1. (a) Find the normalization, the expectation value and the variance of the Marchenko-Pastur distribution
$$P(x) = \alpha \sqrt{\frac{4-x}{x}}\,, \qquad x \in [0,4]\,. \tag{1}$$
   (b) Find the distribution of a random variable $y = x^2$, where x is distributed with (1).

2. A particle jumps from the site 1 to the site 2, or back. The probability of jumping from 1 to 2, or from 2 to 1, in a very short time Δt equals $\lambda \Delta t$. In time $t = 0$ the particle sits in 1.
   (a) Find the probability that the particle sits in 1 at time $t \geqslant 0$.
   (b) Find the conditional probability that the particle sits in 1 at time $t \geqslant 0$ and have never jumped from 1 to 2.

3. A particle can move with unit lenght jumps. A jump is taken every 1 sec, with equal probability of jumping to the left or to the right (the particle cannot stay where it is).
   (a) Verify that this process forms a Markov chain. Find its transfer matrix in a single jump.
   (b) Find the transfer matrix $P_{kj}(n)$ in n jumps.
   (c) Find the mean square displacement of the particle in n jumps.

   $P_{kj}(n)$ is the probability of finding the particle in the site j after n seconds, provided it was in the site k at time zero.

4. The particle from Problem 3 moves with jumps of the length l and a jump is taken every τ seconds.
   (a) Find the probability distribution $P(x,t)$ that the particle is in the location x at time t, provided it has started from $x = 0$ at $t = 0$.
   (b) Use the Stirling formula to verify that if $\tau, l \to 0$ in such a manner $\frac{l^2}{\tau} = 2D = \text{const}$, then
$$P(x,t) = \frac{1}{\sqrt{4\pi D t}} \exp\left(-\frac{x^2}{4Dt}\right). \tag{2}$$
   (c) Verify that the time-dependent probability distribution (2) satisfies the diffusion equation:
$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial x^2}\,. \tag{3}$$

5. A particle can be in one of the three states A, B, C. The probability of jumping in a single discrete step (form brevity, in the following I will omit this clause) from A to B equals 2/3. The probability of jumping from A to C equals $1/3$. The probabilities of jumping from B to A or to C, or staying in B, are equal. The probability of jumping from C to A equals 2/3, and the probability of jumping from C to B equals $1/3$. The particle is in the state A at $n = 0$.
   (a) Find the transfer matrix for this process.
   (b) What are the probabilities of being in A, B, C after 2 steps? After m steps? After $m \gg 1$ steps?
   (c) Do the probabilities after $m \gg 1$ depend on the initial state?
   (d) Do these probabilities satisfy the detailed balance conditions?

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