Gravitational Lensing from clusters of galaxies to test disformal coupling theories

Saboura Zamani

Institute of Physics, University of Szczecin, Poland

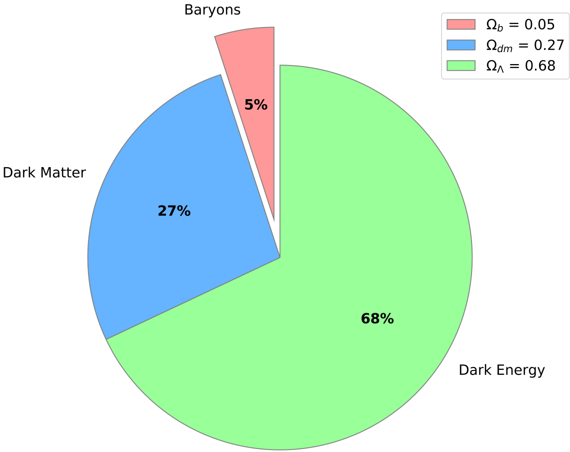
Cracow School of Theoretical Physics
15 - 23 June 2024, Poland

The research is funded by the Polish National Science Centre grant

SZCZECIN COSMOLOGY GROUP

UNIWERSYTET SZCZECIŃSKI

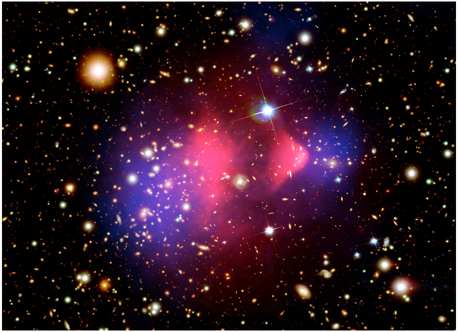
NATIONAL SCIENCE CENTRE POLAND



95% of the Energy-Mass budget of the Universe:
Dark matter + Dark Energy

Dark Matter: Observational Evidence

- Rotational curve
- Mass tracers (X-rays, Sunyaev–Zeldovich, strong & weak lensing)
- Bullet Cluster
- Cosmic Microwave Background (CMB) anisotropies
- Large Scale Structure (LSS)



Credits: ESA

$\boldsymbol{\Lambda}$CDM: $\boldsymbol{\Lambda}$-Cold Dark Matter

- Abundance
  DM $\Rightarrow \sim 27\%$ of the universe's total mass-energy budget
- Cold
  DM particles should be non-relativistic (cold)
- Feebly Interacting
  DM is thought to interact via gravity and the weak nuclear force
- Invisible
  It does not interact with light
- Distribution
  A web-like structure throughout the universe

Einstein-Hilbert Action:

$$S = \int d^4x\sqrt{-g}\, R + S_{\text{fluid}} \tag{1}$$

Our NMC Action: (D. Bettoni, S. Liberati, 2015, 1502.06613)

$$S = \frac{M_{\text{Pl}}^2}{2}\int d^4x\sqrt{-g}\left[R + \boldsymbol{\alpha_{\mathrm{d}}\rho_{\mathrm{DM}}R_{\mu\nu}u^{\mu}u^{\nu}}\right] + S_{\text{fluid}} \tag{2}$$

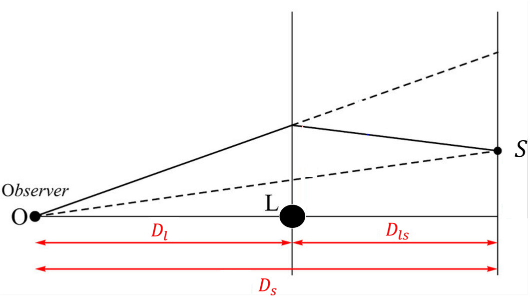
Our DM variable couple to the contracted Ricci tensor with the fluid four-vector velocity.

Modified Poisson equation:

$$\nabla^2\Phi = 4\pi\, G_N\, [\rho - \epsilon\, L^2\, \nabla^2\rho_{\rm DM}] \tag{3}$$

The source of gravity is not only density but also on how the matter is distributed.

- Density $\rho = \rho_{\rm DM} + \rho_{gas}$
- Polarity $\Rightarrow \epsilon = -1$
- Characteristic Length of NMC model $\Rightarrow L \propto \alpha_d$

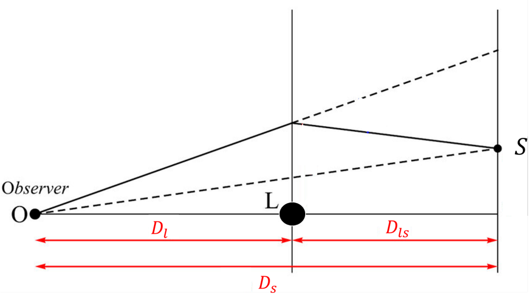


Effective lensing potential:

$$\Phi_{\text{lens}}(R) = \frac{2}{c^2}\frac{D_{ls}}{D_l D_s}\int_{-\infty}^{+\infty}\Phi(R,z)\,dz \tag{4}$$

$$\kappa(R) = \frac{1}{c^2}\frac{D_{ls}D_l}{D_s}\int_{-\infty}^{+\infty}\Delta_r\Phi(R,z)\,dz \tag{5}$$

