Gravitational wave resonance in ultralight dark matter halos

Based on *Phys.Rev.D* 108 (2023) 12, 123539

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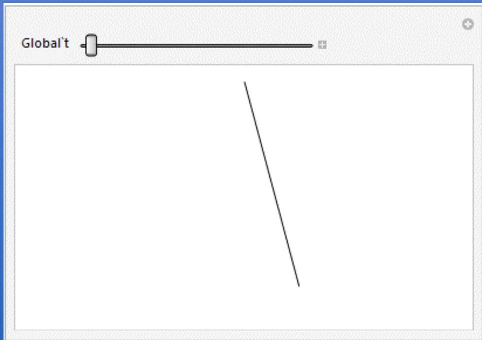
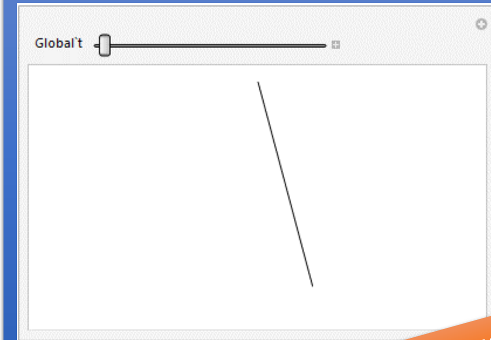
Image: geralt (pixabay.com)

Parametric resonance

$\ddot{x}(t) + Ax(t) = 0$



$\ddot{x}(t) + Ax(t) - 2q\cos(2t)\,x(t) = 0$



Mathieu equation

Why does this exponential instability take place?

$$\ddot{x}(t) + Ax(t) - 2q\cos(2t)\,x(t) = 0$$

$$\pi \equiv \dot{x} \qquad X \equiv (x, \pi)^T \qquad \dot{X} = UX$$

$$U \equiv \begin{pmatrix} 0 & 1 \\ -A + 2q\cos(2t) & 0 \end{pmatrix}$$

Fundamental matrix of solutions: $O(t, t_0)$

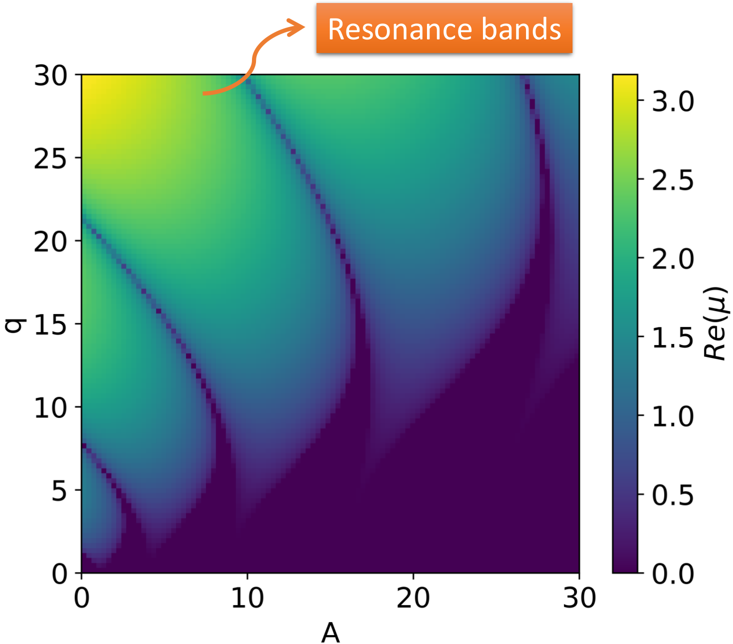
Solve $\dot{O}(t, t_0) = UO(t, t_0)$ from t_0 to $t_0 + T$
$O(t_0, t_0) = I$

Eigenvalues $o^{\pm}$ $\rightarrow$

$$Re(\mu^{\pm}) = \frac{1}{T}\ln|o^{\pm}|$$

$$x(t) \propto \exp(\mu t)$$

Image: Freepik



Bands centered around:

