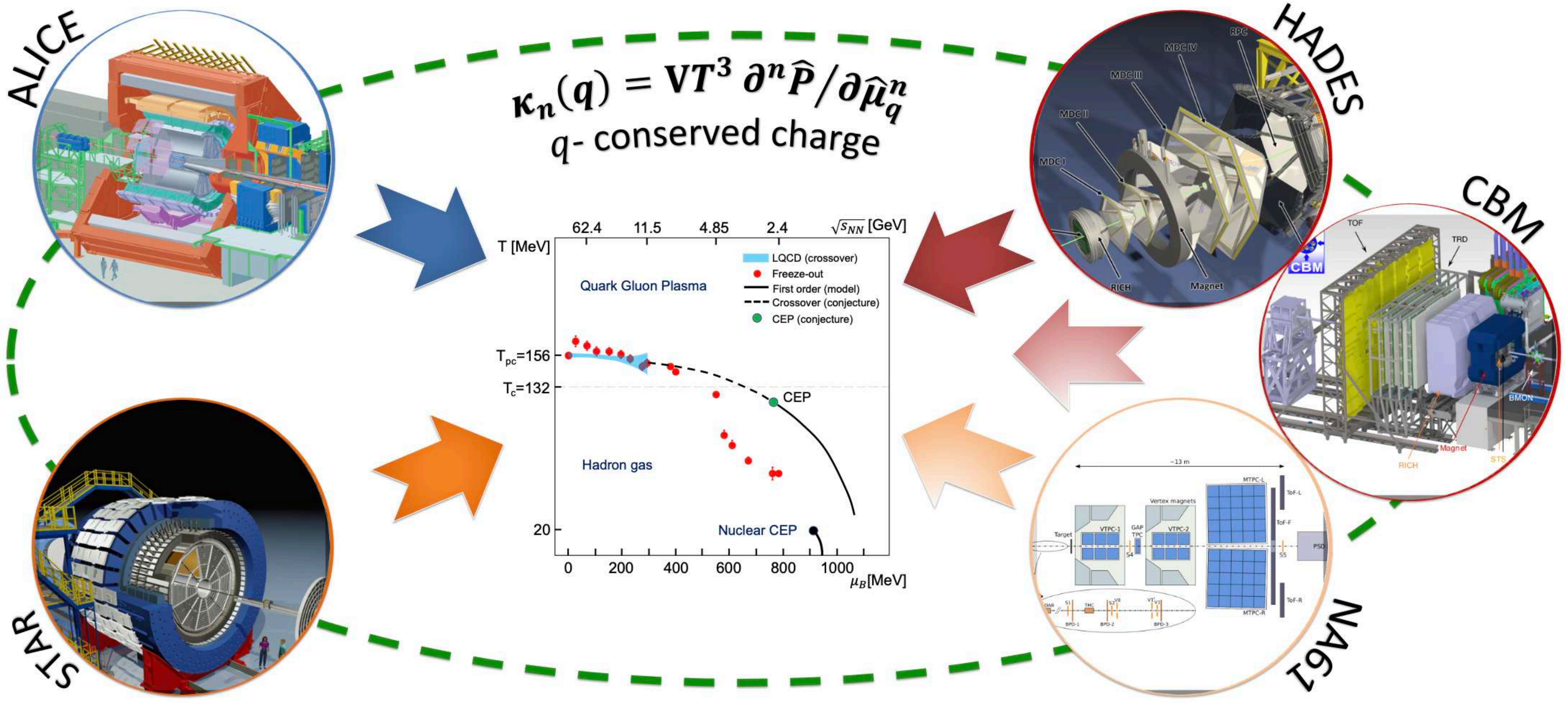
Deciphering the Phase Structure of QCD via Event-by-Event Particle Number Fluctuations

Anar Rustamov



a.rustamov@gsi.de

Outline

Electromagnetically interacting matter

- Ultimate temperature
- Liquid-gas phase transition
- Critical point discoveries

