A QFT based study of transport coefficients of rotating, hot and dense spin 1/2 fermions at large angular velocities

63rd Cracow School of Theoretical Physics : Nuclear Matter at Extreme Densities and High Temperatures

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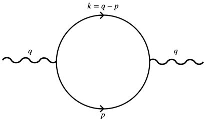
CONTENTS

- Kubo formalism & recent developments
- Transport coefficients in Spin Hydrodynamics
- Computing correlation function of fields in a rotating medium
- Shear & Bulk viscosity of rotating spin $1/2$ fermions

One-loop Kubo estimation of transport coefficients and historical development

Linear response theory : A formalism employed to derive the effect of dissipative forces on $T^{\mu\nu}$

- By defining a statistical operator at equilibrium and expanding it from equilibrium, one defines a nonequilibrium statistical operator.
- Thermal field theory : calculation of the ensemble average of fields
- Two-point function of Noether currents (J^{μ}) and Energy-momentum tensors ($T^{\mu\nu}$)



- Dissipative quantities $(\eta, \zeta, \kappa, \sigma) \xrightarrow{\text{expressed as}}$ Correlation functions in thermal equilibrium, Kubo formulas .
- First calculation of shear viscosity in relativistic hydrodynamics by A. Hosoya *et.al* [1]
- Kubo formulas in magnetic field provided by X.G. Huang [2]
- Strong Magnetic field : bulk viscosity Hattori *et.al* [3], electrical conductivity Hattori *et.al* [4]
- Generic magnetic fields : shear and bulk viscosities Ghosh *et.al* [5], electrical conductvity Satapathy *et.al* [6, 7]
- Under strong rotation attempted : bulk viscosity Satapathy [8], shear viscosity Satapathy *et.al* [9]

Transport coefficients in first order spin hydrodynamics

$|L| \sim 10^5 \hbar$, Production of high angular velocity 10^{21} sec^{-1}:L. Adamczyk [10]
Hydrodynamics : $\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0, \quad \partial_\lambda \Sigma^{\lambda\mu\nu} = 0$

- Extra transport coefficients apart from η, ζ, κ expected Becattini & Tinti :
  F. Becattini, L. Tinti, "Nonequilibrium thermodynamical inequivalence of quantum stress-energy and spin tensors," Phys. Rev. D 87, 025029, (2013) [11].
- Kinetic Theory : Estimated to exist in Bhadury *et.al*
  S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar and R. Ryblewski, "Relativistic dissipative spin dynamics in the relaxation time approximation," Phys. Lett. B 814, 136096 (2021) [12]
- Kubo formalism : Kubo formulas provided by Jin Hu
  J. Hu, "Kubo formulae for first-order spin hydrodynamics," Phys. Rev. D 103, no.11, 116015 (2021) [13]
  - η : Symmetric EMT
  - ζ : Symmetric EMT
  - κ : Symmetric EMT
  - γ : Antisymmetric EMT
  - λ : Antisymmetric EMT

I will discuss ζ & η at large Ω from Kubo formalism which is based on quantum field theory.

Recovering Translational Invariance, computing correlation functions at large Ω

Background fields break translational symmetry :

- Background magnetic fields : Chyi [14], Kuznetsov [15, 16]: coupling of external gauge fields with fermionic fields ($\widetilde{A}^{\mu} = A^{\mu} + A^{\mu}_{\text{ext}}$)
- Fluids in a Rotating environment : Hongo *et.al* [17]
  M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," JHEP 11, 150 (2021)
  Coupling of torsion with fermionic fields $\longrightarrow$ Torsion acts as a background field

Background magnetic field $\rightarrow$ Appearance of "Schwinger phase factor" $\Phi(x, y)$ in coordinate space propagator

$$\text{i.e } D(x, y) = \Phi(x, y)D(x - y), \quad \Phi(x, y) = \exp\left[ie \int_x^y dx'_{\mu} A^{\mu}_{\text{ext}}(x')\right]$$

Two-Point function of $T^{\mu\nu}(x)$, $J^{\mu}(x)$ for generic magnetic fields is unaffected : Ref. Satapathy *et.al* [6, 7]

$S(r, r') \xrightarrow{\text{Large } \Omega} S(r - r')$: Ayala *et.al,* : **Phys. Rev. D 104, no.3, 039901 (2021)**[18]

Recovery of translational invariance of the propagator allows Fourier transformation, computation becomes easier.

$$\downarrow$$

$$S(r) \xrightarrow{\text{F.T}} \widetilde{S}(k)$$

Does such a cancellation occur for a generic case of all strengths of Ω ?

