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SFB | TR12

Doorway States Coupled to a Background: Fidelity and Survival Probability

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Outline

- what is a doorway ?
- examples in molecules, nuclei, billiards
- statistics of coupling coefficients, experimental and analytical
- prepared states in quantum information
- exact results on fidelity and survival probability
- realization with atomic interferometry in experiment

- unrelated: supersymmetry without supersymmetry

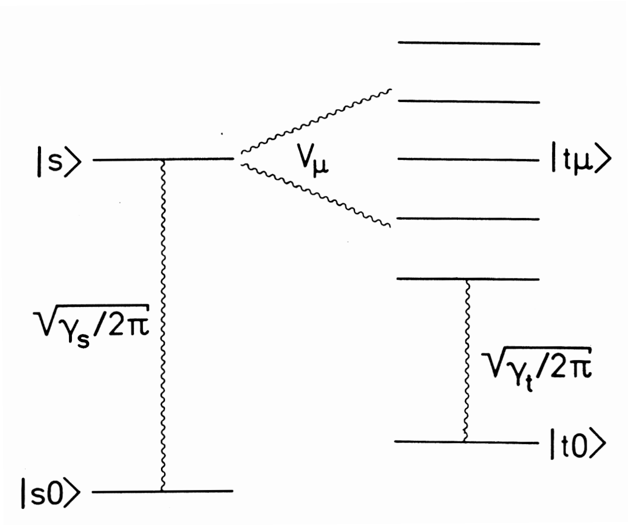
What is a Doorway ?

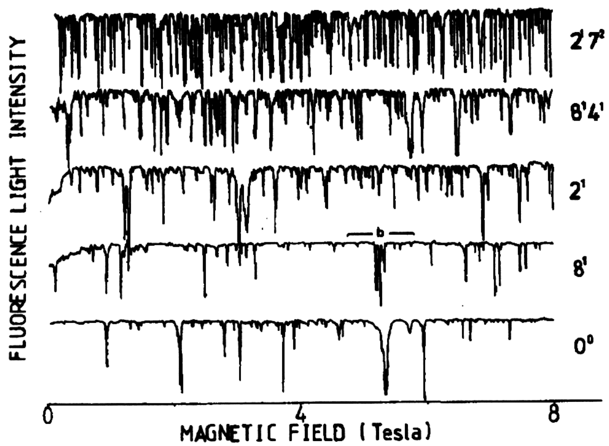
... here comes Naftali Auerbach's explanation:

$\longrightarrow$ through the doorway you can reach other states

Anticrossing Spectroscopy in Molecular Physics

doorway mechanism: singlet state $|s\rangle$ couples with matrix elements V_μ to background of triplet states $|t\mu\rangle$





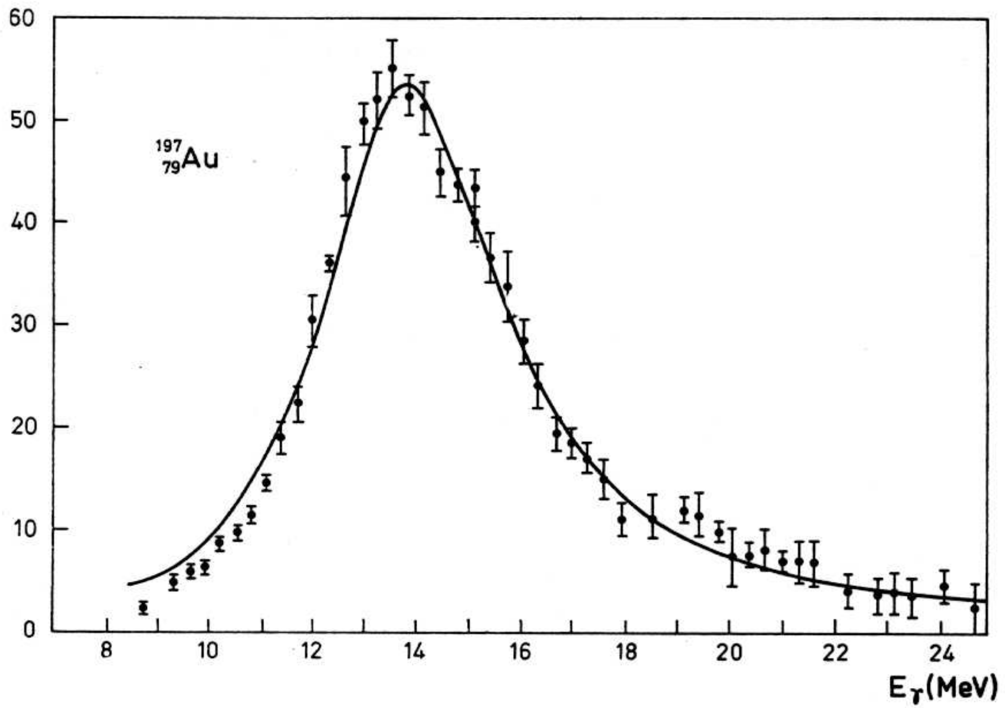
shift triplets with magnetic field $\rightarrow$ resonance fluorescence yield

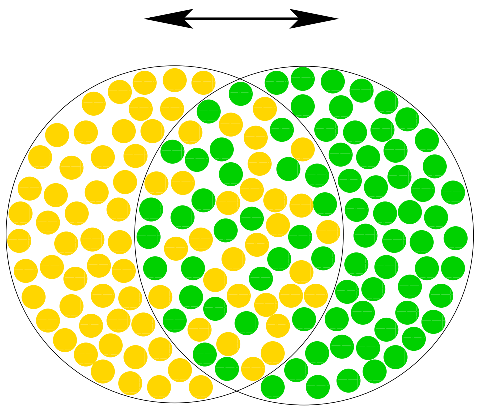
root mean square coupling V_{st} enters statistical observables

Pebay Peyroula, Jost, Lombardi, Dupré, CP 102 (1986) 417

Electric Giant Dipole Resonance

seen in many nuclei, here for Gold ($A = 197$)





high energies

Fultz, Bramblett, Caldwell, Kerr, PR 127 (1962) 1273

Strength Function — Local Density of States

cross section contains huge number of individual states (fragmentation) which cannot be resolved

strength function is related to this: consider state $|\mathrm{GDR}\rangle$ with resonance energy E_{GDR} and couple many states to it