A=1

A=4

A=9

...

$$\mu \propto \begin{cases} q \; if \; A \subset (1-q, 1+q) \\ q^2 if \; A \subset (4-q^2, 4+q^2) \end{cases}$$

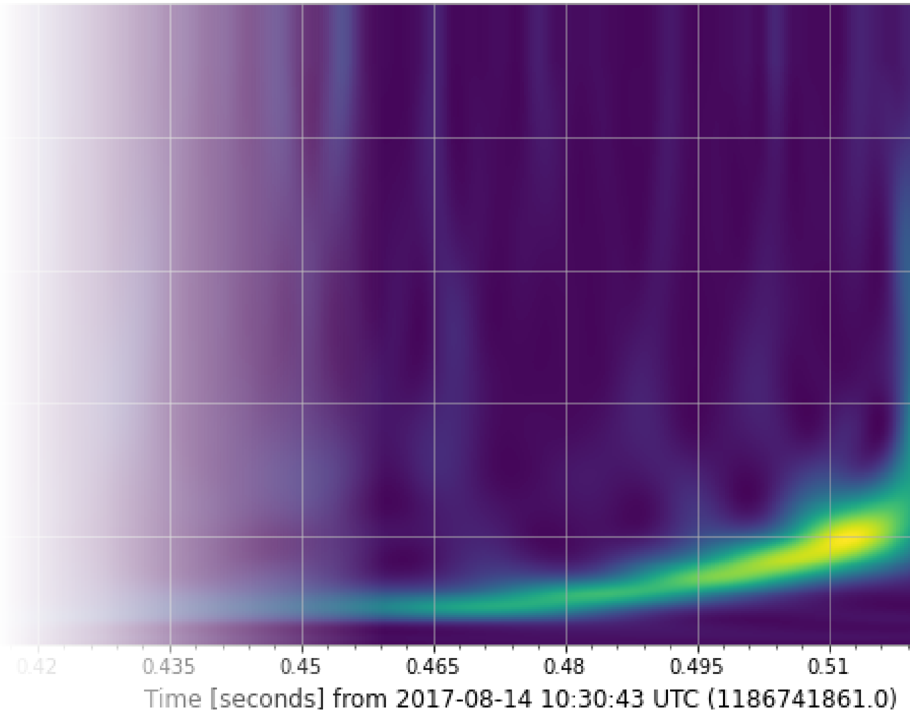
q ≪ 1: narrow band resonance

$$x(t) \propto \exp(\mu t)$$

In light of GW physics,

- Non-linear order: **interactions** between cosmological perturbations might lead to resonance.
- GWs are damped via resonance with photons: Phys.Dark Univ. 40 (2023) 101202, R. Brandenberger, PCMD, A. Ganz, C. Lin.

Are there scenarios where gravitational waves are amplified via parametric resonance?



ULDM halo

Why Ultra-Light Axions (ULAs) as dark matter?

- Incompatibilities between the CDM description and the observed data on sub-galactic scales.

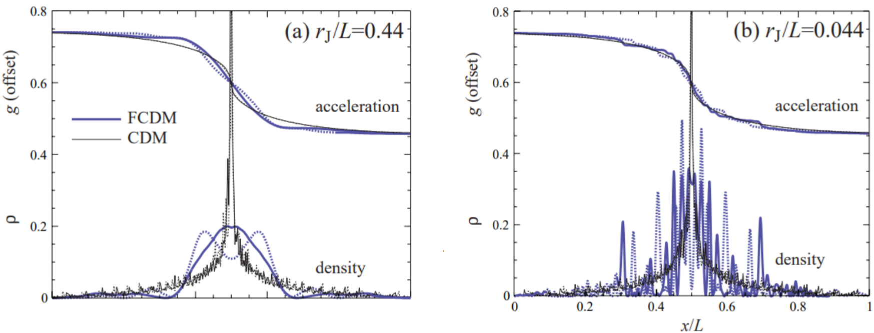


Figure from arXiv:astro-ph/0003365, Wayne Hu, Rennan Barkana, Andrei Gruzinov.