Strongly interacting matter

- Ultimate temperature
- QGP-hadron gas phase transition

Ideal Gas baselines

- Minimal baseline
- Conservation laws

Challenges in measurements

- Particle misidentification issues
- Participant fluctuations

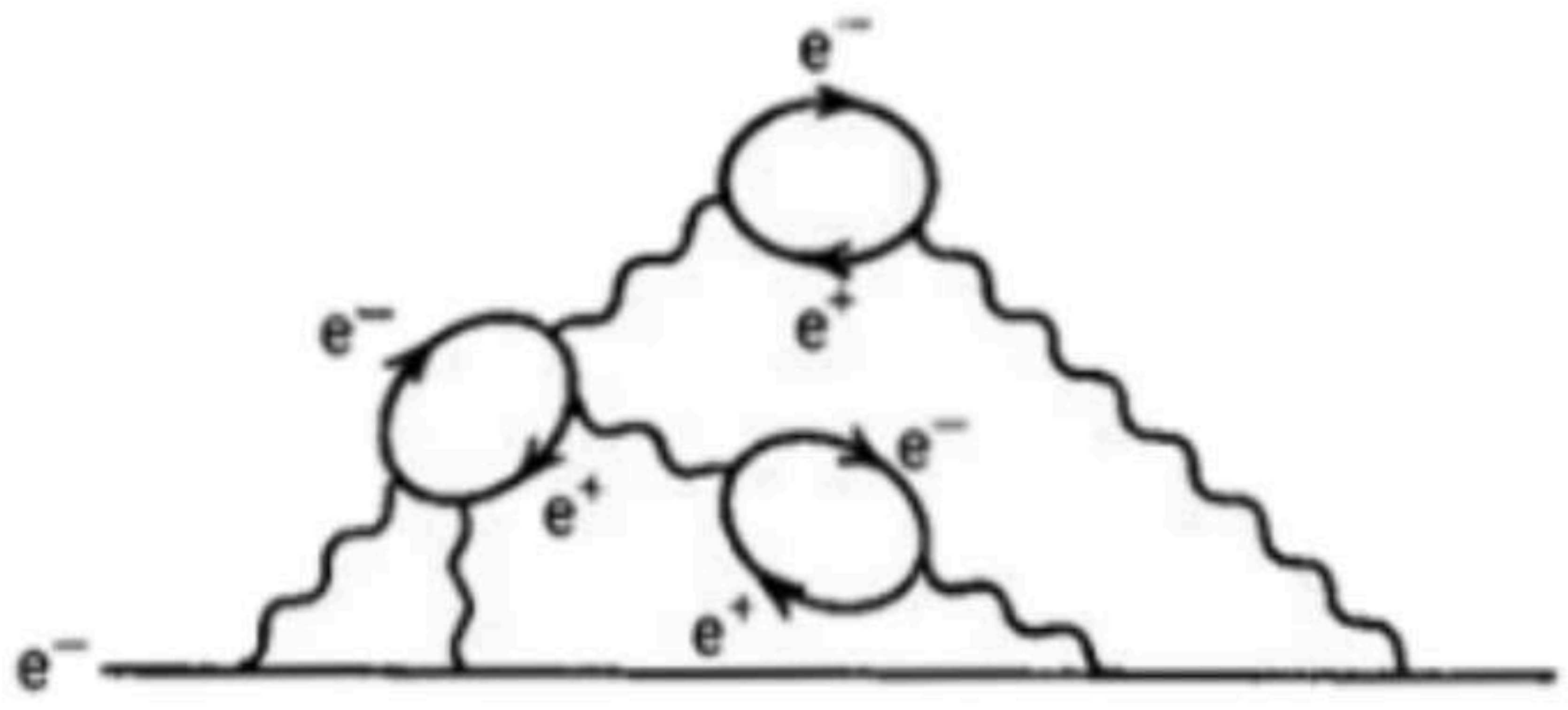
Experimental results