$\longrightarrow$ density around $|\mathrm{GDR}\rangle$

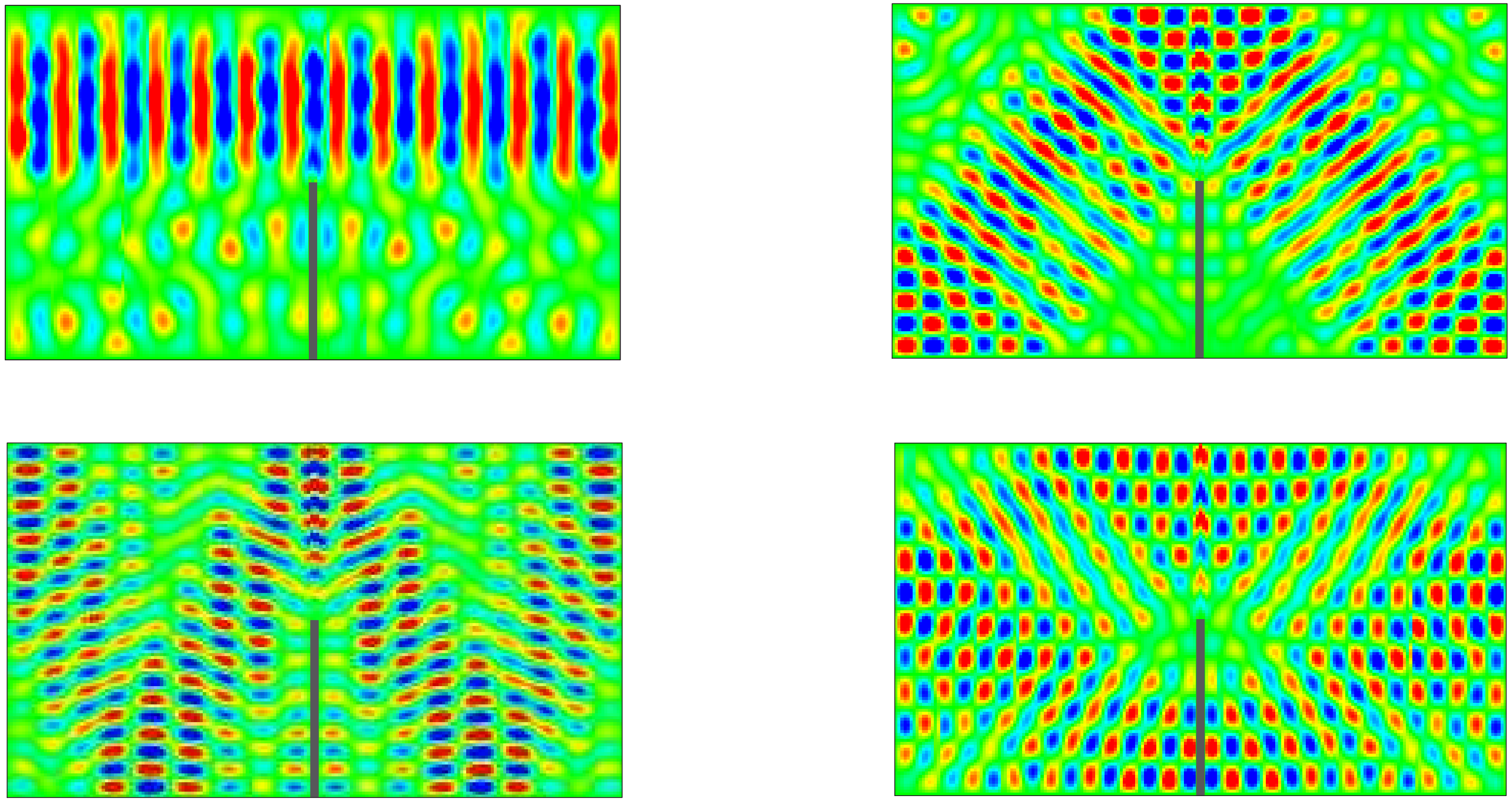
$$\varrho_{\mathrm{GDR}}(E) = \frac{1}{\pi} \frac{\Gamma/2}{(E - E_{\mathrm{GDR}})^2 + \Gamma^2/4} \qquad \text{(Breit–Wigner)}$$

under rather general conditions!

$\longrightarrow$ strictly, one cannot conclude chaotic fluctuations, but at these GDR energies one certainly expects them

Bohr, Mottelson, Nuclear Structure (1969); Zelevinsky, Annu Rev Nucl Part Sci 46 (1996) 237

Superscars in a Pseudointegrable Barrier Billiard



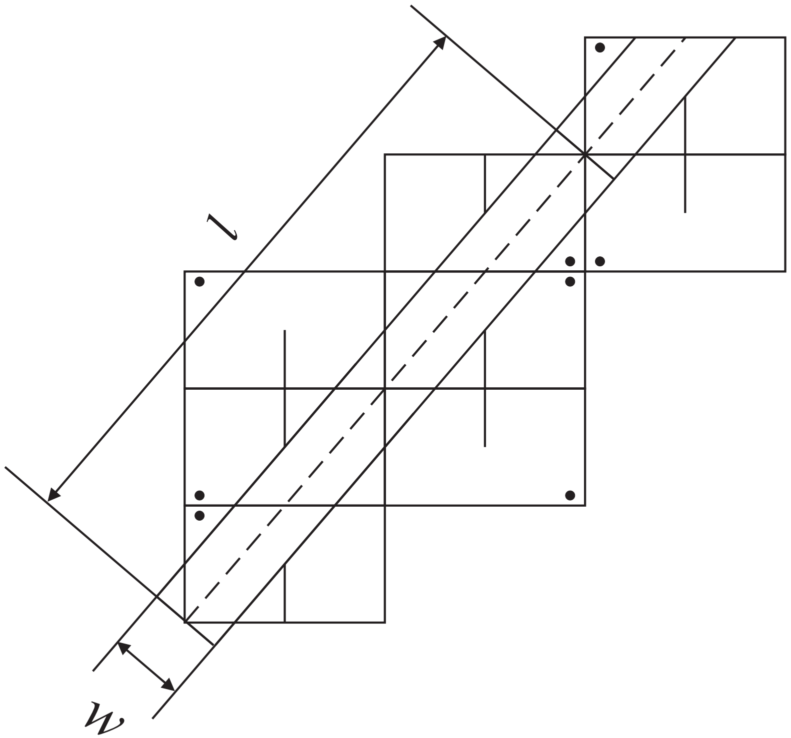
ordinary scars "vanish" at high energies, superscars do not !

