SPEED OF SOUND IN A DYNAMICAL QUARK MODEL

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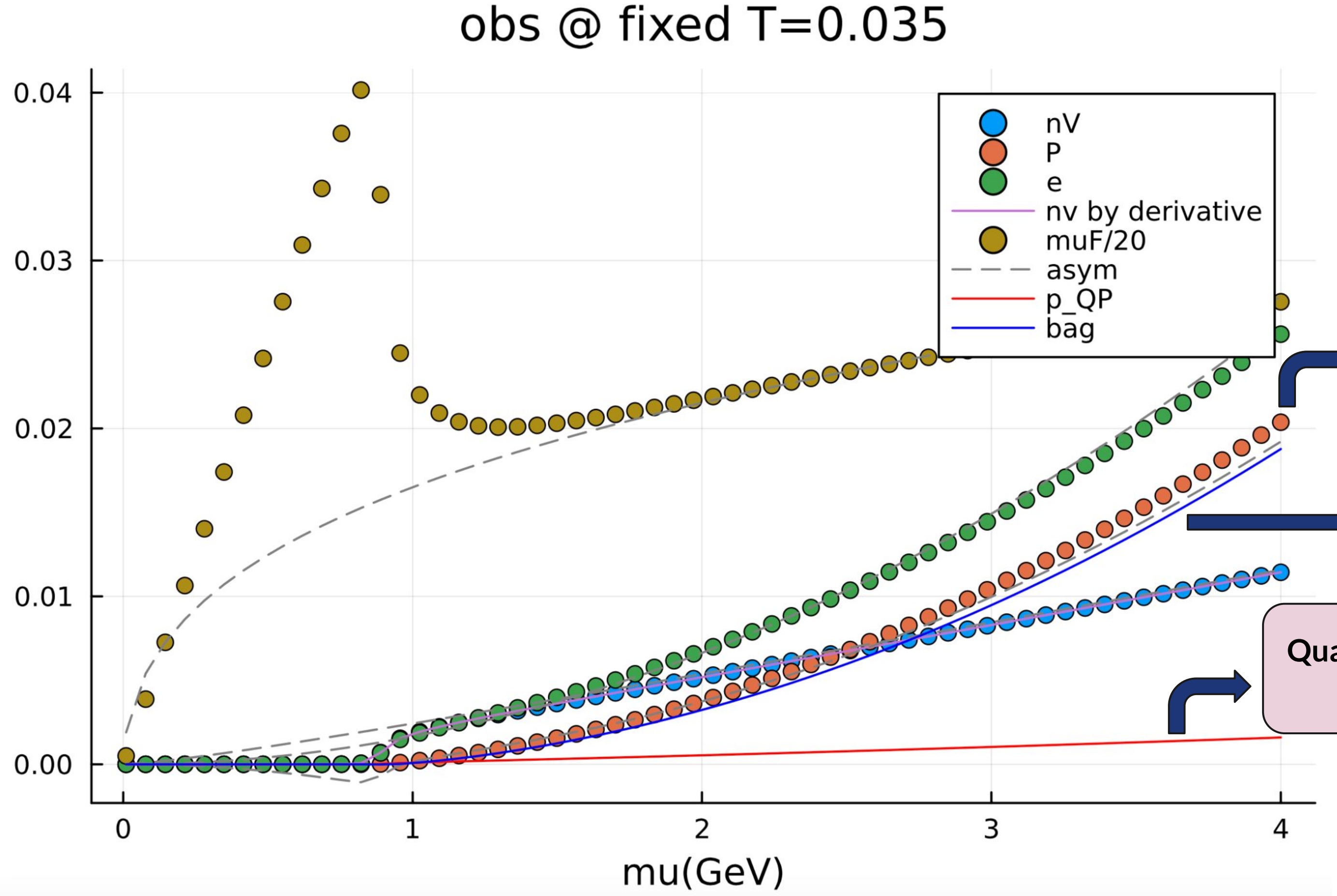
CONTENTS

Walecka (mean field, Low T (units in GeV))



Total Pressure

Mean field Potential energy contribution to total pressure

Quasi-particle contribution to total pressure

QUASIPARTICLE

MEAN FIELD POTENTIAL ENERGY

$$P(\mu, T) = P_{\mathrm{FG}} + \frac{1}{2}\left(\frac{g_\omega^2}{m_\omega^2}\right) n^2 - \frac{1}{2} m_\sigma^2 \bar{\sigma}^2$$

$$P_{\mathrm{FG}} = 4T \int \frac{d^3p}{(2\pi)^3} \left[\ln\left(1 + \mathrm{e}^{-\beta(E^* - \mu^*)}\right) + \ln\left(1 + \mathrm{e}^{-\beta(E^* + \mu^*)}\right)\right]$$

$$n = 4 \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{e^{\beta(E^* - \mu^*)} + 1} - \frac{1}{e^{\beta(E^* + \mu^*)} + 1}\right)$$

