Equation of State of Strongly Interacting Matter for Hydrodynamical Simulations of Heavy-Ion Collisions

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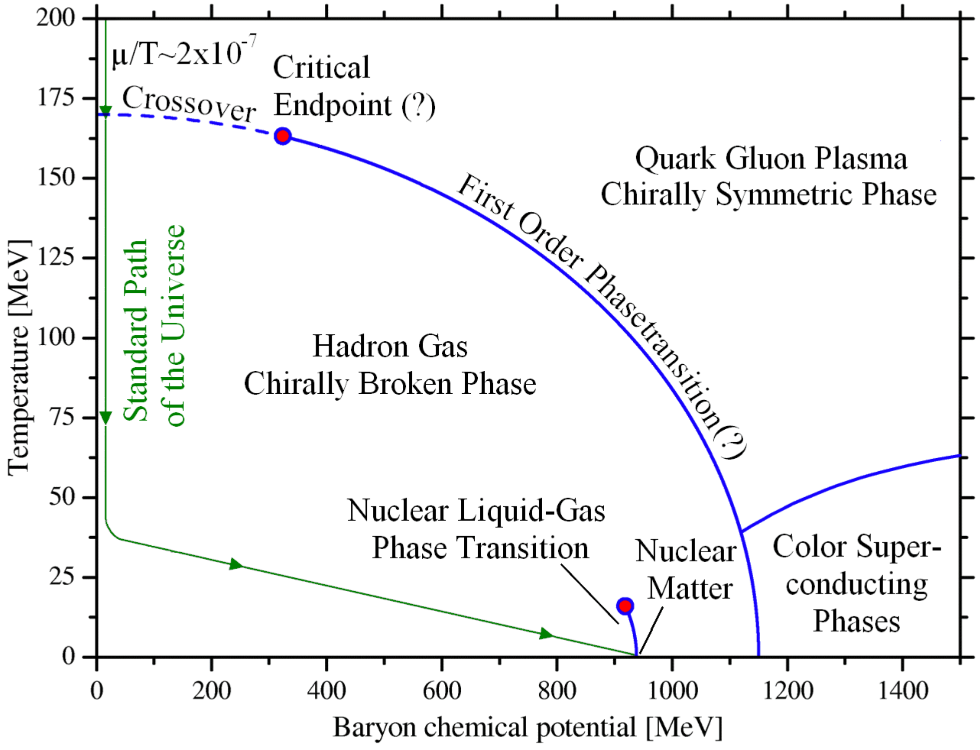
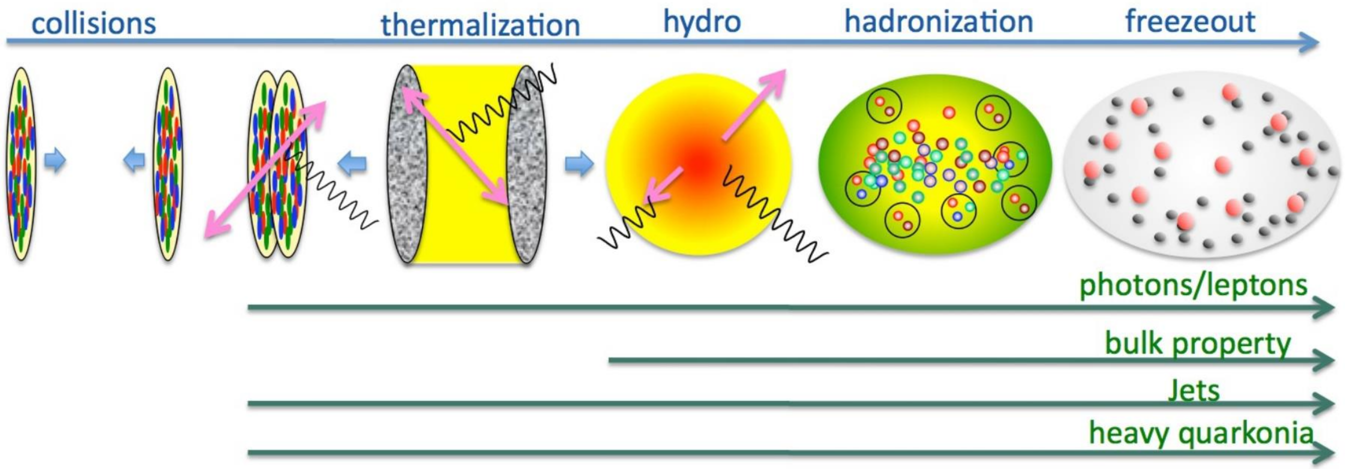
Motivation



