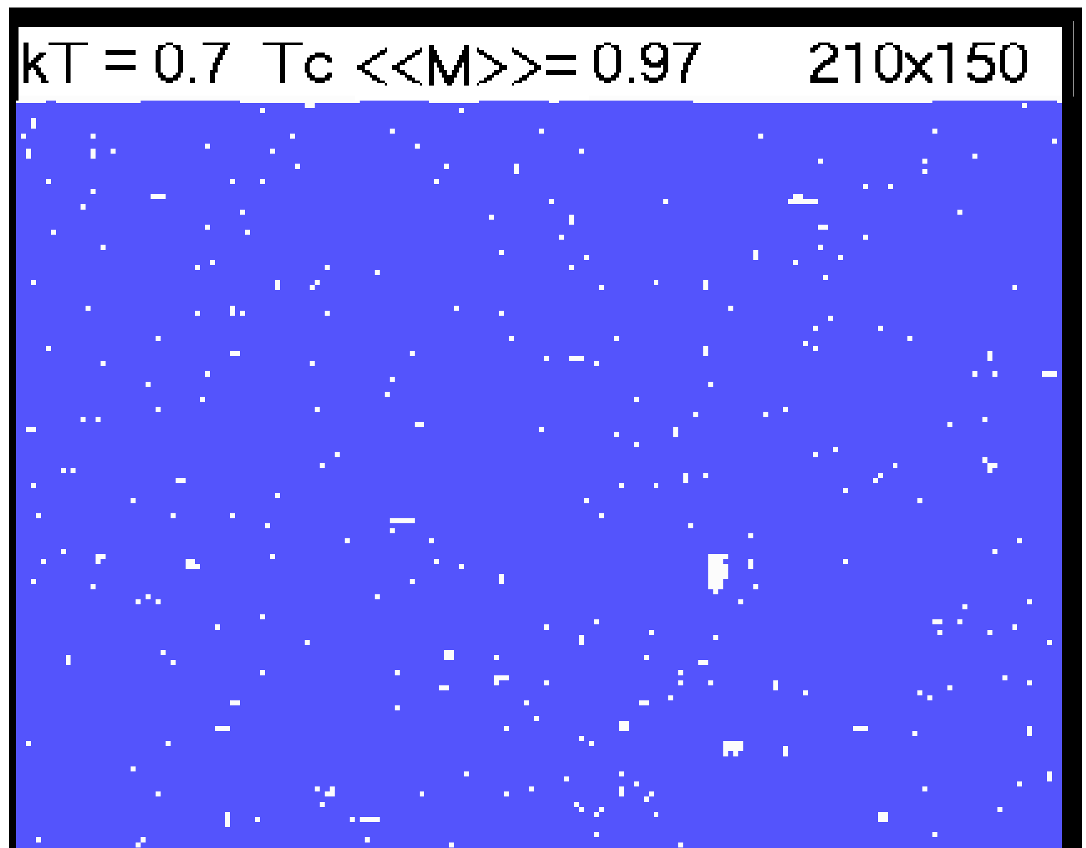


# Example 1

**Ising model: (2d)**

$$\mathcal{H} = -J \sum_{<i,j>} s_i s_j; \qquad s = \pm 1$$



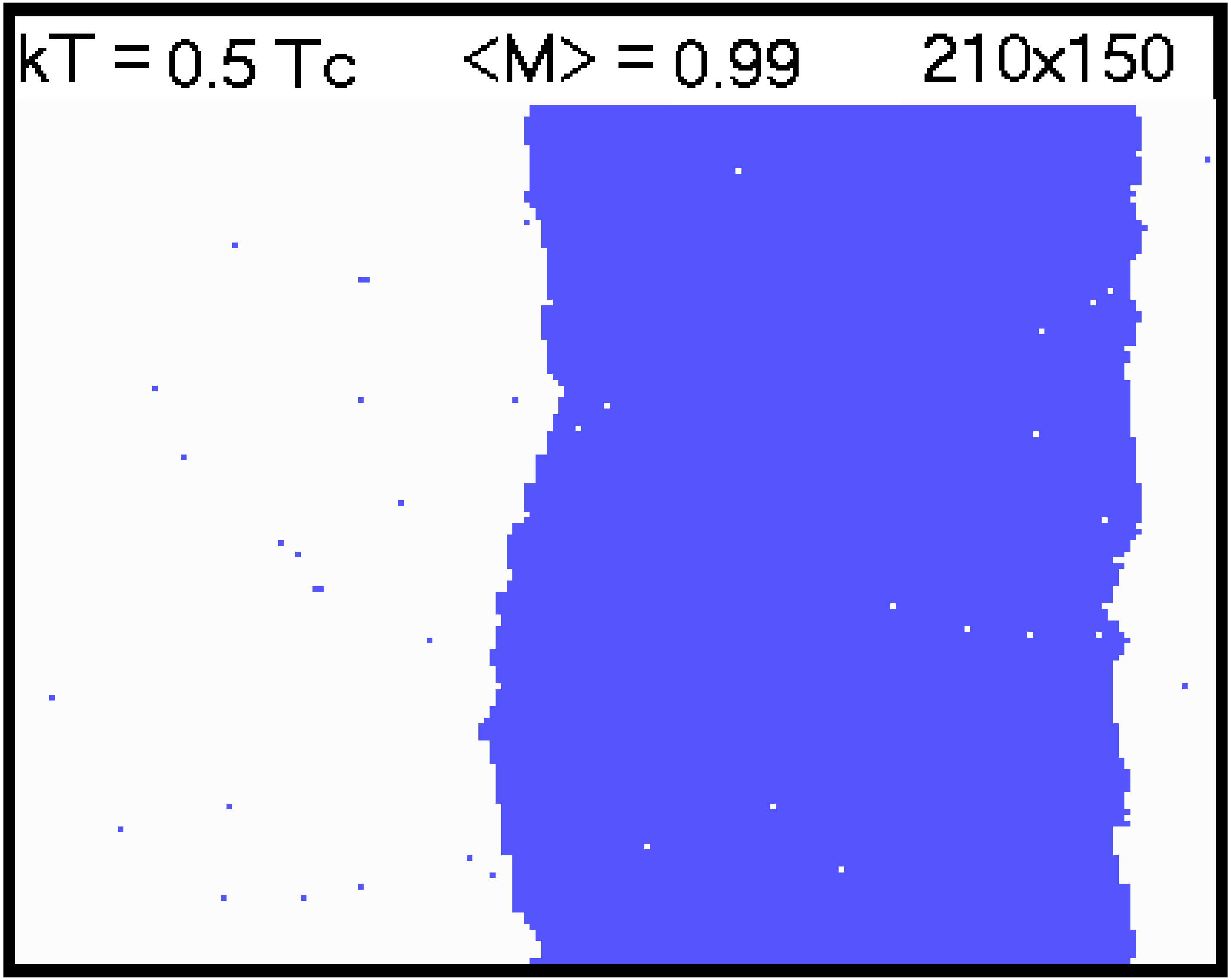


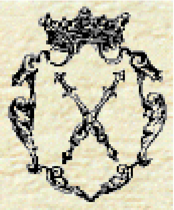

○ ↓ **(s = +1)**

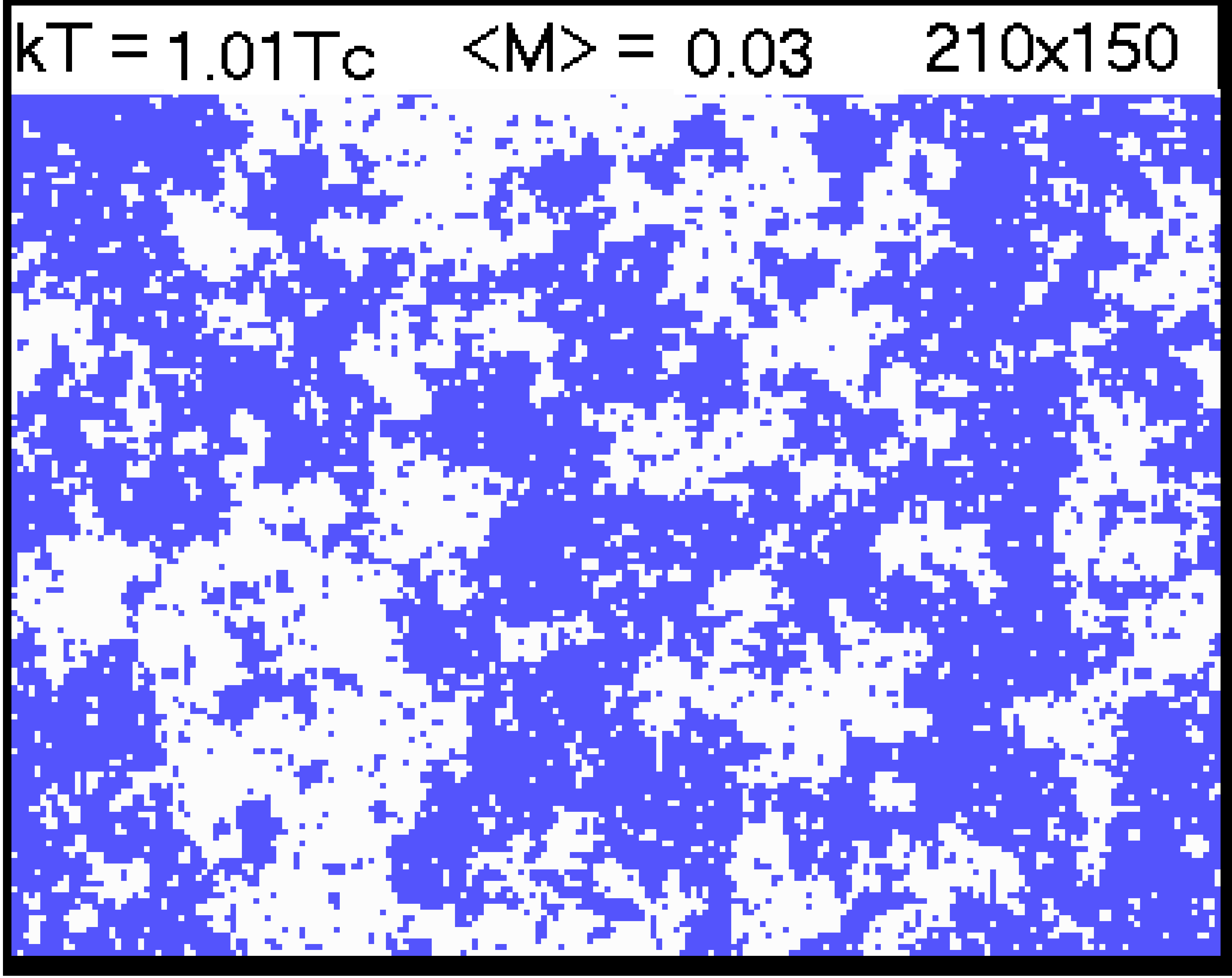
● ↑ **(s = - 1)**

**Ising model: (2d)**

$$\mathcal{H} = -J \sum_{<i,j>} s_i s_j; \qquad s = \pm 1$$

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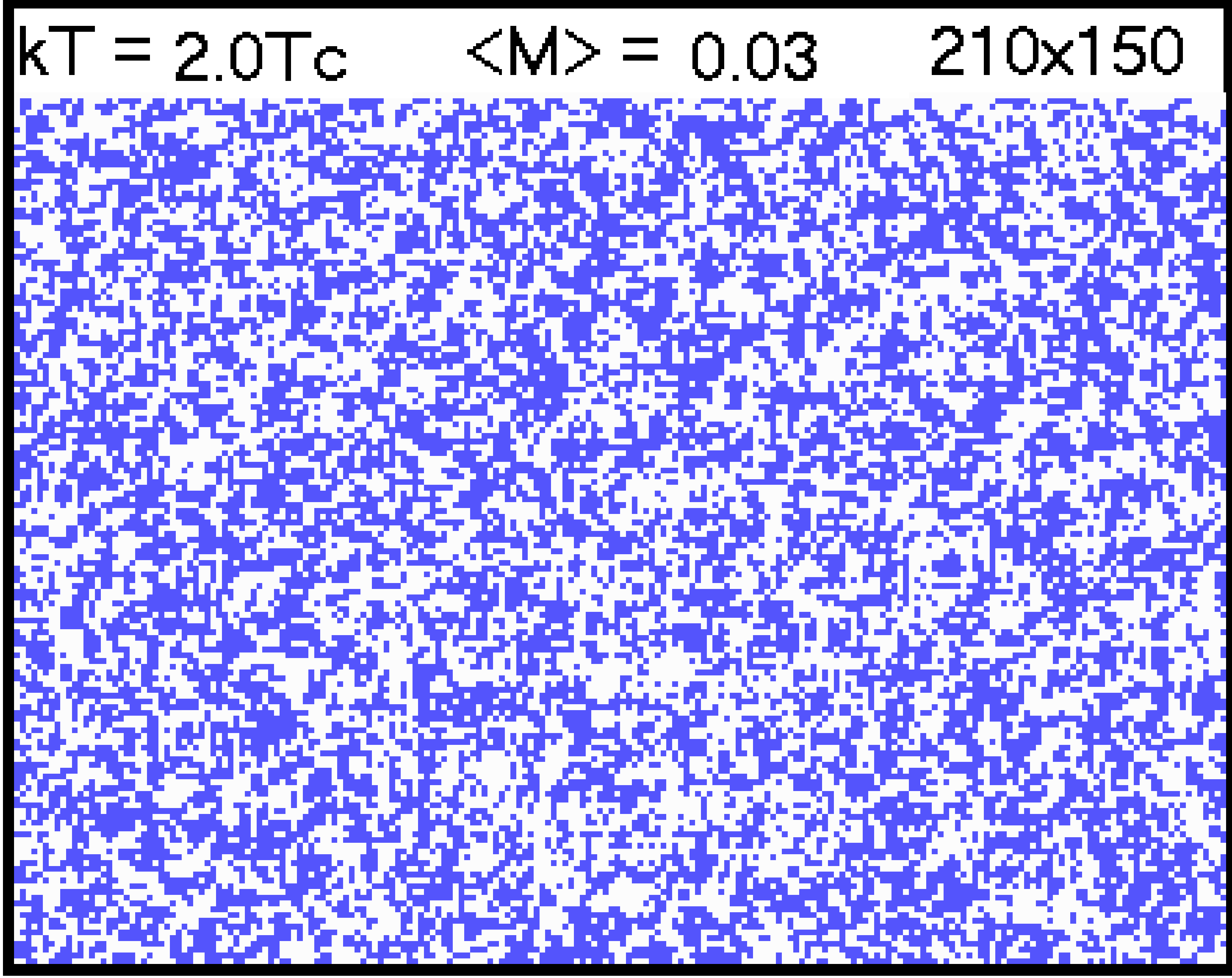
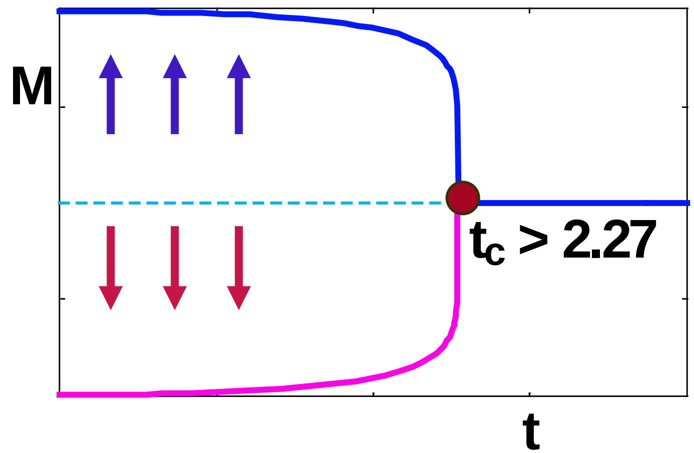
<M> = 0.03
210x150


kT = 2.0Tc
<M> = 0.03
210x150

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} s_i s_j; \qquad s = \pm 1$$

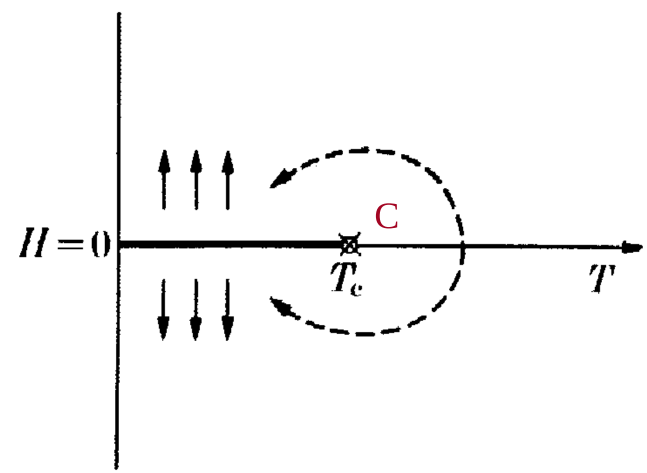



$$\mathcal{H}(\{s_i\})) = \mathcal{H}(\{-s_i\})$$

Global $\mathbb{Z}_2$ symmetry

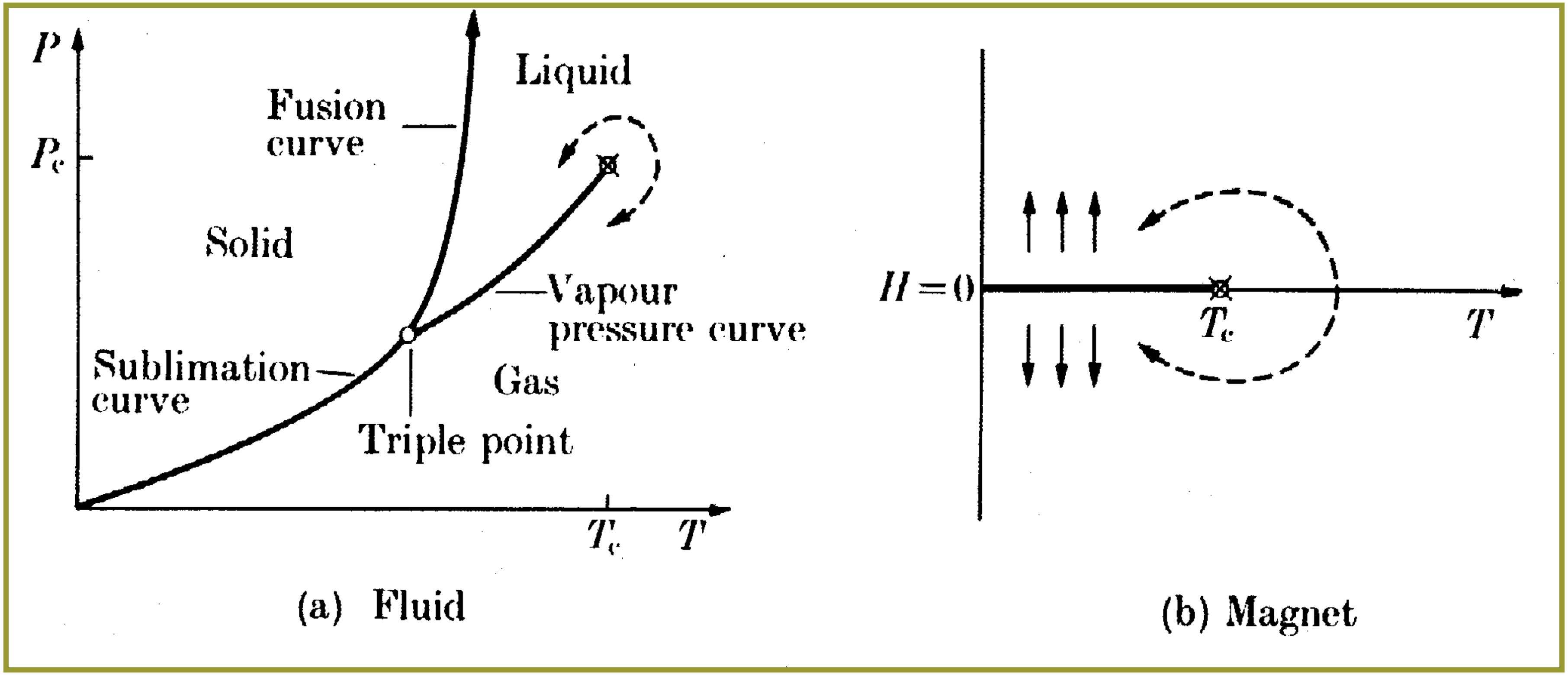
**This discrete symmetry is broken below tc**

Phase diagram of an Ising magnet. All the phase transitions are first-order except at the critical point C. Beyond C it is possible to move continuously from a paramagnetic to a ferromagnetic phase.