The halo description (ground state):

$$ds^2 = -(1+2U)dt^2 + (1-2\bar{U})(dx^2+dy^2+dz^2) \qquad U, \bar{U} \ll 1$$

$$\phi(t) = \phi_0 \cos(mt)$$

$$\rho = \frac{1}{2}m^2\phi_0^2,$$
$$p = -\rho\cos(2mt)$$

→

$$\begin{aligned} T &= T_0 + \delta T \\ U &= U_0 + \delta U \\ \bar{U} &= \bar{U}_0 + \delta\bar{U} \\ R &= R_0 + \delta R \end{aligned}$$

$$R = -6\ddot{\bar{U}} + 2\nabla^2(2\bar{U} - U)$$

Einstein equations →

$$\bar{U}_0 = U_0$$

$$2\nabla^2 U_0 = \rho \rightarrow U_0 \propto \frac{\rho}{k_a^2}$$

$$\delta T = 6\delta\ddot{\bar{U}} \rightarrow \delta\bar{U} = \frac{\rho}{8m^2}\cos(2mt), \quad \delta U = -\delta\bar{U}$$

Oscillating gravitational potentials 👀

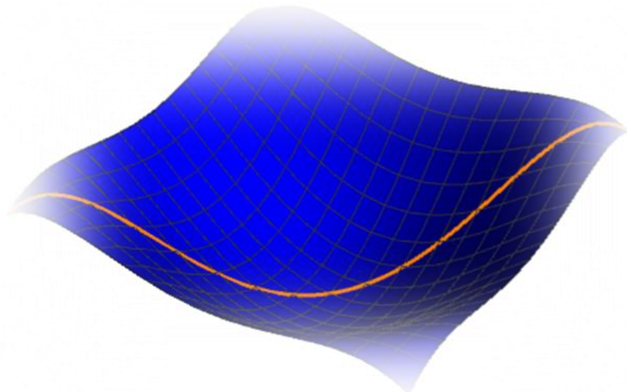
Gravitational Wave and ULDM halo interaction

Gravity bends gravity

From gravitational wave lensing:

$$h_{\mu\nu} = h\epsilon_{\mu\nu}$$

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu h) = 0$$



In the context of ULDM:

$g^{\mu\nu}, g$ → includes oscillating gravitational potentials

h → gravitational wave as the oscillator

Interaction that might lead to a Mathieu equation!

Expanding the equation of motion,

$$\ddot{h} - (1 + 2U + 2\bar{U})\nabla^2 h - \dot{U}\dot{h} - 3\dot{\bar{U}}\dot{h} +$$
$$+\partial_i h \partial_i \bar{U} - \partial_i h \partial_i U = 0$$

$$\tau \equiv mt$$
$$\bar{h} \equiv \exp(\delta U) h$$

To kill friction terms

$$\bar{h}_k'' + \frac{k^2}{m^2}\bar{h}_k - \frac{4}{m^2}\int d^3x \exp(-i\vec{k}\cdot\vec{x}) U_0 \nabla^2 \bar{h} +$$
$$- \frac{1}{2}\frac{\rho}{m^2}\cos(2\tau)\bar{h}_k = 0$$

$$A\bar{h}_k \equiv \frac{k^2}{m^2}\bar{h}_k - \frac{4}{m^2}\int d^3x \exp(-i\vec{k}\cdot\vec{x}) U_0 \nabla^2 \bar{h}$$
$$\simeq \frac{k^2}{m^2}\bar{h}_k$$

$$(\rho/k_a^2)(k^2/m^2)\bar{h}_k$$

$$\bar{h}_k'' + A\bar{h}_k - 2q\cos(2\tau)\bar{h}_k = 0$$

$$q \equiv \rho/m^2/4 \quad \ll 1$$

$$A\bar{h}_k \equiv \frac{k^2}{m^2}\bar{h}_k - \frac{4}{m^2}\int d^3x \exp(-i\vec{k}\cdot\vec{x})U_0\nabla^2\bar{h}$$