The main goal of heavy-ion collisions experiments is to gain a better understanding of the theory of strong interactions – QCD, by detecting critical phenomena.

Exploring of the QCD phase diagram:

- Detect signals of deconfinement PT
- Detect signals of (partial) chiral symmetry restoration
- Locate (tri)critical endpoint(s) if such exists

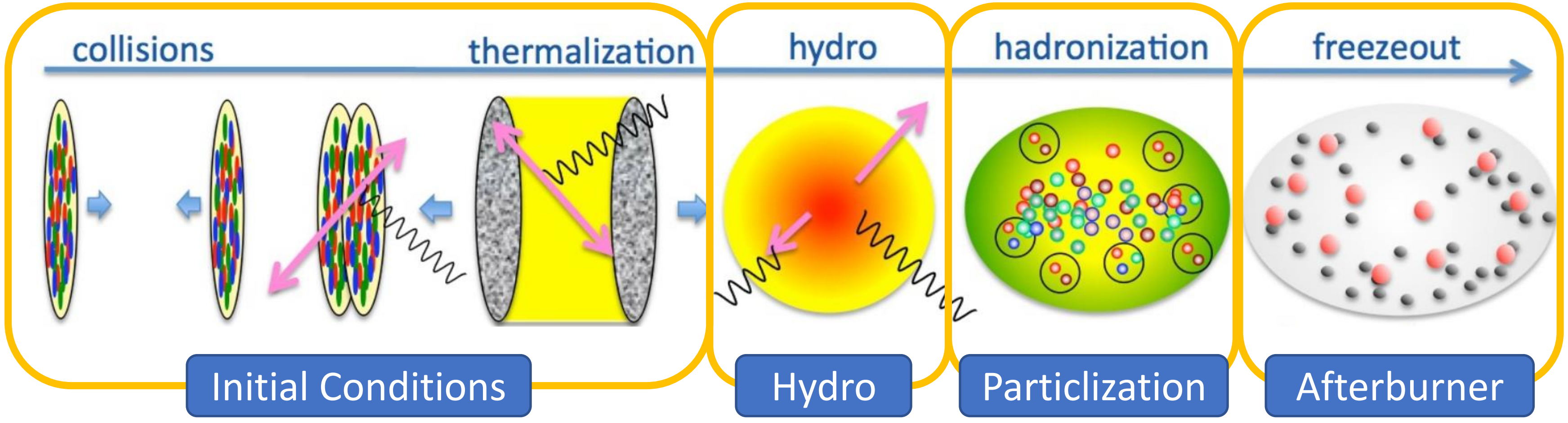


We need good models and tools for simulations, comparison with experimental data and analysis!

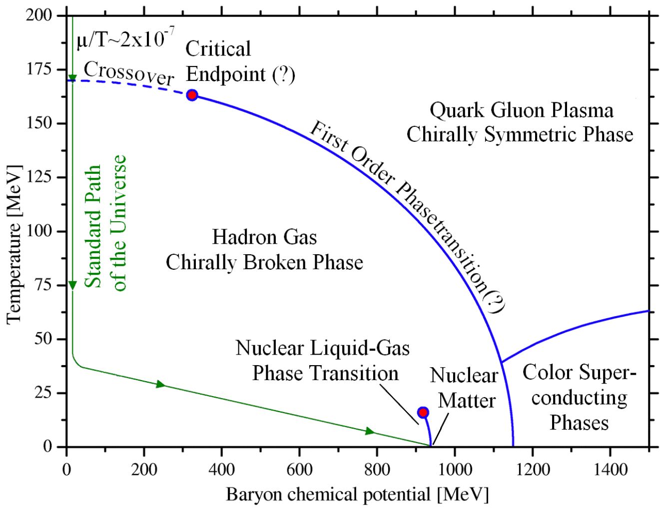
[Figures: Universe 4 (2018) 52 & PoS (KMI 2013) 025]