Fermi functions
(T–>0) → Theta

$$m_N^* = m_N - g_\sigma \bar{\sigma}$$

$$\mu^* = \mu - \left(\frac{g_\omega^2}{m_\omega^2}\right) n$$

DRESSED CHEMICAL POTENTIAL
DOMINATES
ASYMPTOTIC
THERMODYNAMICS
IN DENSE MATTER !!!

$$\mu = \mu_{eff} - \alpha\ \mu_{eff}^{3\eta}$$

$$\mu \to \ \infty$$

$$\mu \to \mu_{eff}^{3\eta}$$

$$\boxed{\mu_{eff} \to \mu^{\frac{1}{3\eta}}}$$

$$n_v \xrightarrow{\text{T=0}} p_f^3 \to \mu_{eff}^3 \to \mu^{\frac{1}{\eta}}$$

$$c_s^2 = \frac{\frac{n_v}{\mu}}{\frac{dn_v}{d\mu}} = \eta$$

GENERALIZED QUASIPARTICLE DENSE MATTER (ASYMPTOTIC) SPEED OF SOUND

MEAN FIELDS

$$\mu' = \mu - 2G_V \int \frac{d^3q}{(2\pi)^3} \, (n_F - \bar{n}_F)$$

$$\propto \mu'^3$$

$$\mu' \propto \mu^{\frac{1}{3}} \longrightarrow c_S^2 \to 1$$

$$c_s^2 = \frac{n_v}{\mu} / \frac{dn_v}{d\mu}$$

$$\frac{\delta\Omega}{\delta m^*} = 0 = \frac{\delta\Omega}{\delta\mu^*}$$

WALECKA

$$\mu^* = \mu - g_\omega\,\omega$$

$$m^* = m - g_\sigma\,\sigma$$

$$\sigma = -\left(\frac{g_\sigma}{m_\sigma^2}\right)\frac{\partial P}{\partial m^*}$$

$$\omega = -\left(\frac{g_\omega}{m_\omega^2}\right)\frac{\partial P}{\partial \mu^*}$$

NJL

$$\mu^* = \mu - 4N_f N_c G_v \int \frac{d^3p}{(2\pi)^3}\left(n(E_p) - \bar{n}(E_p)\right)$$

$$m^* = m + 4N_f N_c G_v \int \frac{d^3p}{(2\pi)^3}\frac{m^*}{E_p}\left(1 - n(E_p) - \bar{n}(E_p)\right)$$

QUASIPARTICLE MODELS

DYNAMICAL CHIRAL QUARK MODEL

$$\mathscr{L} = \bar{\psi}(x)(i\not{\partial}_x - m)\psi(x) - \frac{1}{2}\int d^4y\, \rho^a(x) V^{ab}(x,y) \rho^b(y)$$

Dynamical effect Gluon exchange

$$\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x)$$

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2}(n(E_q) - \bar{n}(E_q))$$

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{A(q)\,\hat{p}\cdot\hat{q}}{2E_q}(1 - n(E_q) - \bar{n}(E_q))$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B}{2E_q}(1 - n(E_q) - \bar{n}(E_q))$$

$$Eq = \sqrt{A_q^2\, q^2 + B_q^2}$$

Natural medium dependence

Quasiparticle property modified

Not non-local NJL

P.M. Lo & E.S. Swanson, Phys. Rev. D81, 034030(2010)

DYNAMICAL (SEPARABLE) MODEL

Gap equations

QUASIPARTICLE PROPERTY MODIFIED

$$\mu(p) = \mu - \omega\, \gamma(p)$$

$$M(p) = m + \sigma\, \gamma(p)$$

DIQUARK <qq> →

$$\Delta(p) = \Delta\, \gamma(p)$$

Dispersion relations

$$E_p = \sqrt{p^2 + M_p^2}$$

$$E_p^{\pm} = E_p \pm \mu_p$$

QUASIPARTICLE PROPERTY MODIFIED BY <qq> →

$$\epsilon_p^{\pm} = \sqrt{(E_p^{\pm})^2 + \Delta_p^2}$$

$$n(x) = \frac{1}{1 + \exp(x/T)}$$

$$\sigma = f(\sigma, \omega, \Delta)$$

$$\sigma = \frac{4 * 2\, G_s}{2\pi^2} \int dq\, q^2\, \gamma(q) \frac{M_q}{2E_q} \left[1 - n(E_q^+) - n(E_q^-) + \frac{E_q^+}{\epsilon_q^+}\left(1 - 2n(\epsilon_q^+)\right) + \frac{E_q^-}{\epsilon_q^-}\left(1 - 2n(\epsilon_q^-)\right)\right]$$

