

Time Series Analysis:

9. Financial time series

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Volatility

In analysing financial time series, it is frequently more important not to predict future values of an asset, but to predict the future *variability* of the asset. Investors realize and accept that future values of an asset are random variables. However, if these random variables come from a narrow distribution, the associated risk is small. Contrariwise, if the random variables come from a wide distribution, the associated risk is large. This is particularly true with assets like options, future contracts and other derivatives.

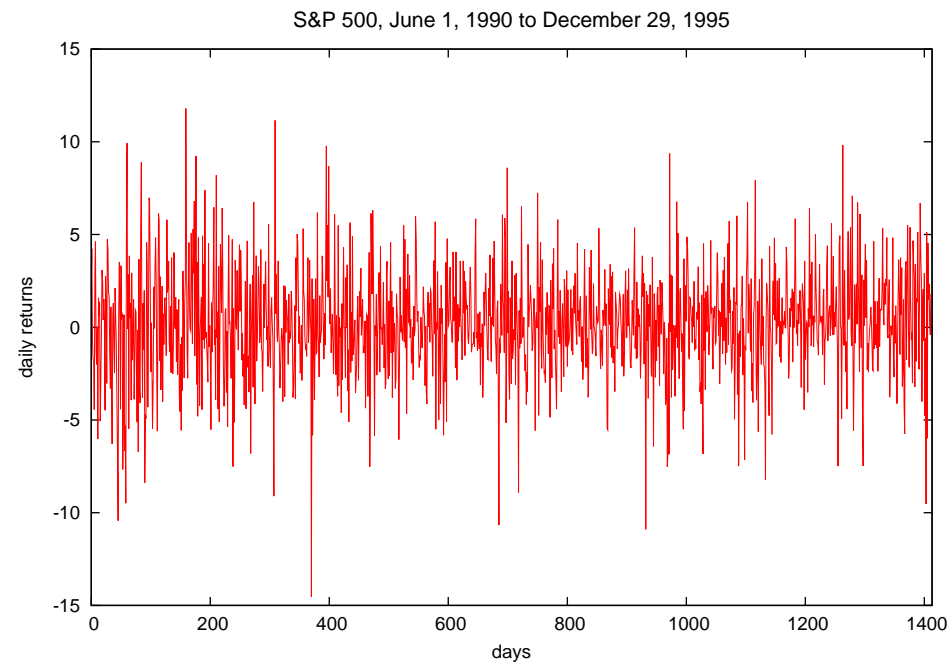
The point is, the width of the distribution can change over time.

Frequently, such series are modelled as

$$\epsilon_n = \sqrt{h_n} \eta_n \quad (1)$$

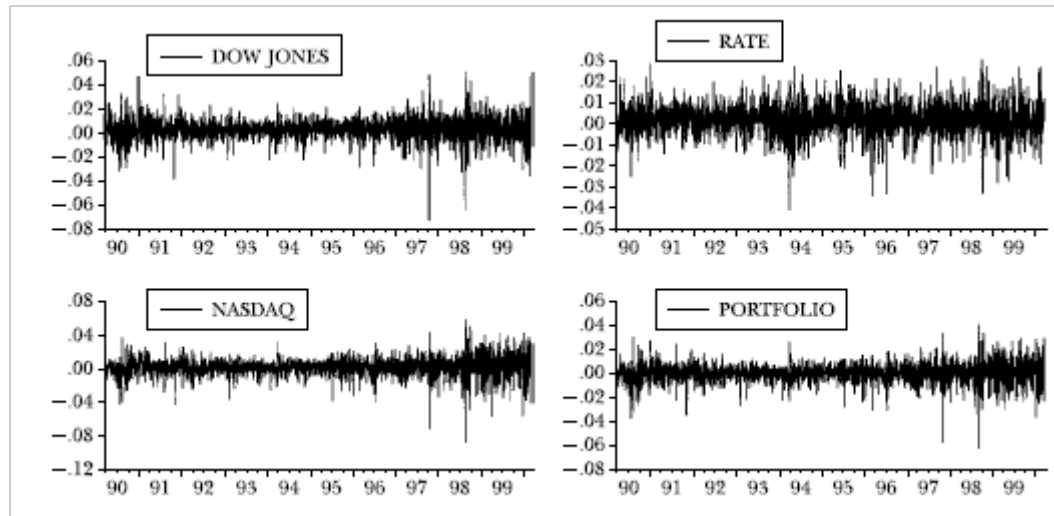
$\{\eta_n\}$ is GWN. $h_n > 0$ is called *volatility*. The presence of an index, h_n , indicates that volatility changes in time