Gravitational Lensing



$$\kappa(R) = \frac{4\pi G_N}{c^2} \frac{D_{ls} D_l}{D_s} \int_{-\infty}^{+\infty} \left[\rho(R, z) - \epsilon L^2 \Delta_r \rho_{DM}(R, z)\right] \, \mathrm{d}z \qquad (6)$$

- $\Delta_r = \frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial^2 r}$ (Spherical coordinates)

CLASH[1] survey programme: 19 clusters (Postman et al. 2011, 1106.3328)

- X-ray $\Rightarrow$ hot gas
- strong and weak gravitational lensing $\Rightarrow$ DM $\Rightarrow$ NFW model

Hot gas = modified β-model (Donahue M. et al. 2015, 1405.7876)

$$\rho_{gas}(r) = \rho_{e,0}\left(\frac{r}{r_0}\right)^{-\alpha}\left[1+\left(\frac{r}{r_{e,0}}\right)^2\right]^{-3\beta_0/2} + \rho_{e,1}\left[\left(\frac{r}{r_{e,1}}\right)^2\right]^{-3\beta_1/2}$$

$\{\rho_{e,0}, r_{e,0}, r_{e,1}, r_0, \alpha, \beta_0, \beta_1\}$ fixed by preliminary fit of X-ray data

- Tension between X-ray and lensing data $\Rightarrow$ Not including X-ray data directly in modeling our cluster

[1] Cluster Lensing And Supernova survey with Hubble

CLASH[1] survey programme: 19 clusters (Postman et al. 2011, 1106.3328)

- X-ray $\Rightarrow$ hot gas
- strong and weak gravitational lensing $\Rightarrow$ DM $\Rightarrow$ NFW model

DM Model $\Rightarrow$ Navarro-Frenk-White Profile (J. F. Navarro, C. S. Frenk, and S. D. M. White, 1996)

$$\rho_{\rm NFW}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

$$\rho_s = \frac{\Delta}{3}\frac{c_\Delta^3}{log(1+c_\Delta) - \frac{c_\Delta}{1+c_\Delta}}\rho_c, \quad c_\Delta = \frac{r_\Delta}{r_s}, \quad \Delta = 200$$

[1] Cluster Lensing And Supernova survey with Hubble

- χ^2 function

$$\chi^2 = \left(\boldsymbol{\kappa}^{\boldsymbol{theo}}(\boldsymbol{\theta}) - \boldsymbol{\kappa}^{\boldsymbol{obs}}\right) \cdot \mathbf{C}^{-1} \cdot \left(\boldsymbol{\kappa}^{\boldsymbol{theo}}(\boldsymbol{\theta}) - \boldsymbol{\kappa}^{\boldsymbol{obs}}\right) \tag{7}$$

$\boldsymbol{\theta} = \{c_{200},\, M_{200},\, L\}$: NMC model parameters

$\mathbf{C}$: Covariance error matrix

- Bayes Theorem - Deriving the Posterior Distribution using MCMC

$$\mathcal{P}(\boldsymbol{\theta}, \mathcal{M}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta}, \mathcal{M})\pi(\boldsymbol{\theta}, \mathcal{M})}{\mathcal{E}(D|\mathcal{M})}$$

- Bayesian Evidence

$$\mathcal{E}(D|\mathcal{M}) = \int d\boldsymbol{\theta}\mathcal{L}(D|\boldsymbol{\theta}, \mathcal{M})\pi(\boldsymbol{\theta}, \mathcal{M})$$

- Bayesian Evidence

$$\mathcal{E}(D|\mathcal{M}) = \int d\boldsymbol{\theta} \mathcal{L}(D|\boldsymbol{\theta}, \mathcal{M}) \pi(\boldsymbol{\theta}, \mathcal{M})$$

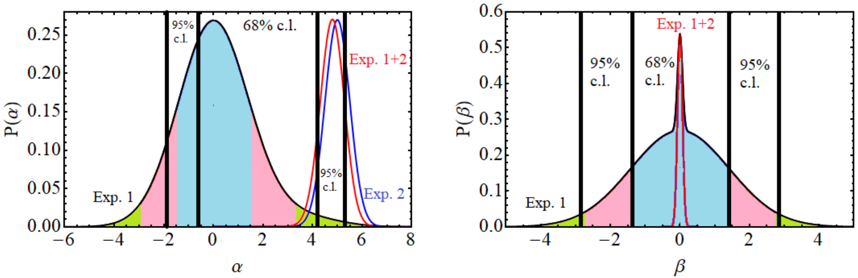
- Bayes Factor

$$\mathcal{B}_j^i = \frac{\mathcal{E}(M_i)}{\mathcal{E}(M_j)}$$

- Jeffrey Scale $\Longrightarrow \mathcal{B}_j^i$ interpretation

$\log \mathcal{B}_j^i < 1$	Disfavoured
$1 \leq \log \mathcal{B}_j^i < 2.5$	Substantial
$2.5 \leq \log \mathcal{B}_j^i < 5$	Strong
$\log \mathcal{B}_j^i \geq 5$	Decisive

When Marginalisation Is Not Enough: An Example



- **Marginalisation**:
  - Simply “reading” the posteriors which are produced as output by the MCMCs.

