Thermodynamics 1

1. Let $f(x, y, z) = 0$, where f is sufficiently smooth. Show that

$$\left.\frac{\partial x}{\partial y}\right|_z \cdot \left.\frac{\partial y}{\partial z}\right|_x \cdot \left.\frac{\partial z}{\partial z}\right|_y = -1\,. \tag{1a}$$

Assuming that there exist an equation of state for a system described by state variables p, V, T, show that

$$\left.\frac{\partial T}{\partial p}\right|_V \cdot \left.\frac{\partial p}{\partial V}\right|_T \cdot \left.\frac{\partial V}{\partial T}\right|_p = -1\,. \tag{1b}$$

2. Show that if the equation of state takes the form

$$p = f(V)T\,, \tag{2}$$

where $f(\cdot)$ is a function, p is the pressure, T the temperature, and V the volume, the internal energy does not depend on the volume.

3. Using the equation of state for the perfect gas, find the expression for the internal energy and the entropy of the gas.

4. The perfect gas expands adiabatically to the vacuum (the external pressure acting on the piston is zero). The initial volume is V_0, the final volume is V. Find the change in the entropy of the gas.

5. A system is described by state variables p, V, T (pressure-volume-temperature) that are related to each other by an equation of state. Show the following identity for the molar heat capacities:

$$C_p = C_V + \left\{ \left.\frac{\partial U}{\partial V}\right|_T + p \right\} \left.\frac{\partial V}{\partial T}\right|_p\,, \tag{3}$$

where U is the internal energy. In particular, show that for the perfect gas $C_p = C_V + R$.

6. Show that for the gas from the previous problem,

$$C_p - C_V = -\left\{ p \left.\frac{\partial V}{\partial p}\right|_T + \left.\frac{\partial U}{\partial p}\right|_T \right\} \left.\frac{\partial p}{\partial t}\right|_V\,, \tag{4a}$$

$$\left.\frac{\partial H}{\partial p}\right|_T - V = \left.\frac{\partial U}{\partial p}\right|_T + p \left.\frac{\partial V}{\partial p}\right|_T\,, \tag{4b}$$

H is the enthalpy.

7. In the Joule-Thompson effect gas that is initially confined in a volume V_1 under the pressure p_1 is slowly pushed through a porous diaphragm to a volume V_2 under the pressure p_2 (the diaphragm prevents the gas from gaining kinnetic energy). Show that the specific ethalpy of the gas is constant in such a process. Express the Joule-Thompson coefficient

$$\mu_J = \left.\frac{\partial T}{\partial p}\right|_H \tag{5}$$

by C_p and $\left.\frac{\partial V}{\partial T}\right|_p$. What is μ_J for

(a) the perfect gas $pV = nRT$,

(b) the Van der Waals gas $\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT\,.$

n is the molar number, a, b are constant parameters. Calculate the inversion temperature and plot it as a function of the pressure.

PFG