A typical time series in finances



Regions of high and low volatility can easily be seen. High volatilities and low volatilities tend to cluster.

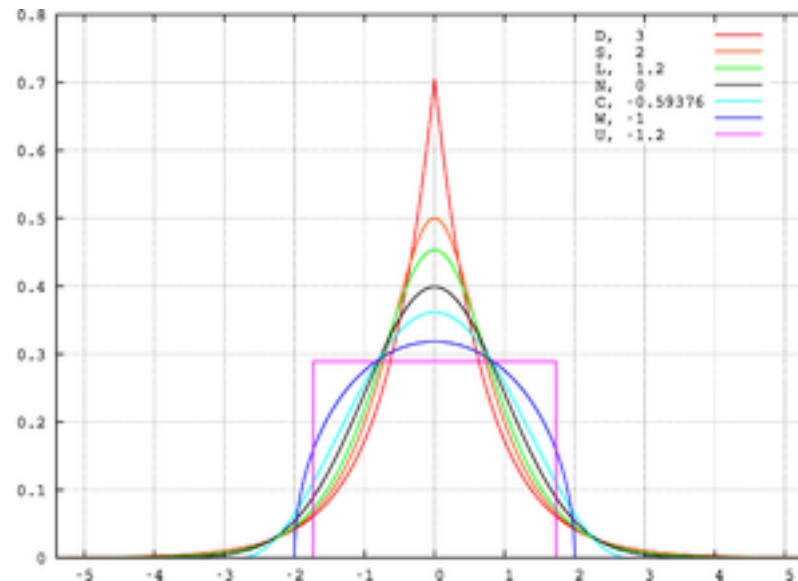
A classic example



From the seminal paper by Robert Engle,
Journal of Economic Perspectives **15**, 157-168 (2001).

Features of *real life* financial time series

- Time-varying variance
- Regions with large variances have a tendency to cluster
- Non-zero kurtosis
- Heavy (fat) tails



In finances, we call the variance *volatility*.

Predicting **volatility** is *essential* in assessing future financial risks.

Heteroscedasticity

In statistics, if there are subpopulations within a collection of random variables that have different variabilities than the others, *especially* if those subpopulations can be associated with a quantity that sets some sort of “order” or “sequence”, we say that these random variables are **heteroskedastic** (or **heteroscedastic**). The “variability” can be quantified as the variance or any other measure of statistical dispersion.

If the variability is constant within the collection of random variables, we say that these variables are **homoscedastic**.

- **Classic example 1:** Babies are born with similar body weights: the variance of body weights in newborn babies is small. However, the variance of body weights in children just a few years old is considerably larger. Thus, the population of young children, from newborns to pre-school or introductory

school ages is heteroscedastic with respect to body weight. The age of children acts as an “ordering quantity” here.

- **Classic example 2:** A classic example of heteroscedasticity is that of income versus expenditure on meals. As one’s income increases, the variability of food consumption will increase. A poorer person will spend a rather constant amount by always eating inexpensive food; a wealthier person may occasionally buy inexpensive food and at other times eat expensive meals. Those with higher incomes display a greater variability of food consumption. The income level, or some other measure of personal wealth, is the “ordering quantity”.

In Time Series Analysis we usually talk about a *conditional* volatility and a *conditional* heteroscedasticity, as we do not know their *absolute*, “ideal” values. Rather than that, we only know them with respect to the actual initial data.