Ising magnet

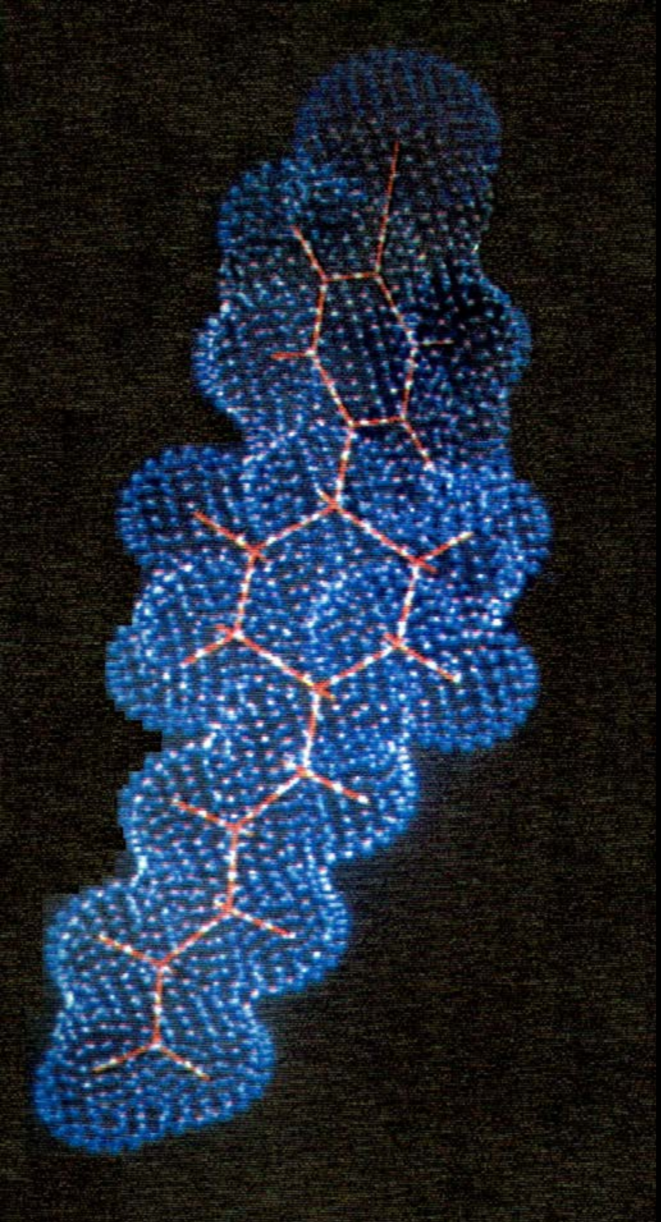
# Analogy to liquid - gas phase transition