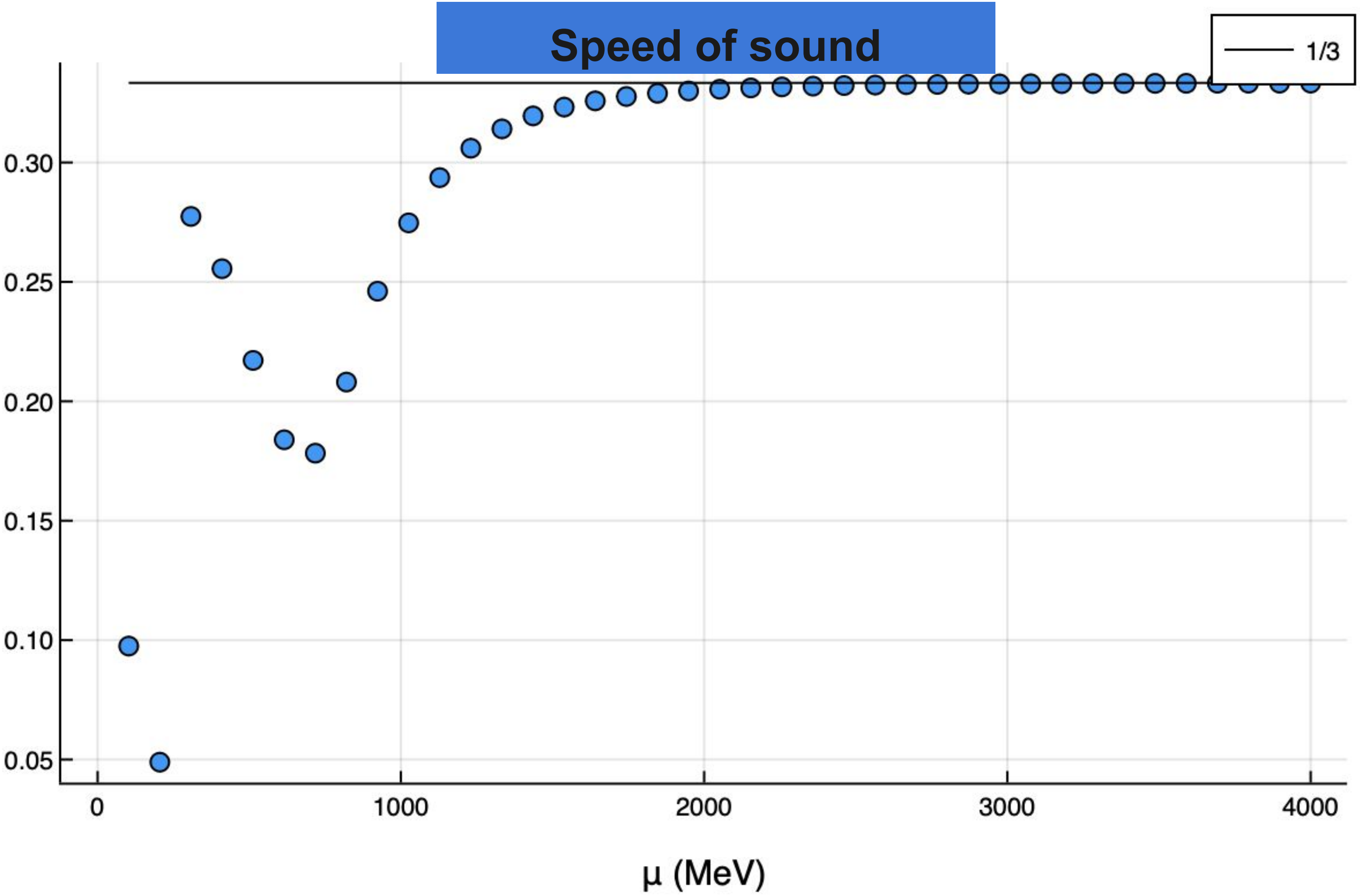
$$\omega = f(\sigma, \omega, \Delta)$$

$$\omega = \frac{4 * 2\, G_v}{2 * 2\pi^2} \int dq\, q^2\, \gamma(q) \left[n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_q^+}\left(1 - 2n(\epsilon_q^+)\right) - \frac{E_q^-}{\epsilon_q^-}\left(1 - 2n(\epsilon_q^-)\right)\right]$$