- **Profile distribution**:
  - An extension of the profile likelihood
  - It highlights the behavior of the posterior around the maximum of the likelihood. (A. Gómez-Valent, 2022, 2203.16285)

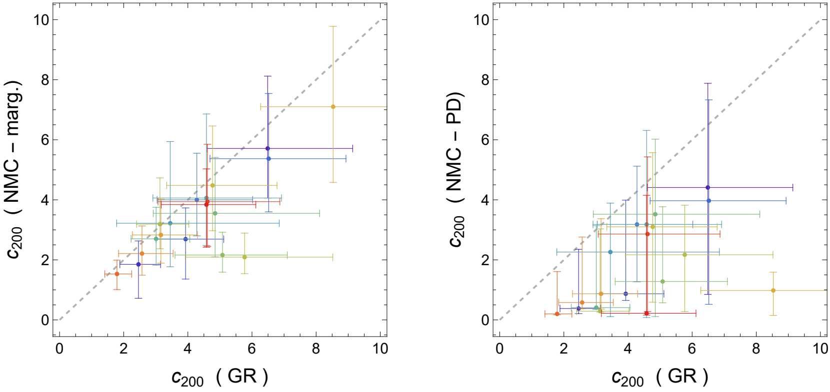


Figure 1: Comparison between values of concentration parameter c_{200} in GR and NMC model considering Marg. and PD.

Results: GR vs Modified

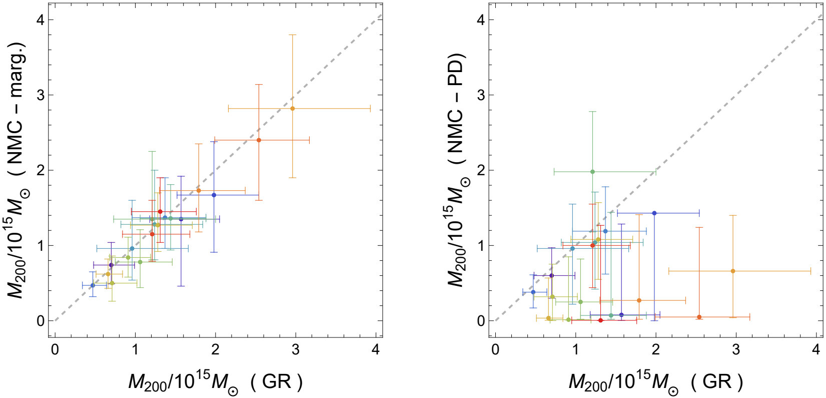


Figure 2: Comparison between values of mass M_{200} in GR and NMC model considering Marg. and PD.

Results

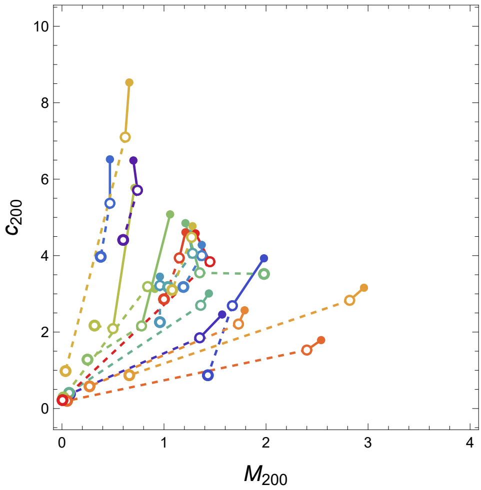
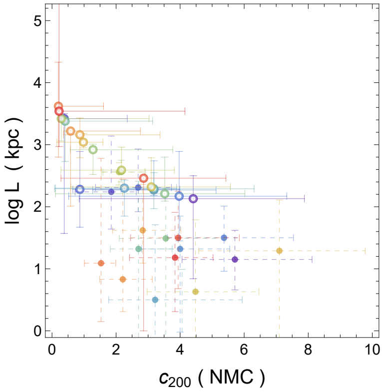
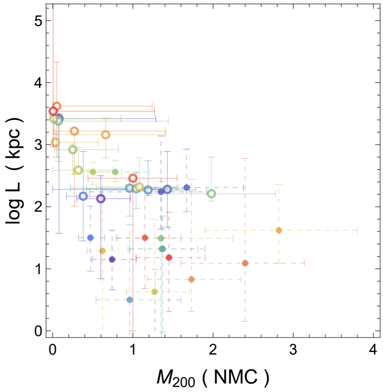


Figure 3: Comparison of dark matter parameters c_{200}, M_{200}, obtained from GR and from the NMC model considered in this work.