Motivation

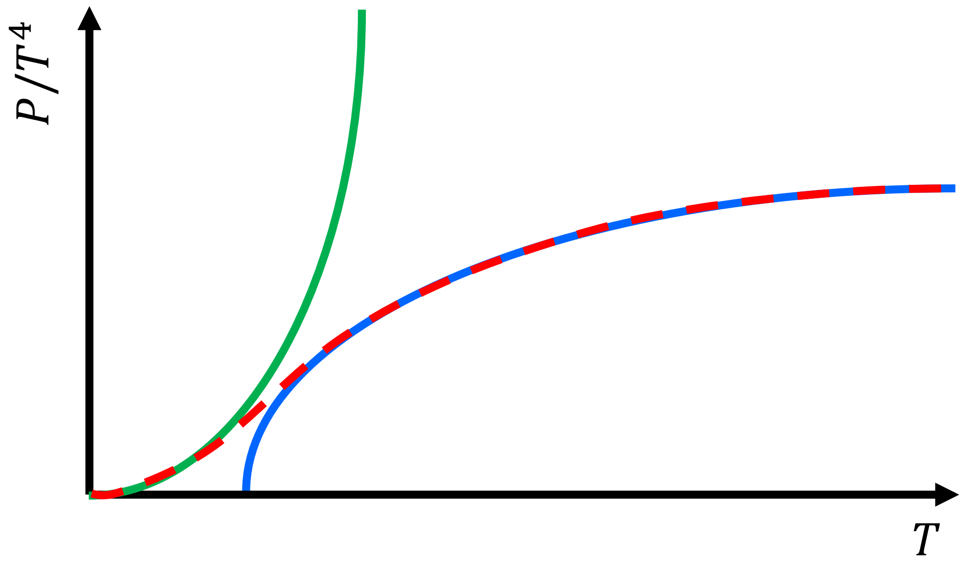
Physical Input into Hydro
- Initial state
- Equation of State
- Transport coefficients
- Freezeout



EoS
- Is needed to build closed system of equations
- Contains all the information about the thermodynamic properties of the system
- Modelling heavy-ion collisions with different EoS, one can perform a-posteriori analyses of the QCD matter properties in the unsolvable regions of the phase diagram



EoS Construction: Idea



Goal: construct the model that effectively describes the transition from hadronic matter to quark-gluon plasma

To achieve this, we use the switching function approach [PRC 90, 024915 (2014)]

$$P = f P_{HRG} + (1-f) P_{pQCD}$$

$$0 \le f \le 1, \lim_{T\to 0} f = 1, \lim_{T\to\infty} f = 0$$

HRG – constant part

$$P_{HRG} = \sum_i p_i$$

$$p_i = g_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_i} \frac{1}{e^{(E_i-\mu)/T} \pm 1}$$

Perturbative QCD

$$P_{pQCD} = \sum_{i=0}^{6} g^i F_i(T,\mu)$$

$$g = \sqrt{4\pi\alpha_s}$$

Switching function

$$f = \left[e^{(T-F(T,\mu))/\Delta T} + 1\right]^{-1}$$

$$F(T,\mu_B) = ?$$

EoS Construction: pQCD EoS

pQCD EoS known up to the $O(g^6 \ln g)$ [PRD 68, 054017 (2003)]

$$P_{pQCD} = \sum_{i=0}^{6} g^i F_i(T,\mu)$$

g^6 contribution is a free parameter [PRD 67, 105008 (2003)]

$$P_{pQCD} = \sum_{i=0}^{6} g^i F_i(T,\mu) \Rightarrow P_{pQCD} = \sum_{i=0}^{6} g^i F_i(T,\mu) + g^6 C_6$$