Bogomolny, Schmit, PRL 92 (2004) 244102

Bogomolny, Dietz, Friedrich, Miski–Oglu, Richter, Schäfer, Schmit, PRL 97 (2006) 254102

Constructed Superscars

integrable approximation in Periodic Orbit Channels



inside POC:

$$\Psi^{(\mathsf{F})}_{m,n}(\vec{r}) \sim \sin\left(\frac{\pi m \xi}{l} + \delta\right) \sin\left(\frac{\pi n \eta}{w}\right)$$

outside POC:

$$\Psi^{(\mathsf{F})}_{m,n}(\vec{r}) = 0$$

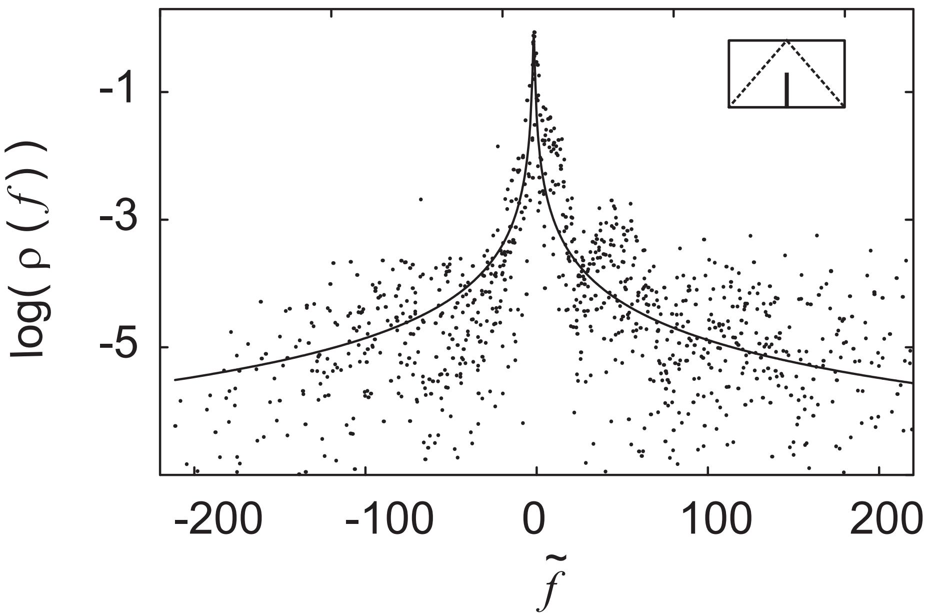
families $\mathsf{F} \in \{\mathsf{BB}, \mathsf{V}, \mathsf{D}, \mathsf{W}\}$

Local Density of States

sum over measured states ν, average over m quantum number

$$\rho_n(\tilde{f}) = \left\langle \sum_\nu |c_{m,n}(\tilde{f}_\nu)|^2 \delta\left(\tilde{f} - \tilde{f}_\nu + \tilde{f}_{m,n}\right) \right\rangle_m$$

overlap $\quad c_{m,n}(\tilde{f}_\nu) = \langle \Psi^{(\mathrm{F})}_{m,n} | \Psi_{\tilde{f}_\nu} \rangle$



Breit–Wigner shape

doorway mechanism

Modeling the Doorway Mechanism

total Hamilonian $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{V}$

one doorway $\mathcal{H}_S|s\rangle = E_s|s\rangle$, background $\mathcal{H}_B|b\rangle = E_b|b\rangle$

orthogonality $\langle s|b\rangle = 0$

coupling $\langle s|\mathcal{V}|s\rangle = 0 = \langle b|\mathcal{V}|b'\rangle$ and $\langle s|\mathcal{V}|b\rangle = V_{bs}$

solve $\mathcal{H}|\nu\rangle = \mathcal{E}_\nu|\nu\rangle$ get $\mathcal{E}_\nu = E_s - \sum_b \frac{V_{bs}^2}{E_b - \mathcal{E}_\nu}$

coupling coefficients $c_{s\nu} = \langle \nu|s\rangle = \left(1 + \sum_b \frac{V_{bs}^2}{(E_b - \mathcal{E}_\nu)^2}\right)^{-1/2}$

previous example: $|s\rangle = |\Psi_{m,n}^{(\mathsf{F})}\rangle$

Matrix Representation for Statistical Model

N background states, matrix H_B, vector V

$$H = \begin{bmatrix} E_s & V^T \\ V & H_B \end{bmatrix}$$

total Hamiltonian has $N+1$ states, eventually $N \to \infty$

average over random H_B (mean level spacing D)