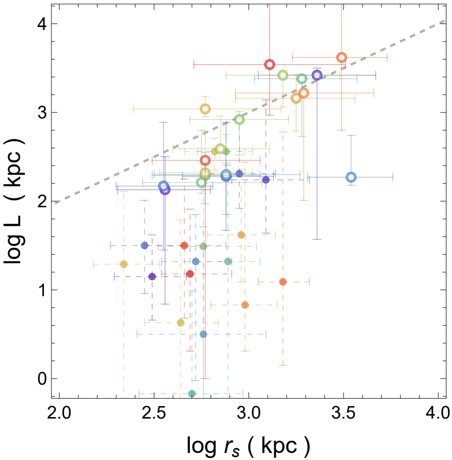




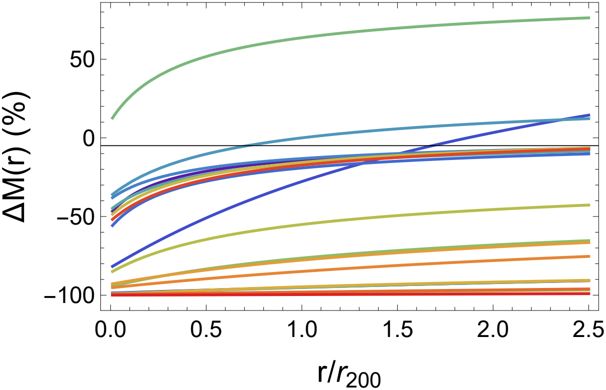
$$\nabla^2\Phi = 4\pi G_N\left[\rho - \epsilon\, L^2\, \nabla^2\rho_{\rm NFW}\right]$$

$$\rho_{\rm NFW}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}, \quad \rho_s = \frac{\Delta}{3}\,\frac{c_\Delta^3}{log(1+c_\Delta) - \frac{c_\Delta}{1+c_\Delta}}\rho_c$$

Results: Closer Look!



$$\rho_{\mathrm{NFW}}(x) \propto \frac{1}{x(1+x)^2} + \frac{L^2}{r_s^2}\frac{6}{x(1+x)^4}, \ (x = r/r_s)$$



$$\Delta M_{200} \equiv \frac{M_{200,\mathrm{NMC}} - M_{200,\mathrm{GR}}}{M_{200,\mathrm{GR}}}$$