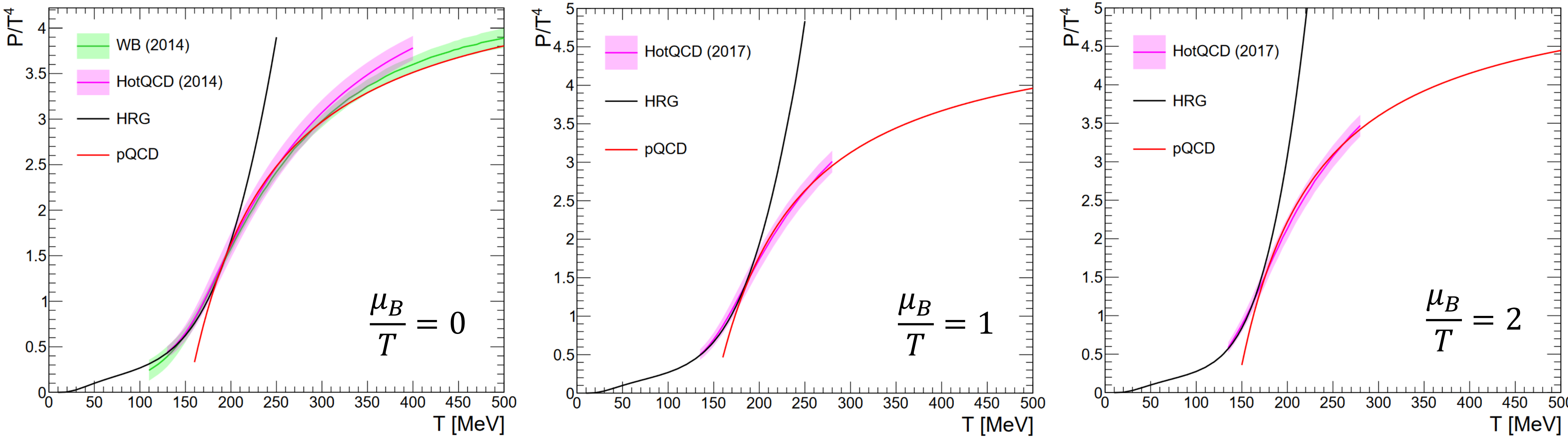
Fastest apparent convergence renormalization scale:

$$\Lambda = 1.8688 \times C_\Lambda \pi T e^{-\frac{1}{27}\sum_f \aleph(\hat{\mu}_f)}, \qquad \hat{\mu}_f = \frac{\mu_f}{2\pi T}, \qquad \aleph(\hat{\mu}_f) = \Psi\left(\frac{1}{2} + i\hat{\mu}_f\right) + \Psi\left(\frac{1}{2} - i\hat{\mu}_f\right)$$

Three-loop running coupling: $t = \ln \Lambda^2/\Lambda_{MS}^2$

$$\alpha_s = \frac{1}{b_0 t}\left(1 - \frac{b_1 \ln t}{b_0^2 t} - \frac{b_1^2(\ln^2 t - \ln t - 1) + b_0 b_2}{b_0^4 t^2} - \frac{b_1^3(\ln^3 t - 2.5\ln^2 t - 2\ln t + 0.5) + 3 b_0 b_1 b_2 \ln t}{b_0^6 t^3}\right)$$