The ARCH model

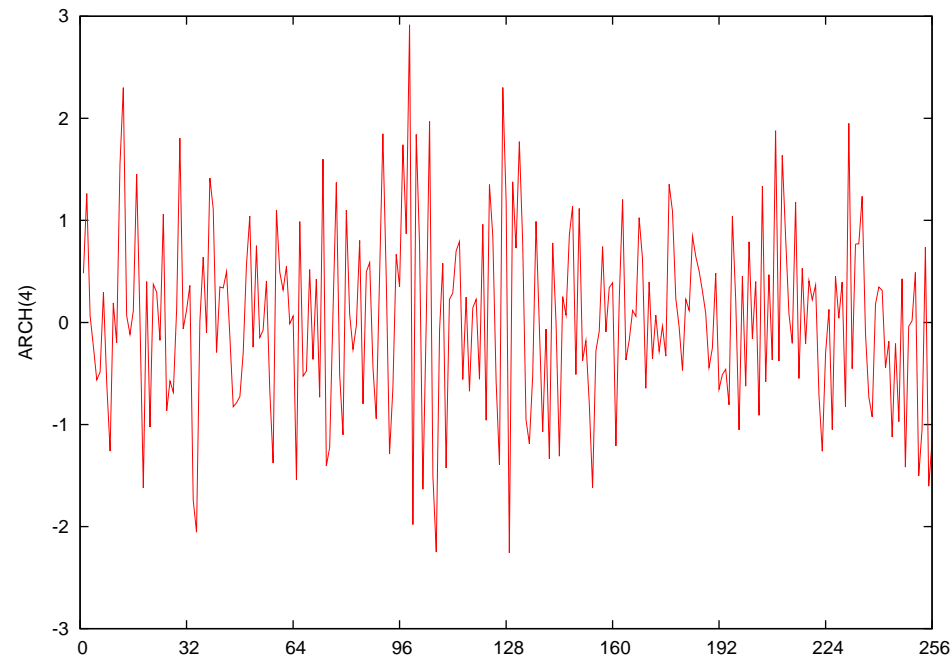
AutoRegressive Conditional Heteroscedasticity (Robert Engle, 1982). The **ARCH(q)** model is of the form (1) with

$$h_n = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{n-i}^2, \quad (2)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, 2, \dots, q-1$, $\alpha_q > 0$. q is the order of ARCH(q) and therefore α_q must be strongly greater than zero.

The question *why* such models describe financial time series correctly is beyond the scope of this lecture.

An example



ARCH(4) with $\alpha_0 = 0.5$, $\alpha_1 = 0.25$, $\alpha_2 = 0.0625$, $\alpha_3 = 0.0$, $\alpha_4 = 0.125$.

Fitting parameters to an ARCH model

1. Find the best fitting AR(q) model of the original time series. Thus

$$y_n = a_0 + \sum_{i=1}^q a_i y_{n-i} + \epsilon_n .$$

2. We now have the order, q , and errors, ϵ_n . Using least squares, fit parameters $\hat{\alpha}_i$

$$\epsilon_n^2 = \hat{\alpha}_0 + \sum_{i=1}^q \hat{\alpha}_i \epsilon_{n-i}^2 .$$

3. Using an χ^2 statistics, verify whether any of the $\hat{\alpha}_i$'s, $i = 1, \dots, q$, is significantly greater than zero (these are parameters of the ARCH model), the null hypothesis being that there are no ARCH contributions.

Note: "Significantly greater than zero" does not mean that the value is large, but that the difference from zero is **statistically significant**.