Shear and bulk viscosity from correlation functions

arxiv:2307.09953 & arxiv:2309.05284

- $\eta = -\frac{1}{10} \lim_{\vec{q}=\vec{0}, q_0 \to 0} \frac{\rho_\eta(q)}{q_0}, \quad \rho_\eta(q) = \text{Im } \Pi_\eta(q) = \text{Im} i \int d^4 r \; e^{iq\cdot r} \langle \pi^{\mu\nu}(0) \pi_{\mu\nu}(r) \rangle_R,$
- $\zeta = - \lim_{\vec{q}=\vec{0}, q_0 \to 0} \frac{\rho_\zeta(q)}{q_0}, \quad \rho_\zeta(q) = \text{Im } \Pi_\zeta(q) = \text{Im } i \int d^4 r \; e^{iq\cdot r} \langle \mathcal{P}^*(r) \mathcal{P}^*(0) \rangle_R$

Two-point function of EMTs :

$$\langle T^{\mu\nu}(0) T^{\alpha\beta}(r) \rangle = -\frac{1}{16} \Big[\text{Tr} \{ \widetilde{\gamma}^{\{\mu} \widetilde{D}^{\nu\}} S(0,r) \widetilde{\gamma}^{\{\alpha} \widetilde{D}^{\beta\}} S(r,0) \} - \text{Tr} \{ \widetilde{\gamma}^{\{\mu} \widetilde{D}^{\nu\}} \widetilde{D}^{\{\alpha} S(0,r) \widetilde{\gamma}^{\beta\}} S(r,0) \}$$
$$- \text{Tr} \{ \widetilde{\gamma}^{\{\mu} S(0,r) \widetilde{\gamma}^{\{\alpha} \widetilde{D}^{\beta\}} \widetilde{D}^{\nu\}} S(r,0) \} + \text{Tr} \{ \widetilde{\gamma}^{\{\mu} \widetilde{D}^{\{\alpha} S(0,r) \widetilde{\gamma}^{\beta\}} \widetilde{D}^{\nu\}} S(r,0) \} \Big]$$

A. Ayala, L. A. Hernández, K. Raya and R. Zamora, **"Fermion propagator in a rotating environment," Phys. Rev. D 103, no.7, 076021 (2021) [erratum: Phys. Rev. D 104, no.3, 039901 (2021)]**.

$$\widetilde{S}(k) = \frac{(k_0 + \frac{\Omega}{2} - k_z + ik_\perp)(\gamma_0 + \gamma_3) + m(1+\gamma_5)}{(k_0 + \frac{\Omega}{2})^2 - \vec{k}^2 - m^2 + i\epsilon} \mathcal{O}^+ + \frac{(k_0 - \frac{\Omega}{2} + k_z - ik_\perp)(\gamma_0 - \gamma_3) + m(1+\gamma_5)}{(k_0 - \frac{\Omega}{2})^2 - \vec{k}^2 - m^2 + i\epsilon} \mathcal{O}^-$$

Matsubara frequencies at finite T, μ and Ω

Formalisms :

- Real-Time Formalism (Equilibrium & Nonequilibrium)
- Imaginary-Time Formalism (Equilibrium : Makes use of Matsubara Frequency sums)

  Saclay method of the frequency sum evaluation :

  Mikko Laine & Aleksi Vuorinen, **"Basics of Thermal Field Theory," Lect. Notes Phys. 925, pp.1-281 (2016) Springer, 2016.** [19]

$$(a)\ T \sum_{\{p_N\}} \frac{1}{\left[i\widetilde{\omega}_N + \frac{\Omega}{2} + \mu\right]^2 - \vec{p}^2 - m^2} \frac{1}{\left[i\nu_N - i\widetilde{\omega}_N + \frac{\Omega}{2} - \mu\right]^2 - \vec{k}^2 - m^2}$$
$$= \frac{1}{4E_pE_k}\left\{ \frac{n_F(E_p+\mu+\Omega/2) + n_F(E_k-\mu+\Omega/2) - 1}{q_0 - E_p - E_k} + \frac{n_F(E_k+\mu+\Omega/2) - n_F(E_p+\mu+\Omega/2)}{q_0 + E_k - E_p} \right.$$
$$\left. + \frac{n_F(E_p-\mu-\Omega/2) - n_F(E_k-\mu-\Omega/2)}{q_0 + E_p - E_k} + \frac{1 - n_F(E_p-\mu-\Omega/2) - n_F(E_k+\mu-\Omega/2)}{q_0 + E_k + E_p} \right\} \quad (1)$$