Results

	GR		MOD (marg.)			MOD (PD)			$\log \mathcal{B}_j^i$	$\log \mathcal{S}_j^i$
Cluster	c_{200}	M_{200}	c_{200}	M_{200}	$\log L$	c_{200}	M_{200}	$\log L$		
		$(10^{15}\,\mathrm{M}_\odot)$		$(10^{15}\,\mathrm{M}_\odot)$	(kpc)		$(10^{15}\,\mathrm{M}_\odot)$	(kpc)		
A383	$6.49^{+2.65}_{-1.89}$	$0.70^{+0.29}_{-0.22}$	$5.71^{+2.41}_{-1.65}$	$0.74^{+0.30}_{-0.25}$	$\mathbf{1.15^{+0.47}_{-0.49}}$	$4.41^{+3.47}_{-3.56}$	$0.60^{+0.37}_{-0.30}$	$(0.84, 2.13, 2.50)$	$0.005^{+0.024}_{-0.023}$	$-0.010^{+0.035}_{-0.038}$
A209	$2.46^{+0.69}_{-0.58}$	$1.57^{+0.48}_{-0.39}$	$1.85^{+0.78}_{-1.13}$	$1.35^{+0.57}_{-0.89}$	$2.24^{+0.90}_{-0.60}$	$(0.21, 0.38, 2.35)$	$(0.005, 0.078, 1.285)$	$(1.57, 3.42, 3.50)$	$0.006^{+0.022}_{-0.022}$	$-0.025^{+0.038}_{-0.052}$
A2261	$3.93^{+1.19}_{-0.92}$	$1.98^{+0.56}_{-0.46}$	$2.69^{+1.04}_{-1.33}$	$1.67^{+0.71}_{-0.76}$	$2.31^{+0.62}_{-0.39}$	$(0.65, 0.87, 3.99)$	< 1.43	$2.28^{+0.61}_{-0.61}$	$-0.0005^{+0.0190}_{-0.0202}$	$0.011^{+0.034}_{-0.037}$
RXJ2129	$6.52^{+2.41}_{-1.83}$	$0.47^{+0.17}_{-0.13}$	$5.37^{+2.17}_{-1.77}$	$0.47^{+0.18}_{-0.15}$	$1.50^{+0.51}_{-0.54}$	$3.97^{+3.36}_{-3.45}$	$0.38^{+0.23}_{-0.21}$	$2.17^{+0.72}_{-0.72}$	$0.014^{+0.022}_{-0.023}$	$0.005^{+0.040}_{-0.037}$
A611	$4.28^{+1.74}_{-1.24}$	$1.37^{+0.51}_{-0.41}$	$4.00^{+1.55}_{-1.20}$	$1.37^{+0.53}_{-0.41}$	$1.32^{+0.53}_{-1.34}$	$3.18^{+1.94}_{-1.91}$	$1.19^{+0.59}_{-0.57}$	$(2.18, 2.27, 2.74)$	$-0.013^{+0.021}_{-0.022}$	$-0.021^{+0.046}_{-0.041}$
MS2137	$3.45^{+3.40}_{-1.67}$	$0.96^{+0.70}_{-0.44}$	$3.22^{+2.72}_{-1.45}$	$0.96^{+0.64}_{-0.42}$	$0.50^{+1.21}_{-2.49}$	$(0.11, 2.26, 3.89)$	$0.96^{+0.59}_{-0.74}$	$(1.85, 2.30, 2.43)$	$0.057^{+0.021}_{-0.025}$	$0.084^{+0.031}_{-0.043}$
RXJ2248	$4.58^{+2.34}_{-1.67}$	$1.24^{+0.60}_{-0.42}$	$4.06^{+2.80}_{-1.58}$	$1.28^{+0.72}_{-0.47}$	$-0.17^{+1.16}_{-1.73}$	$3.18^{+3.13}_{-3.10}$	$1.04^{+0.67}_{-0.62}$	$(1.97, 2.29, 2.55)$	$-0.049^{+0.017}_{-0.025}$	$-0.077^{+0.035}_{-0.044}$
MACSJ1115	$3.01^{+1.05}_{-0.78}$	$1.44^{+0.44}_{-0.38}$	$2.70^{+1.05}_{-0.88}$	$1.36^{+0.45}_{-0.42}$	$1.32^{+1.09}_{-3.06}$	$(0.35, 0.41, 3.14)$	$(0.02, 0.07, 1.44)$	$(2.73, 3.38, 3.42)$	$0.014^{+0.023}_{-0.022}$	$0.008^{+0.032}_{-0.037}$
MACSJ1931	$4.85^{+3.26}_{-1.93}$	$1.21^{+0.79}_{-0.48}$	$3.55^{+1.86}_{-1.45}$	$1.35^{+0.90}_{-0.54}$	$1.49^{+0.81}_{-2.44}$	$(0.11, 3.52, 6.02)$	$1.98^{+0.80}_{-0.80}$	$(2.09, 2.21, 2.80)$	$-0.025^{+0.027}_{-0.020}$	$-0.056^{+0.038}_{-0.037}$