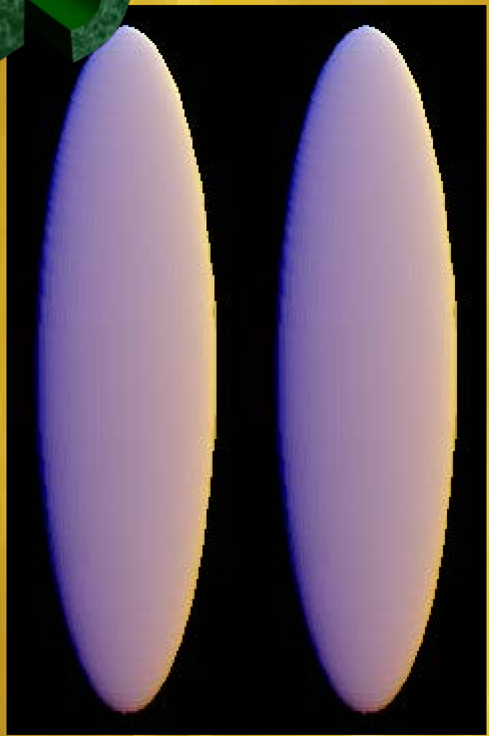


# Example 2:

# Liquid Crystals (breaking of global continuous Symmetry and Goldstone modes)

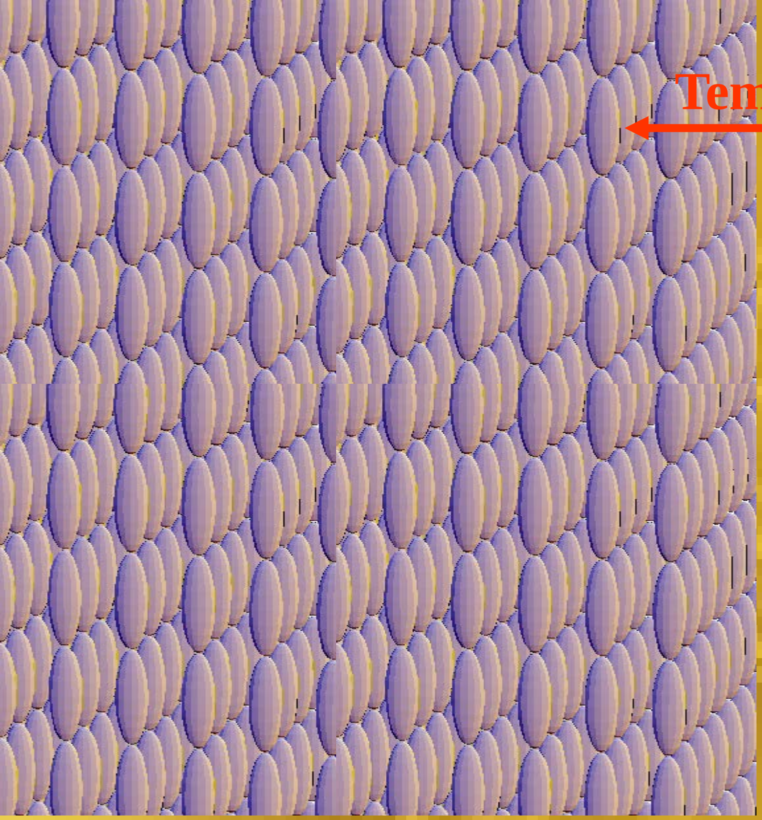
# Liquid crystals



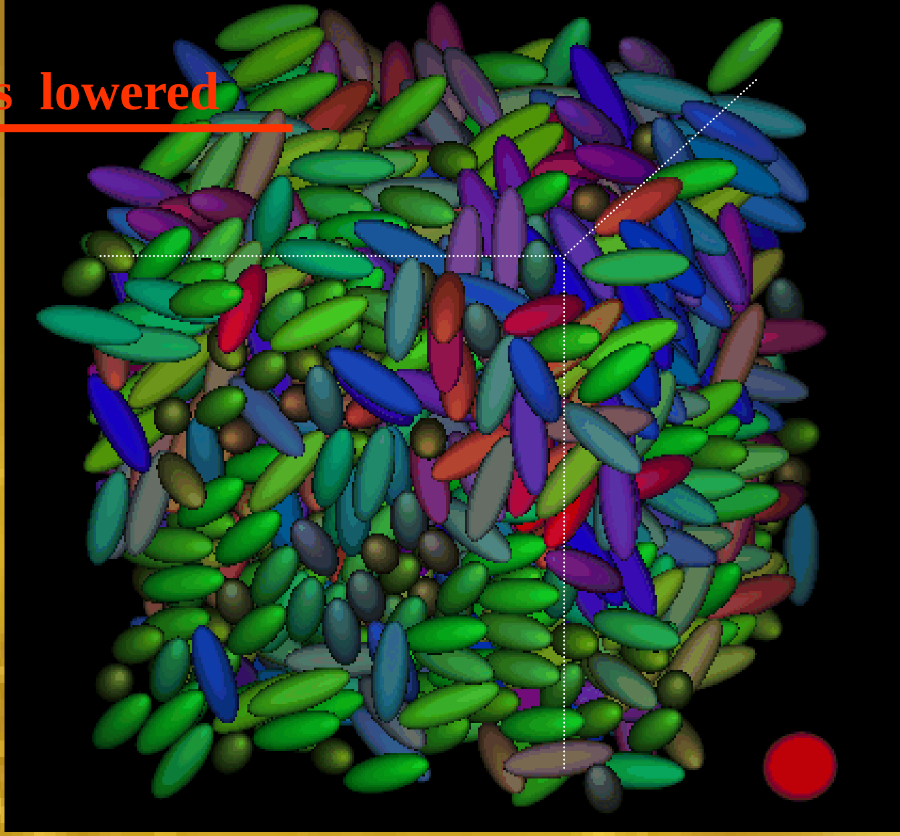



Ground state of
two liquid crystalline
molecules

Temperature is lowered

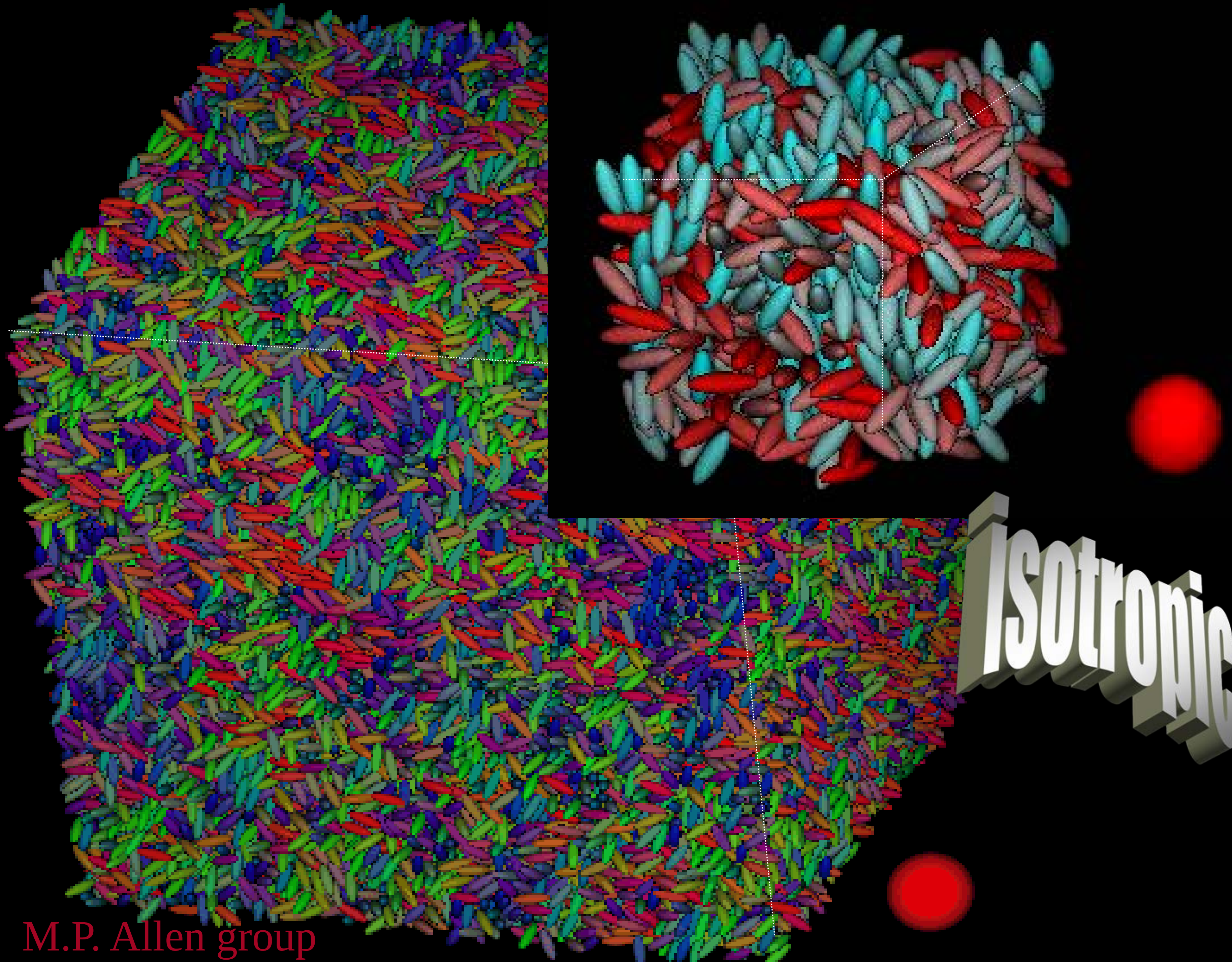


A possible crystalline phase at low temperatures

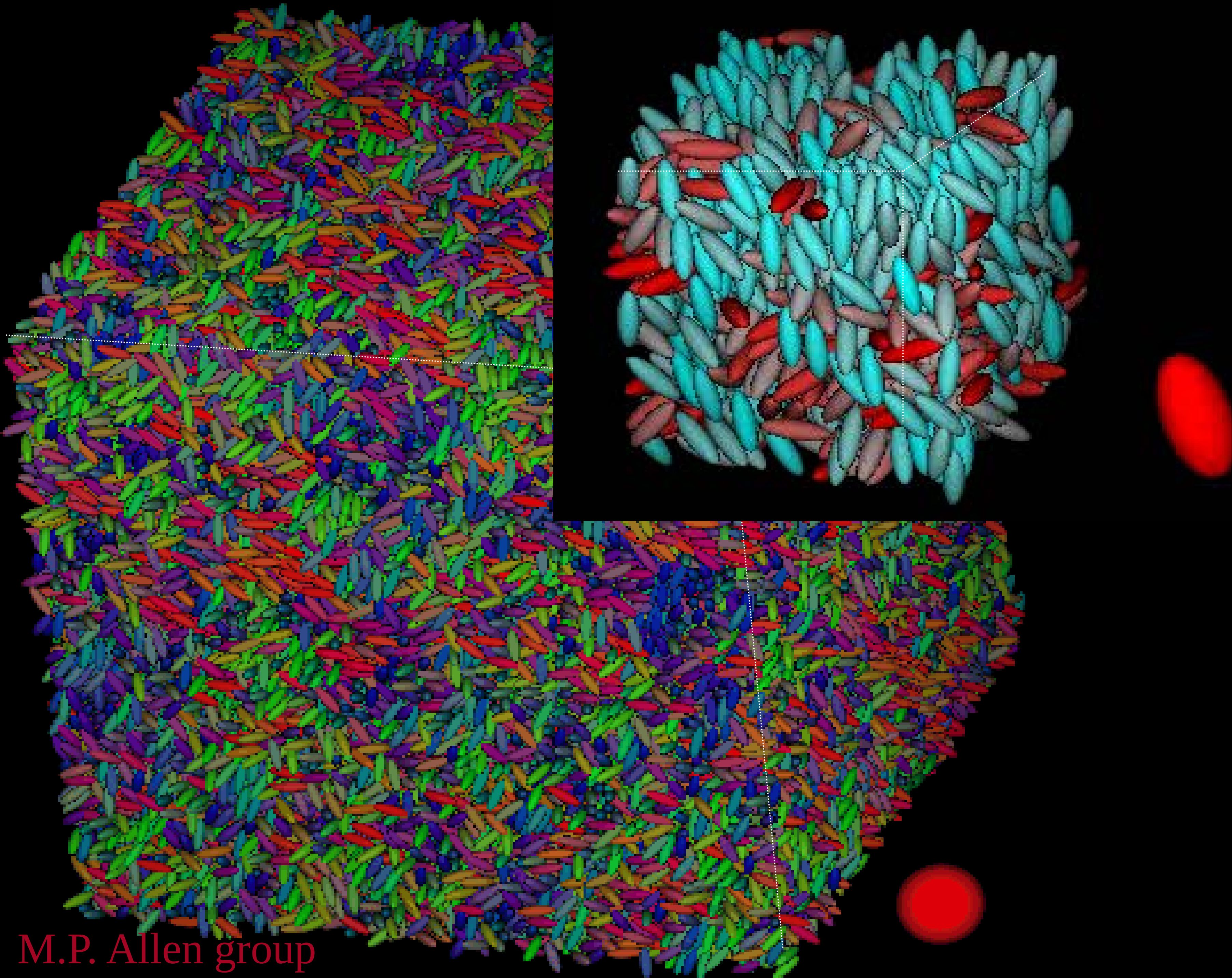


Isotropic liquid at high temperatures

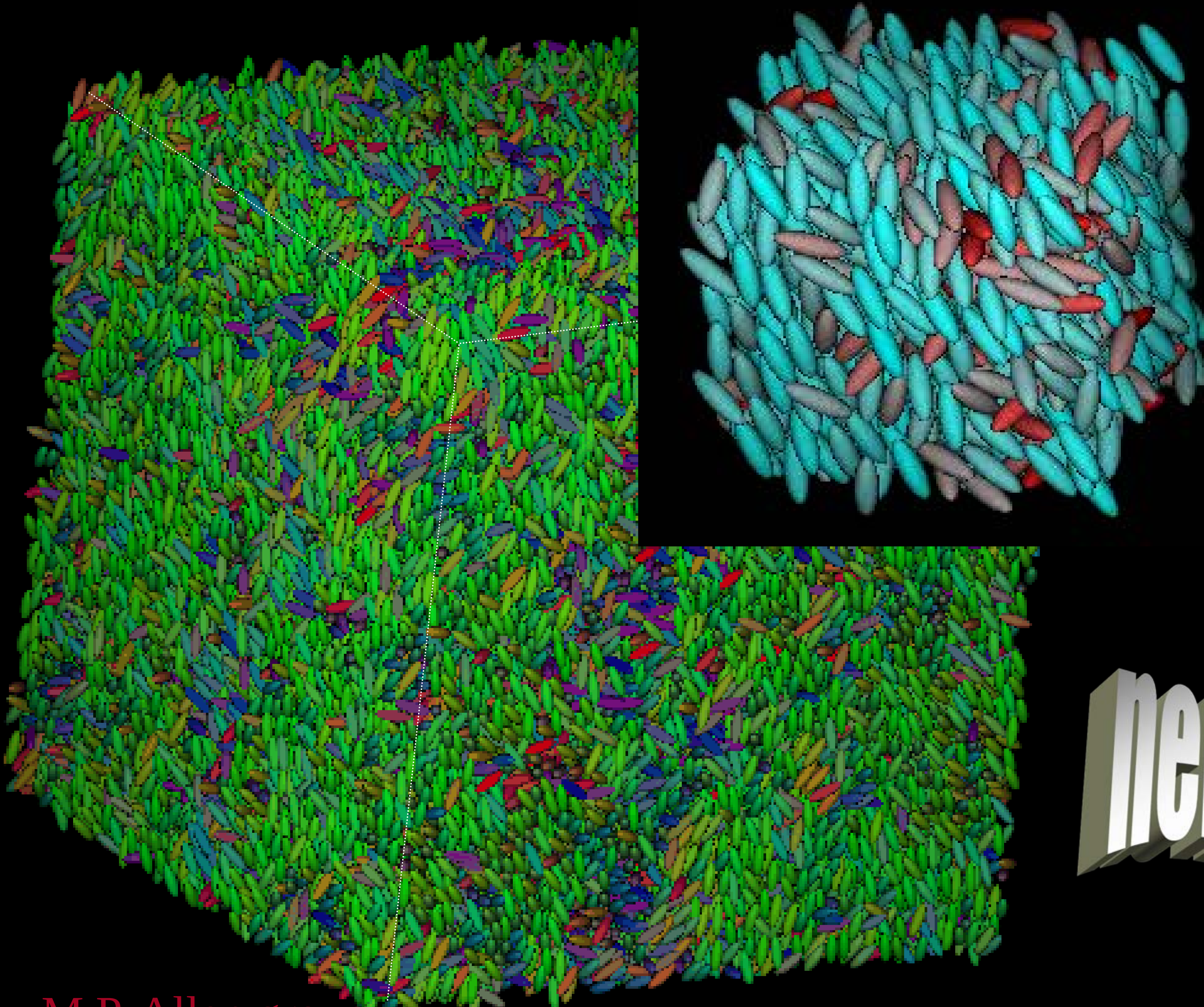
**Intermediate structures:**
**liquids with partial orientational order connected (or not) with a partial translational order**

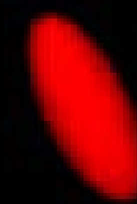


M.P. Allen group



M.P. Allen group





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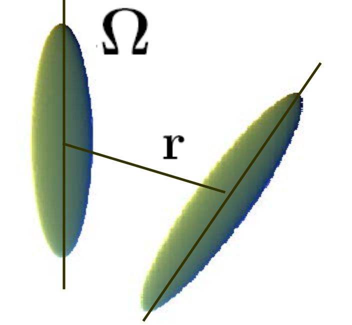
M.P. Allen group

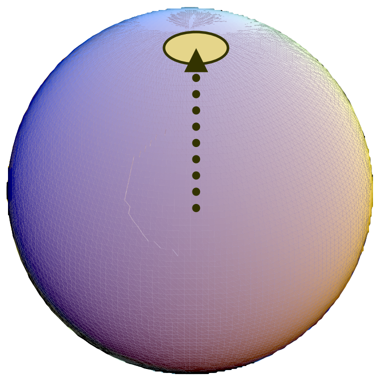


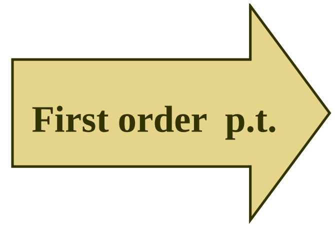

$$\mathcal{H}(\{\Omega_i\},\{r_{kj}\}) = \mathcal{H}(\{\mathcal{R}\Omega_i\},\{r_{kj}\})$$