$$(b)\ T \sum_{\{p_N\}} \frac{1}{\left[i\widetilde{\omega}_N + \frac{\Omega}{2} + \mu\right]^2 - \vec{p}^2 - m^2} \frac{1}{\left[i\nu_N - i\widetilde{\omega}_N - \frac{\Omega}{2} - \mu\right]^2 - \vec{k}^2 - m^2}$$
$$= \frac{1}{4E_pE_k}\left\{ \frac{n_F(E_p+\mu+\Omega/2) + n_F(E_k-\mu-\Omega/2) - 1}{q_0 - E_p - E_k} + \frac{n_F(E_k+\mu+\Omega/2) - n_F(E_p+\mu+\Omega/2)}{q_0 + E_k - E_p} \right.$$
$$\left. + \frac{n_F(E_p-\mu-\Omega/2) - n_F(E_k-\mu-\Omega/2)}{q_0 + E_p - E_k} + \frac{1 - n_F(E_p-\mu-\Omega/2) - n_F(E_k+\mu+\Omega/2)}{q_0 + E_k + E_p} \right\} \quad (2)$$

Matsubara Frequencies continued

$$(c)\ T\sum_{\{p_N\}} \frac{1}{[i\widetilde{\omega}_N - \frac{\Omega}{2} + \mu]^2 - \vec{p}^2 - m^2} \frac{1}{[i\nu_N - i\widetilde{\omega}_N + \frac{\Omega}{2} - \mu]^2 - \vec{k}^2 - m^2}$$

$$= \frac{1}{4E_pE_k}\Bigg\{ \frac{n_F(E_p+\mu-\Omega/2)+n_F(E_k-\mu+\Omega/2)-1}{q_0-E_p-E_k} + \frac{n_F(E_k+\mu-\Omega/2)-n_F(E_k+\mu-\Omega/2)}{q_0+E_k-E_p}$$
$$+\frac{n_F(E_p-\mu+\Omega/2)-n_F(E_k-\mu+\Omega/2)}{q_0+E_p-E_k} + \frac{1-n_F(E_p-\mu+\Omega/2)-n_F(E_k+\mu-\Omega/2)}{q_0+E_k+E_p}\Bigg\} \quad (3)$$

$$(d)\ T\sum_{\{p_N\}} \frac{1}{[i\widetilde{\omega}_N - \frac{\Omega}{2} + \mu]^2 - \vec{p}^2 - m^2} \frac{1}{[i\nu_N - i\widetilde{\omega}_N - \frac{\Omega}{2} - \mu]^2 - \vec{k}^2 - m^2}$$

$$= \frac{1}{4E_pE_k}\Bigg\{ \frac{n_F(E_p+\mu-\Omega/2)+n_F(E_k-\mu-\Omega/2)-1}{q_0-E_p-E_k} + \frac{n_F(E_k+\mu-\Omega/2)-n_F(E_k+\mu-\Omega/2)}{q_0+E_k-E_p}$$
$$+\frac{n_F(E_p-\mu+\Omega/2)-n_F(E_k-\mu+\Omega/2)}{q_0+E_p-E_k} + \frac{1-n_F(E_p-\mu+\Omega/2)-n_F(E_k+\mu-\Omega/2)}{q_0+E_k+E_p}\Bigg\}. \quad (4)$$

