Home Assignment 8

The file nile.txt provides the historical data on the Nile minima.

A. Do the Detrended Fluctuation Analysis on these data and estimate the Hurst exponent of this series.

The simplest DFA algorithm requires the series to have 2^s elements. You need to either truncate the series or try to develop an algorithm that will work on the whole series, or at least on a series of length $2^s + 2^{s'}$.

B. Go back to the original data. There are 663 data points in the series. Let the series be $\{x_i\}_{i=1}^{663}$. For $n = 2, 3, \ldots, 663$ do the following:

1. Let
$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \cdots + x_n) \tag{1}$$
2. Calculate the standard deviation, d_n, of the subseries $\{x_i\}_{i=1}^{n}$:
$$d_n = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{2}$$
3. Calculate the rescaled range
$$r_n^{**} = \max_{1 \leqslant i \leqslant n} \frac{x_1 + x_2 + \cdots + x_i - i \cdot \bar{x}}{d_n} - \min_{1 \leqslant i \leqslant n} \frac{x_1 + x_2 + \cdots + x_i - i \cdot \bar{x}}{d_n} \tag{3}$$

Plot r_n^{**} as a function of n, $\log r_n^{**}$ as a function of $\log n$ and fit
$$\log r_n^{**} = k \cdot \log \frac{n}{2} \tag{4}$$

This was the original approach of Harold Hurst ☺.

C. Try interpreting the differences between **A** and **B**.

Have fun,
PFG