MACSJ1720	$5.08^{+2.02}_{-1.48}$	$1.06^{+0.40}_{-0.31}$	$2.16^{+0.76}_{-0.57}$	$0.78^{+0.43}_{-0.34}$	$2.56^{+0.19}_{-0.16}$	$(0.57, 1.28, 3.77)$	$(0.02, 0.25, 0.82)$	$(2.52, 2.92, 3.01)$	$0.009^{+0.023}_{-0.022}$	$-0.016^{+0.046}_{-0.032}$
MACSJ0416	$3.13^{+0.90}_{-0.73}$	$0.91^{+0.28}_{-0.23}$	$3.19^{+1.54}_{-0.82}$	$0.84^{+0.27}_{-0.26}$	$-1.89^{+1.47}_{-3.43}$	$(0.26, 0.29, 3.03)$	$(0.010, 0.014, 0.850)$	$(3.06, 3.42, 3.44)$	$-0.075^{+0.026}_{-0.022}$	$-0.143^{+0.046}_{-0.037}$
MACSJ0429	$5.77^{+2.75}_{-1.85}$	$0.71^{+0.31}_{-0.23}$	$2.09^{+0.80}_{-0.55}$	$0.50^{+0.36}_{-0.24}$	$2.56^{+0.15}_{-0.21}$	$(0.28, 2.17, 3.82)$	$(0.002, 0.319, 0.752)$	$(2.58, 2.59, 2.96)$	$-0.083^{+0.020}_{-0.025}$	$-0.154^{+0.038}_{-0.046}$
MACSJ1206	$4.77^{+2.01}_{-1.43}$	$1.28^{+0.43}_{-0.34}$	$4.48^{+1.98}_{-1.51}$	$1.27^{+0.43}_{-0.35}$	$0.63^{+1.16}_{-1.23}$	$3.10^{+2.47}_{-2.51}$	$1.08^{+0.49}_{-0.53}$	$(2.26, 2.32, 2.79)$	$0.004^{+0.020}_{-0.022}$	$-0.0004^{+0.0355}_{-0.0402}$
MACSJ0329	$8.53^{+2.71}_{-2.26}$	$0.66^{+0.18}_{-0.15}$	$7.10^{+2.68}_{-2.52}$	$0.62^{+0.20}_{-0.19}$	$1.29^{+0.82}_{-2.30}$	$0.98^{+0.61}_{-0.83}$	$(0.006, 0.035, 0.568)$	$(2.95, 3.04, 3.18)$	$0.009^{+0.020}_{-0.024}$	$0.0004^{+0.0374}_{-0.0455}$
RXJ1347	$3.16^{+1.14}_{-0.89}$	$2.96^{+0.97}_{-0.80}$	$2.83^{+1.19}_{-0.94}$	$2.82^{+0.98}_{-0.92}$	$1.62^{+0.74}_{-0.53}$	$(0.36, 0.87, 3.37)$	$(0.04, 0.66, 1.40)$	$(2.79, 3.16, 3.43)$	$0.008^{+0.029}_{-0.026}$	$0.005^{+0.053}_{-0.045}$
MACSJ1149	$2.57^{+0.97}_{-0.73}$	$1.79^{+0.58}_{-0.49}$	$2.21^{+0.92}_{-0.72}$	$1.73^{+0.62}_{-0.55}$	$0.83^{+1.61}_{-0.52}$	$(0.37, 0.58, 2.76)$	$(0.02, 0.27, 1.41)$	$(2.01, 3.22, 3.23)$	$0.033^{+0.023}_{-0.021}$	$0.042^{+0.048}_{-0.029}$
MACSJ0717	$1.79^{+0.46}_{-0.38}$	$2.54^{+0.63}_{-0.55}$	$1.53^{+0.46}_{-0.52}$	$2.40^{+0.74}_{-0.80}$	$1.09^{+1.69}_{-0.94}$	$(0.17, 0.20, 1.61)$	$(0.02, 0.05, 1.24)$	$3.62^{+0.71}_{-0.82}$	$0.008^{+0.022}_{-0.017}$	$0.002^{+0.037}_{-0.030}$
MACSJ0647	$4.61^{+2.26}_{-1.54}$	$1.21^{+0.47}_{-0.37}$	$3.94^{+1.91}_{-1.49}$	$1.15^{+0.45}_{-0.36}$	$1.50^{+0.75}_{-0.82}$	$2.86^{+2.57}_{-2.58}$	$1.00^{+0.55}_{-0.56}$	> 2.46	$0.041^{+0.023}_{-0.020}$	$0.046^{+0.038}_{-0.034}$
MACSJ0744	$4.58^{+2.09}_{-1.41}$	$1.31^{+0.45}_{-0.36}$	$3.84^{+1.19}_{-1.42}$	$1.45^{+0.45}_{-0.41}$	$1.18^{+0.73}_{-0.87}$	$(0.16, 0.22, 4.15)$	$(0.002, 0.005, 1.267)$	$(2.97, 3.54, 3.62)$	$-0.002^{+0.021}_{-0.023}$	$-0.003^{+0.032}_{-0.048}$