Ω enters into the distribution function, competes with μ

Spectral function of bulk viscosity

$$
\begin{aligned}
&\rho_\zeta(q) \\
&= -\pi \int \frac{d^3p}{(2\pi)^3}\left\{\frac{\mathcal{A}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) + (c_s^2/3)\mathcal{C}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) + (c_s^4/18)\mathcal{E}(p_\perp, p_z, p_0, q_\perp, q_z, q_0)}{4E_pE_{q-p}}\right\} \\
&\quad \Big[\Big\{n_F(E_{q-p}+\widetilde{\mu}) - n_F(E_p+\widetilde{\mu})\Big\}\delta(q_0+E_{q-p}-E_p) \\
&\quad +\Big\{n_F(E_p-\widetilde{\mu}) - n_F(E_{q-p}-\widetilde{\mu})\Big\}\delta(q_0+E_p-E_{q-p}) \\
&\quad +\Big\{n_F(E_{q-p}+\widetilde{\mu}) + n_F(E_p-\widetilde{\mu}) - 1\Big\}\delta(q_0-E_{q-p}-E_p) \\
&\quad +\Big\{1 - n_F(E_{q-p}-\widetilde{\mu}) - n_F(E_p+\widetilde{\mu})\Big\}\delta(q_0+E_{q-p}+E_p)\Big] \\
&-\pi \int \frac{d^3p}{(2\pi)^3}\left\{\frac{\mathcal{B}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) + (c_s^2/3)\mathcal{D}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) + (c_s^4/18)\mathcal{F}(p_\perp, p_z, p_0, q_\perp, q_z, q_0)}{4E_pE_{q-p}}\right\} \\
&\quad \Big[\Big\{n_F(E_{q-p}+\bar{\mu}) - n_F(E_p+\bar{\mu})\Big\}\delta(q_0+E_{q-p}-E_p) \\
&\quad +\Big\{n_F(E_p-\bar{\mu}) - n_F(E_{q-p}-\bar{\mu})\Big\}\delta(q_0+E_p-E_{q-p}) \\
&\quad +\Big\{n_F(E_{q-p}+\bar{\mu}) + n_F(E_p-\bar{\mu}) - 1\Big\}\delta(q_0-E_{q-p}-E_p) \\
&\quad +\Big\{n_F(E_{q-p}-\bar{\mu}) + n_F(E_p+\bar{\mu}) - 1\Big\}\delta(q_0+E_{q-p}+E_p)\Big],
\end{aligned}
$$

$$
\boxed{\bar{\mu} = \mu + \Omega/2, \qquad \widetilde{\mu} = \mu - \Omega/2}
$$

$$
\begin{aligned}
\mathcal{A}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) &= -4\{(q_z - p_z)^2 + (q_z - p_z)p_z + p_z^2\}\{(q_0 - p_0 + q_z - p_z)(4p_0 + 4p_z - 2\Omega) \\
&\quad -4(q_\perp - p_\perp)p_\perp - 2(p_0 + p_z)\Omega + \Omega^2\} \\
\mathcal{B}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) &= -4\{(q_z - p_z)^2 + (q_z - p_z)p_z + p_z^2\}\{(q_0 - p_0 + q_z - p_z)(4p_0 - 4p_z + 2\Omega) \\
&\quad -4(q_\perp - p_\perp)p_\perp + 2(p_0 - p_z)\Omega + \Omega^2\} \\
\mathcal{C}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) &= \{4(q_0 - p_0)(q_z) - (q_z - p_z)\Omega - 4p_0 p_z\}\{(q_z - p_z + q_0 - p_0)(4p_0 + 4p_z - 2\Omega) \\
&\quad -4(q_\perp - p_\perp)p_\perp - 2(p_0 + p_z)\Omega + \Omega^2\} \\
\mathcal{D}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) &= \{4(q_0 - p_0)(3q_z - 3p_z) - p_z\Omega + 4p_0 p_z\}\{(q_z - p_z - q_0 + p_0)(4p_0 - 4p_z + 2\Omega) \\
&\quad +4(q_\perp - p_\perp)p_\perp - 2(p_0 - p_z)\Omega - \Omega^2\} \\
\mathcal{E}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) &= -\frac{1}{2}\{2(q_0 - p_0)p_0 - 3(q_0 - p_0)\Omega - 4p_0^2\}\{(q_0 - p_0 - q_z + p_z)(4p_0 - 4p_z + 2\Omega) \\
&\quad -4(q_\perp - p_\perp)p_\perp + 2(p_0 - p_z)\Omega + \Omega^2\} \\
\mathcal{F}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) &= -\frac{1}{2}\{2(q_0 - p_0)p_0 + 3(q_0 - p_0)\Omega - 4p_0^2\}\{(q_0 - p_0 + q_z - p_z)(4p_0 + 4p_z - 2\Omega) \\
&\quad -4(q_\perp - p_\perp)p_\perp - 2(p_0 + p_z)\Omega + \Omega^2\}.
\end{aligned}
$$

