1. From the set of $\{1, 2, 3, 4, 5\}$ we choose randomly, without repetitions, two numbers. Find the probability that
   (a) the product of the chosen numbers is greater than 10,
   (b) an odd number has been chosen first,
   (c) the product of the chosen numbers is greater than 10 and an odd number has been chosen first,
   (d) the product of the chosen numbers is greater than 10, provided we know that an odd number has been chosen first.
2. We roll two perfect dice. What is the probability that the sum of the pips equals 6, provided at least one of the dice shows a prime number of pips?
3. What is the probability that the birthdays of at least two students in a class of $n = 20$ coincide?
4. What is more probable:
   (a) scoring at least one "6" in 6 rolls of dice, or
   (b) scoring at least two "6" in 12 rolls of dice, or
   (c) scoring at least three "6" in 18 rolls of dice?
5. Statistically, one in 1000 people suffers from a certain disease. A test for this disease gives the following results:
   (a) Positive with the probability of 95% if the person is sick,
   (b) Negative with the probability of 5% if the person is sick,
   (c) Positive with the probability of 5% if the person is healthy,
   (d) Negative with the probability of 95% if the person is healthy.

   A person tested themselves and the result was positive. What is the probability that the person is actually sick?
6. The person from the Problem 5 repeated the test and the result was again positive. What is the probability that the person is sick? What would the probability be after a third positive test?
7. A random variable is drawn from the probability density

$$f(x) = \begin{cases} 0 & \text{if } x < 0\,, \\ a\sin(x) & \text{if } 0 \leqslant x \leqslant \pi\,, \\ 0 & \text{if } x > \pi\,. \end{cases} \tag{1}$$

   (a) Calculate a.
   (b) Find the cumulative distribution function (pol. dystrybuantę) of this probability density and its inverse.
   (c) Find the probability of $0 \leqslant x \leqslant \frac{\pi}{4}$.
8. The examiner ask a student a string of questions. The probability of answering a single question correctly equals 90%. The exam stops after the student fails to give a correct answer. What is the probability distribution of the number of questions asked, the most probable number of questions asked and the expectation value of the number of questions asked?

9. A cannon sits in the plane (x, y), at the point $(0, -1)$. The barrel of the cannon can rotate freely in the plane (x, y). Let θ be the angle between the barrel and the y axis. This angle is chosen randomly with the uniform distribution on $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Find the probability distribution of the crossing points between the line of the shot and the x axis.

10. A random variable X is distributed uniformly over $[-2, 2]$. What is the probability distribution of $Y = X^2$?

11. Let two independent random variables X, Y come from a uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$. What is the probability distribution of $Z = Y + Y$, its expectation value and the variance?

12. Find the characteristic function of the normal distribution $N(\bar{x}, \sigma)$.

13. Let two independent random variables come from the normal distribution $N(\bar{x}, \sigma)$. Find the probability distribution of the sum of these two variables.

14. Let two independent random variables come from the Cauchy distribution:

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1 + x^2} \,. \tag{2}$$

Find the probability distribution of the sum of these two variables.

The probability that we manage to discuss all these problems in a single class is low.
PFG