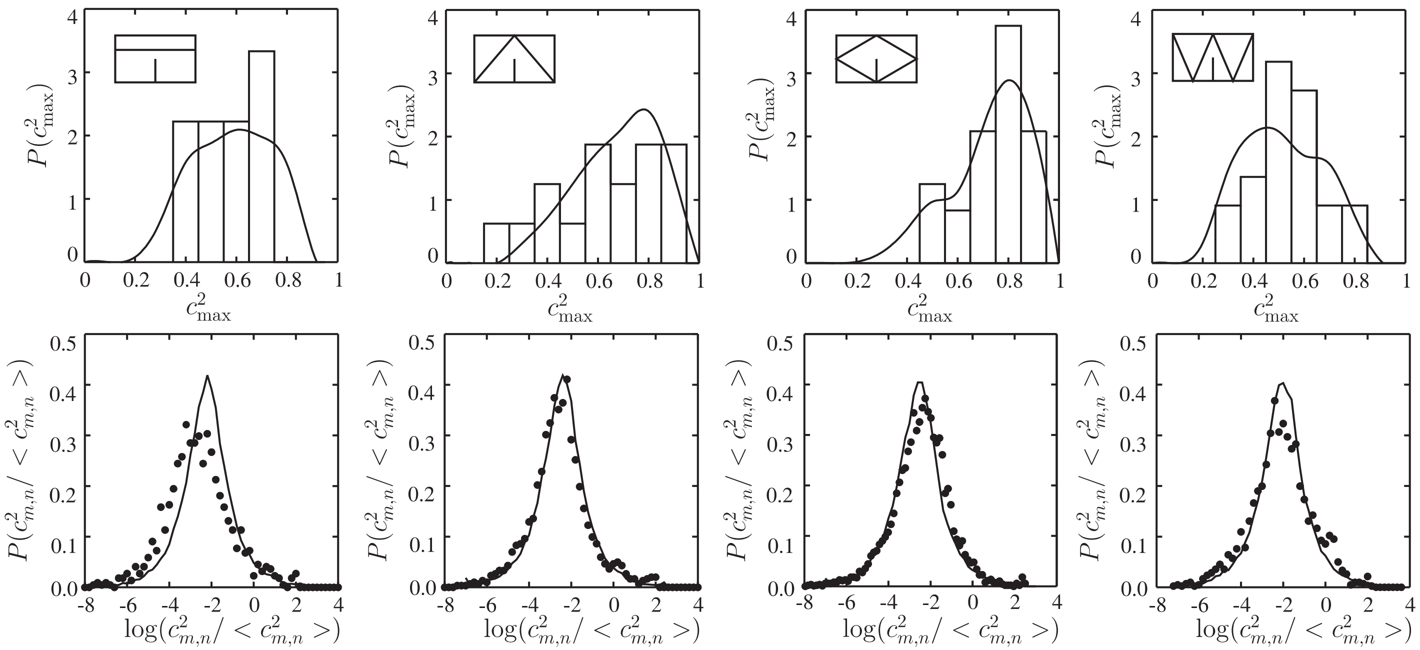
observables only depend on $\langle V^2 \rangle = \frac{1}{N} V^T V \; , \quad \lambda = \frac{\sqrt{\langle V^2 \rangle}}{D}$

Local Density is Breit–Wigner with width $\Gamma = 2\pi \frac{\langle V^2 \rangle}{D} = 2\pi \lambda^2 D$

Experimental Coefficient Statistics versus Numerics

distribution of squared maximum coefficient $c^2_{\max} = \max\left(c^2_{m,n}(\tilde{f}_\nu)\right)$

distribution of all coefficients $c^2_{m,n}(\tilde{f}_\nu)$



chaotic background, extract the only parameter $\lambda = \sqrt{\langle V^2 \rangle}/D$

Åberg, Guhr, Miski–Oglu, Richter, PRL 100 (2008) 204101

Analytical Calculation for Coefficient Distribution

maximum modulus coefficient $c_{\max} = \max\left(|c_{s\nu}|\right)$

distribution $$p_{\max}(c) \;=\; \Big\langle \delta(c - c_{\max}) \Big\rangle_{H_B,V}$$

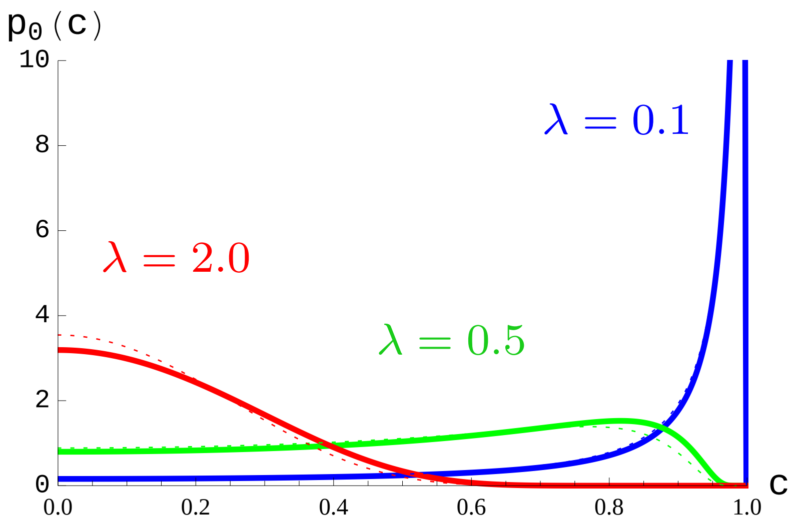
for small coupling λ, the overlap c_{s0} between the unperturbed doorway state $|s\rangle$ (at $\lambda = 0$) and the evolved doorway state $|0\rangle$ (at $\lambda > 0$) should be largest

$\longrightarrow$ approximately $$p_{\max}(c) \;\approx\; p_0(c) \;=\; \Big\langle \delta(c - |c_{s0}|) \Big\rangle_{H_B,V}$$

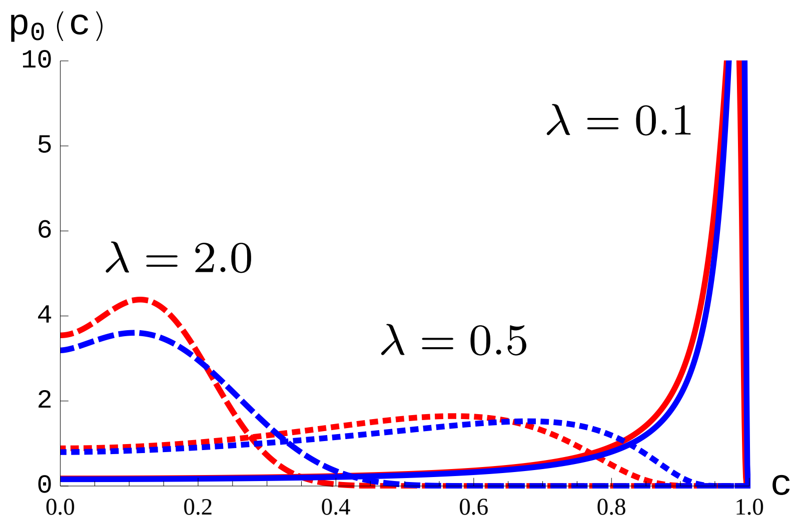
using supersymmetry, we find complicated, but closed form results