EoS Construction

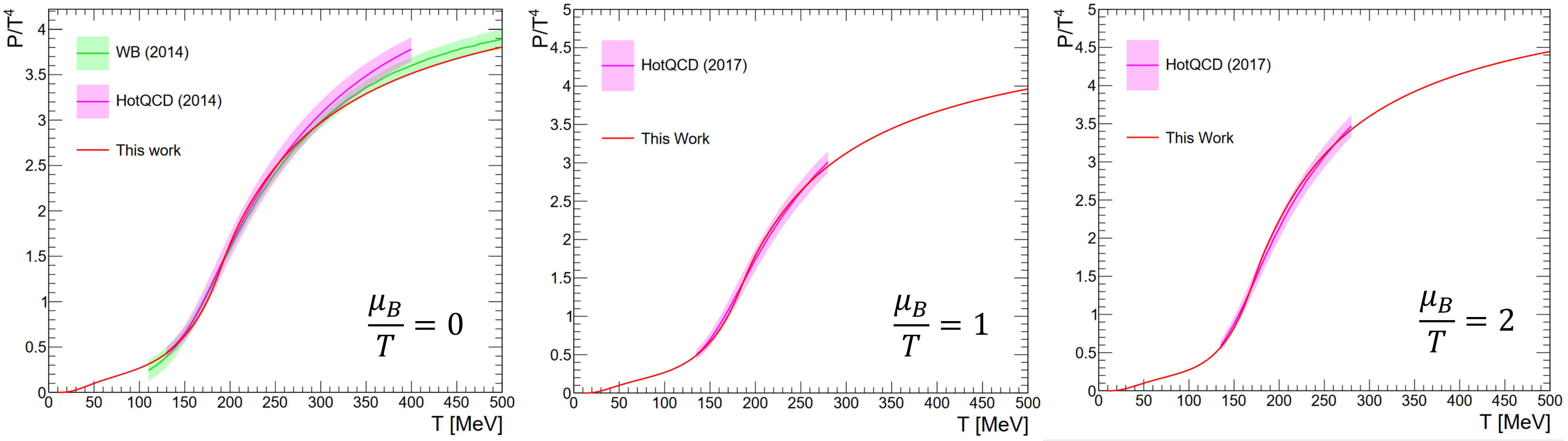


pQCD and HRG EoS are very close to each other over the wide range of chemical potentials

➡ Extract “distance of closest approach” as the function of μ_B ➡

$$F(T, \mu_B) = T_0 - a^2\mu_B^2 - b^4\mu_B^4$$
$$T_0 = 190.7\ \text{MeV}, a = 0.00115\ \text{MeV}^{-1/2},$$
$$b = 0.000256\ \text{MeV}^{-3/4}, \Delta T = 7.5\ \text{MeV}$$