The GARCH model

Generalized AutoRegressive Conditional Heteroscedasticity (Tim Bollerslev, 1986). The **GARCH(p,q)** model is again of the form (1) with

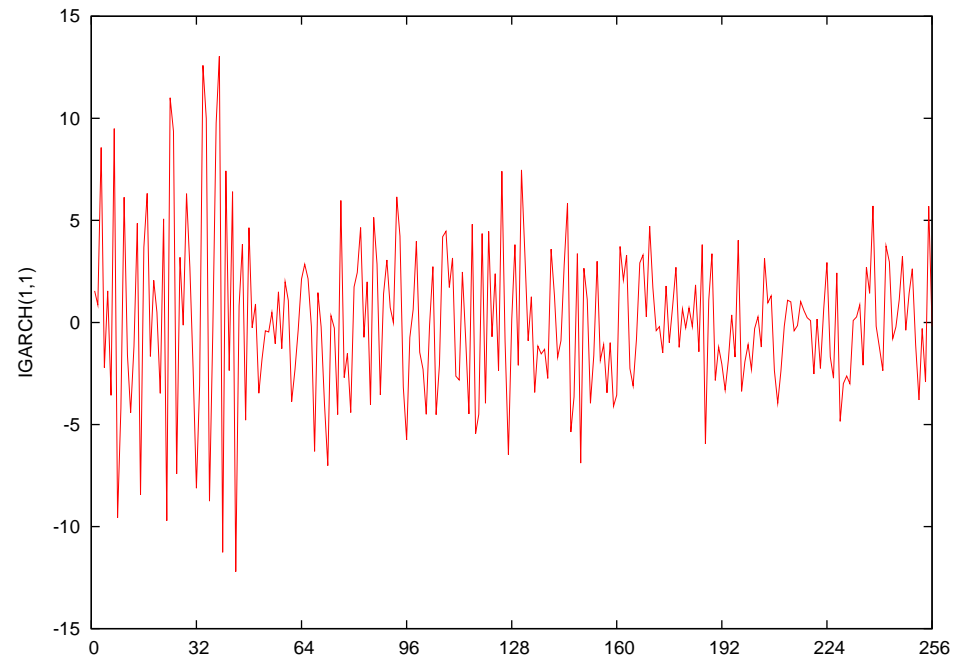
$$h_n = \alpha_0 + \sum_{i=1}^q \alpha_i y_{n-i}^2 + \sum_{i=1}^p \beta_i h_{n-i}, \quad \alpha_i, \beta_i \geq 0. \quad (3)$$

If

$$\sum_{i=1}^q \alpha_i + \sum_{i=1}^p \beta_i = 1, \quad (4)$$

there can appear persistent trends in the volatility. We call such a model IGARCH.

An example



IGARCH(1,1) with $\alpha_0 = 0.5$, $\alpha_1 = 0.25$, $\beta_1 = 0.75$

FIGARCH

Often the IGARCH model seems to be too restrictive as it implies infinite persistence of a volatility shock. On the other hand, long-range correlations in volatility are frequently present. In such situation a *fractionally integrated* GARCH model comes in handy.

A GARCH(p,q) model has the ARMA(m,p) representation

$$[1 - \alpha(Z) - \beta(Z)] \epsilon_n^2 = \alpha_0 + [1 - \beta(Z)] \nu_n, \quad (5a)$$

where $m = \max(p, q)$, Z is the time-shift operator, $\alpha(Z), \beta(Z)$ are the usual polynomials with roots outside the unit circle and

$$\nu_n = \epsilon_n^2 - h_n \quad (5b)$$

is called a *volatility shock*.

In this notation, an IGARCH is represented as

$$[1 - \alpha(Z) - \beta(Z)] (1 - Z)\epsilon_n^2 = \alpha_0 + [1 - \beta(Z)] \nu_n. \quad (6)$$

If we replace the first difference operator $1 - Z$ by its fractional counterpart, we get the FIGARCH(p,d,q):

$$[1 - \alpha(Z) - \beta(Z)] (1 - Z)^d \epsilon_n^2 = \alpha_0 + [1 - \beta(Z)] \nu_n \quad (7)$$

with $0 < d < 1$. Note: we calculate h_n from (7) and then use Eq. (1) which then becomes a nonlinear equation for ϵ_n .