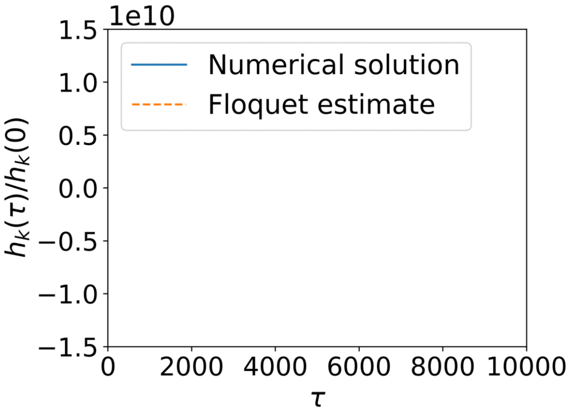
$$\simeq \frac{k^2}{m^2}\bar{h}_k$$

$$k^2 = m^2$$

Floquet instability theory:

$$h_k \simeq \bar{h}_k \propto \exp(q\tau/2)$$

$$\exp(\delta U) \simeq 1$$



$$m = 10^{-22}\text{eV}$$

$$\rho = 10^{16} \times 0.4\text{GeV/cm}^3$$

Parametric resonance

Amplification estimates

$\rho = f\rho_{DM}$ $f = 1$ → ULAs constitute the totality of dark matter

density in the solar region ←

$$\rho = 0.4\text{GeV}/\text{cm}^3$$
$$m \simeq 10^{-22}\text{eV}$$
→ 3.9×10^{17} years

time estimated via Floquet theory to achieve O(1) amplification

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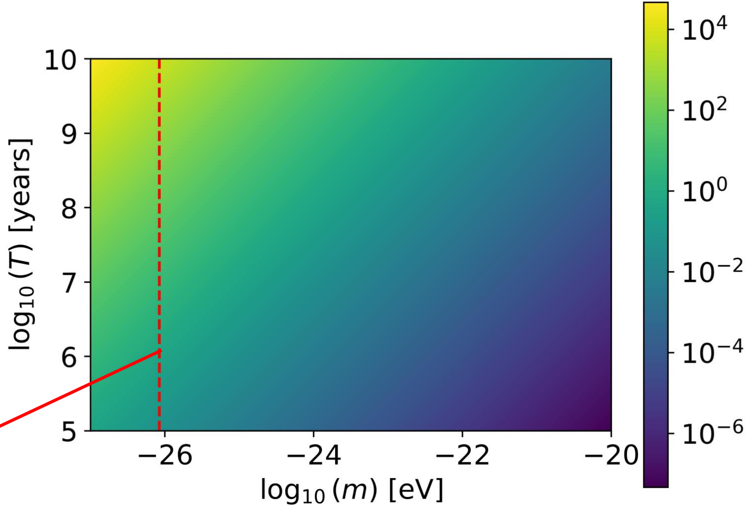
time estimated via Floquet theory to achieve O(1) amplification

→ **Higher densities** are required to reduce the time scale, e.g.

- arXiv:2212.05664 [astro-ph.HE], Man Ho Chan, Chak Man Lee.
- arXiv:2103.12439 [astro-ph.HE], Sourabh Nampalliwar, Saurabh K., Kimet Jusufi, Qiang Wu, Mubasher Jamil, Paolo Salucci.