Results

Cluster	GR		MOD (marg.)			MOD (PD)			$\log \mathcal{B}_j^i$	$\log \mathcal{S}_j^i$
	c_{200}	M_{200} (10^{15} $\mathrm{M}_\odot$)	c_{200}	M_{200} (10^{15} $\mathrm{M}_\odot$)	$\log L$ (*kpc*)	c_{200}	M_{200} (10^{15} $\mathrm{M}_\odot$)	$\log L$ (*kpc*)		
A383	$6.49^{+2.65}_{-1.89}$	$0.70^{+0.29}_{-0.22}$	$5.71^{+2.41}_{-1.65}$	$0.74^{+0.30}_{-0.25}$	$\mathbf{1.15^{+0.47}_{-0.49}}$	$4.41^{+3.47}_{-3.56}$	$0.60^{+0.37}_{-0.30}$	$(0.84, 2.13, 2.50)$	$0.005^{+0.024}_{-0.023}$	$-0.010^{+0.035}_{-0.038}$
A209	$2.46^{+0.69}_{-0.58}$	$1.57^{+0.48}_{-0.39}$	$1.85^{+0.78}_{-1.13}$	$1.35^{+0.57}_{-0.89}$	$2.24^{+0.90}_{-0.60}$	$(0.21, 0.38, 2.35)$	$(0.005, 0.078, 1.285)$	$(1.57, 3.42, 3.50)$	$0.006^{+0.022}_{-0.022}$	$-0.025^{+0.038}_{-0.052}$
A2261	$3.93^{+1.19}_{-0.92}$	$1.98^{+0.56}_{-0.46}$	$2.69^{+1.04}_{-1.33}$	$1.67^{+0.71}_{-0.76}$	$2.31^{+0.62}_{-0.39}$	$(0.65, 0.87, 3.99)$	< 1.43	$2.28^{+0.61}_{-0.61}$	$-0.0005^{+0.0190}_{-0.0202}$	$0.011^{+0.034}_{-0.037}$
RXJ2129	$6.52^{+2.41}_{-1.83}$	$0.47^{+0.17}_{-0.13}$	$5.37^{+2.17}_{-1.77}$	$0.47^{+0.18}_{-0.15}$	$1.50^{+0.51}_{-0.54}$	$3.97^{+3.36}_{-3.45}$	$0.38^{+0.23}_{-0.21}$	$2.17^{+0.72}_{-0.72}$	$0.014^{+0.022}_{-0.023}$	$0.005^{+0.040}_{-0.037}$
A611	$4.28^{+1.74}_{-1.24}$	$1.37^{+0.51}_{-0.41}$	$4.00^{+1.55}_{-1.20}$	$1.37^{+0.53}_{-0.41}$	$1.32^{+0.53}_{-1.34}$	$3.18^{+1.94}_{-1.91}$	$1.19^{+0.59}_{-0.57}$	$(2.18, 2.27, 2.74)$	$-0.013^{+0.021}_{-0.022}$	$-0.021^{+0.046}_{-0.041}$
MS2137	$3.45^{+3.40}_{-1.67}$	$0.96^{+0.70}_{-0.44}$	$3.22^{+2.72}_{-1.45}$	$0.96^{+0.64}_{-0.42}$	$0.50^{+1.21}_{-2.49}$	$(0.11, 2.26, 3.89)$	$0.96^{+0.59}_{-0.74}$	$(1.85, 2.30, 2.43)$	$0.057^{+0.021}_{-0.025}$	$0.084^{+0.031}_{-0.043}$
RXJ2248	$4.58^{+2.34}_{-1.67}$	$1.24^{+0.60}_{-0.42}$	$4.06^{+2.80}_{-1.58}$	$1.28^{+0.72}_{-0.47}$	$-0.17^{+1.16}_{-1.73}$	$3.18^{+3.13}_{-3.10}$	$1.04^{+0.67}_{-0.62}$	$(1.97, 2.29, 2.55)$	$-0.049^{+0.017}_{-0.025}$	$-0.077^{+0.035}_{-0.044}$
MACSJ1115	$3.01^{+1.05}_{-0.78}$	$1.44^{+0.44}_{-0.38}$	$2.70^{+1.05}_{-0.88}$	$1.36^{+0.45}_{-0.42}$	$1.32^{+1.09}_{-3.06}$	$(0.35, 0.41, 3.14)$	$(0.02, 0.07, 1.44)$	$(2.73, 3.38, 3.42)$	$0.014^{+0.023}_{-0.022}$	$0.008^{+0.032}_{-0.037}$
MACSJ1931	$4.85^{+3.26}_{-1.93}$	$1.21^{+0.79}_{-0.48}$	$3.55^{+1.86}_{-1.45}$	$1.35^{+0.90}_{-0.54}$	$1.49^{+0.81}_{-2.44}$	$(0.11, 3.52, 6.02)$	$1.98^{+0.80}_{-0.80}$	$(2.09, 2.21, 2.80)$	$-0.025^{+0.027}_{-0.020}$	$-0.056^{+0.038}_{-0.037}$
MACSJ1720	$5.08^{+2.02}_{-1.48}$	$1.06^{+0.40}_{-0.31}$	$2.16^{+0.76}_{-0.57}$	$0.78^{+0.43}_{-0.34}$	$2.56^{+0.19}_{-0.16}$	$(0.57, 1.28, 3.77)$	$(0.02, 0.25, 0.82)$	$(2.52, 2.92, 3.01)$	$0.009^{+0.023}_{-0.022}$	$-0.016^{+0.046}_{-0.032}$
MACSJ0416	$3.13^{+0.90}_{-0.73}$	$0.91^{+0.28}_{-0.23}$	$3.19^{+1.54}_{-0.82}$	$0.84^{+0.27}_{-0.26}$	$-1.89^{+1.47}_{-3.43}$	$(0.26, 0.29, 3.03)$	$(0.010, 0.014, 0.850)$	$(3.06, 3.42, 3.44)$	$-0.075^{+0.026}_{-0.022}$	$-0.143^{+0.046}_{-0.037}$
MACSJ0429	$5.77^{+2.75}_{-1.85}$	$0.71^{+0.31}_{-0.23}$	$2.09^{+0.80}_{-0.55}$	$0.50^{+0.36}_{-0.24}$	$2.56^{+0.15}_{-0.21}$	$(0.28, 2.17, 3.82)$	$(0.002, 0.319, 0.752)$	$(2.58, 2.59, 2.96)$	$-0.083^{+0.020}_{-0.025}$	$-0.154^{+0.038}_{-0.046}$
MACSJ1206	$4.77^{+2.01}_{-1.43}$	$1.28^{+0.43}_{-0.34}$	$4.48^{+1.98}_{-1.51}$	$1.27^{+0.43}_{-0.35}$	$0.63^{+1.16}_{-1.23}$	$3.10^{+2.47}_{-2.51}$	$1.08^{+0.49}_{-0.53}$	$(2.26, 2.32, 2.79)$	$0.004^{+0.020}_{-0.022}$	$-0.0004^{+0.0355}_{-0.0402}$
MACSJ0329	$8.53^{+2.71}_{-2.26}$	$0.66^{+0.18}_{-0.15}$	$7.10^{+2.68}_{-2.52}$	$0.62^{+0.20}_{-0.19}$	$1.29^{+0.82}_{-2.30}$	$0.98^{+0.61}_{-0.83}$	$(0.006, 0.035, 0.568)$	$(2.95, 3.04, 3.18)$	$0.009^{+0.020}_{-0.024}$	$0.0004^{+0.0374}_{-0.0455}$
RXJ1347	$3.16^{+1.14}_{-0.89}$	$2.96^{+0.97}_{-0.80}$	$2.83^{+1.19}_{-0.94}$	$2.82^{+0.98}_{-0.92}$	$1.62^{+0.74}_{-0.53}$	$(0.36, 0.87, 3.37)$	$(0.04, 0.66, 1.40)$	$(2.79, 3.16, 3.43)$	$0.008^{+0.029}_{-0.026}$	$0.005^{+0.053}_{-0.045}$
MACSJ1149	$2.57^{+0.97}_{-0.73}$	$1.79^{+0.58}_{-0.49}$	$2.21^{+0.92}_{-0.72}$	$1.73^{+0.62}_{-0.55}$	$0.83^{+1.61}_{-0.52}$	$(0.37, 0.58, 2.76)$	$(0.02, 0.27, 1.41)$	$(2.01, 3.22, 3.23)$	$0.033^{+0.023}_{-0.021}$	$0.042^{+0.048}_{-0.029}$
MACSJ0717	$1.79^{+0.46}_{-0.38}$	$2.54^{+0.63}_{-0.55}$	$1.53^{+0.46}_{-0.52}$	$2.40^{+0.74}_{-0.80}$	$1.09^{+1.69}_{-0.94}$	$(0.17, 0.20, 1.61)$	$(0.02, 0.05, 1.24)$	$3.62^{+0.71}_{-0.82}$	$0.008^{+0.022}_{-0.017}$	$0.002^{+0.037}_{-0.030}$
MACSJ0647	$4.61^{+2.26}_{-1.54}$	$1.21^{+0.47}_{-0.37}$	$3.94^{+1.91}_{-1.49}$	$1.15^{+0.45}_{-0.36}$	$1.50^{+0.75}_{-0.82}$	$2.86^{+2.57}_{-2.58}$	$1.00^{+0.55}_{-0.56}$	> 2.46	$0.041^{+0.023}_{-0.020}$	$0.046^{+0.038}_{-0.034}$
MACSJ0744	$4.58^{+2.09}_{-1.41}$	$1.31^{+0.45}_{-0.36}$	$3.84^{+1.19}_{-1.42}$	$1.45^{+0.45}_{-0.41}$	$1.18^{+0.73}_{-0.87}$	$(0.16, 0.22, 4.15)$	$(0.002, 0.005, 1.267)$	$(2.97, 3.54, 3.62)$	$-0.002^{+0.021}_{-0.023}$	$-0.003^{+0.032}_{-0.048}$