- Search for a crossover transition
- Search for a critical point

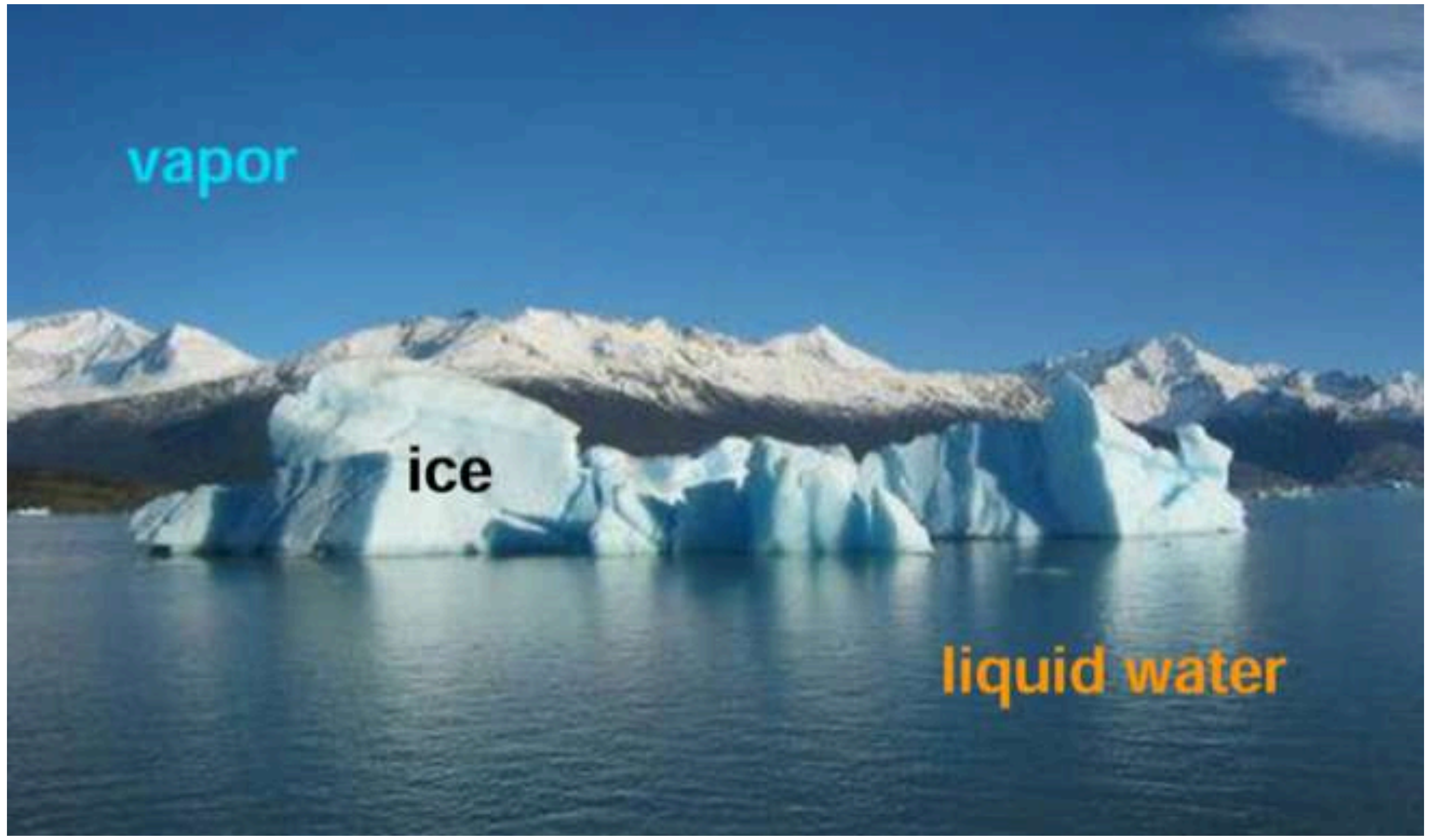
Future prospects

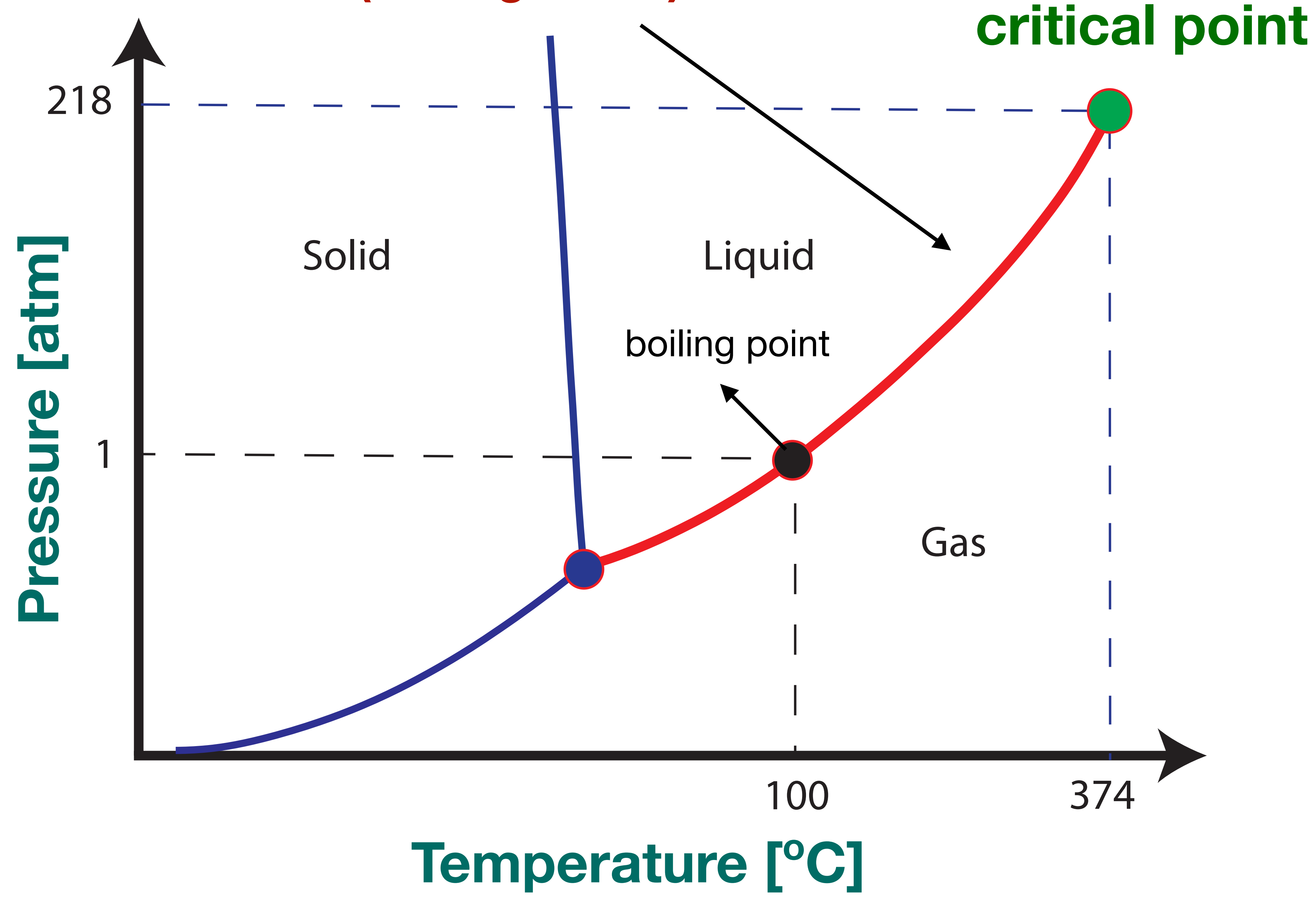
Electromagnetically interacting matter

- Ultimate temperature
- Liquid-gas phase transition
- Critical point discoveries