In **dense regions** the amplifications might reach very high values: $\rho \simeq 1.4 \times 10^7 GeV/cm^3$

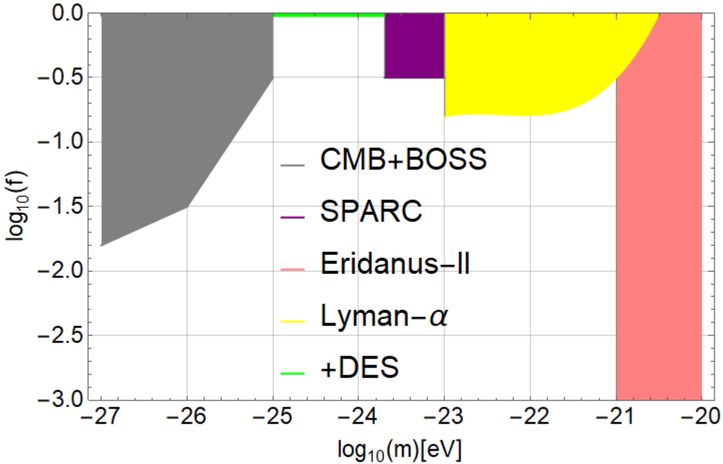


$\rho/m^2 \simeq 0.01$

$h_k \simeq \bar{h}_k \propto \exp{(q\tau/2)}$

PTA frequency range

What about the constraints already imposed to the ULA fraction as dark matter?

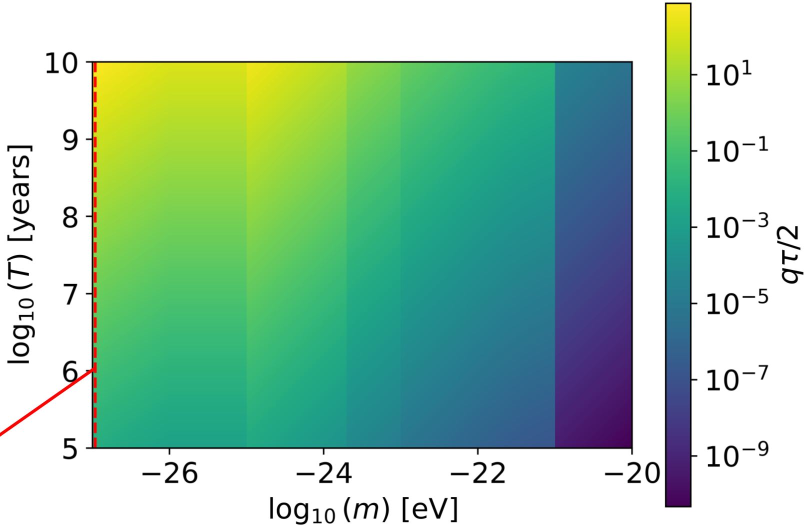


- **CMB+BOSS**: Planck and LSS bounds from galaxy clustering.
- **SPARC**: bounds from galaxy rotation curves.
- **Eridanus-II**: bounds from Ultrafaint Dwarf Galaxy Eridanus II.
- **Lyman-α**: bounds from Lyman-α forest.
- **+DES**: bounds from galaxy weak lensing and Planck.

Implicitly considered:

- bounds from the UV luminosity function and optical depth to reionization.
- bounds from the M87 black hole spin.

Considering **constraints:** $\rho = f\rho_{DM}$ $\rho \simeq 1.4 \times 10^7 GeV/cm^3$



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PTA frequency range

Summary and future work

- Gravitational waves are **amplified** via parametric resonance with the oscillating gravitational potentials of ULDM halos.
- Significant amplifications nowadays can be achieved in **dense regions** in the halo.
- Possible **GW sources**: primordial perturbations and supermassive black hole binaries (10^{-8}Hz to 10^{-13}Hz).
- Upper bound on the amplifications (**$h \ll 1$**).

- Can **modified gravity** or the **GW background** boost the resonance?
- Can we **detect** the resonant amplification (e.g. PTA) to test ULDM?

