Statistical Physics 2

1. Find the changes in the density profile of a perfect gas in a uniform gravity field. The temperature of the gas is fixed.

2. Monomers of a 2-D linear polymer are rectalgles $L \times l$, $L > l$. If the shorter side of a monomer is aligned with the axis of the polymer (the monomer "stands"), this monomer has the energy E_1. If the longer side is aligned with the axis (the monomer "lies"), this monomer has the energy E_2. There are no gaps between the monomers. Find the expectation value of the lenght of a N-particle polymer at temperature T.

3. A diatomic perfect gas has the Hamiltonian

$$H = \frac{1}{2m} \sum_{i=1}^{N} \left(\vec{p}_{x,i}^{\,2} + \vec{p}_{y,i}^{\,2} \right) + \frac{1}{2} g \sum_{i=1}^{N} (\vec{x}_i - \vec{y}_i)^2 \,.$$

   The particles do not interact with each other (the gas is perfect).

   $\vec{x}_i, \vec{y}_i$ are the positions of the atoms that build the i-th particle of the gas, $i = 1, \ldots, n$, $\vec{p}_{x,i}, \vec{p}_{y,i}$ are the corresponding momenta, $g > 0$ is the elastic constant. The temperature, T, is fixed. Find the Helmholtz free energy of the system, the internal energy, the entropy and the heat capacity under a constant volume.

4. There is a system of N atoms, each of them has an internal magnetic moment μ. The Hamiltonian of the system in ax external magnetic field has the form

$$H = H(p, q) - \mu B \sum_{i=1}^{N} \cos \alpha_i \,,$$

   where $B = \left| \vec{B} \right|$, $H(p, q)$ is the Hamiltonian without the external magnetic field, and α_i is the angle between $\vec{B}$ and the magnetic moment of the i-th atom. Show that

   (a) The induced magnetic moment is

$$M = N\mu \left(\operatorname{ctgh} \theta - \frac{1}{\theta} \right)$$

   where $\theta = \mu B / k_B T$,

   (b) the magnetic susceptibility per atom is

$$\chi = \frac{\mu^2}{k_B T} \left(\frac{1}{\theta^2} - \operatorname{cosech}^2 \theta \right)$$

   (c) χ satisfies the Curie law at large temperatures, $\chi \sim T^{-1}$. Find the proportionality constant (the Curie constant).

PFG