Ultimate temperature



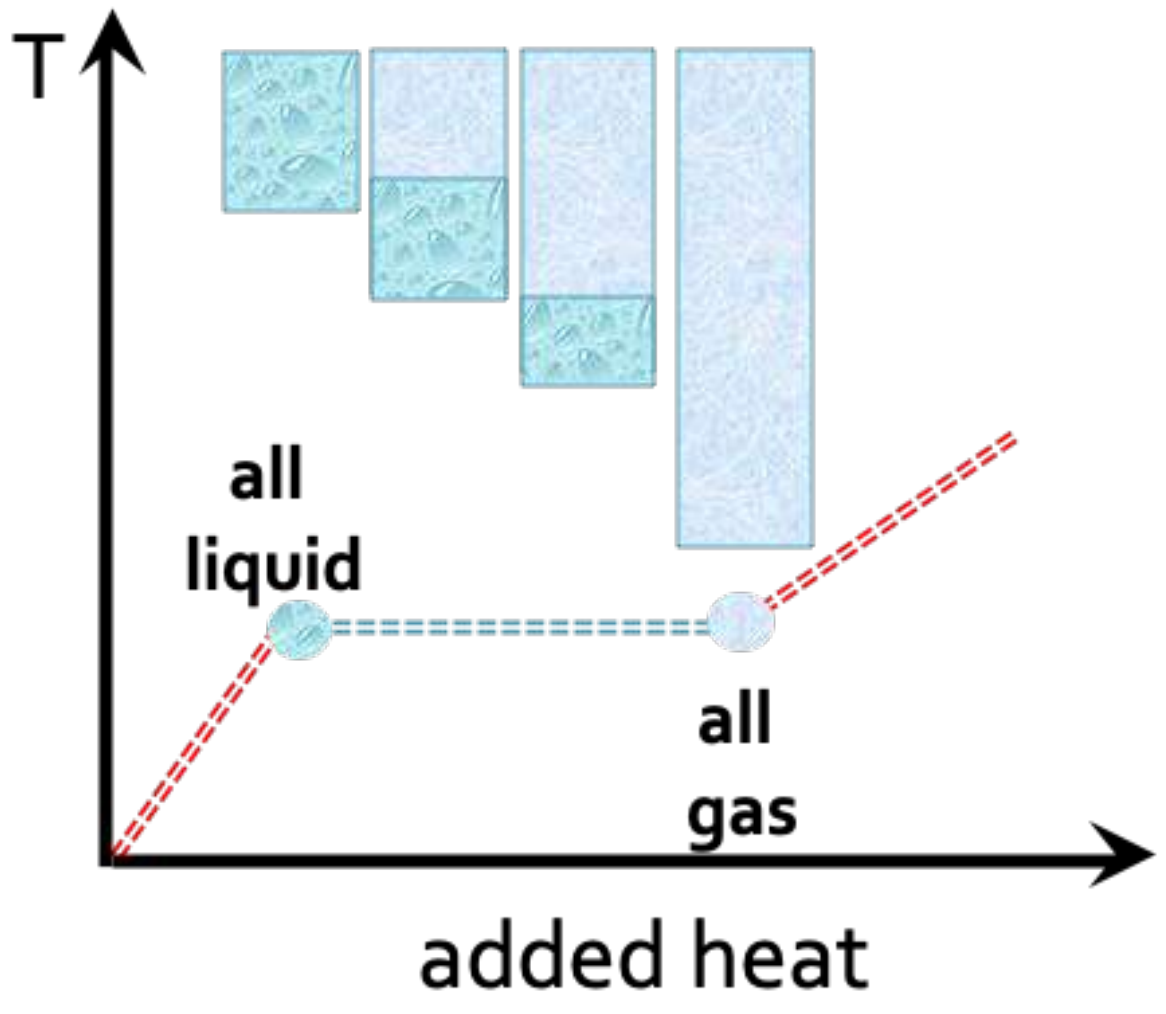


Relevant for this presentation:

- **boiling point at a given pressure:**
  - **ultimate temperature at which stable liquid can exist at that pressure**
- **boiling curve (first order phase transition) terminates at the critical point (second order phase transition)**

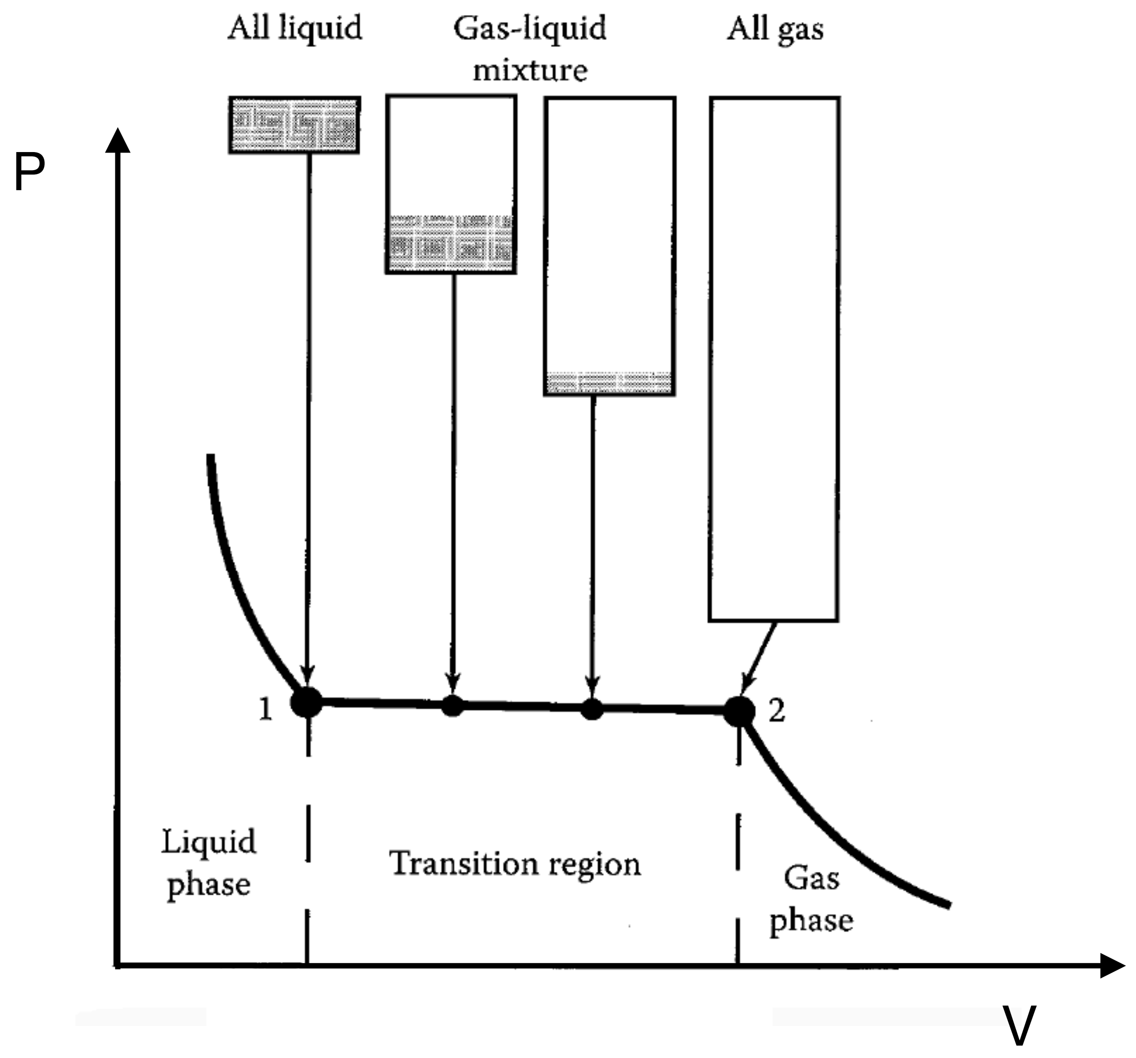
Caveat: Existence of ultimate temperature!