More information of FIGARCH can be found for example in Maryam Tayefi and T. V. Ramanathan, *An Overview of FIGARCH and Related Time Series Models*, *Austrian Journal of Statistics*, **41**, 175–196 (2012).

EGARCH

All of the models: ARCH, GARCH, IGARCH, FIGARCH are *symmetric*: As they are on ϵ_n^2 , they react the same to negative and positive innovations of the same magnitude. However, it is known that volatilities after a negative innovation increase more than after a positive innovation of the same magnitude. To accommodate for this type of behaviour, the EGARCH, or exponential GARCH model has been introduced. In this model the $\ln h_n$ instead of pure h_n is used. In its simplest form an EGARCH(1,1) is:

$$\ln h_n = \alpha_0 + g_1(\eta_{n-1}) + \beta_1 \ln h_{n-1} \quad (8a)$$

where

$$g_1(\eta_{n-1}) = \alpha_1 \eta_{n-1} + \gamma_1 (|\eta_{n-1}| - \langle |\eta_{n-1}| \rangle). \quad (8b)$$

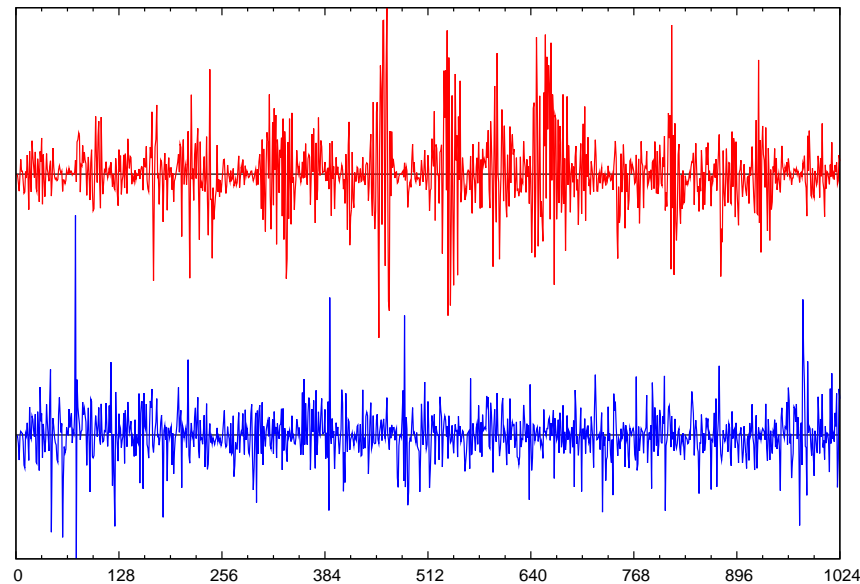
$\langle |\eta_{n-1}| \rangle$ is the *weighted innovation* or the average of $|\eta_j|$ up to η_{n-1} . $\alpha_0, \alpha_1, \beta_1, \gamma_1$ are constants (no restrictions on the sign of these constants because the *logarithm* of h_n is calculated from the model). The model responds differently to “good news” and “bad news”. A positive shock results in $(\alpha_1 + \gamma_1)\eta_{n-1}$ and a negative in $(\alpha_1 - \gamma_1)\eta_{n-1}$.

The model can be easily extended to contain more past values of ϵ_{n-j} , $\ln h_{n-j}$.

More complicated models like FIEGARCH* or models with seasonalities can be built.

*Hojatallah Goudarzi, *Modeling Long Memory in The Indian Stock Market using Fractionally Integrated Egarch Model*, International Journal of Trade, Economics and Finance, **1**, 2010-023X (2010).

Example



Two EGARCH(1,1), Eq. (8), processes.

Upper curve: $\alpha_0 = 0.25, \alpha_1 = 0.25, \beta_1 = 0.85, \gamma_1 = +0.95$.

Lower curve: $\alpha_0 = 0.25, \alpha_1 = 0.25, \beta_1 = 0.85, \gamma_1 = -0.95$.

The underlying GWN noises are **identical** in both cases. The curves have been shifted vertically for better visibility.