Summary and future work

- Gravitational waves are **amplified** via parametric resonance with the oscillating gravitational potentials of ULDM halos.
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Thank you for your attention!

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Gravitational wave and gauge field interaction

Set-up: gravitational wave in a medium with subluminal speed of light interacting with an electromagnetic field.

$$g_{\mu\nu} = \tilde{\eta}_{\mu\nu} + h_{\mu\nu}$$

$$\tilde{\eta}_{\mu\nu} = (-1, 1/c_s^2, 1/c_s^2, 1/c_s^2)$$

$$\begin{aligned} 0 &= g_{i\alpha}\partial_\mu \left(F_{\rho\sigma} g^{\alpha\rho} g^{\mu\sigma}\right) \\ &= \partial_t^2 A_i - c_s^2 \partial_t h_{ij} \cdot \partial_t A_j + c_s^2 \partial_j F_{ij} \\ &\quad - c_s^4 h_{jk} \partial_j F_{ik} - c_s^4 F_{kj} \partial_j h_{ik} \end{aligned}$$

$$h_{ij} = h_0 \cos\omega t \cos\omega z \cdot \epsilon_{ij}$$

$$\begin{aligned}
\mathcal{A}_x (t, k_x, k_y, k_z) &\equiv A_x (t, k_x, k_y, k_z) + A_x (t, k_x, k_y, -k_z) \\
\mathcal{A}_y (t, k_x, k_y, k_z) &\equiv A_y (t, k_x, k_y, k_z) + A_y (t, k_x, k_y, -k_z) \\
\mathcal{A}_z (t, k_x, k_y, k_z) &\equiv A_z (t, k_x, k_y, k_z) - A_z (t, k_x, k_y, -k_z)
\end{aligned}$$

$$k_z = \omega/2$$

$$\boxed{\ddot{\mathcal{Y}} + c_s^2 \mathcal{G}\mathcal{Y} + c_s^2 \mathcal{F}\dot{\mathcal{Y}} + c_s^4 \mathcal{M}\mathcal{Y} \simeq 0}$$

$$\mathcal{Y} = \begin{pmatrix} \mathcal{A}_x \\ \mathcal{A}_y \\ \mathcal{A}_z \end{pmatrix}$$

$$\mathcal{G} = \begin{pmatrix} k_y^2 + \frac{\omega^2}{4} & -k_x k_y & -k_x k_z \\ -k_x k_y & k_x^2 + \frac{\omega^2}{4} & -k_y k_z \\ -k_x k_z & -k_y k_z & k_x^2 + k_y^2 \end{pmatrix}$$

→ gradient matrix

$$\mathcal{F} = \frac{1}{2} h_0 \omega \sin\omega t \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

→ friction matrix

oscillating mass matrix →

$$\mathcal{M} = \frac{1}{4} h_0 \cos\omega t \begin{pmatrix} -2k_x k_y + 2k_y^2 + \omega^2 & 2k_x^2 - 2k_x k_y + \omega^2 & -2\omega (k_x + k_y) \\ 2k_x k_y + 2k_y^2 + \omega^2 & -2k_x^2 - 2k_x k_y - \omega^2 & -2\omega (k_x - k_y) \\ -(k_x + k_y)\omega & (k_y - k_x)\omega & 2k_\epsilon^2 \end{pmatrix}$$

$$\mathcal{S} = \begin{pmatrix} \frac{k_x}{k_z} & \frac{k_y}{k_z} & 1 \\ -\frac{k_z}{k_x} & 0 & 1 \\ -\frac{k_y}{k_x} & 1 & 0 \end{pmatrix} \qquad \mathcal{S}\mathcal{Y} \equiv \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