Kohler, Guhr, Sommers, NJP 12 (2010) 073026

Distribution: Regular and Chaotic Background



regular (Poisson)



chaotic (GOE, GUE)

similar for regular and chaotic $\longrightarrow$ safe extraction of λ

Prepared States in Quantum Information

wish to know how well a prepared state can be isolated and how the unavoidable mixing with the surrounding behaves

maps one–to–one onto doorway mechanism:

prepared state = doorway (no eigenstates!)

surrounding = background, mixing = coupling (statistical)

interesting objects here: averaged fidelity and survival probability

Fidelity Amplitude and Local Density of States

fidelity amplitude $f(t)$ is overlap between time evolved states

$$\exp\left(-\frac{i}{\hbar}(\mathcal{H}_S + \mathcal{H}_B)t\right)|s\rangle \quad \text{and} \quad \exp\left(-\frac{i}{\hbar}(\mathcal{H}_S + \mathcal{H}_B + \mathcal{V})t\right)|s\rangle$$

for doorway energy $E_s = 0$ we have with $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{V}$

$$f(t) = \langle s|\exp\left(-\frac{i}{\hbar}\mathcal{H}t\right)|s\rangle = \sum_\nu |c_{s\nu}|^2 \exp\left(-\frac{i}{\hbar}E_\nu t\right)$$

Fourier transform is local density of states

$$\rho(E) = \sum_\nu |c_{s\nu}|^2 \delta(E - E_\nu)$$

Fidelity and Survival Probability

fidelity equals survival probability $F(t) = |f(t)|^2$

decomposition in diagonal and off–diagonal part

$$F(t) = \sum_{\mu,\nu} |c_{s\mu}|^2 |c_{s\nu}|^2 \exp\left(\frac{i}{\hbar}(E_\mu - E_\nu)t\right) = \mathrm{IPR} + F_{\mathrm{fluc}}(t)$$

“inverse” participation ratio $\mathrm{IPR} = \sum_{\nu} |c_{s\nu}|^4$

fluctuations $F_{\mathrm{fluc}}(t) = 2 \sum_{\mu \neq \nu} |c_{s\mu}|^2 |c_{s\nu}|^2 \cos\left(\frac{E_\mu - E_\nu}{\hbar} t\right)$

Approximations for Averaged Survival Probability

"Drude approximation" gives a Fermi's Golden Rule result

$$P(t) = \langle F(t) \rangle = \langle |f(t)|^2 \rangle \approx |\langle f(t) \rangle|^2 = \exp(-\Gamma t)$$

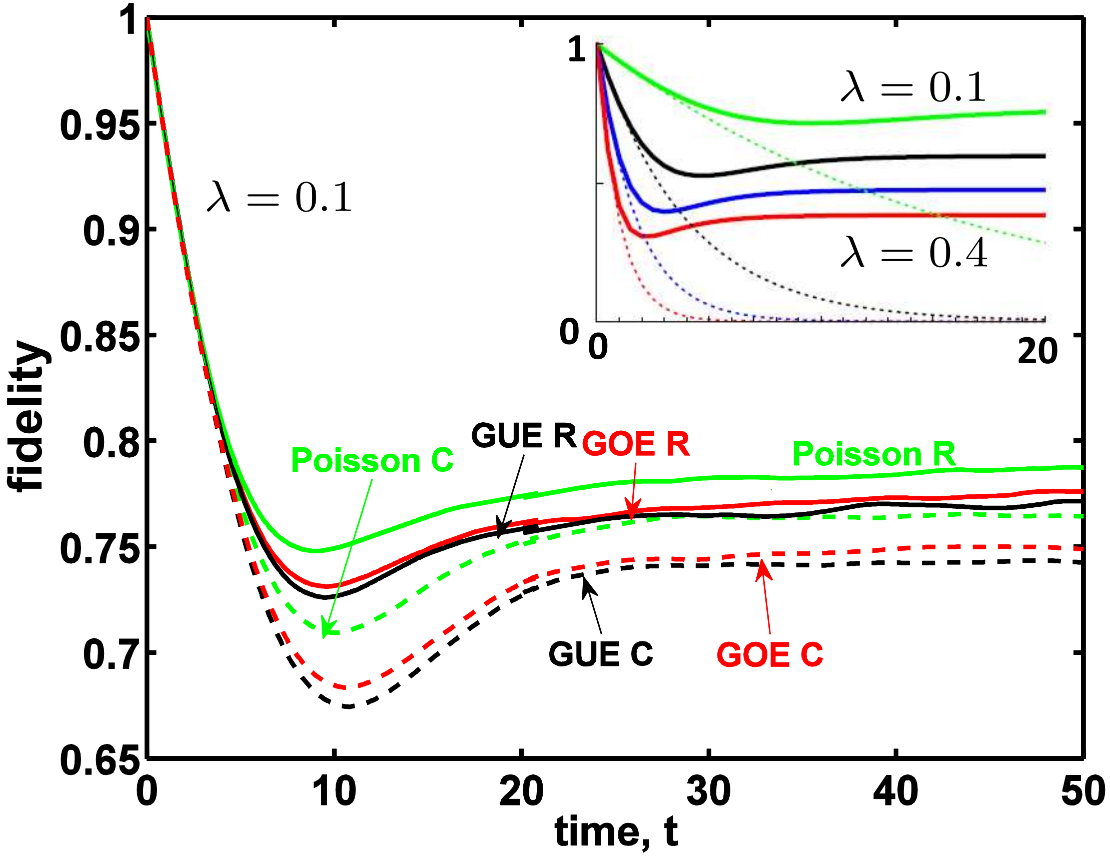
correction $\langle \mathrm{IPR} \rangle = \dfrac{D}{\pi \Gamma}$

further approximative correction due to correlations yields revival !

