Krakow, October 2007

SMOLUCHOWSKI LECTURE

PROBLEMS 3 – 5

3. Problem : Give the proof of the K-theorem ($K > 0$, $dK / dt < 0$) for several special stochastic processes :

(i) the Smoluchowski equation,
(ii) the Fokker-Planck equation,
(iii) the Pauli equation,
(iv) the Boltzmann equation in relaxation approximation.

Discuss the meaning of the K-functional.

4. Problem : Demonstrate by analytical considerations and/or by computer simulations for simple examples that the trajectories fill – under certain conditions - densely the phase space. Study (following Lecture Notes 3-4) the tent map and the Hamilton dynamics $H = -a\, q + b\, p$ or look for other simple cases. Discuss the conditions for ergodic behaviour and master equations.

5. Problem : Study stochastic properties - similar as in problem 4 - of the two-dimensional Hamiltonian map, the Chirikov-Taylor standard map

$x(t+1) = x(t) – (K/ 2 pi) \sin (2 pi\, y(t))$,

$y(t+1) = y(t) + x(t+1) \pmod 1$.

Try first to find a Perron-Frobenius equation for the phase-space density $f(x,y,t)$ (or better for its Fourier series) and then a diffusion-like equation for the density $n(x, t)$ neglecting memory effects.