Spectral function of shear viscosity

$$
\begin{aligned}
\rho_\eta(q) \quad = \quad & -\pi \int \frac{d^3p}{(2\pi)^3} \Big(\frac{\mathcal{C}(p_\perp, p_z, p_0, q_\perp, q_z, q_0)}{4E_p E_{q-p}} \Big) \Bigg[\Big\{ n_F(E_{q-p} + \bar{\mu}) - n_F(E_p + \bar{\mu}) \Big\} \Big\{ \delta(q_0 + E_{q-p} - E_p) \Big\} \\
& + \Big\{ n_F(E_p - \bar{\mu}) - n_F(E_{q-p} - \bar{\mu}) \Big\} \Big\{ \delta(q_0 + E_p - E_{q-p}) \Big\} \\
& + \Big\{ n_F(E_{q-p} + \bar{\mu}) + n_F(E_p - \bar{\mu}) - 1 \Big\} \Big\{ \delta(q_0 - E_{q-p} - E_p) \Big\} \\
& + \Big\{ n_F(E_{q-p} - \bar{\mu}) + n_F(E_p + \bar{\mu}) - 1 \Big\} \Big\{ \delta(q_0 + E_{q-p} + E_p) \Big\} \Bigg] \\
& - \pi \int \frac{d^3p}{(2\pi)^3} \Big(\frac{\mathcal{D}(p_\perp, p_z, p_0, q_\perp, q_z, q_0)}{4E_p E_{q-p}} \Big) \Bigg[\Big\{ n_F(E_{q-p} + \widetilde{\mu}) - n_F(E_p + \widetilde{\mu}) \Big\} \Big\{ \delta(q_0 + E_{q-p} - E_p) \Big\} \\
& + \Big\{ n_F(E_p - \widetilde{\mu}) - n_F(E_{q-p} - \widetilde{\mu}) \Big\} \Big\{ \delta(q_0 + E_p - E_{q-p}) \Big\} \\
& + \Big\{ n_F(E_{q-p} + \widetilde{\mu}) + n_F(E_p - \widetilde{\mu}) - 1 \Big\} \Big\{ \delta(q_0 - E_{q-p} - E_p) \Big\} \\
& + \Big\{ n_F(E_{q-p} - \widetilde{\mu}) + n_F(E_p + \widetilde{\mu}) - 1 \Big\} \Big\{ \delta(q_0 + E_{q-p} + E_p) \Big\} \Bigg] ,
\end{aligned}
$$

$$
\boxed{\bar{\mu} = \mu + \Omega/2, \qquad \widetilde{\mu} = \mu - \Omega/2}
$$

$$
\mathcal{C}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) = -\frac{1}{480}\Big\{2(p_z - q_z)\big(q_0 - p_0 + 8p_z - 2p_0 + \Omega\big) + 8(q_z - p_z)^2 + 3(q_0 - p_0)\Omega
+ p_z\left(8p_z - 2p_0 + \Omega\right)\Big]\Big[(q_z - p_z)\big(4p_z + 4p_0 - 2\Omega\big) - 4(q_\perp - p_\perp)p_\perp + \big(2q_0 - 2p_0 - \Omega\big)\big(2p_z + 2p_0 - \Omega\big)\Big]
+ \frac{1}{10}\left[p_z\left(q_z - 2p_z\right)\right]\Big[(q_z - p_z)\big(4p_z - 4p_0 - 2\Omega\big) - 4(q_\perp - p_\perp)p_\perp + \big(2q_0 - 2p_0 - \Omega\big)\big(2p_z + 2p_0 + \Omega\big)\Big\},
$$

$$
\mathcal{D}(p_\perp, p_z, p_0, q_\perp, q_z, q_0) = -\frac{1}{480}\Big\{2(p_z - q_z)\big(q_0 - p_0 + 8p_z + 2p_0 + \Omega\big) + 8(q_z - p_z)^2 + 3(q_0 - p_0)\Omega
+ p_z\left(8p_z + 2p_0 + \Omega\right)\Big]\Big[-k_z\big(-4p_z + 4p_0 + 2\Omega\big) - 4(q_\perp - p_\perp)p_\perp + \big(2q_0 - 2p_0 + \Omega\big)\big(-2p_z + 2p_0 + \Omega\big)\Big]
+ \frac{1}{10}\left[p_z\left(q_z - 2p_z\right)\right]\Big[(q_z - p_z)\big(4p_z + 4p_0 + 2\Omega\big) - 4(q_\perp - p_\perp)p_\perp + \big(2q_0 - 2p_0 + \Omega\big)\big(-2p_z + 2p_0 + \Omega\big)\Big\}.
$$