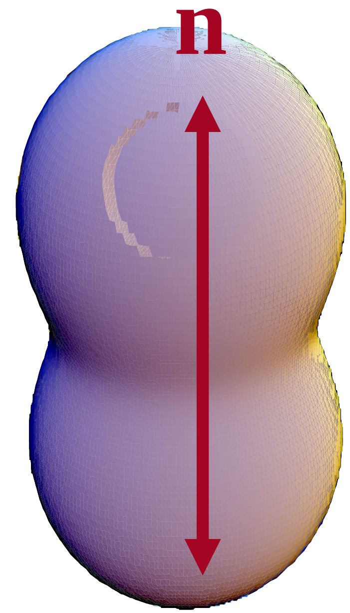
Global $\mathbb{SO}(3)$ symmetry

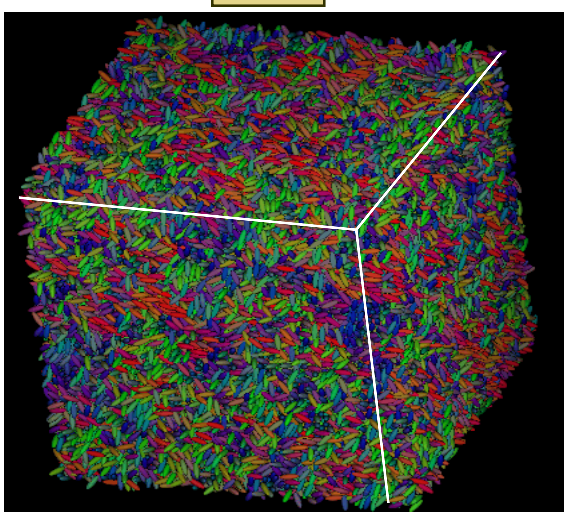
$$\mathbb{SO}(3) \Rightarrow \mathbb{D}_{\infty h}$$