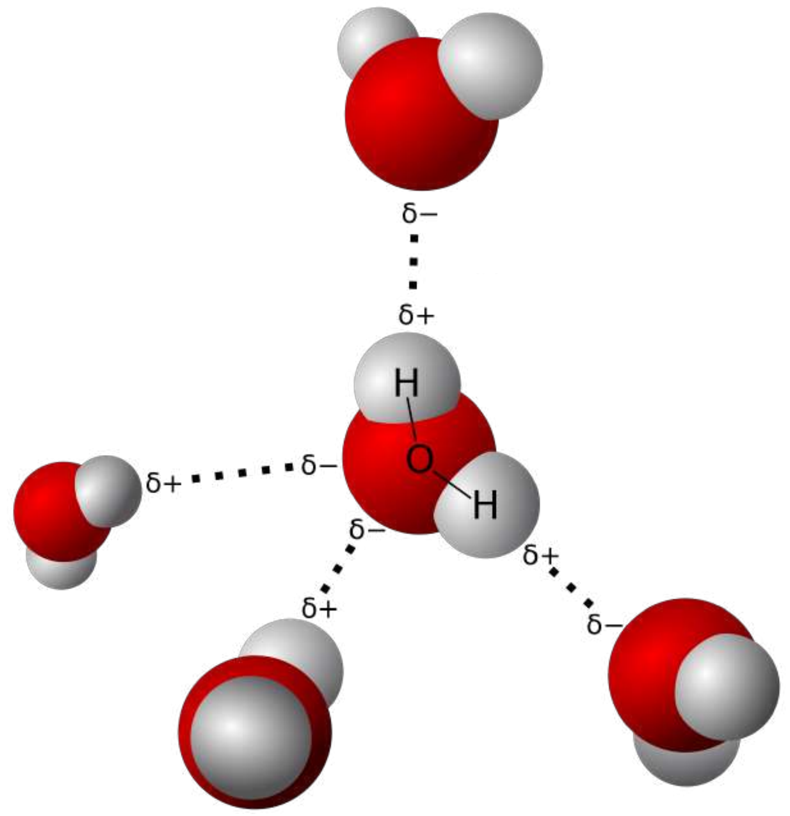
Phase transitions, the role of interactions; Episode I



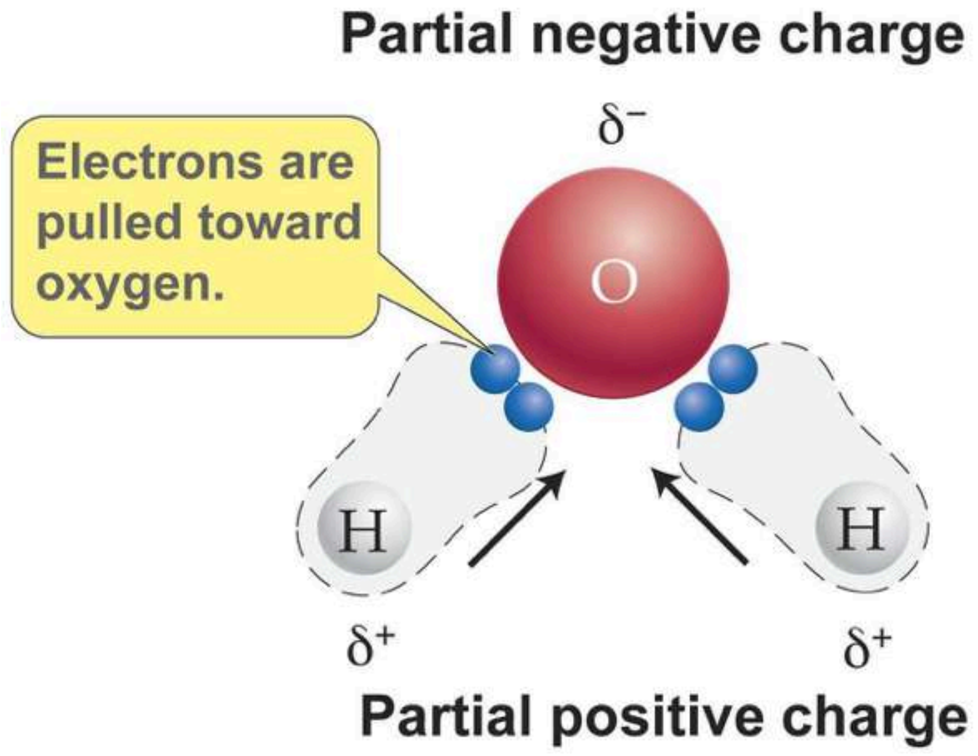
Signatures for first order phase transition

- Constant temperature (latent heat)
- Constant pressure
- Existence of a mixed phase





electronegativity



- Starting from boiling point the supplied heat is spent to break hydrogen bonds
- Interactions are important for phase transitions

Van der Waals, Nobel Prize (1910)

Van der Waals Equation of State (EoS)