Bulk viscosity from Kubo formalism

$$
\begin{aligned}
&\zeta(T,\mu,\Omega)\\
&= \frac{1}{8T}\Bigg\{\int \frac{d^3p\ p_z^2}{(2\pi)^3\ 4E_p^2\Gamma}\Big[\Big\{4\big(E_p+p_z\big)^2-4p_\perp^2-\Omega^2\Big\}\mathcal{N}(\pm\mu,\pm\Omega/2)\\
&+\Big\{4\big(E_p-p_z\big)^2-4p_\perp^2-\Omega^2\Big\}\mathcal{N}(\pm\mu,\mp\Omega/2)\Big]\Bigg\}\\
&+\frac{c_s^2}{12T}\Bigg\{\int \frac{d^3p}{(2\pi)^3\ 4E_p^2\Gamma}\Big[\Big\{-2p_z\Big(-2E_p^3+E_p^2(\Omega-8p_z)+2E_p(\Omega^2+p_\perp^2+2p_z(\Omega-p_z))\\
&+2\Omega(p_z^2-p_\perp^2)\Big)\Big\}\mathcal{N}(\pm\mu,\pm\Omega/2)-\frac{\Omega p_z}{2}\Big\{8E_p^2-4p_\perp^2+8E_pp_z+3p_z^2\Big\}\mathcal{N}(\pm\mu,\mp\Omega/2)\Big]\Bigg\}\\
&+\frac{c_s^4\Omega}{144T}\Bigg\{\int \frac{d^3p}{(2\pi)^3\ 4E_p^2\Gamma}\Big[\Big\{\Omega^3-4\Omega\Big(E_p^2+p_\perp^2-p_z^2\Big)+E_p\Big(3\Omega^2+4p_z(\Omega+p_z)\Big)\Big\}\mathcal{N}(\pm\mu,\pm\Omega/2)\\
&+\Big\{\Omega^3+8\Omega(p_\perp^2-2E_p^2)-4(2\Omega+E_p)p_z^2\Big\}\mathcal{N}(\pm\mu,\mp\Omega/2)\Big]\Bigg\}
\end{aligned}
$$

where $\mathcal{N}(\pm\mu,\pm\Omega/2)$ and $\mathcal{N}(\pm\mu,\mp\Omega/2)$ are given by

$$
\mathcal{N}(\pm\mu,\pm\Omega/2)=\sum_{s=\pm1} n_F(E_p+s\mu+s\Omega/2)\big(1-n_F(E_p+s\mu+s\Omega/2)\big)
$$

$$
\mathcal{N}(\pm\mu,\mp\Omega/2)=\sum_{s=\pm1} n_F(E_p+s\mu-s\Omega/2)\big(1-n_F(E_p+s\mu-s\Omega/2)\big),
$$

Shear viscosity from Kubo formalism

$$\eta(T,\mu,\Omega) = \frac{1}{240}\int \frac{d^3p}{(2\pi)^3}\Big(\frac{1}{4E_p^2\Gamma T}\Big)$$

$$\Big\{\Big\{160p_z^4 + 4p_\perp^2\Omega^2 - 3p_z\Omega^3 + 8p_\perp^2 p_z(\Omega - 4E_p) + \Omega^3(\Omega - E_p)$$

$$+32p_z^3(\Omega + 5E_p) - 8p_z^2\Omega(2E_p + 3\Omega)\Big\}\mathcal{N}(\pm\mu, \mp\Omega/2)$$

$$+\Big\{160p_z^4 + 4p_\perp^2\Omega^2 - 3p_z\Omega^3 + 8p_\perp^2 p_z(\Omega - 4E_p) + \Omega^3(\Omega - E_p)$$

$$+32p_z^3(\Omega - 5E_p) - 8p_z^2\Omega(2E_p - 3\Omega)\Big\}\mathcal{N}(\pm\mu, \pm\Omega/2)\Big\}$$

where $\mathcal{N}(\pm\mu, \pm\Omega/2)$ and $\mathcal{N}(\pm\mu, \mp\Omega/2)$ are given by

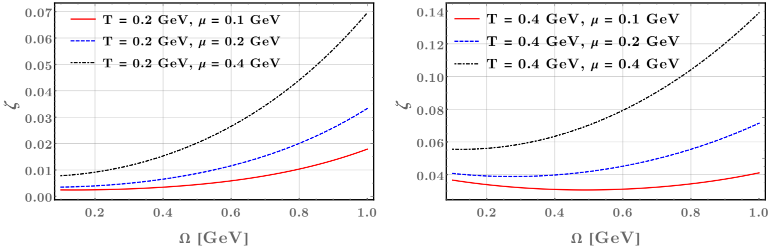
$$\mathcal{N}(\pm\mu, \pm\Omega/2) = \sum_{s=\pm1} n_F(E_p + s\mu + s\Omega/2)\{1 - n_F(E_p + s\mu + s\Omega/2)\}$$

$$\mathcal{N}(\pm\mu, \mp\Omega/2) = \sum_{\lambda=\pm1} n_F(E_p + \lambda\mu - \lambda\Omega/2)\{1 - n_F(E_p + \lambda\mu - \lambda\Omega/2)\},$$