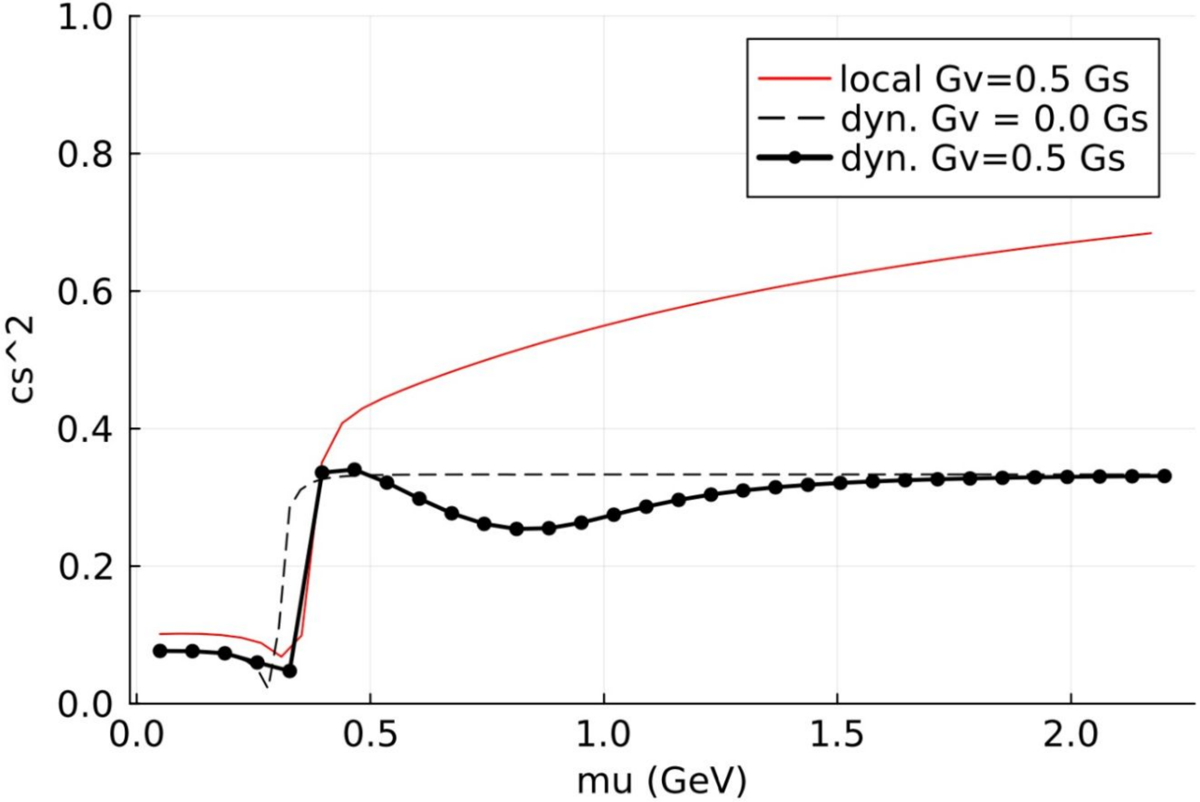
$$\Delta = f(\sigma, \omega, \Delta)$$

$$\Delta = \frac{4 * 2\, G_d}{2 * 2\pi^2} \int dq\, q^2\, \gamma(q) \Delta \left[\frac{1}{\epsilon_q^+}\left(1 - 2n(\epsilon_q^+)\right) + \frac{1}{\epsilon_q^-}\left(1 - 2n(\epsilon_q^-)\right)\right]$$