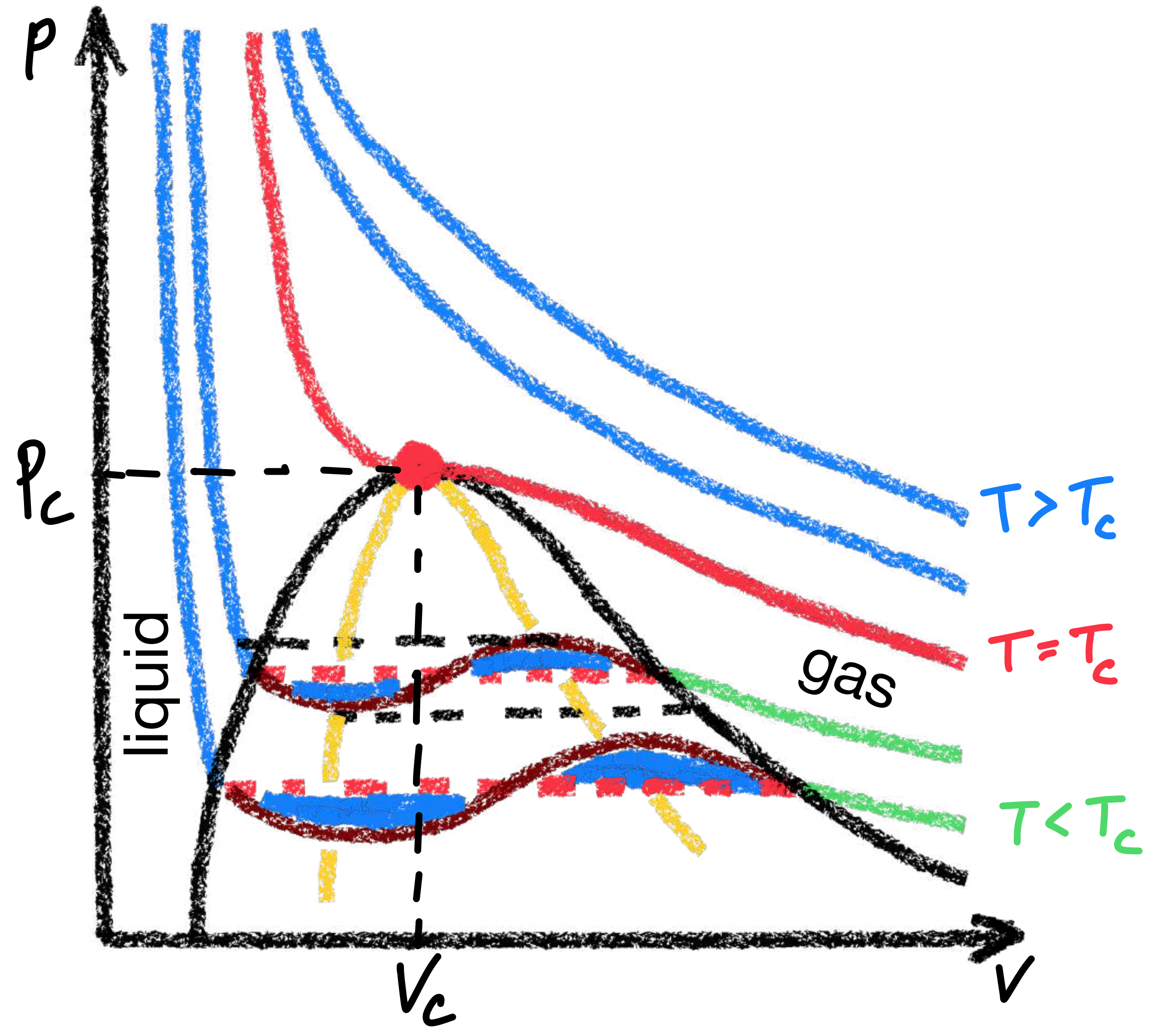
$$(V-b)\left(P+\frac{a}{V^2}\right)=RT$$

a - attraction

b - repulsion

from statistical mechanics in GCE

$$\frac{\langle\rho^2\rangle-\langle\rho\rangle^2}{\langle\rho\rangle^2}=\frac{T\chi_T}{V} \qquad \chi_k=-\frac{1}{V\left(\frac{\partial P}{\partial V}\right)_T}$$



- χ_T - diverges at critical point
- enhanced density fluctuations

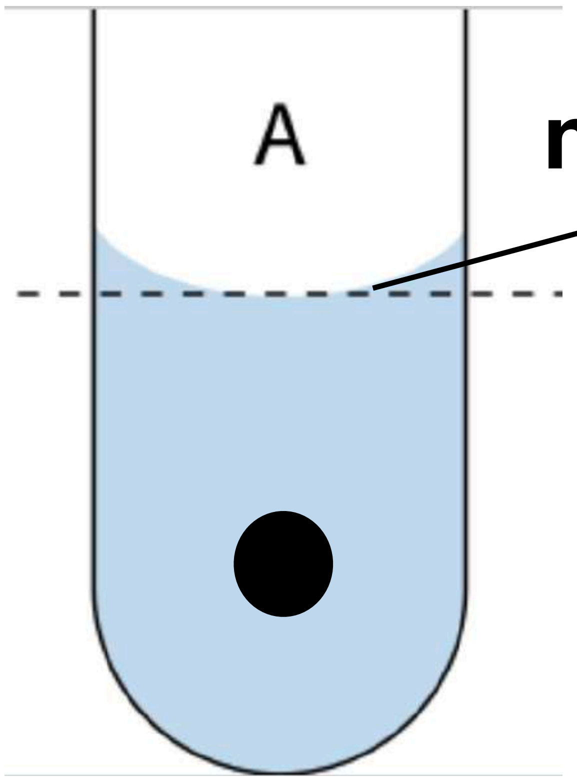
Signatures for second order phase transition

- Enhanced density fluctuations
- Increased compressibility
- Increase in density-density correlations

- Interactions are important for phase transitions

Critical point discoveries

discovered ~ 200 years ago



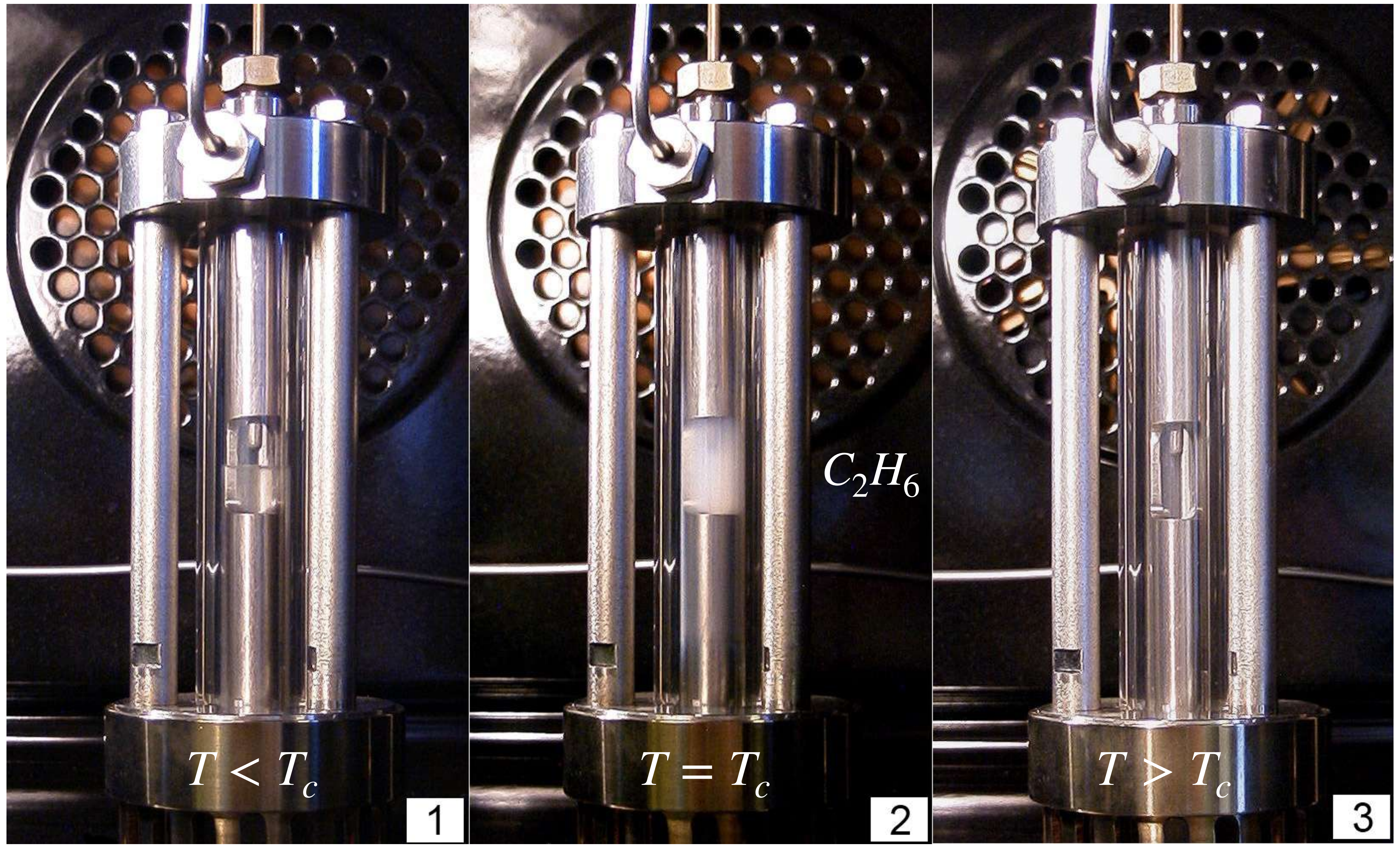
Cagniard de la Tour (1777-1859)