$\zeta(\Omega)$ vs Ω

S. Satapathy, *Bulk viscosity of rotating, hot and dense spin 1/2 fermionic systems from correlation functions*, arxiv:2307.09953

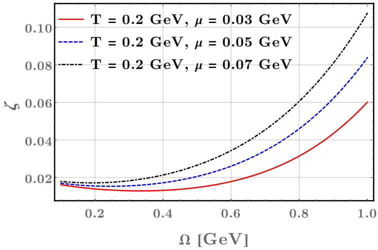
Ω : 0.1 - 1.0 GeV, μ : 0.1, 0.2, 0.4 GeV

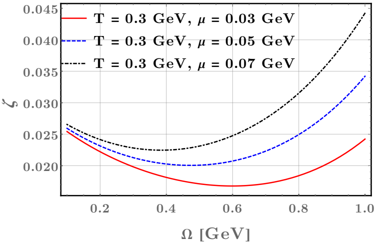


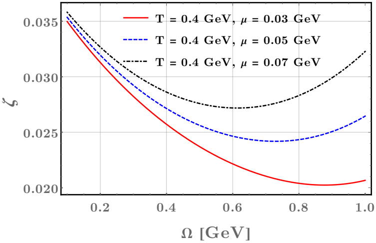
ζ increases with T, μ, Ω

$\zeta(\Omega)$ vs Ω at Low μ

Ω : 0.1 - 1.0 GeV, μ : 0.03, 0.05, 0.07 GeV



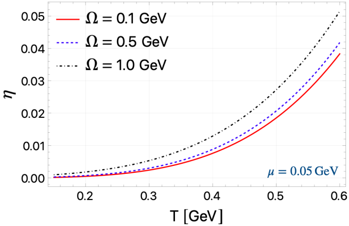


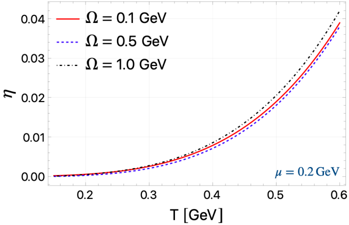


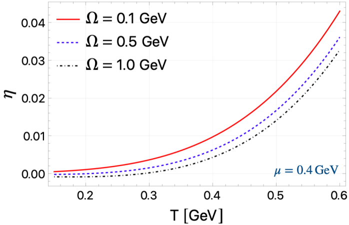
- Decreases in the region of low Ω. Anticipated to approach "zero" i.e conformality
- The increase at large Ω is due to the escaping of fermions from the cylinder which violate causality
- Ω has to be restricted between a certain range : Not too high or low

$\eta(T)$ vs T , μ vs Ω

S. Satapathy, R. Singh, P. Panday, S.A. Khan, D. Dey, *Shear viscosity of rotating, hot, and dense spin-half fermionic systems from quantum field theory* , arxiv:2309.05284







Limitations

• One should also consider the effect of magnetic field as well for charged spin 1/2 particles which could be produced in off-central collisions. Inclusion should be at the propagator level.

• Not applicable to smaller angular velocities. Limitation of the propagator.

• Not applicable to QGP. Gauge bosons have not been studied.

• A complete study should involve "Ladder Diagrams".

1-loop : $1/\Gamma$, N-loop ladder : g^{2N}/Γ^{N+1}, Resummation required

P. Romatschke, "Shear Viscosity at Infinite Coupling: A Field Theory Calculation," Phys. Rev. Lett. **127**, no.11, 111603 (2021). [arXiv:2104.06435 [hep-th]].

Thank you all for your patience & attention!!!