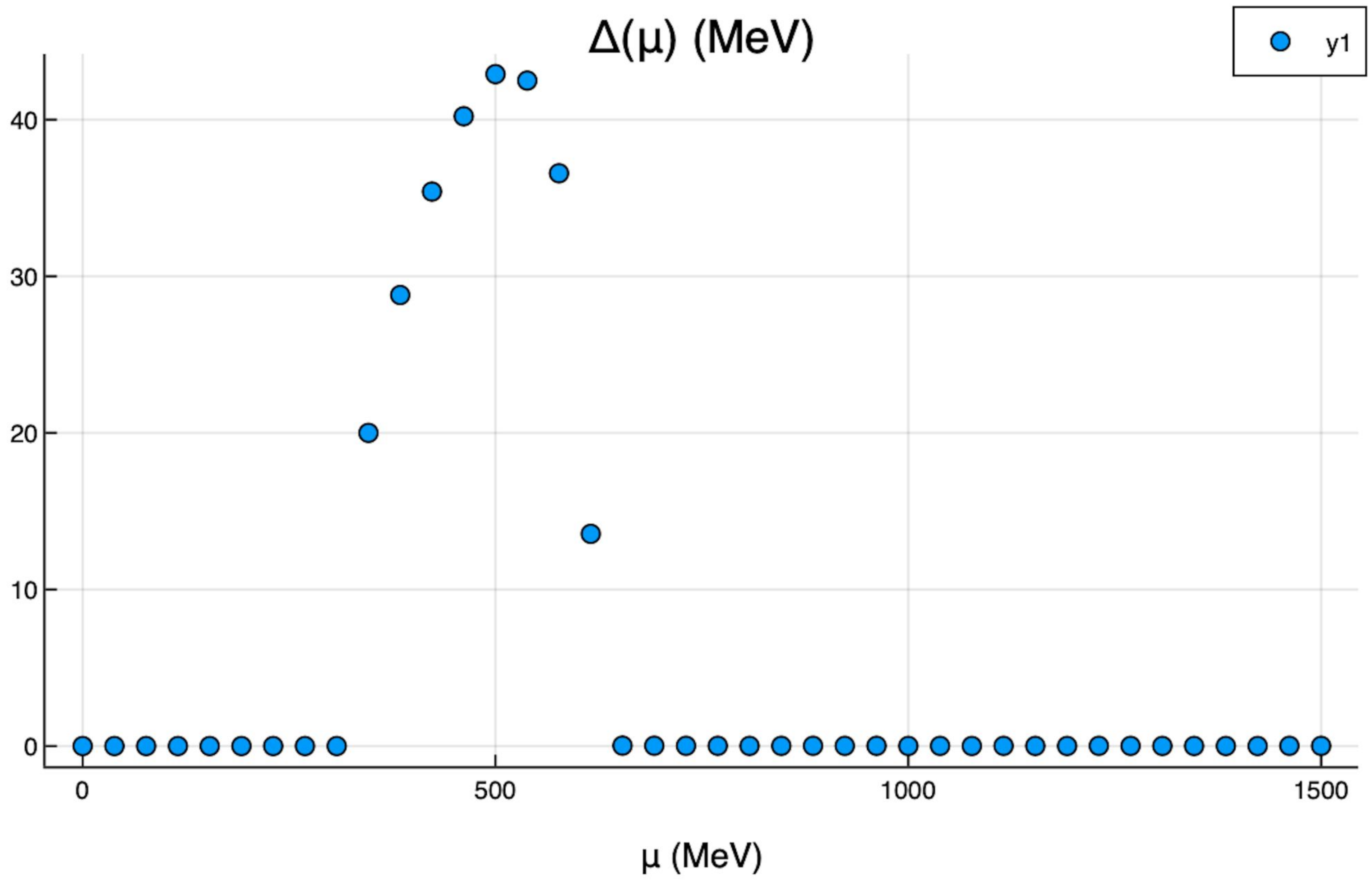
Speed of sound

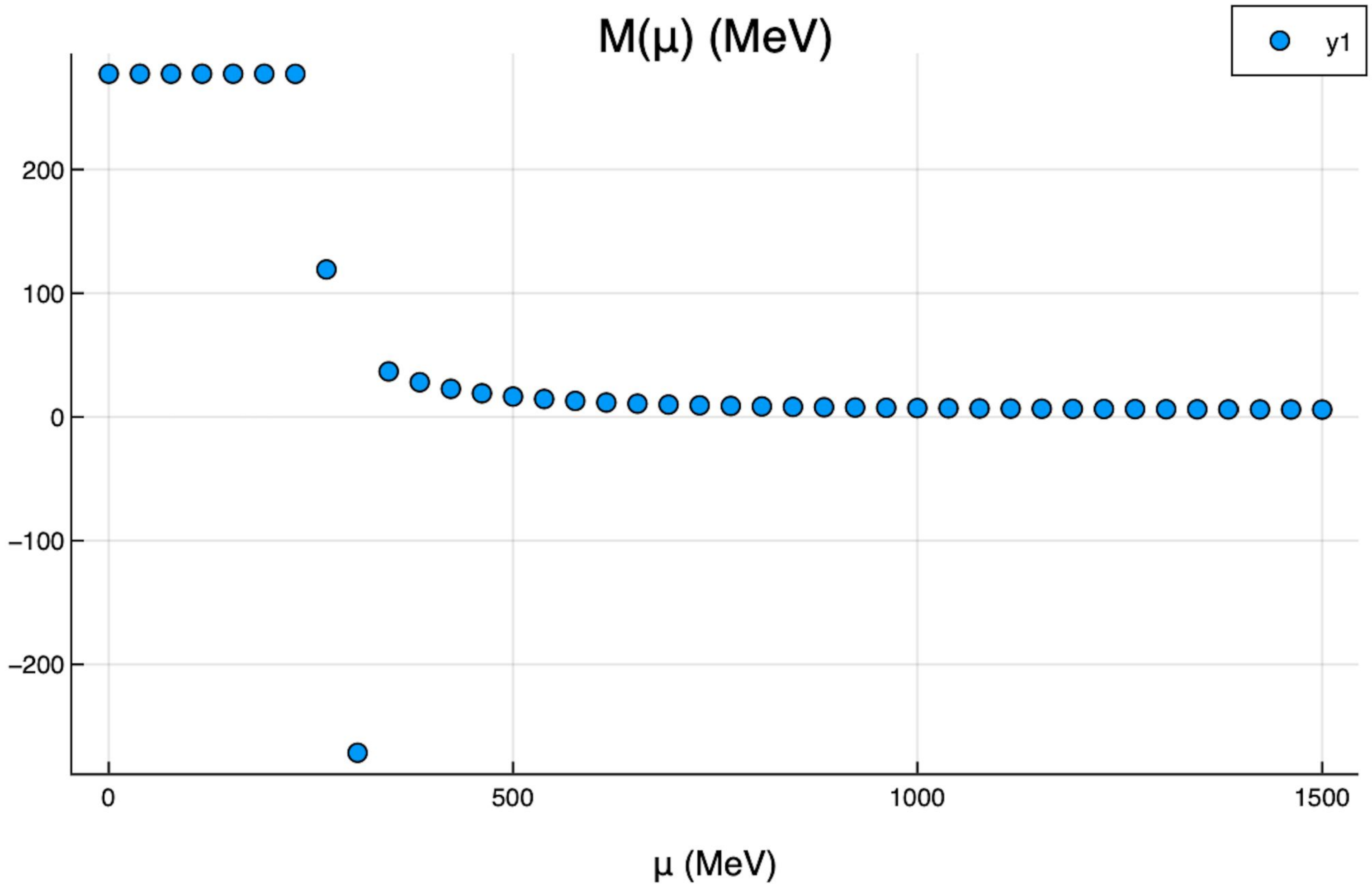


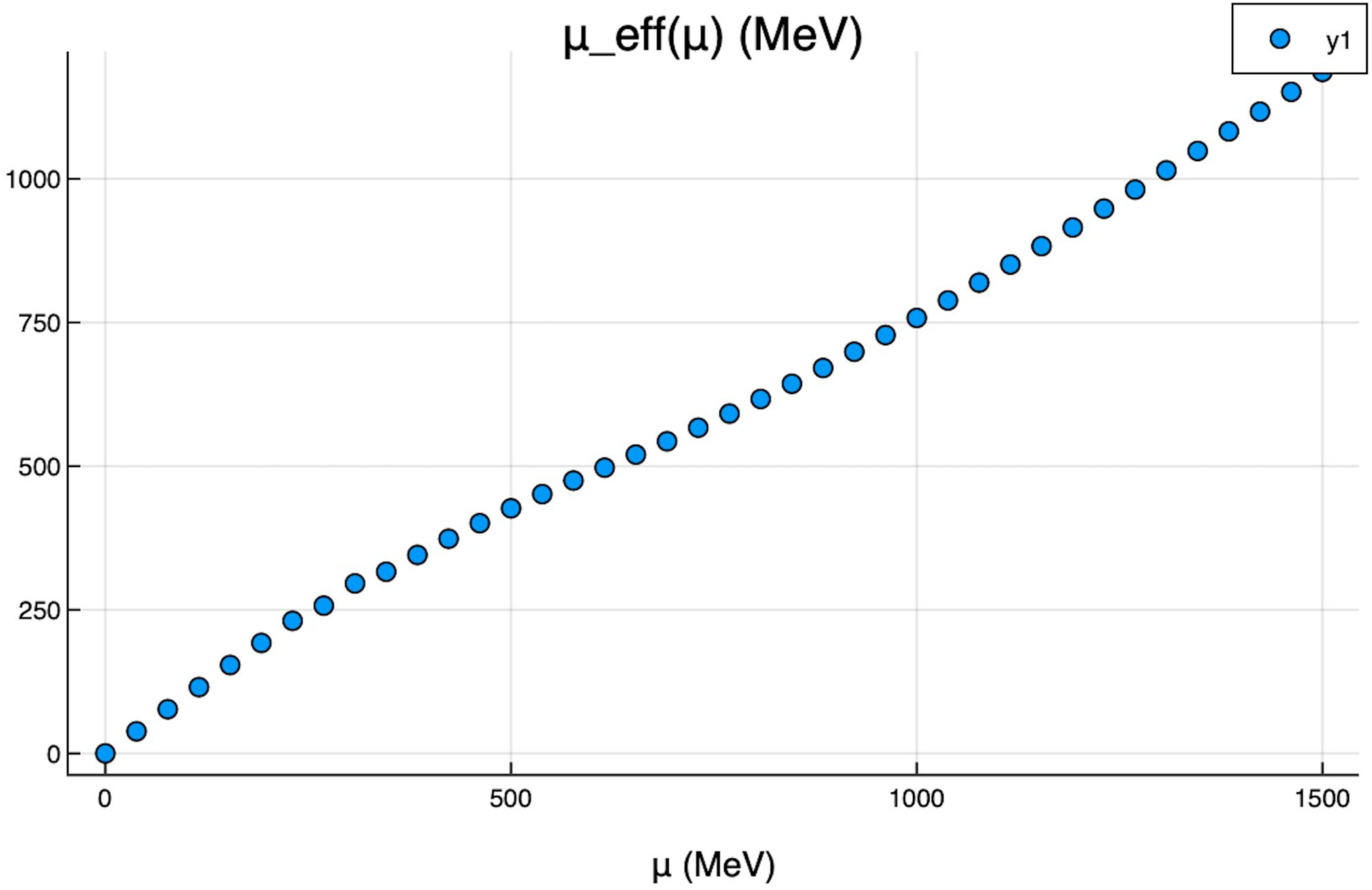
SPEED OF SOUND – DENSE MATTER DOMAIN



NJL fails conformal limit


Δ(μ) (MeV)
y1
0
10
20
30
40
0
500
1000
1500
μ (MeV)


M(μ) (MeV)
y1
200
100
0
−100
−200
0
500
1000
1500
μ (MeV)


μ_eff(μ) (MeV)
y1
1000
750
500
250
0
0
500
1000
1500
μ (MeV)