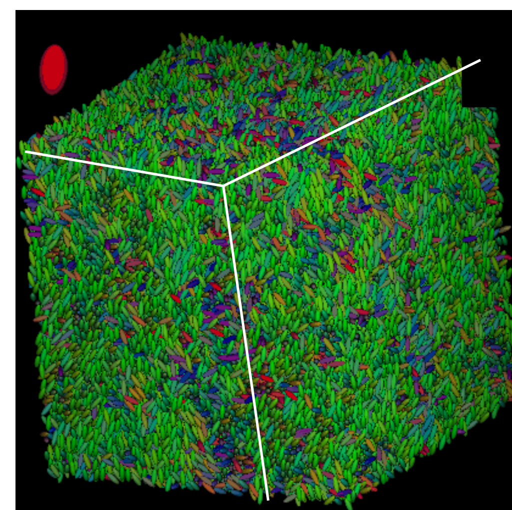
**Director**

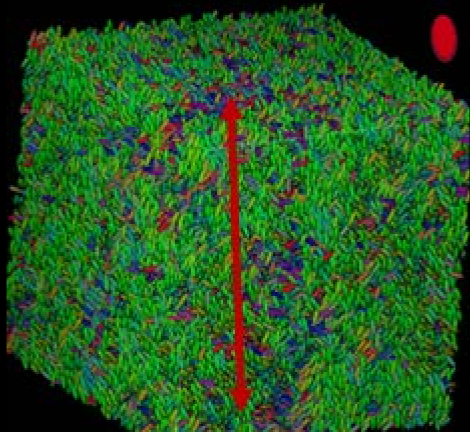


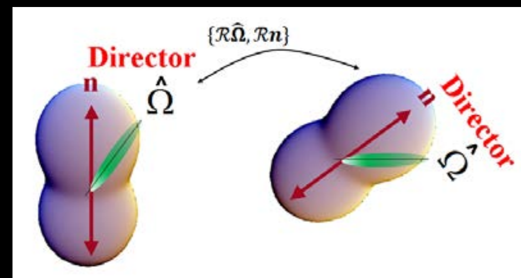


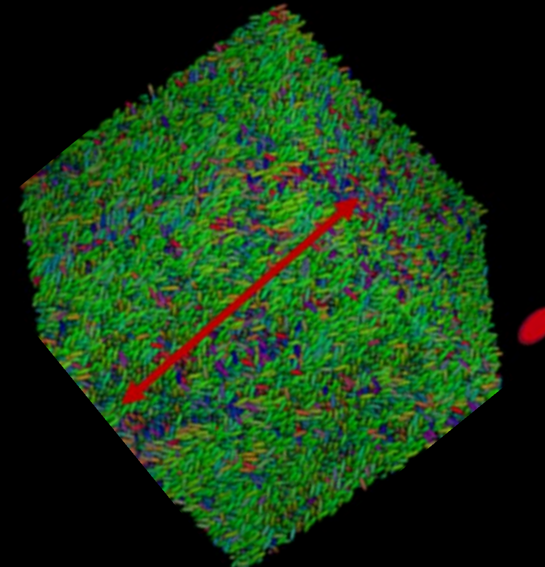



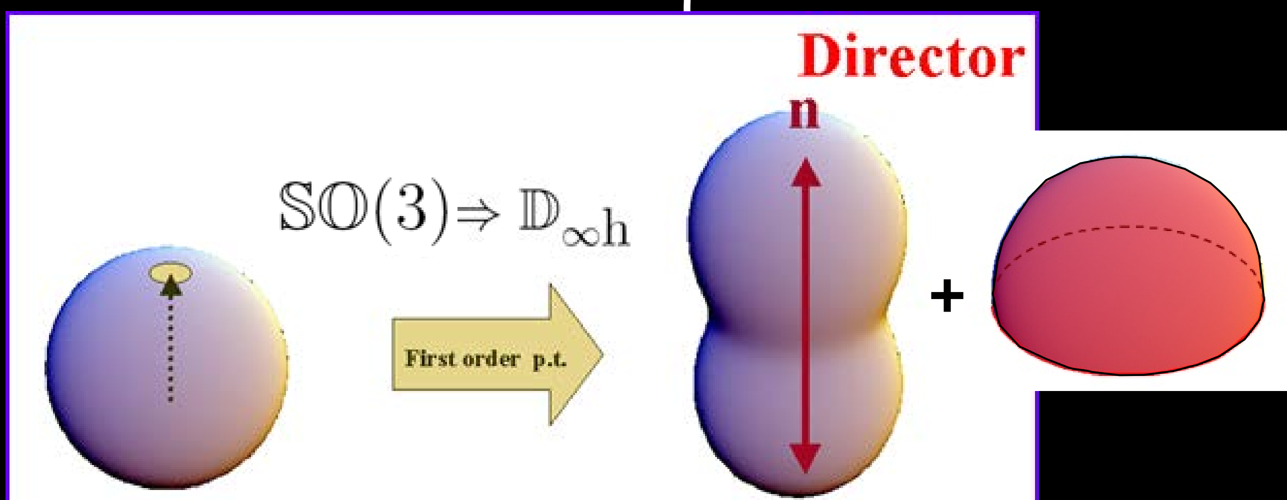




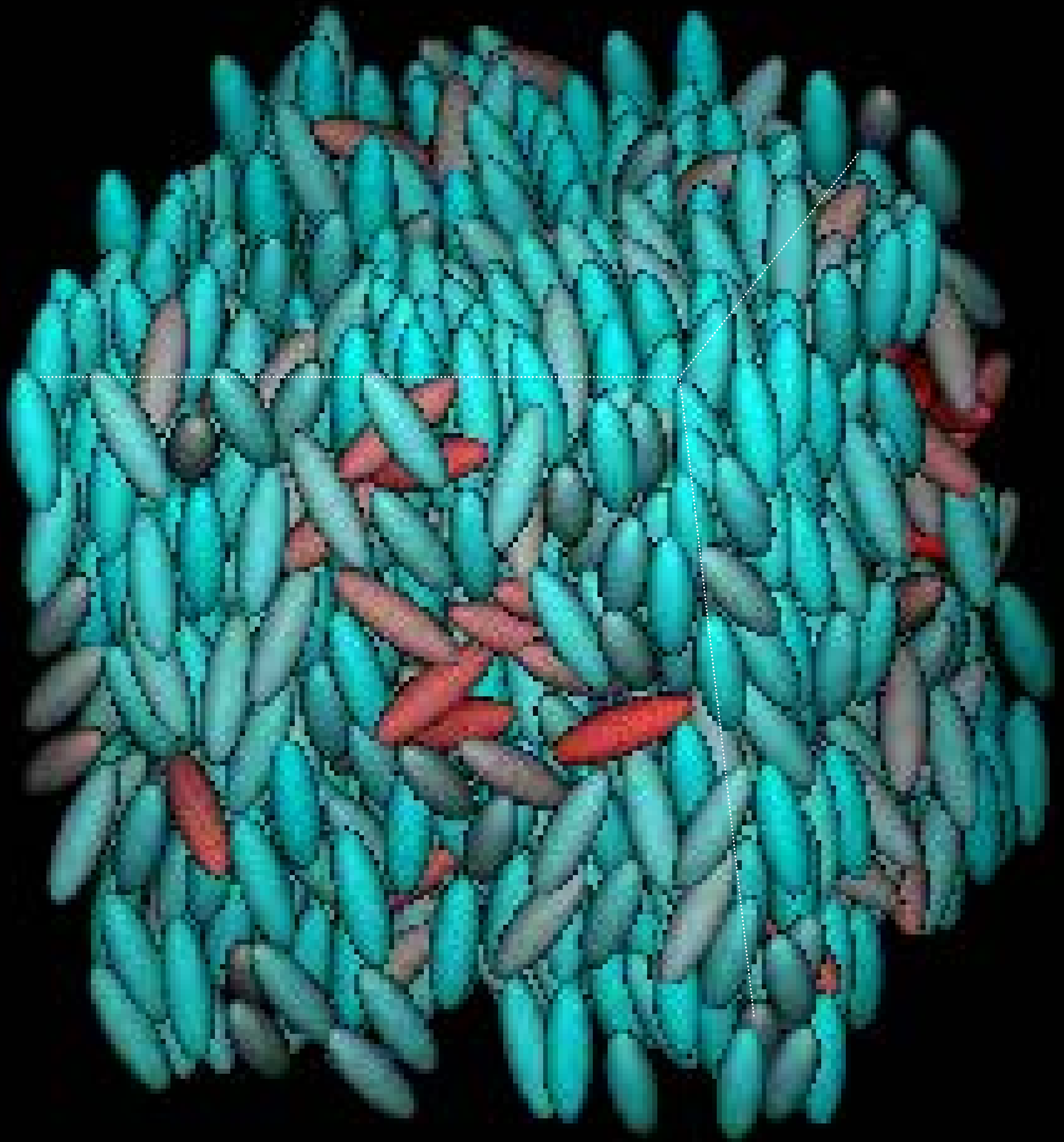


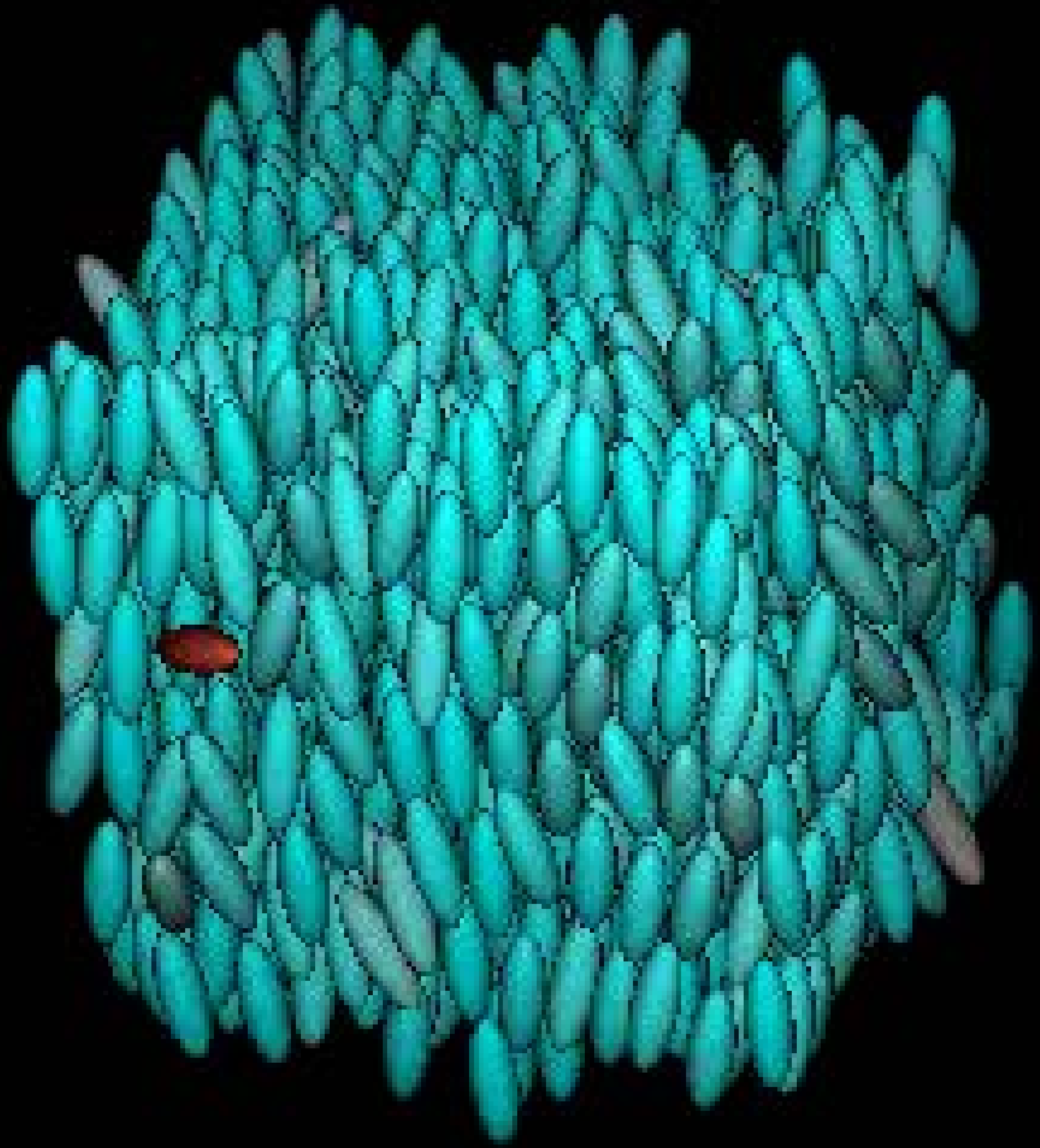
This pair together recovers back the symmetry of the Hamiltonian

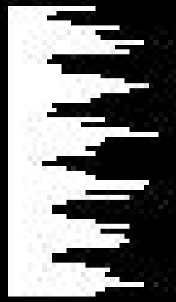


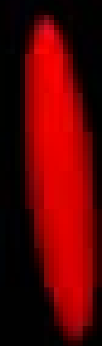

Space of degeneracy of the director




M.P. Allen group







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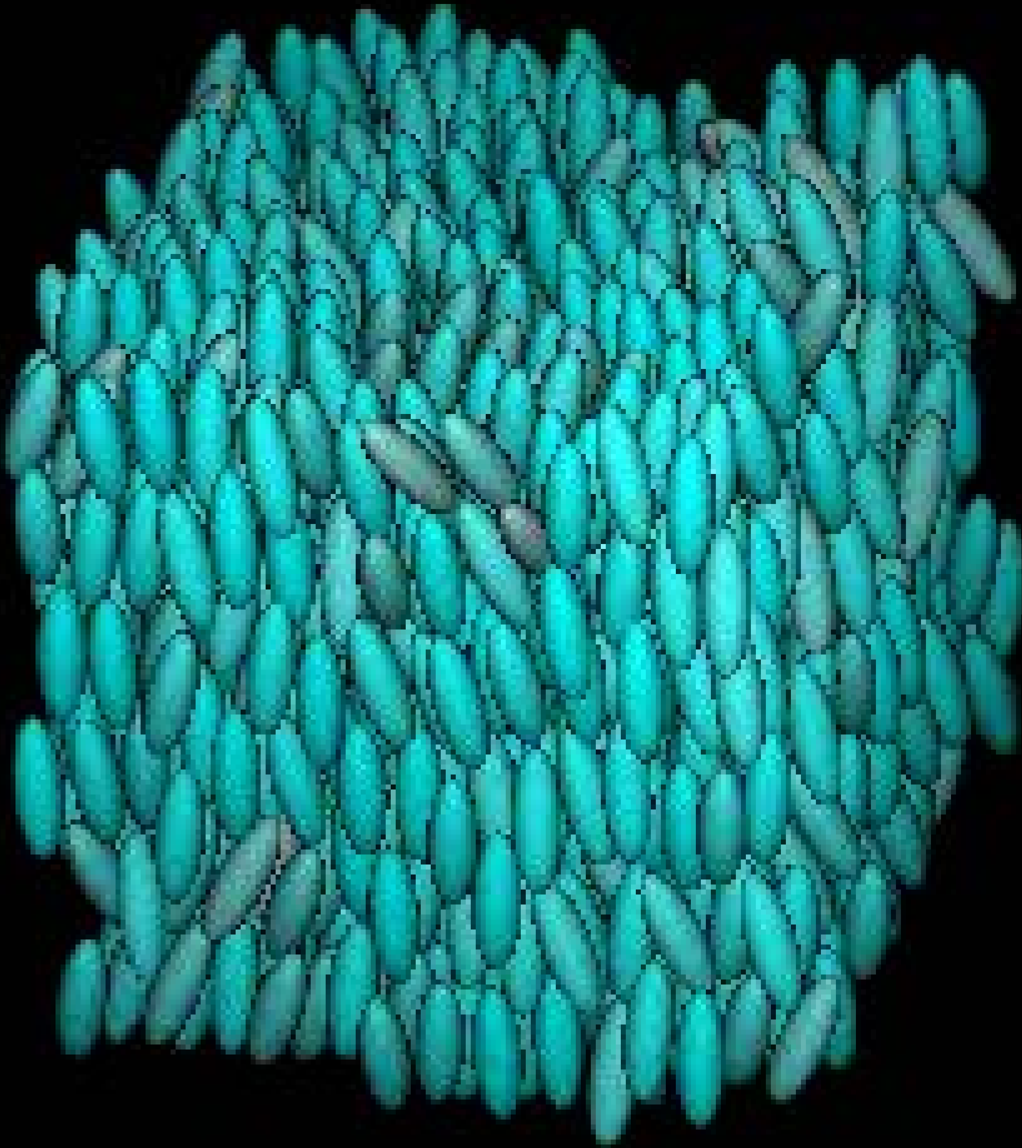