Ann. Chim. Phys., 21 (1822) 127-132

using steam digester
invented by Denis Papin in 1679

T_{cp}^{water} = **362 °C (today: 374 °C)**

critical opalescence



in statistical mechanics (GCE)

$$\frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle^2} = \frac{T\chi_T}{V}$$

density fluctuations

A. Einstein, Annalen der Physik, Volume 338, Issue 16, 1910: $h \sim \frac{1}{\lambda^4}\chi_T$

By listening to the system — **discovering critical point** — **By watching the system**

Interpretation of critical opalescence

11. *Theorie der Opaleszenz von homogenen Flüssigkeiten und Flüssigkeitsgemischen in der Nähe des kritischen Zustandes;*
von A. Einstein.

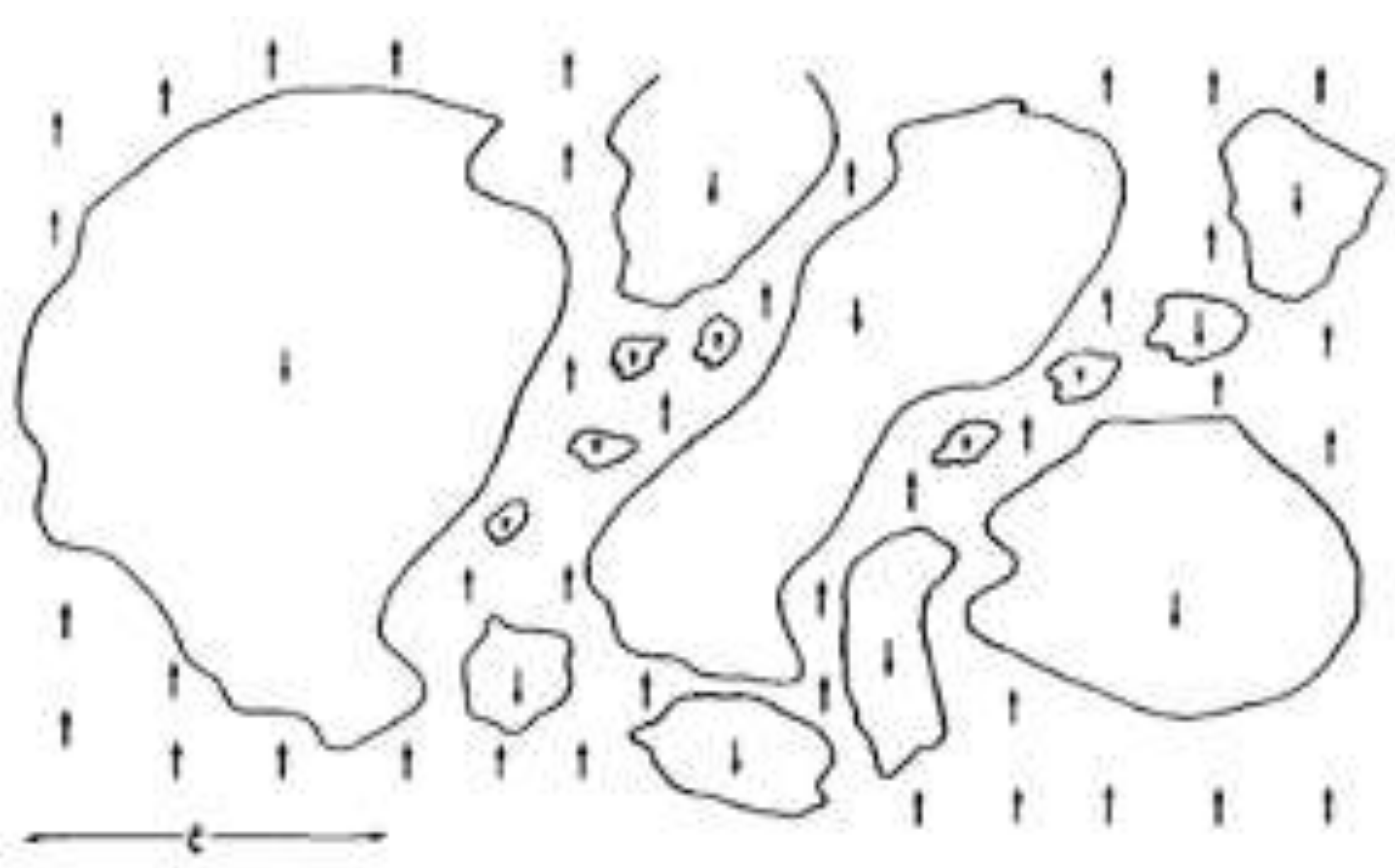
Smoluchowski hat in einer wichtigen theoretischen Arbeit[1]) gezeigt, daß die Opaleszenz bei Flüssigkeiten in der Nähe des kritischen Zustandes sowie die Opaleszenz bei Flüssigkeitsgemischen in der Nähe des kritischen Mischungsverhältnisses und der kritischen Temperatur vom Standpunkte der Molekulartheorie der Wärme aus in einfacher Weise erklärt werden kann. Jene Erklärung beruht auf folgender allgemeiner Folgerung aus Boltzmanns Entropie — Wahrscheinlichkeitsprinzip: Ein nach außen abgeschlossenes physikalisches System durchläuft im Laufe unendlich langer Zeit alle Zustände, welche mit dem (konstanten) Wert seiner Energie vereinbar sind. Die statistische Wahrscheinlichkeit eines Zustandes ist hierbei aber nur dann merklich von Null verschieden, wenn die Arbeit, die man nach der Thermodynamik zur Erzeugung des Zustandes aus dem Zustande idealen thermodynamischen Gleichgewichtes aufwenden müßte, von derselben Größenordnung ist, wie die kinetische Energie eines einatomigen Gasmoleküls bei der betreffenden Temperatur.