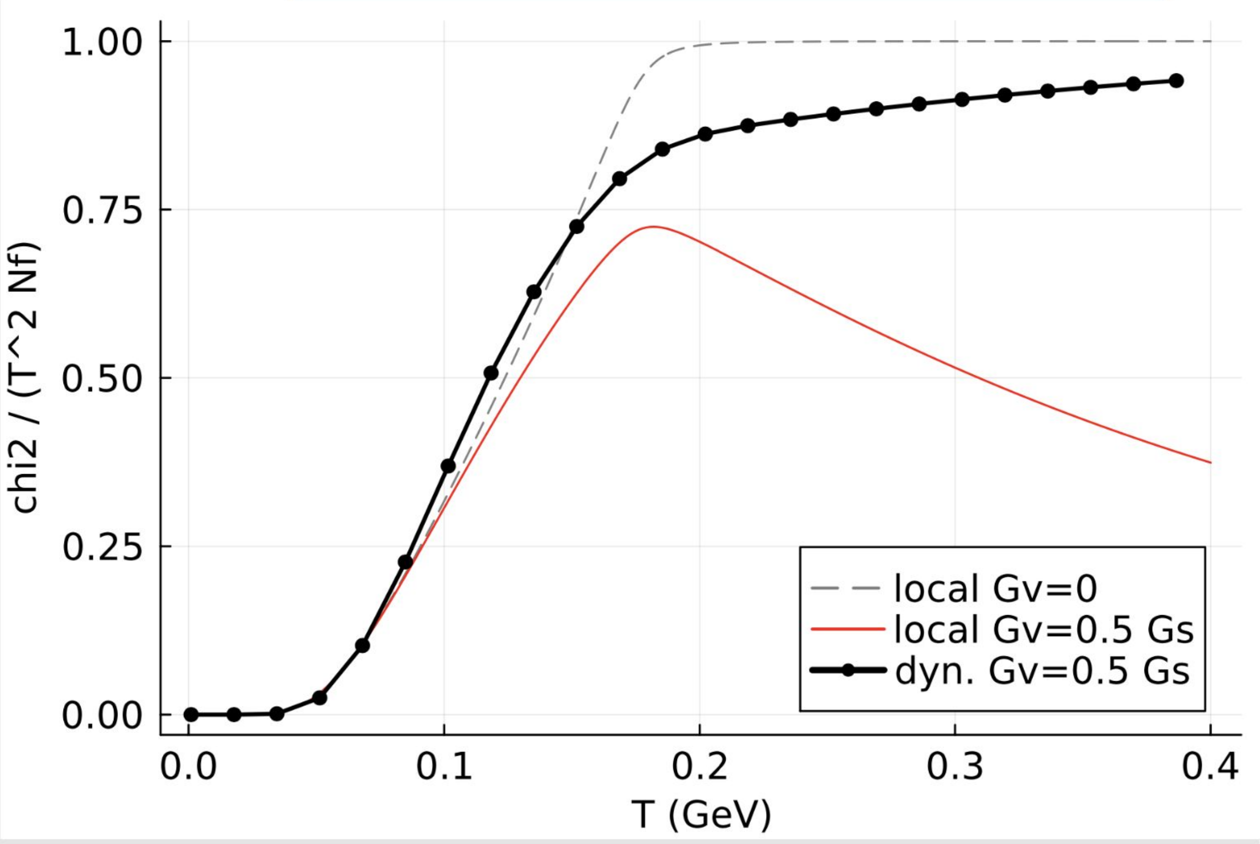
QUARK NUMBER SUSCETIBILITY (QNS)

$$\chi_2 = \frac{\partial n_v^{QP}}{\partial \mu} = \frac{\partial n_v^{QP}}{\partial \mu'} \times \frac{\partial \mu'}{\partial \mu}$$

$$\chi_2 = \frac{\chi_2^{(0)}}{1 + 2G_v \chi_2^{(0)}}$$

$$\chi_2^{(0)} = \frac{\partial n_v^{QP}}{\partial \mu'}$$

QNS- FINITE TEMPERATURE DOMAIN



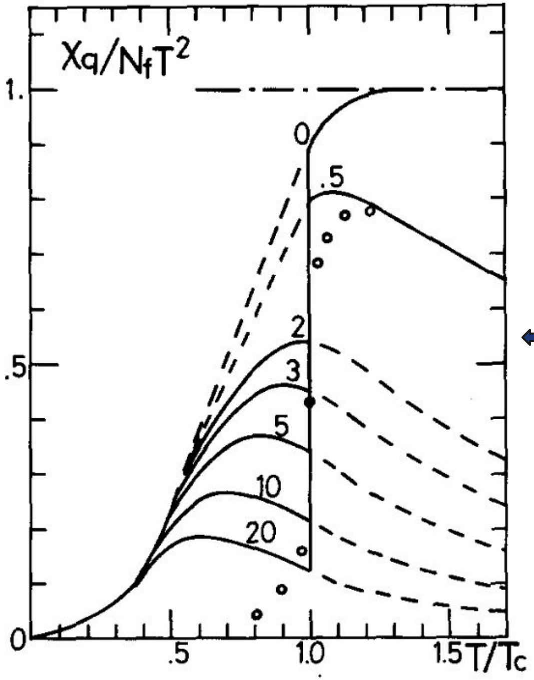


Fig. 1. The temperature dependence of the quark-number susceptibility χ_q in the unit of N_fT^2 with some of the vector coupling $g_V\Lambda^2$: $g_V\Lambda^2=0$, 0.5, 2, 3, 5, 10, 20, which are indicated with the numbers attached to the respective curves. The dash-dotted line shows the free massless case. The small circles are the lattice result on an $8^3\times4$ lattice with the quark mass $m/T=0.2$ [7] compiled in ref. [9].