← to get the 2 dynamical dof

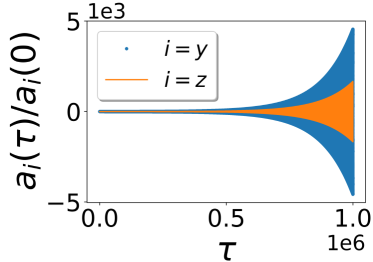
$$\mathcal{S}\left(\ddot{\mathcal{Y}} + c_s^2 \mathcal{G}\mathcal{Y} + c_s^2 \mathcal{F}\dot{\mathcal{Y}} + c_s^4 \mathcal{M}\mathcal{Y}\right) = 0$$

$$\boxed{y'' + c_s^2 \tilde{\mathcal{F}} y' + c_s^2 \tilde{k}^2 y + c_s^4 \tilde{\mathcal{M}} y = 0}$$

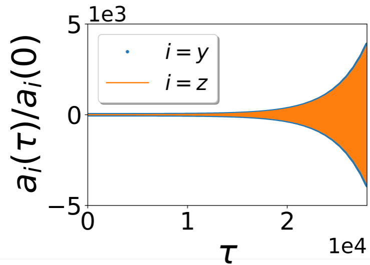
$$y = (a_y, a_z)^T \qquad \tau \equiv \frac{\omega t}{2} \qquad \tilde{k}^2 \equiv 4k^2/\omega^2$$

$$\tilde{\mathcal{F}} = h_0 \sin 2\tau \begin{pmatrix} \frac{\omega^2(k_x+k_y)}{4k_x k^2} & -\frac{\omega(k_x^2 - k_x k_y + k_z^2)}{2k_x k^2} \\ \frac{\omega(-k_x^2 + 2k_x k_y + k_y^2)}{2k_x k^2} & \frac{-4k_x k_\epsilon^2 - \omega^2(k_x+k_y)}{4k_x k^2} \end{pmatrix}$$

$$\tilde{\mathcal{M}} = h_0 \cos 2\tau \begin{pmatrix} \frac{2k_\epsilon^2}{\omega^2} + 1 + \frac{k_y}{k_x} & \frac{-2k_x + 2k_y}{\omega} - \frac{\omega}{2k_x} \\ \frac{2k_y^2}{\omega k_x} - \frac{2k_x - 4k_y}{\omega} & \frac{-2\epsilon_{ij}k_i k_j}{\omega^2} - 1 - \frac{k_y}{k_x} \end{pmatrix}$$