Summary & Conclusion

- Disformally NMC Dark matter
- Modified Poisson equation $\Rightarrow$ depend to gradiant of the density
- CLASH $\Rightarrow$ NFW + Modified β-model
- Volume effects $\Rightarrow$ Narrow minimum $\Rightarrow$ Marg. + PD

- Marginal analysis results $\Rightarrow$ more close to GR
- L $\Rightarrow$ 0.1 - 10^2 kpc $\Rightarrow$ average value $\sim$ 10 kpc

- PD: c_{200} , M_{200} $<$ GR
- Marginalised to PD $\Rightarrow$ $L \ll r_s$ to $L \sim r_s$

Thank you!

Theoretical Background: NMC - Newtonian Limit

Modified Poisson equation:

$$\nabla^2\Phi = 4\pi\, G_N \left[\rho - \epsilon\, L^2\, \nabla^2 \rho_{\mathrm{DM}}\right] \tag{8}$$

$$\nabla^2\Psi_{ij} = 4\pi\, G\eta_{ij} \left[\rho - \epsilon\, L^2\, \nabla^2 \rho_{\mathrm{DM}}\right] \tag{9}$$

- $\Phi = \Psi \Rightarrow$ no anisotropic stress
- Density $\rho = \rho_{\mathrm{DM}} + \rho_{gas}$
- Polarity $\Rightarrow \epsilon = -1$
- characteristic Length of NMC model $\Rightarrow L$

Dark Matter

- Fritz Zwicky $\Rightarrow$ 1930s - Dark Matter
- 95% of Universe
  $\Rightarrow$ DE: $\sim 68\%$
  $\Rightarrow$ DM: $\sim 27\%$

Statistics: The Bayesian Approach

- Evidence: Highly prior dependent (Nesseris & Bellido 2013, 1210.7652)
- Kullback-Leibler divergence (KL) (Kullback & Leibler 1951)

$$\mathcal{D}_{KL,i} = \int d\boldsymbol{\theta}\, \frac{\mathcal{L}(d|\boldsymbol{\theta}, \mathcal{M}_i)}{\mathcal{E}(d|\mathcal{M}_i)} \log \frac{\mathcal{L}(d|\boldsymbol{\theta}, \mathcal{M}_i)}{\mathcal{E}(d|\mathcal{M}_i)}$$

$\mathcal{D}_{KL,i}$: prior-dependent as $\mathcal{B}_{ij}$

- Suspiciousness (Handley & Lemos 2019, 1903.06682, Handley & Lemos 2019, 1902.04029, Joackimi et al. 2021, 2102.09547)

$$\log \mathcal{S}_{ij} = \log \mathcal{B}_{ij} + \mathcal{D}_{KL,i} - \mathcal{D}_{KL,j}$$

$\log \mathcal{S}_{ij}$ prior-independent

$\log \mathcal{S}_{ij} < 0$	Tension
$\log \mathcal{S}_{ij} > 0$	Consistency