Results



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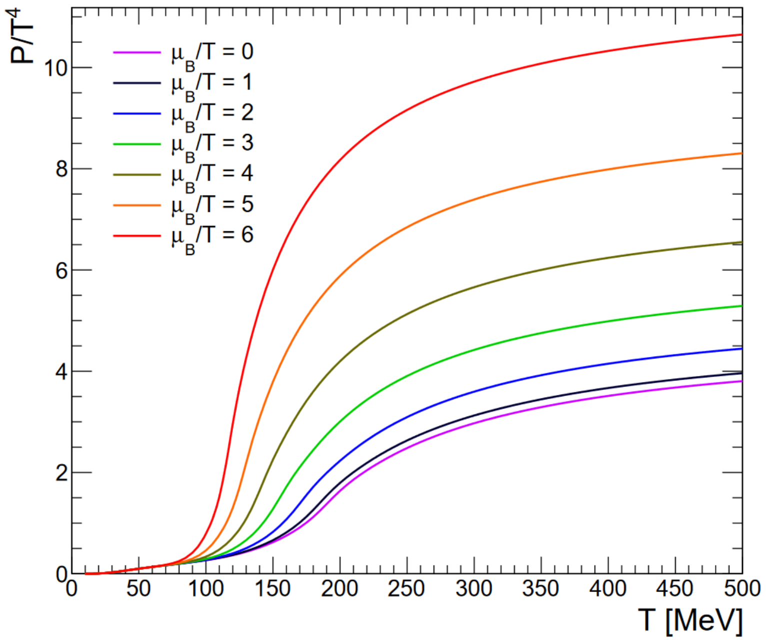
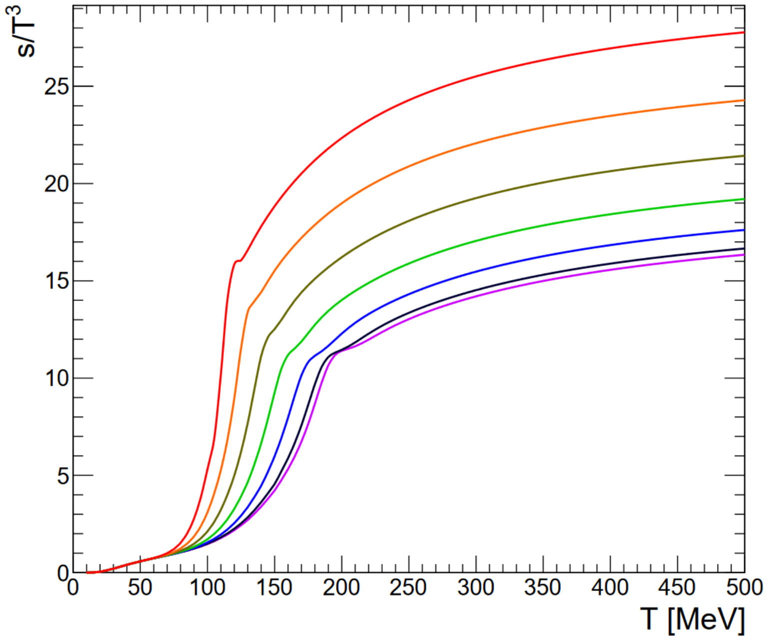
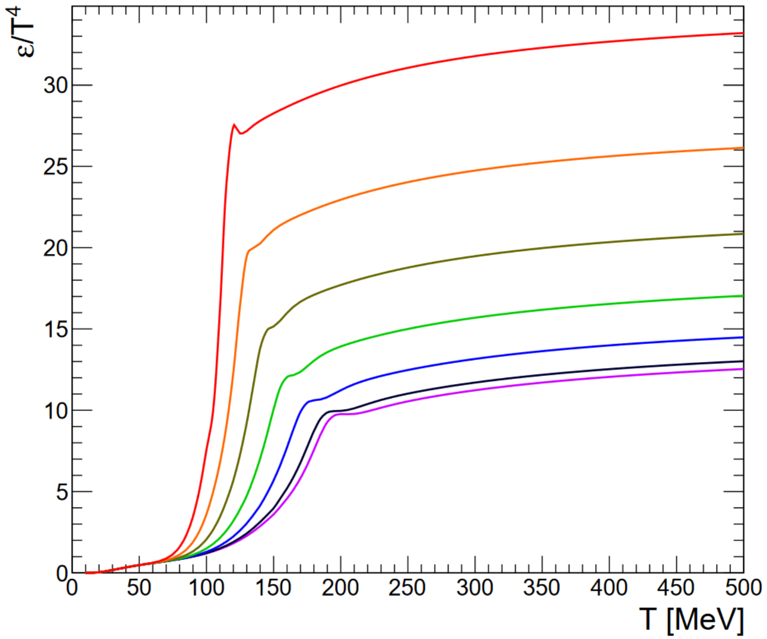


P/T^4
μ_B/T = 0
μ_B/T = 1
μ_B/T = 2
μ_B/T = 3
μ_B/T = 4
μ_B/T = 5
μ_B/T = 6
0 2 4 6 8 10
0 50 100 150 200 250 300 350 400 450 500
T [MeV]



s/T^3
0 5 10 15 20 25
0 50 100 150 200 250 300 350 400 450 500
T [MeV]



ε/T^4
0 5 10 15 20 25 30
0 50 100 150 200 250 300 350 400 450 500
T [MeV]

EoS Construction: TZIS

$$P = \begin{cases} \tilde{P}_h, & \mu_h < \mu_B < \mu_c \\ \tilde{P}_q, & \mu_c < \mu_B < \mu_q \\ P_{Smooth}, & \text{otherwise} \end{cases}$$

Taylor expansion:

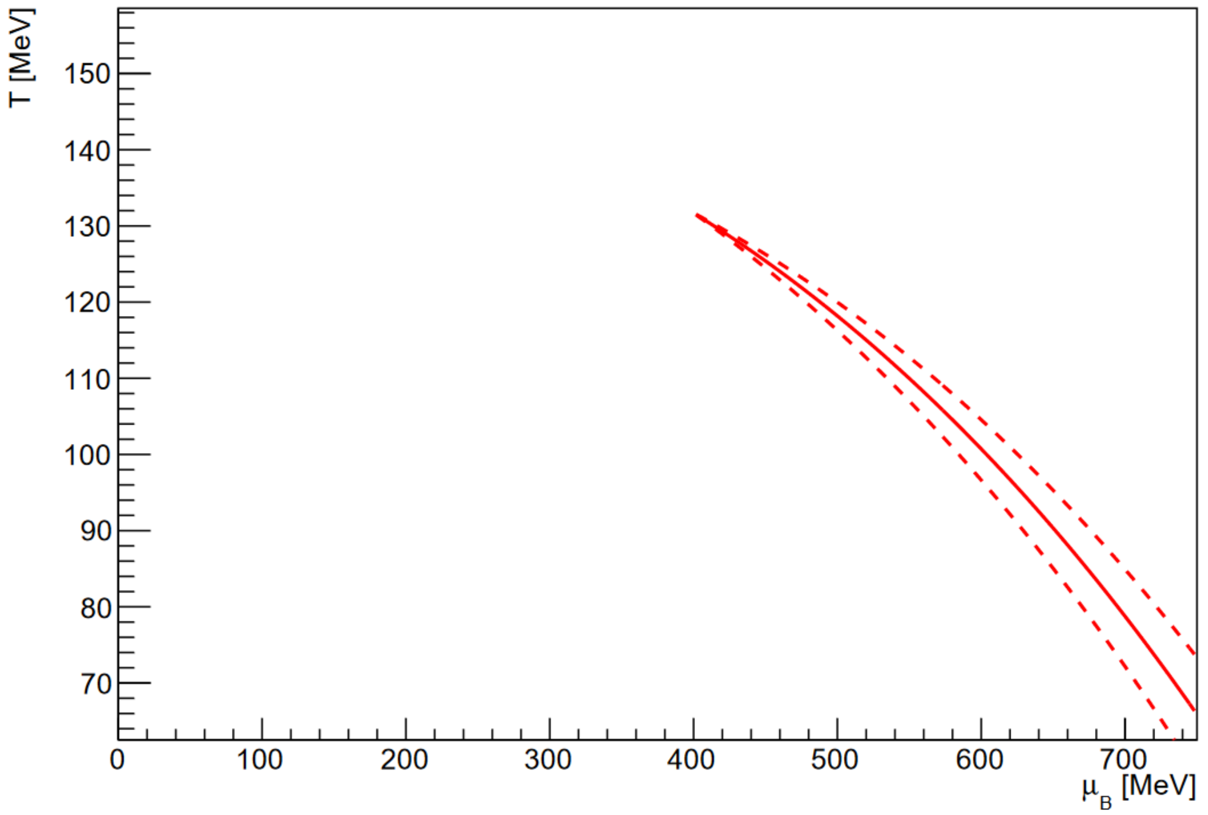
$$\tilde{P}_i \approx a_i + (\mu_B - \mu_i)b_i + (\mu_B - \mu_i)^2 c_i,$$

$$\tilde{n}_i \approx b_i + 2(\mu_B - \mu_i)c_i, \qquad i = h, q$$

Boundary conditions:

$$\tilde{P}_i(\mu_i) = P_i(\mu_i) \Rightarrow a_i = P_i(\mu_i) = P_i$$

$$\tilde{n}_i(\mu_i) = n_i(\mu_i) \Rightarrow b_i = n_i(\mu_i) = n_i$$



1st order phase transition:

$$\tilde{P}_h(\mu_c) = \tilde{P}_q(\mu_c) \Rightarrow (\mu_c - \mu_h)^2 c_h - (\mu_c - \mu_q)^2 c_q = P_q - P_h + (\mu_c - \mu_q)n_q - (\mu_c - \mu_h)n_h$$

$$\tilde{n}_h(\mu_c) = \tilde{n}_q(\mu_c) - \Delta n \Rightarrow 2(\mu_c - \mu_h)c_h - 2(\mu_c - \mu_q)c_q = n_q - n_h - \Delta n \qquad \Delta n = n_0 \left|\frac{T - T_c}{T_c}\right|^{0.3265}$$