Gruver, Aliagla, Cerdeira, Mello, Proto, PRE 55 (1997) 6370

Exact Survival Probability and Numerics

supersymmetry for complex (C), numerics for real (R) couplings



revival and
λ dependent
saturation

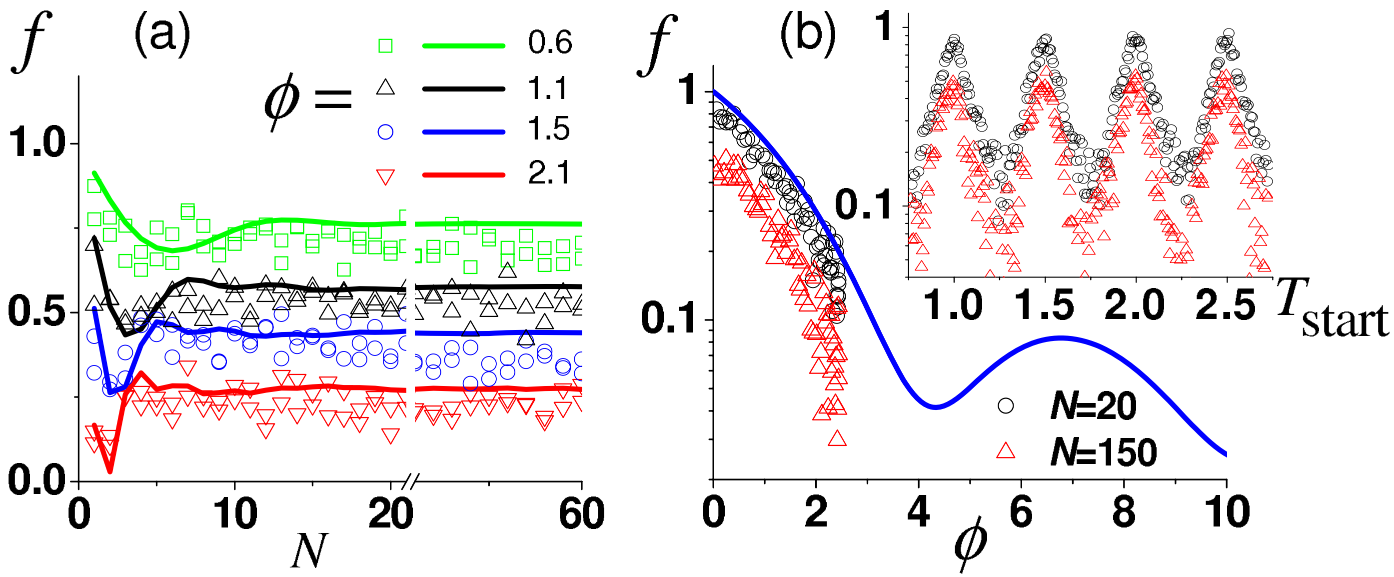
$$P(\infty) = \langle \mathrm{IPR} \rangle$$

type of coupling (R,C) more significant than type of chaos

Kohler, Sommers, Åberg, Guhr (2010); Kohler, Sommers, Åberg (2010)

Experimental Realization of Quantum Kicked Rotor

atomic interferometry of matter waves in periodically pulsed optical standing wave

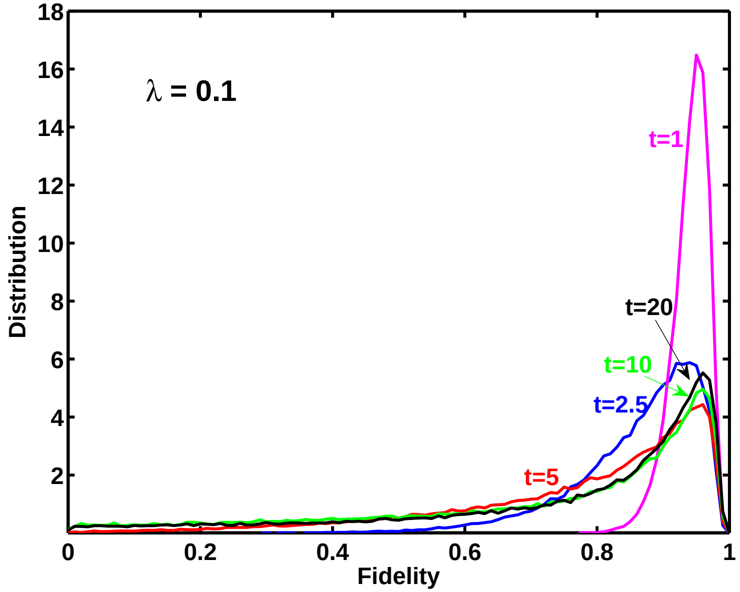


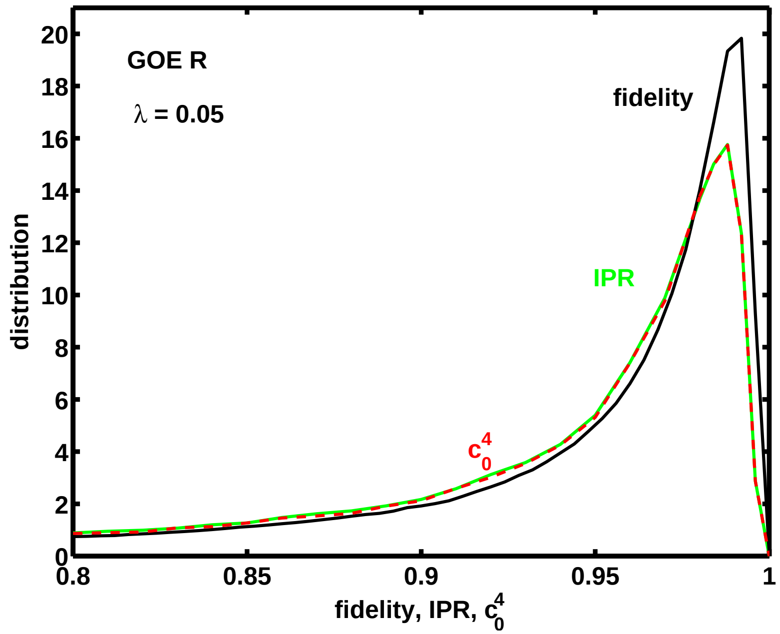
number of kicks N correponds to time t, perturbation strength ϕ to coupling λ

