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Comment

Comment on "Stochastic resonance in a linear system with signal-modulated noise" by L. Cao and D. J. Wu

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Abstract. – The system discussed recently by Cao and Wu in *Europhys. Lett.*, **61** (2003) 593, merely reproduces properties of the input signal.

Stochastic resonance (SR) is an example of a constructive role of noise, where the noise and a dynamical system act together to reinforce the output of the latter. Recently, Cao and Wu in ref. [1] discussed an overdamped, linear system with purely additive, correlated, coloured noises, partially modulated by a periodic signal, and observed a phenomenon which they described as SR. This effect indeed fits into a broad definition of SR that some parameters of the system are optimized by a specific level of the stochastic forcing [2,3], but the authors of ref. [1] do not provide any qualitative interpretation of their findings. The dynamical system is governed by the following equation:

$$\dot{x} = -\alpha x + f(t) = -\alpha x + A_0 \xi(t) \cos(\Omega t + \phi) + \zeta(t)\,, \tag{1}$$

where $\zeta(t) = \xi(t) + \eta(t)$ and $\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0$, $\langle \xi(t)\xi(t') \rangle = Qu(|t-t'|)$, $\langle \eta(t)\eta(t') \rangle = Du(|t-t'|)$, $\langle \xi(t)\eta(t') \rangle = 2\lambda\sqrt{QD}u(|t-t'|)$. In ref. [1], $u(|t-t'|) = \exp[-|t-t'|/\tau]/\tau$, but, as we shall see, the choice of u is not particularly important. Cao and Wu calculate $\langle x(t)x(t+t') \rangle$ (but do not provide the final formula for it), use the Wiener-Khinchin theorem to calculate the power spectrum, and conclude that SR is present in the system.

While the authors of ref. [1] do not state that explicitly, we assume that the correlation function has been averaged over the initial phase, ϕ, as otherwise it would produce a time-dependent spectrum whose interpretation is not obvious [4,5]. These authors also use a rather non-standard measure of the SR. But the crucial observation is that, regardless of whether we admit time-dependent power spectra or not and what definition of SR we use, the signal-to-noise ratio (SNR) for the system (1) with *any* correlation function $u(|t-t'|)$ has the form

$$\mathrm{SNR} = \frac{p\,Q}{Q + D + 2\lambda\sqrt{QD}}, \tag{2}$$

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where the coefficient p may depend on time, the characteristic frequency Ω and other factors, cf. eqs. (14)-(16) in ref. [1]. If we now calculate the correlation function of the input in (1) and average it over the initial phase ϕ:

$$\frac{1}{2\pi}\int\limits_0^{2\pi} \langle f(t)f(t+t')\rangle \, \mathrm{d}\phi = \frac{1}{2}A_0^2 Q u(|t'|)\cos\Omega t' + (Q + D + 2\lambda\sqrt{QD})u(|t'|)\,, \qquad (3)$$

we get for the SNR *of the input signal* an expression of precisely the same form as (2) but with p replaced by some other coefficient $\tilde{p}$. The same result may be obtained directly from (3) by comparing the amplitudes of the oscillatory term and the noisy background. Since there is no feedback, features of the input signal are oblivious to any dynamics that the system (1) may display. Any resonant behaviour corresponding to changes in Q can be present in the output if and only if it is present in the input. Thus, the resonant behaviour of the output solely reflects changes in spectral properties of the input signal.

For $\lambda < 0$, the function (2) displays a maximum and even diverges for $\lambda = -1$ and $Q = D$, which is unphysical. The reason for this behaviour of SNR is that for negative λ the variance of the unmodulated noise $\zeta(t)$ may decrease as the variance of the stochastic amplitude of the periodic signal increases —observe that Q enters the variances of both the noises. The fact that one cannot independently manipulate various noise strengths is unwelcome and we should think of a better parameterisation of the problem. If, instead of (1), we write

$$\dot{x} = -\alpha x + \sqrt{Q}\,\xi(t)\cos(\Omega t + \phi) + \sqrt{D}\,\zeta(t) \qquad (4)$$

where $\zeta(t) = \lambda\,\xi(t) + \sqrt{1-\lambda^2}\,\eta(t)$, $\lambda \in [-1, 1]$ and

$$\langle \xi(t)\rangle = \langle \eta(t)\rangle = \langle \xi(t)\eta(t')\rangle = 0\,, \qquad (5a)$$

$$\langle \xi(t)\xi(t')\rangle = \langle \eta(t)\eta(t')\rangle = u(|t-t'|)\,, \qquad (5b)$$

we capture the most essential features of the system. Now the variances of the unmodulated and signal-modulated noises are independent, we do not risk any divergences, but we do not have any resonant behaviour in the input signal, either.

In conclusion, we have shown that the dynamics of the system described in ref. [1] merely reproduces properties of the input signal. Moreover, the resonant properties of the input signal are an artifact of a not particularly successful parameterisation of the noise.

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