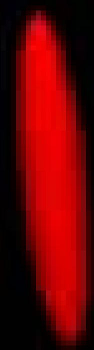




M.P. Allen group

$$\boxed{\mathcal{H}(\{\Omega_i\},\{\mathbf{r}_k-\mathbf{r}_j\})=\mathcal{H}(\{\mathcal{R}\Omega_i\},\{\mathcal{T}\mathbf{r}_k-\mathcal{T}\mathbf{r}_j\})}$$

$\mathbb{D}_{\infty h}\times\mathbb{T}_2\times\mathbb{T}_1$ (d) I-$S_A$

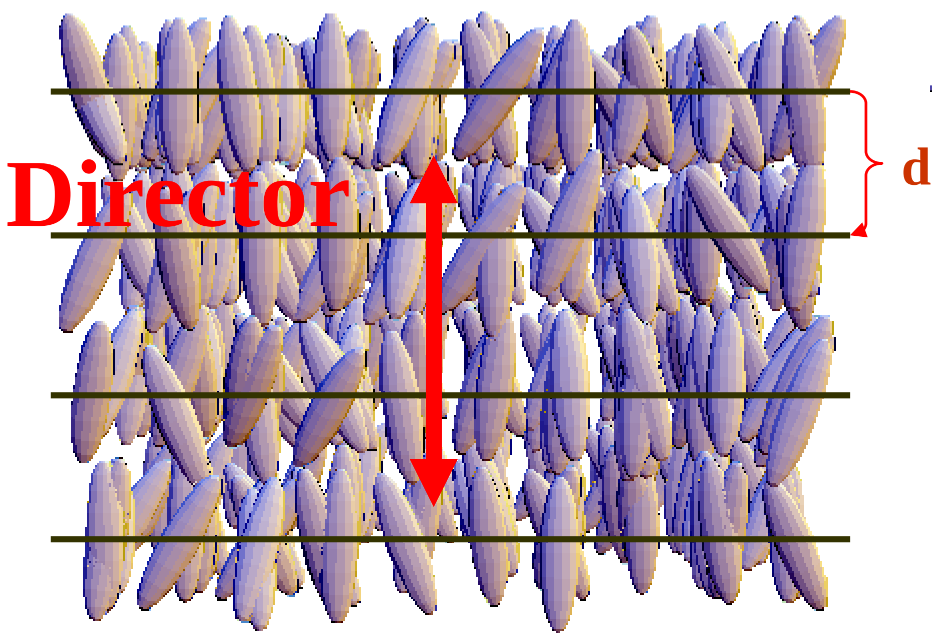
Global $\mathbb{SO}(3)\times\mathbb{T}(3)$ symmetry

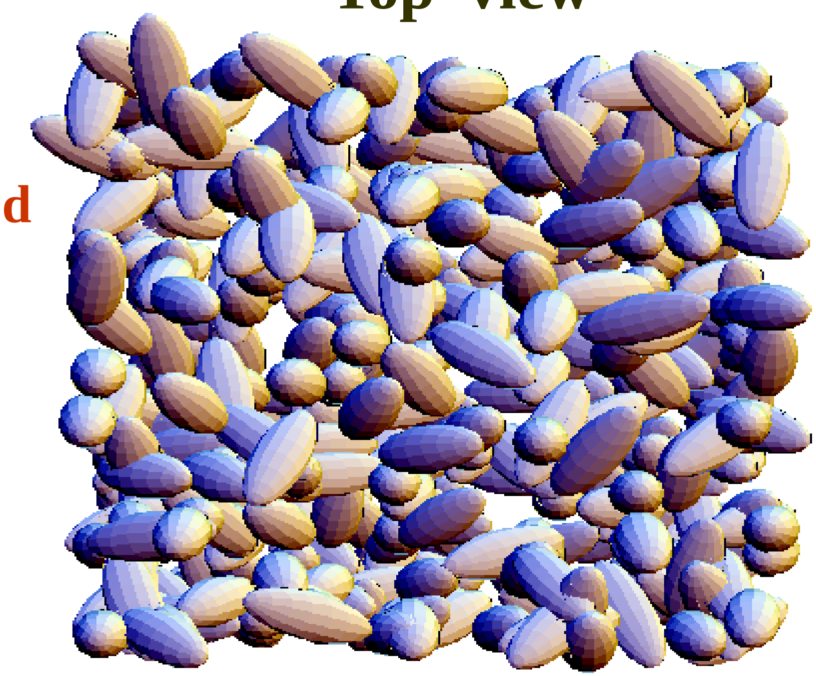
(I- N) N-$S_A$

$\mathbb{D}_{\infty h}\times\mathbb{T}_3 \Longrightarrow \mathbb{D}_{\infty h}\times\mathbb{T}_2\times\mathbb{T}_1$ (d)