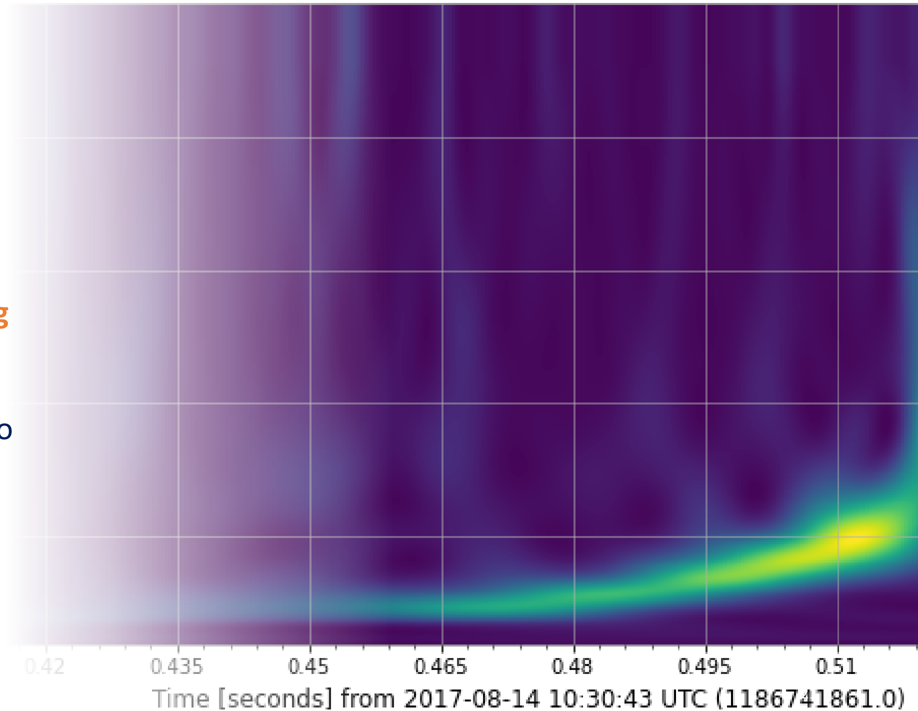
$c_s = 1$



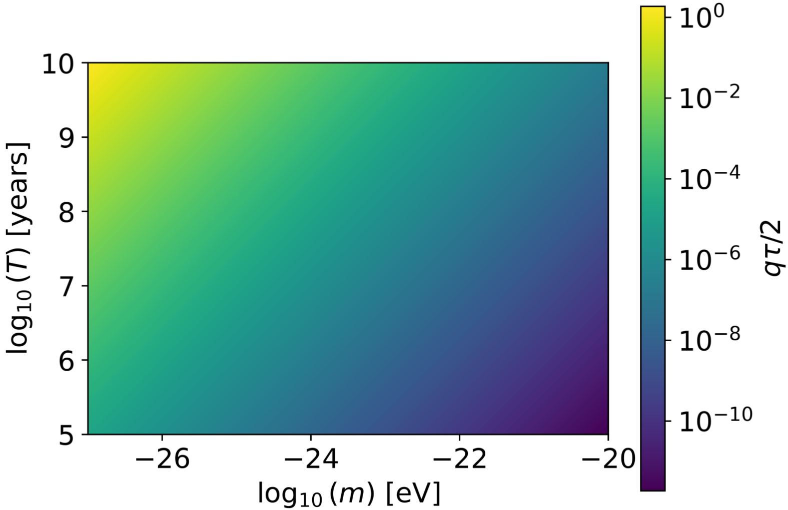
$c_s = 1/1.333$

Therefore,

- The GW is converted to photons (**damping**).
- The extension to **many GW modes** boosts the resonance.
- Challenges: **high refractive index** and **long time intervals**.
- Can we use this conversion mechanism to **detect GWs**?



Less dense regions are possible for the smaller masses: $\rho \simeq 5.9 \times 10^2 GeV/cm^3$



$h_k \simeq \bar{h}_k \propto \exp(q\tau/2)$

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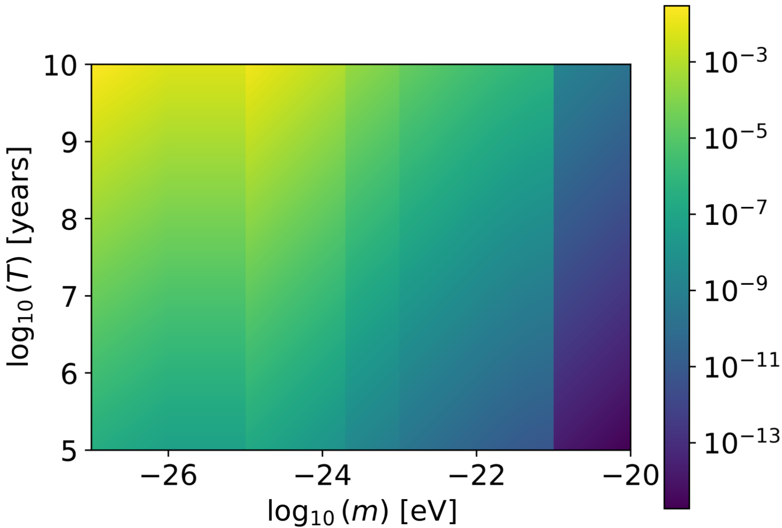
Considering **constraints:** $\rho = f\rho_{DM}$ $\rho \simeq 5.9 \times 10^2 GeV/cm^3$



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PTA frequency range