Wu, Tonyushkin, Prentiss, PRL 103 (2009) 034101

Distributions of Fidelity and Survival Probability

numerics for $\langle \delta(z - F(t)) \rangle$





GOE background
stable limit $t \geq 20$

comparison
fidelity, IPR, c^4

Kohler, Sommers, Åberg, Guhr (2010)

Summary and Conclusions

- doorways are ubiquitous in physics
- experimental, numerical and analytical results for coupling coefficients, fidelity and survival probability
- precise understanding of revival and saturation
- striking difference for real and complex couplings

appendix:

Supersymmetry without Supersymmetry

Mario Kieburg and Thomas Guhr

Random Matrix Kernels

kernels of Random Matrix correlation functions given by lowest order averages of ratios of characteristic polynomials

$$K_N^{(\beta)}(x_q, x_p) \sim \mathrm{Im}\, \frac{Z_1^{(\beta)}(x_p, x_q) - Z_1^{(\beta)}(0,0)}{x_q - x_p}$$

$$Z_1^{(\beta)}(x_p, x_q) \sim \int \exp\left(-\frac{\beta}{2}\mathrm{tr}\, H^2\right) \left(\frac{\det(H - x_q)}{\det(H - x_p^-)}\right)^{|\gamma|} d[H]$$

$$\sim \int \exp\left(-\frac{\beta}{2|\gamma|}\mathrm{str}\, \sigma^2\right) \mathrm{sdet}^{\,-\beta N/2\gamma}(\sigma - x^-) d[\sigma]$$ with

$2 \times 2\ (\beta = 2),\ 4 \times 4\ (\beta = 1, 4)$ supermatrix σ

Grönqvist, Guhr, Kohler (2004)

Borodin, Strahov (2006): arbitrary order, factorizing probability density yields determinants/Pfaffians in terms of $Z_1^{(\beta)}(x_p, x_q)$

GUE Correlation Functions with Supersymmetry

$2k \times 2k$ supermatrix model, eigenvalue–angle coordinates

$$R_k(x_1, \ldots, x_k) = \int d[s] B_{k/k}(s) \exp\left(-\mathrm{str}\,(s-x)^2\right) \mathrm{sdet}^{\,-N}(s - x^-)$$

Berezinian (Jacobian) $\quad B_{k/k}(s) = \det\left[\frac{1}{s_{p1} - i s_{q2}}\right]_{p,q=1,\ldots,k}$

is the reason for $k \times k$ determinant structure

$$R_k(x_1, \ldots, x_k) = \det\left[K_N^{(2)}(x_p, x_q)\right]_{p,q=1,\ldots,k}$$

but: GOE and GSE remained unsolved in this way

Guhr (1991)

Supersymmetry without Mapping onto Superspace

factorization $P(E) = \prod_{n=1}^{N} \widetilde{P}(E_n)$, example: Hermitean matrices

$$Z_k(\kappa) = \int P(H) \prod_{p=1}^{k} \frac{\det(H - \kappa_{p2})}{\det(H - \kappa_{p1})} d[H]$$

$$= \int \prod_{n=1}^{N} \left[\widetilde{P}(E_n) \prod_{p=1}^{k} \frac{E_n - \kappa_{p2}}{E_n - \kappa_{p1}} \right] \Delta_N^2(E) d[E]$$

Vandermonde determinant

$$\Delta_N(E) = \prod_{n,l} (E_n - E_l) = \det \left[E_n^{l-1} \right]_{n,l=1,\ldots,N}$$

Berezinian (Jacobian) in Hermitean Superspace

k eigenvalues s_{p1} in bosonic sector,
m eigenvalues s_{q2} in fermionic sector

$$s = \mathrm{diag}\,(s_1, s_2) = \mathrm{diag}\,(s_{11}, \ldots, s_{k1}, s_{12}, \ldots, s_{m2})$$

Berezinian is $B^2_{k/m}(s)$, for $m \geq k$ we find

$$B_{k/m}(s) = \frac{\Delta_k(s_1)\Delta_m(s_2)}{\prod_{p,q}(s_{p1} - s_{q2})} = \det \begin{bmatrix} \left[\dfrac{1}{s_{p1} - s_{q2}}\right]_{p=1,\ldots,k,q=1,\ldots,m} \\ \left[s_{q2}^{p-1}\right]_{q=1,\ldots,m,p=1,\ldots,m-k} \end{bmatrix}$$

this is a $m \times m$ determinant !

Integrand is Ratio of Berezinians

$$\kappa = \mathrm{diag}\,(\kappa_1, \kappa_2) = \mathrm{diag}\,(\kappa_{11}, \ldots, \kappa_{k1}, \kappa_{12}, \ldots, \kappa_{k2})$$

crucial identity

$$\prod_{n=1}^{N} \prod_{p=1}^{k} \frac{E_n - \kappa_{p2}}{E_n - \kappa_{p1}} \Delta_N(E) \, \frac{B_{k/k}(\kappa)}{B_{k/k}(\kappa)} = \frac{B_{k/k+N}(\kappa, E)}{B_{k/k}(\kappa)}$$

makes integral elementary

$$Z_k(\kappa) = \int \prod_{n=1}^{N} \widetilde{P}(E_n) \frac{B_{k/k+N}(\kappa, E)}{B_{k/k}(\kappa)} \Delta_N(E) d[E]$$

$$= \frac{1}{B_{k/k}(\kappa)} \int \prod_{n=1}^{N} \widetilde{P}(E_n) E_n^{n-1} B_{k/k+N}(\kappa, E) d[E]$$

Supersymmetry Implies Decomposition Formula

straightforward reordering of determinants gives

$$Z_k(\kappa) = \frac{1}{B_{k/k}(\kappa)} \det \left[\frac{Z_1(\kappa_{p1}, \kappa_{q2})}{\kappa_{p1} - \kappa_{q2}} \right]_{p,q=1,\ldots,k}$$