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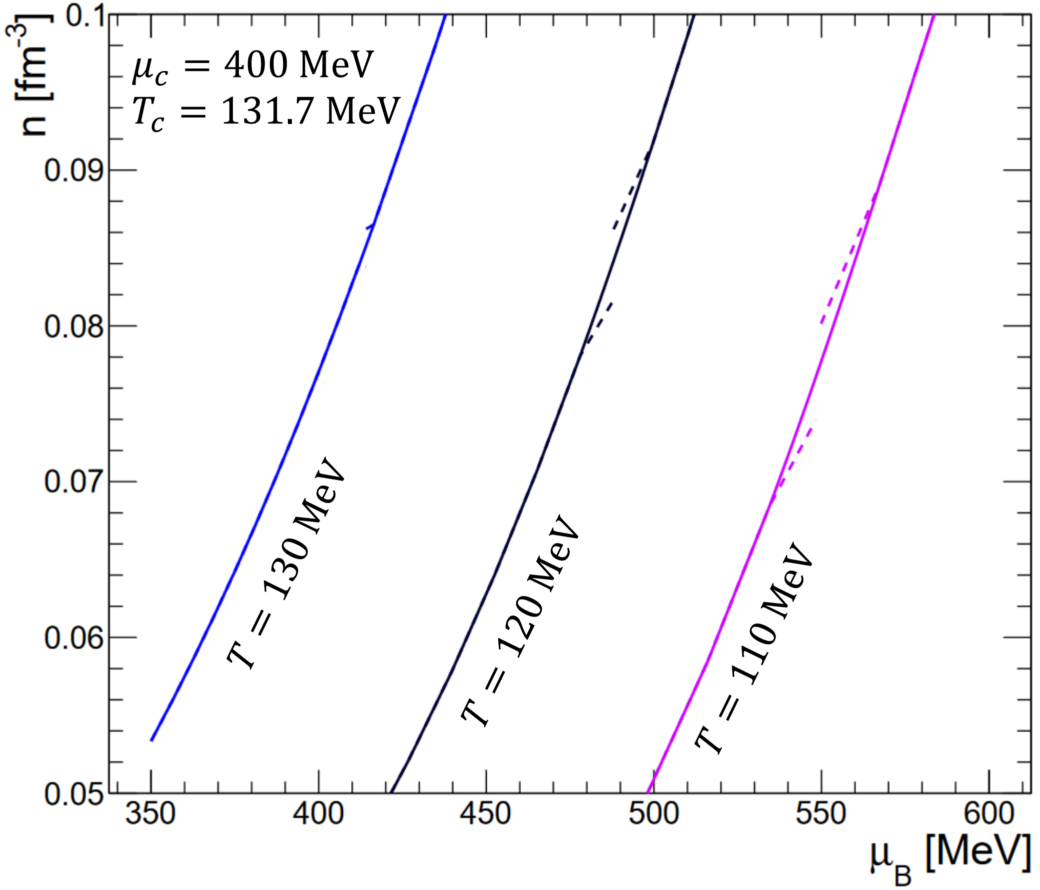

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$$\tilde{n}_h(\mu_c) = \tilde{n}_q(\mu_c) - \Delta n \Rightarrow 2(\mu_c - \mu_h)c_h - 2(\mu_c - \mu_q)c_q = n_q - n_h - \Delta n$$

$$\Delta n = n_0 \left|\frac{T - T_c}{T_c}\right|^{0.3265}$$

Conclusions & Outlook

Conclusions:

- We presented an effective equation of state of strongly interacting matter based on switching function approach
- The transition from hadrons to quark-gluon EoS tuned to describe the lQCD Data
- The obtained equation of state shows good behaviour at sufficiently high values of μ_B and can be used for hydrodynamic simulations of relativistic heavy ion collisions
- 1st order phase transition and critical endpoint was added to background EoS using two zone interpolation scheme

Outlook:

- Goal: Construct a tool to investigate the correlation between the existence and position of the critical endpoint and observables in heavy ion collisions