Thermodynamics 2

1. It has been verified experimentally that in a certain temperature range, the magnetization, M, of a paramagnet depends on the ratio H/T only, or $M = f(H/T)$ (H is the magnetic field and T is the temperature). Show that the internal energy of this paramagnet does not depend on M. What is the functional form of the entropy?

2. Show that under a constant magnetic field, the heat capacity of a uniform magnetic material has the form

$$c_H = \left.\frac{\partial U}{\partial T}\right|_H - H \left.\frac{\partial M}{\partial T}\right|_H , \tag{1}$$

   where U is the internal energy, M is the magnetization, and T the temperature. The changes of the volume resulting from the change of the magnetization are negligible.

3. An elastic spring satisfies the Hooke's law: the extension is proportional to the force. The elastic constant of the spring, k, depends on temperature. Find the Helmholtz free energy, the internal energy and the entropy of the spring. The thermal expansion of the spring is negligible.

4. The internal energy of a unit volume of a photon gas depends on temperature, T, only, $u = u(T)$. The equation of statetakes the form $p = \frac{1}{3}u$. Find $u(T)$, the specific entropy, the specific enthalpy, the specific Helmholtz free energy, and the specific Gibbs free energy. Show the Carnot cycle od this gas on a $p-V$ plot. What is the efficiency of such a "photonic heat engine"?

5. There are two containers. There is a perfect gas in each of the containers. Paramatetrs of the gas in each container are $\{p_1, V_1, T_1\}$ and $\{p_2, V_2, T_2\}$, and the heat capacities under contstant volume of the gas are C_V^1, C_V^2, respectively. The containers are connected by a piston that does not conduct heat. The containers are in equilibrium. Express the differential form of the heat exchange, DQ, as a Pfaffian. Under what circumstances does this Pfaffian have an integrating factor? If the integrating factor exists, find it and integrate the Pfaffian. What is the physical interpretation of the integrating factor?

6. A perfect gas is first isothermally compressed form V_2 to V_1, $V_1 < V_2$, and then adiabatically expanded from V_1 to V_2. After that, the process is repeated several times. What happens with the temperature of this gas after each cycle? Is it possible to reach the absolute zero in a sequence of such cycles?

For future reference, please be advised that problems 3 and 4 might be of particular importance.
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