1. Find the stationary solution of the 1-D diffusion equation in a harmonic potential

$$U(x) = \tfrac{1}{2} g x^2 \,.$$

2. A particle moves on a 1-D, infinite mesh. The disnace between two neighbouring nodes equals a. In the interval τ the particle moves by $n{\neq}0, n \in \mathbb{Z}$ nodes (the particle cannot stay on the same node), and the jumps to the left or to the right are equally probale. The probability of jumping by $|n|$

   (i) is proportional to $|n|^{-1}$;

   (ii) is proportional to do n^{-2};

   (iii) is proportional to n^{-4};

   (iv) is proportional to $q^{-|n|}$, where $0 < q < 1$.

   (a) Are all the above processes well defined?

   (b) Normalize the probabilities if the processes are well defined.

   (c) Which processes approach diffusion in the limit

$$\begin{cases} a \to 0 \\ \tau \to 0 \\ \dfrac{a^2}{\tau} = B = \text{const} \end{cases}$$

   Provide solutions to the corresponding diffusion equations.

   (d) What would change if in 2iv we let $n{=}0$?

PFG