- reminiscent of Wick theorem: $1 \times k \longrightarrow k \times 1$
- $Z_1(\kappa_{p1}, \kappa_{q2})$ explicitly known in terms of $\widetilde{P}(E_n)$
- specific form of $\widetilde{P}(E_n)$ never used
- clear separation of algebraic and analytic features
- applicability is very general

Kieburg, Guhr, JPA 43 (2010) 07520; Kieburg, Guhr, JPA 43 (2010) 135204

Some Matrix Ensembles Yielding Determinants

matrix ensemble	probability density P for the matrices	matrices in the characteristic polynomials	probability density $g(z)$
Hermitian ensemble [57, 31, 62, 32, 34, 35]	$\widetilde{P}\,(\mathrm{tr}\, H^m, m \in \mathbb{N})$ $H = H^\dagger$	H	$P(x)\delta(y)$
circular unitary ensemble (unitary group) [37, 63, 14, 13, 38, 20, 64, 21]	$\widetilde{P}\,(\mathrm{tr}\, U^m, m \in \mathbb{N})$ $U^\dagger U = 1\!\!1_N$	U and $U^\dagger$	$P\left(e^{\imath\varphi}\right)\delta(r-1)$
Hermitian chiral (complex Laguerre) ensemble [65, 66, 67, 7]	$\widetilde{P}\,(\mathrm{tr}(AA^\dagger)^m, m \in \mathbb{N})$ A is a complex $N \times M$ matrix with $N \leq M$	$AA^\dagger$	$P(x)x^{M-N}\Theta(x)\delta(y)$
Gaussian elliptical ensemble [9, 10, 11, 36]; for $\tau = 1$ complex Ginibre ensemble	$\exp\left[-\frac{(\tau+1)}{2}\mathrm{tr}\, H^\dagger H\right] \times$ $\times \exp\left[-\frac{(\tau-1)}{2}\mathrm{Re}\ \mathrm{tr}\, H^2\right]$ H is a complex matrix; $\tau > 0$	H and $H^\dagger$	$\exp\left[-r^2\left(\sin^2\varphi + \tau\cos^2\varphi\right)\right]$
Gaussian complex chiral ensemble [12]	$\exp\left[-\mathrm{tr}\, A^\dagger A - \mathrm{tr}\, B^\dagger B\right]$ $C = \imath A + \mu B$ $D = \imath A^\dagger + \mu B^\dagger$ A and B are complex $N \times M$ matrices with $N \leq M$	CD and $D^\dagger C^\dagger$	$K_{M-N}\left(\frac{1+\mu^2}{2\mu^2} r\right) r^{M-N} \times$ $\times \exp\left(\frac{1-\mu^2}{2\mu^2} r\cos\varphi\right)$

Some Matrix Ensembles Yielding Pfaffians

matrix ensemble	probability density P for the matrices	matrices in the characteristic polynomials	probability densities $g(z_1, z_2)$ and $\tilde{g}(z_1, z_2)$	probability density $h(z)$
real symmetric matrices [31, 24, 18]	$\widetilde{P}\,(\mathrm{tr}\,H^m, m \in \mathbb{N})$ $H = H^T = H^*$	H	$P(x_1)P(x_2)\times$ $\times\delta(y_1)\delta(y_2)\Theta(x_2 - x_1)$	$P(x)\delta(y)$
circular orthogonal ensemble [4]	$\widetilde{P}\,(\mathrm{tr}\,U^m, m \in \mathbb{N})$ $U^\dagger U = 1\!\!1_N$ and $U^T = U$	U and $U^\dagger$	$P(e^{\imath\varphi_1})P(e^{\imath\varphi_2})\times$ $\times\delta(r_1 - 1)\delta(r_2 - 1)\times$ $\times\Theta(\varphi_2 - \varphi_1)$	$P(e^{\imath\varphi})\delta(r - 1)$
real symmetric chiral (real Laguerre) ensemble [21, 32, 33, 34]	$\widetilde{P}\left(\mathrm{tr}(AA^T)^m, m \in \mathbb{N}\right)$ A is a real $N \times M$ matrix with $\nu = M - N \geq 0$	AA^T	$P(x_1)P(x_2)\times$ $\times(x_1x_2)^{(\nu-1)/2}\times$ $\times\delta(y_1)\delta(y_2)\Theta(x_2 - x_1)$	$P(x)\delta(y)x^{(\nu-1)/2}$
Gaussian real elliptical ensemble; for $\tau = 1$ real Ginibre ensemble [10, 15, 35, 16, 36, 25] [37, 38, 23]	$\exp\left[-\frac{(\tau+1)}{2}\,\mathrm{tr}\,H^T H\right]\times$ $\times\exp\left[-\frac{(\tau-1)}{2}\,\mathrm{tr}\,H^2\right]$ $H = H^*$; $\tau > 0$	H	$\prod\limits_{j\in\{1,2\}} \exp\left[-\tau x_j^2\right]\times$ $\times\sqrt{\mathrm{erfc}(\sqrt{2(1+\tau)}y_j)}\times$ $\times[\delta(y_1)\delta(y_2)\Theta(x_2 - x_1)+$ $+2\imath\delta^2(z_1 - z_2^*)\Theta(y_1)]$	$\exp(-\tau x^2)\delta(y)$
Gaussian real chiral ensemble [9, 39]	$\exp\left[-\,\mathrm{tr}\,A^T A - \mathrm{tr}\,B^T B\right]$ $C = A + \mu B$ $D = -A^T + \mu B^T$ A and B are real $N \times M$ matrices with $\nu = M - N \geq 0$	CD	$\prod\limits_{j\in\{1,2\}} \exp\left[-2\eta_- z_j\right]\times$ $\times\lvert z_j\rvert^\nu\sqrt{f(2\eta_+ z_j)}\times$ $\times[\delta(y_1)\delta(y_2)\Theta(x_2 - x_1)+$ $+2\imath\delta^2(z_1 - z_2^*)\Theta(y_1)]$	$x^{\nu/2}\exp\left[-2\eta_- x\right]\times$ $\times K_{\nu/2}(2\eta_+ x)\delta(y)$