**Side view** **Top view**

$S_A$



# Example 3:

# Superconductivity (role of local gauge symmetry)

# Landau-Ginzburg-Abrikosov

$$\{\psi(\boldsymbol{r}), \quad \boldsymbol{A}_\mu(\boldsymbol{r})\}$$

$$\psi \longrightarrow \psi' = \psi \exp\left[i\Lambda(\boldsymbol{r})\right]$$

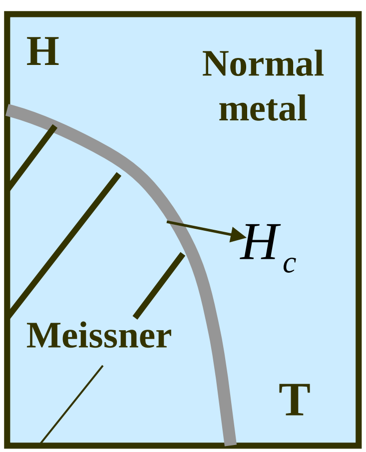
$$\boldsymbol{A} \longrightarrow \boldsymbol{A}' = \boldsymbol{A} + \frac{1}{2e}\boldsymbol{\nabla}\Lambda$$

**Two length scales:**

$\lambda$ : **London penetration depth**

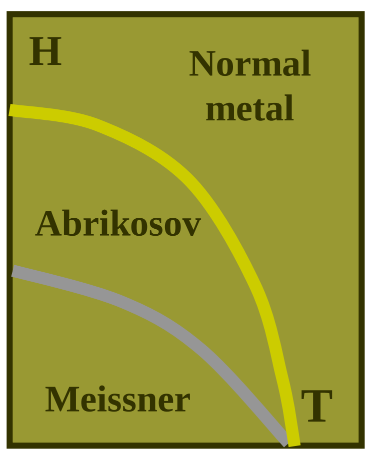
$\xi$ : **coherence length**

$\xi / \lambda > \sqrt{2}$




**Type I**

$\xi / \lambda < \sqrt{2}$



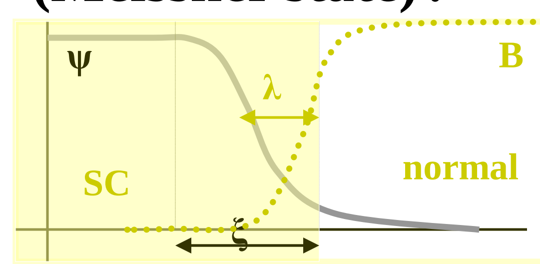

**Type II**

**1. Normal metal :** $\psi = 0$, $\mathbf{B} = \mu_0 \mathbf{H}$,

$\mu_0 \approx 1, \quad (in \;\; CGS)$

**2. Normal superconductor**

**(Meissner state) :**



# Meissner Effect (manifestation of local gauge symmetry – example of Higgs mechanism)

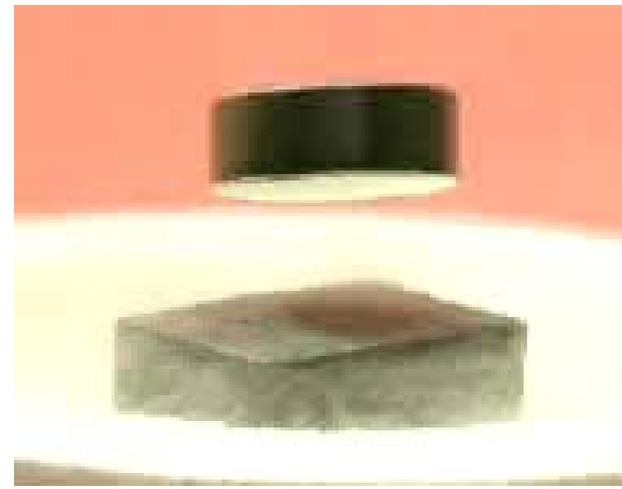
Superconductors expel magnetic field,

and hence repel magnets. This repulsion can be

stronger than gravity, which leads to levitation

- the most fascinating manifestation of

superconductivity.

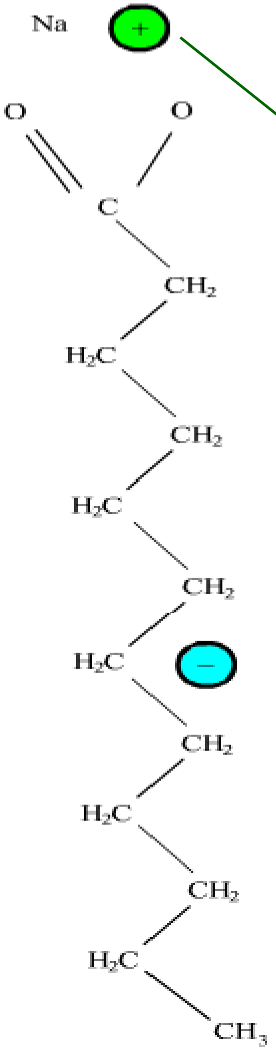


A magnet moving by a conductor induces currents in the conductor. This is the principle upon which the electric generator operates. But, in a superconductor the induced currents exactly mirror the field that would have otherwise penetrated the superconducting material - causing the magnet to be repulsed. This phenomenon is known as diamagnetism and is today often referred to as the "Meissner effect". The Meissner effect is so strong that a magnet can actually be levitated over a superconductive material.

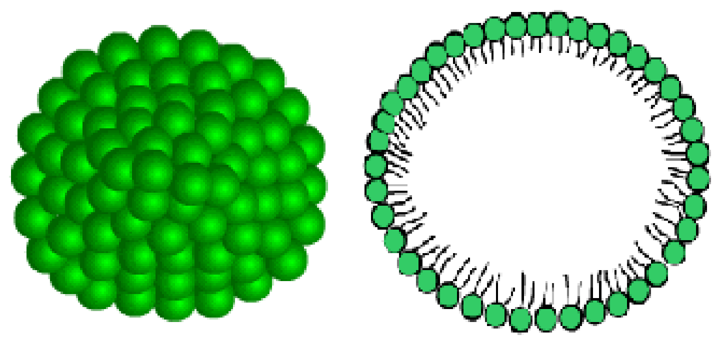
Hierarchical selforganization (without, or) with only a partial involvement of symmetry breaking mechanism

(polimeric, lyotropic, biological, … matter)

