DYNAMICAL TYPICALITY : CONVERGENCE OF TIME EVOLVED MACRO OBSERVABLES TO THEIR MEAN VALUES

- Quantum system ($\mathrm{Dim}(\mathscr{H}) = N$) with Hamiltonian partly deterministic and partly random, $H_0 + V$ ($V_{i,j}$ complex Guassian random variables).
- Observable $\hat{P} \to P(t)$.
- $|\psi_t\rangle$ is random $\to P(t)$ is random.
- Does $P(t) \to \mathbb{E}[P(t)]$?

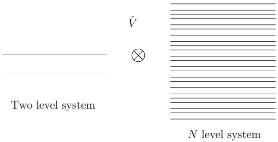

FIGURE: 2 level system (spin) coupled randomly to an N level environment

$$\mathrm{Var}[P(t)] = \mathbb{E}\left[\left(\langle\psi_t|\hat{P}|\psi_t\rangle\right)^2\right] - \mathbb{E}\left[\langle\psi_t|\hat{P}|\psi_t\rangle\right]^2$$

$$\lim_{N\to\infty} \mathrm{Var}[P(t)] \to 0?$$

- The answer: Yes! For any finite time $\text{Var}[P(t)] \rightarrow 0$ when $N \rightarrow \infty$.
- But in order to be meaningful the observable has to comply with some conditions. There can be no λ such that

$$\lim_{N \rightarrow \infty} \frac{\text{Tr}[P - \lambda]}{N} = 0$$

In some sense the operator $\hat{P}$ cannot tend in the limit to $\lambda \times$ Identity.

Examples

- Projection over a state $|\phi\rangle\langle\phi|$ is not a macro observable.
- The sum over a finite fraction of projection operators of the Hilbert space $\sum_{k=1}^{N/a} |\phi_k\rangle\langle\phi_k|$ is a macro observable.
- In the spin model $\sigma_z \otimes I_N$ is a macro observable.

Steps in the analysis

1. Time evolution operator in $|\psi_t\rangle$ is expanded in terms of V
2. Plug into the variance $\mathrm{Var}\left[P(t)\right]$
3. Use Wick's theorem to express the product of random matrices as a sum over graphs.
4. Classify the important graphical contributions.
5. Sum up the contributions and notice that $\mathrm{Var}\left[P(t)\right] \leq c_0 \left(\frac{e^{ct}}{N} + \frac{e^{(ct)^2}}{N^2} \right)$