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PHYSICS LETTERS B

Quark-number susceptibility and fluctuations in the vector channel at high temperatures ☆

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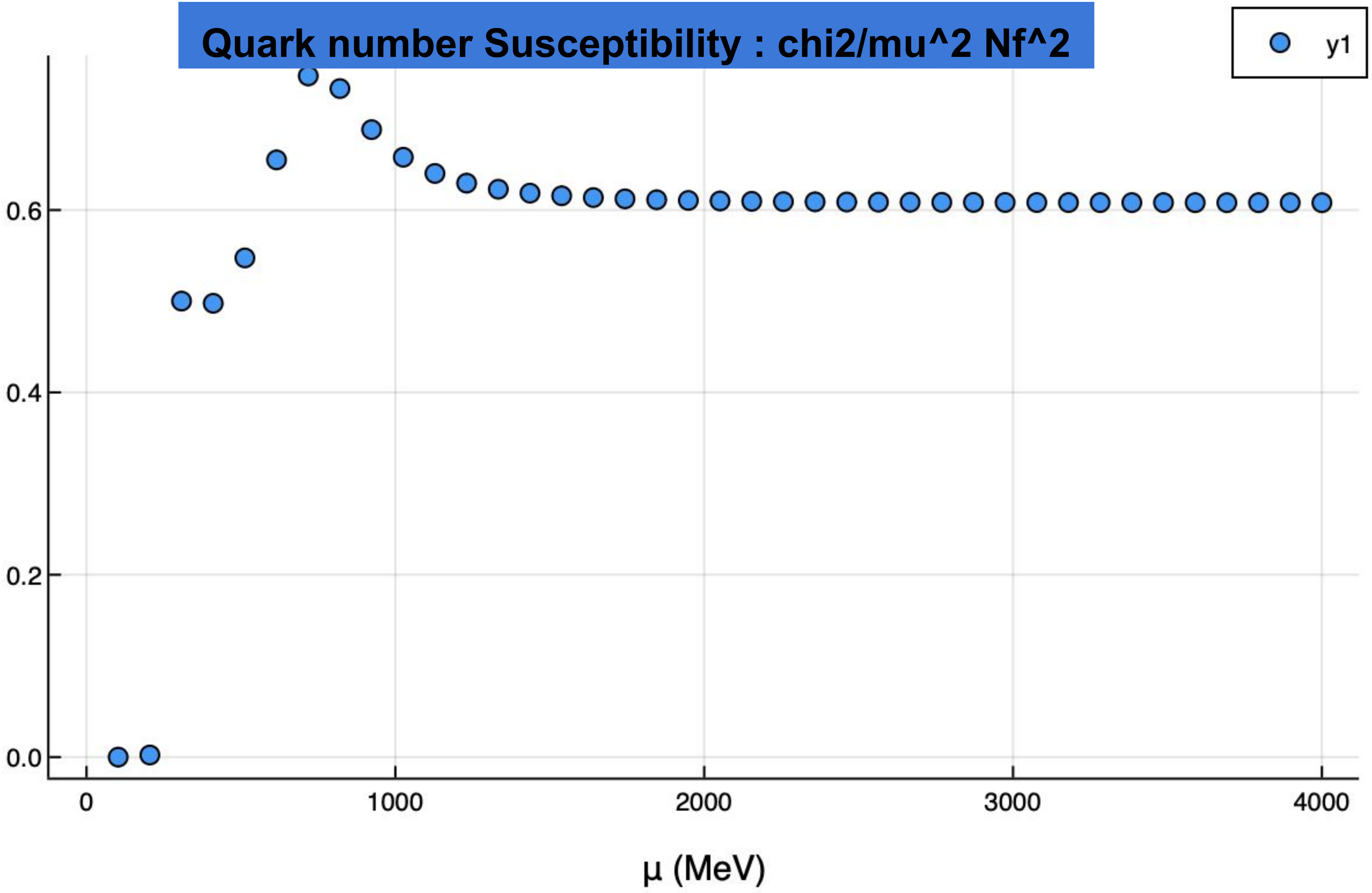
The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

Quark number Susceptibility : chi2/mu^2 Nf^2



SUMMARY & CONCLUSION

- Effective quasiparticle models (eg. NJL, Walecka) translate to non-conformal dense matter asymptotics.
- Speed of sound highly sensitive to dynamical effects.
- Gluon exchange-motivated momentum-dependent potential modifies quasiparticle relation generating natural medium dependence —---> conformal limit.
- Couplings are naturally implemented in dynamical model.
- A dense matter model must ensure correct dressing of chemical potential for correct asymptotics.
- Dynamical model explains QNS in dense & finite T.

OUTLOOK

- So far we only fix UV: conformal limit / perturbative QCD.
- IR: confinement and chiral symmetry <-> formally related to the gluon exchange.
- Can we find a density functional that gives comparable results to a dynamical model?

$$\mu' = \mu + \frac{\partial U}{\partial \bar{n}_V}$$

- Dynamical effect in diquark gap and superconductivity.