# Aphiphilic molecules

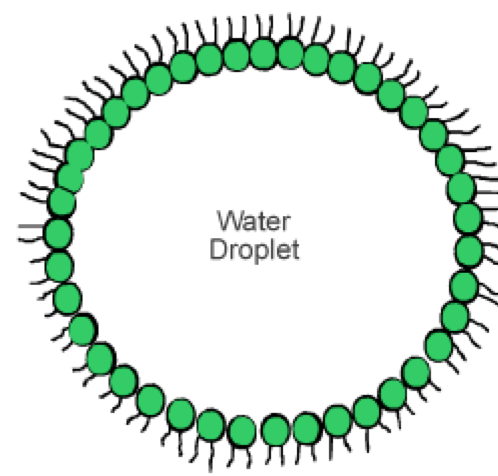


**Polarny (jonowy)  koniec: lubi wodę lub inne polarne środowisko**

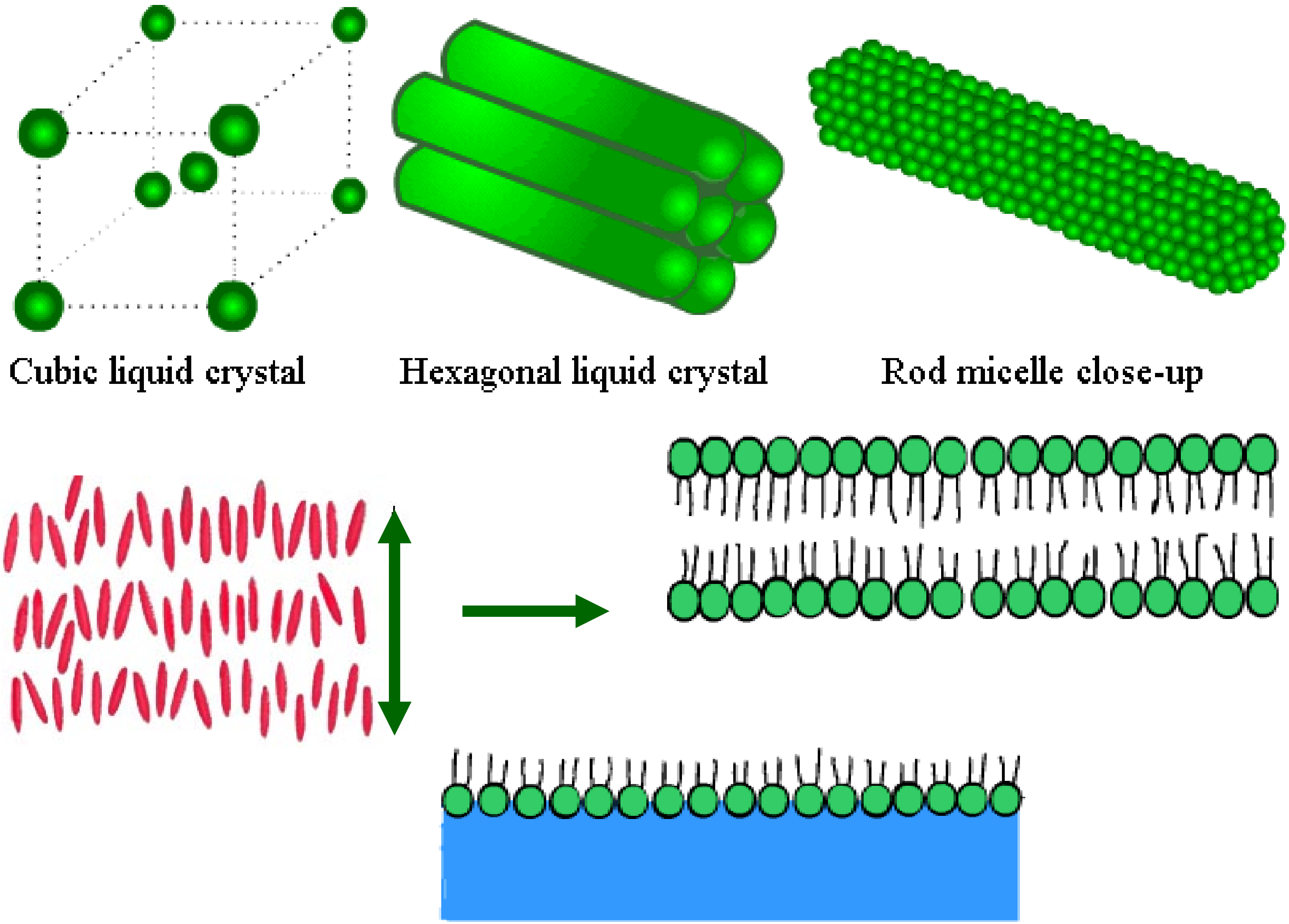


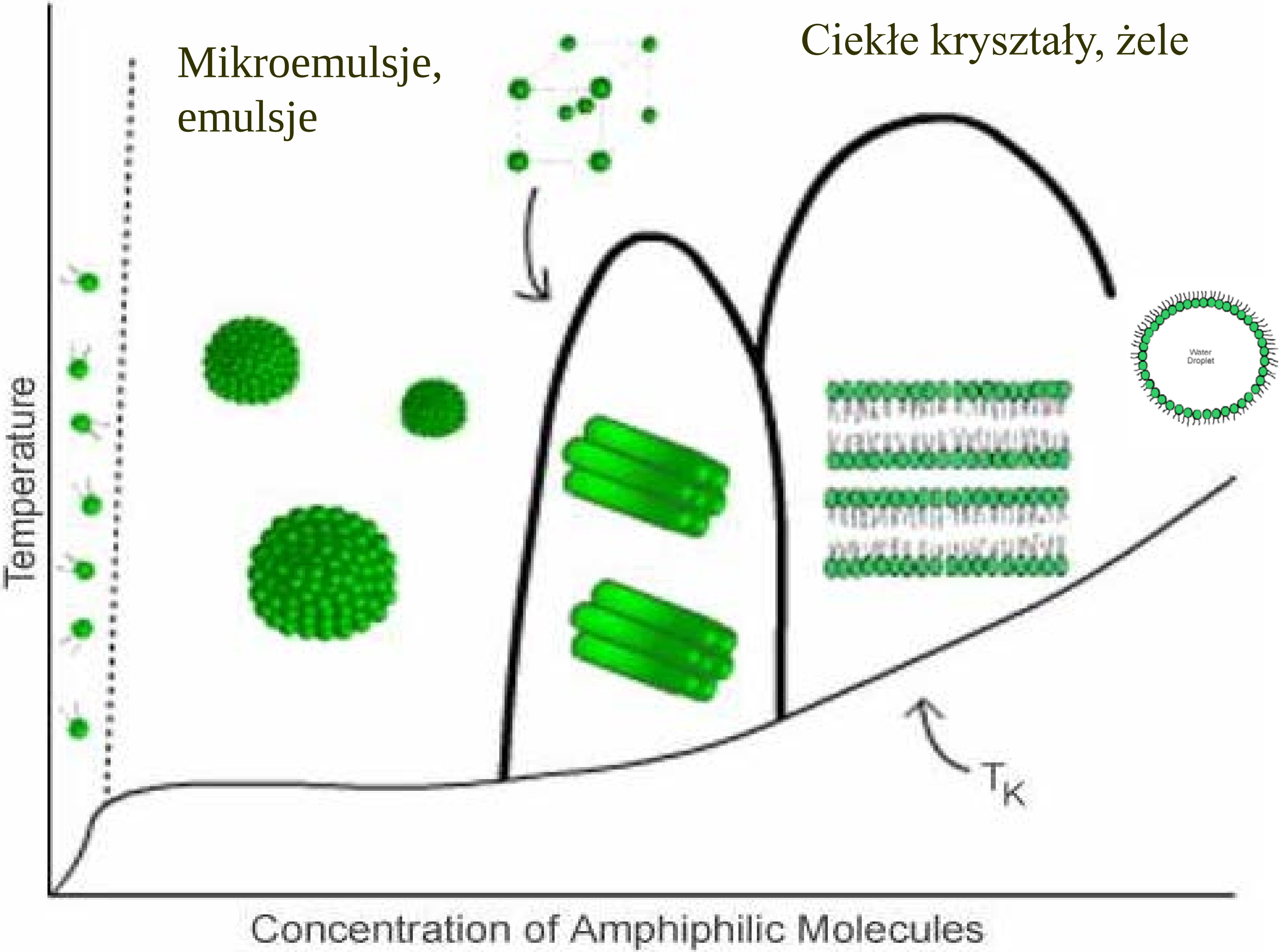
Spherical micelle      Cross-section

**niepolarny (węglowodorowy) koniec: lubi niepolarne środowisko**

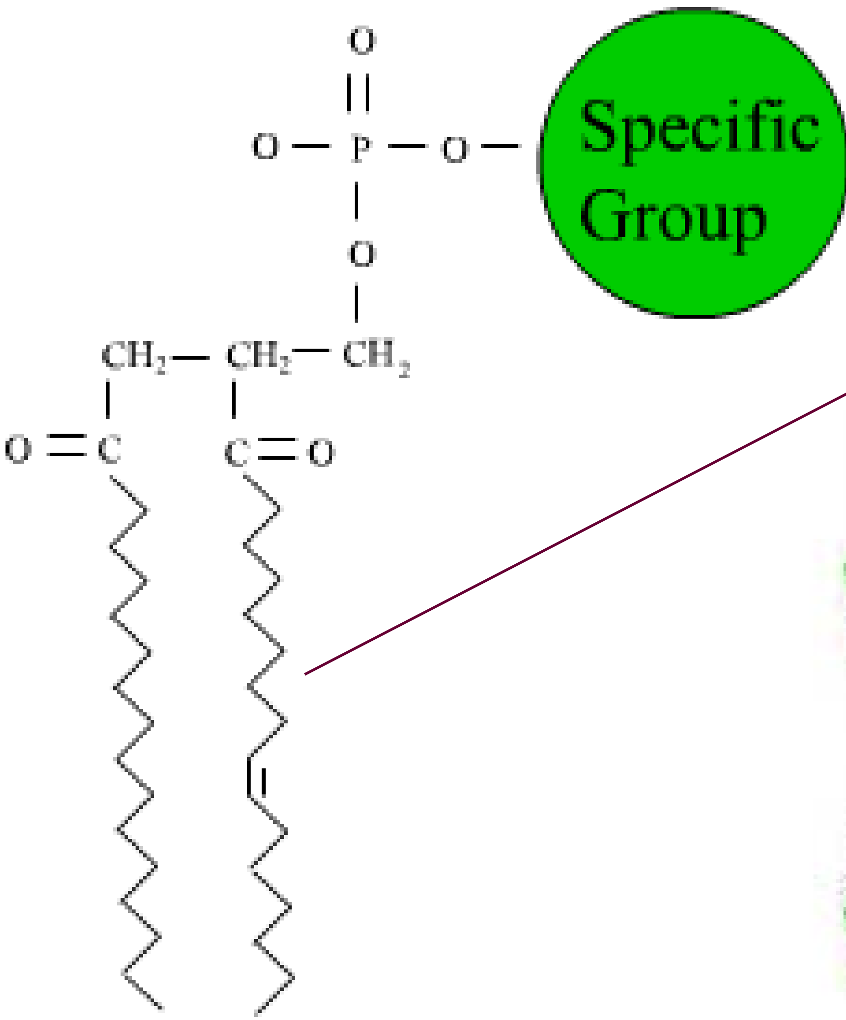


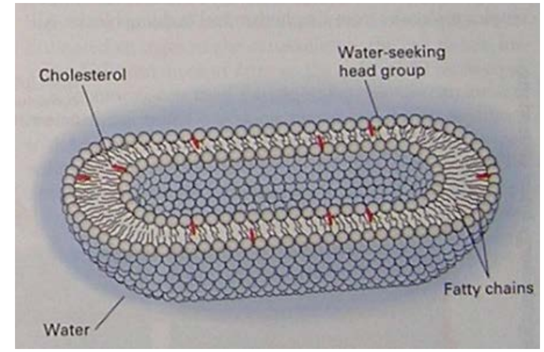

Reverse micelles


Cubic liquid crystal
Hexagonal liquid crystal
Rod micelle close-up

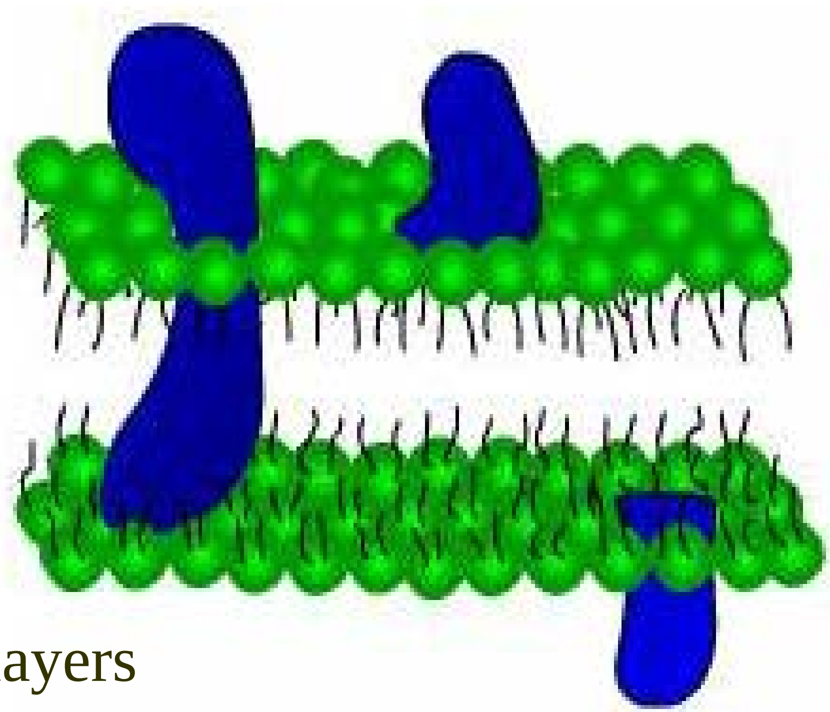

Mikroemulsje,
emulsje
Ciekłe kryształy, żele
Water
Droplet
Temperature
$T_K$
Concentration of Amphiphilic Molecules

# Biological Membranes




phospholipids



Lyotropic bilayers

Ill condensing matter or systems with randomness (glasses, spin glasses), dynamical phases transitions (synchronization), granular media, foams, etc….

end