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References I

- A. Hosoya, M. Sakagami and M. Takao, Annals Phys. **154**, 229 (1984).
- X. G. Huang, A. Sedrakian and D. H. Rischke, "Kubo formulae for relativistic fluids in strong magnetic fields," Annals Phys. **326**, 3075-3094 (2011) doi:10.1016/j.aop.2011.08.001 [arXiv:1108.0602 [astro-ph.HE]].
- K. Hattori, X. G. Huang, D. H. Rischke and D. Satow, "Bulk Viscosity of Quark-Gluon Plasma in Strong Magnetic Fields," Phys. Rev. D **96**, no.9, 094009 (2017) [arXiv:1708.00515 [hep-ph]].
- K. Hattori and D. Satow, "Electrical Conductivity of Quark-Gluon Plasma in Strong Magnetic Fields," Phys. Rev. D **94**, no.11, 114032 (2016) [arXiv:1610.06818 [hep-ph]].
- S. Ghosh and S. Ghosh, "One-loop Kubo estimations of the shear and bulk viscous coefficients for hot and magnetized Bosonic and Fermionic systems," Phys. Rev. D **103**, 096015 (2021) [arXiv:2011.04261 [hep-ph]].
- S. Satapathy, S. Ghosh and S. Ghosh, "Kubo estimation of the electrical conductivity for a hot relativistic fluid in the presence of a magnetic field," Phys. Rev. D **104**, no.5, 056030 (2021) [arXiv:2104.03917 [hep-ph]].
- S. Satapathy, S. Ghosh and S. Ghosh, "Quantum field theoretical structure of electrical conductivity of cold and dense fermionic matter in the presence of a magnetic field," Phys. Rev. D **106**, no.3, 036006 (2022) [arXiv:2112.08236 [hep-ph]].
- S. Satapathy, "Bulk viscosity of rotating, hot and dense spin 1/2 fermionic systems from correlation functions," [arXiv:2307.09953 [nucl-th]].
- S. Satapathy, R. Singh, P. Panday, S. A. Khan and D. Dey, "Shear viscosity of rotating, hot, and dense spin-half fermionic systems from quantum field theory," [arXiv:2309.05284 [hep-ph]].

References II

L. Adamczyk *et al.* [STAR], "Global Λ hyperon polarization in nuclear collisions: evidence for the most vortical fluid," Nature **548**, 62-65 (2017) doi:10.1038/nature23004 [arXiv:1701.06657 [nucl-ex]].

F. Becattini, L. Tinti, "Nonequilibrium thermodynamical inequivalence of quantum stress-energy and spin tensors," Phys. Rev. D **87**, 025029, (2013).

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar and R. Ryblewski, "Relativistic dissipative spin dynamics in the relaxation time approximation," Phys. Lett. B **814**, 136096 (2021) [arXiv:2002.03937 [hep-ph]].

J. Hu, "Kubo formulae for first-order spin hydrodynamics," Phys. Rev. D **103**, no.11, 116015 (2021) [arXiv:2101.08440 [hep-ph]].

T. K. Chyi, C. W. Hwang, W. F. Kao, G. L. Lin, K. W. Ng and J. J. Tseng, "The weak field expansion for processes in a homogeneous background magnetic field," Phys. Rev. D **62**, 105014 (2000) [arXiv:hep-th/9912134 [hep-th]].

A. Kuznetsov and N. Mikheev, "Electroweak processes in external electromagnetic fields," *Springer Tracts Mod. Phys.* **197** (2004) 1–120.

A. Kuznetsov and N. Mikheev, eds., *Electroweak Processes in External Active Media*, vol. 252 of *Springer Tracts in Modern Physics*.
2013.

M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," JHEP **11**, 150 (2021) [arXiv:2107.14231 [hep-th]].

References III

A. Ayala, L. A. Hernández, K. Raya and R. Zamora, "Fermion propagator in a rotating environment," Phys. Rev. D **103**, no.7, 076021 (2021) [erratum: Phys. Rev. D **104**, no.3, 039901 (2021)] [arXiv:2102.03476 [hep-ph]].

M. Laine and A. Vuorinen, "Basics of Thermal Field Theory," Lect. Notes Phys. **925**, pp.1-281 (2016) Springer, 2016.

M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," JHEP **11**, 150 (2021) [arXiv:2107.14231 [hep-th]].

S. Satapathy, S. Ghosh and S. Ghosh, "Kubo estimation of the electrical conductivity for a hot relativistic fluid in the presence of a magnetic field," Phys. Rev. D **104**, no.5, 056030 (2021) [arXiv:2104.03917 [hep-ph]].

S. Satapathy, S. Ghosh and S. Ghosh, Phys. Rev. D **106**, no.3, 036006 (2022) doi:10.1103/PhysRevD.106.036006 [arXiv:2112.08236 [hep-ph]].

R. H. Fang, "Thermodynamics for a Rotating Chiral Fermion System in the Uniform Magnetic Field," Symmetry **14**, no.6, 1106 (2022) doi:10.3390/sym14061106 [arXiv:2112.03468 [hep-th]].