The theory of the Opalescence of homogeneous fluids and liquid mixtures near the critical state

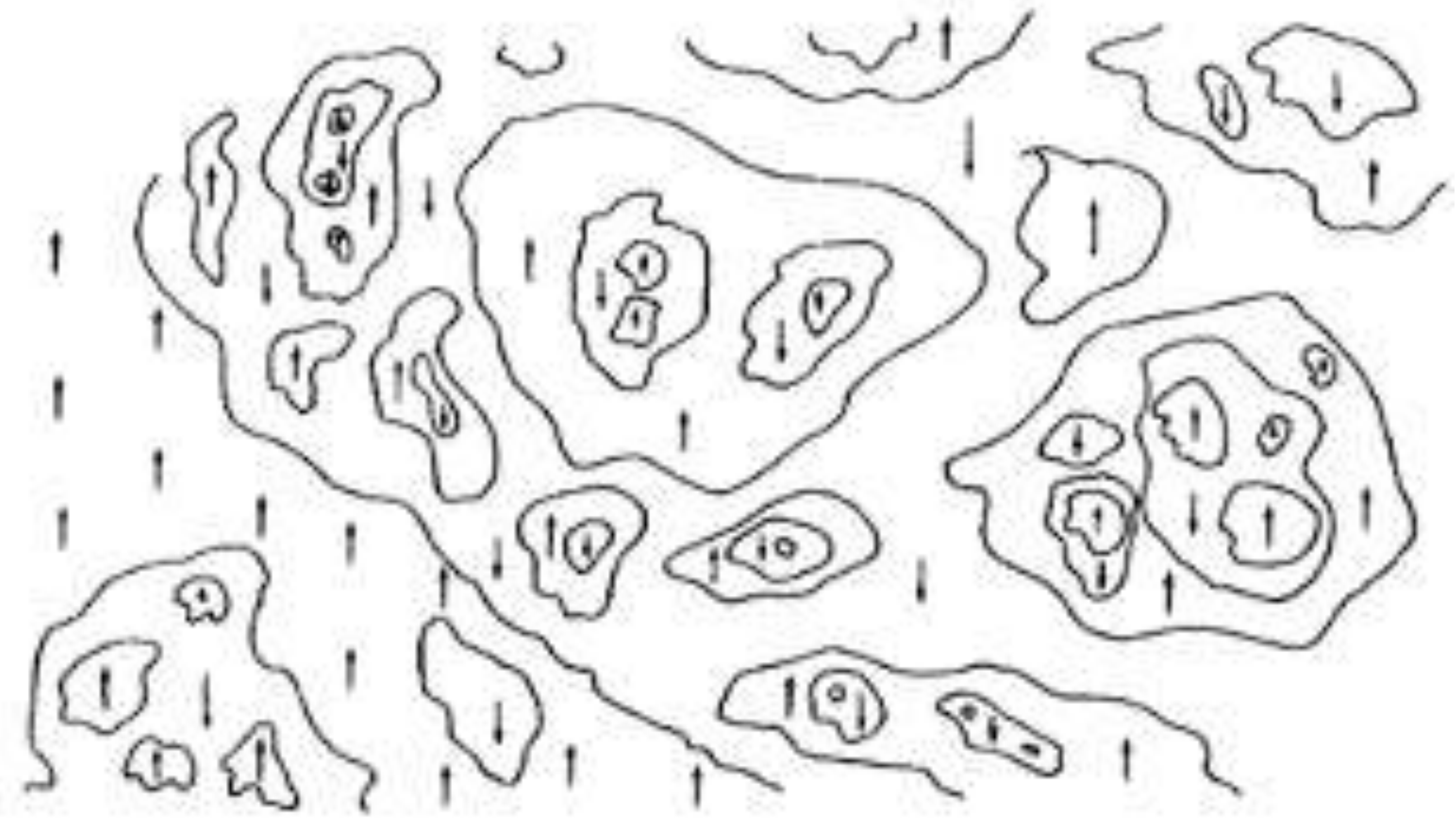
In an important theoretical paper Smoluchowski has shown that the opalescence of fluids near the critical state as well as the opalescence of liquid mixtures near the critical mixing ratio and the critical temperature can be explained in a simple way from the point of view of the molecular theory of heat.

The statistical probability of a state is noticeably different from zero only when the work that would have to be expended according to thermodynamics to produce the state in question from the state of ideal thermodynamic equilibrium is of the same order of magnitude as the kinetic energy of a monatomic gas molecule at the temperature under consideration.

Self-similarity near critical point



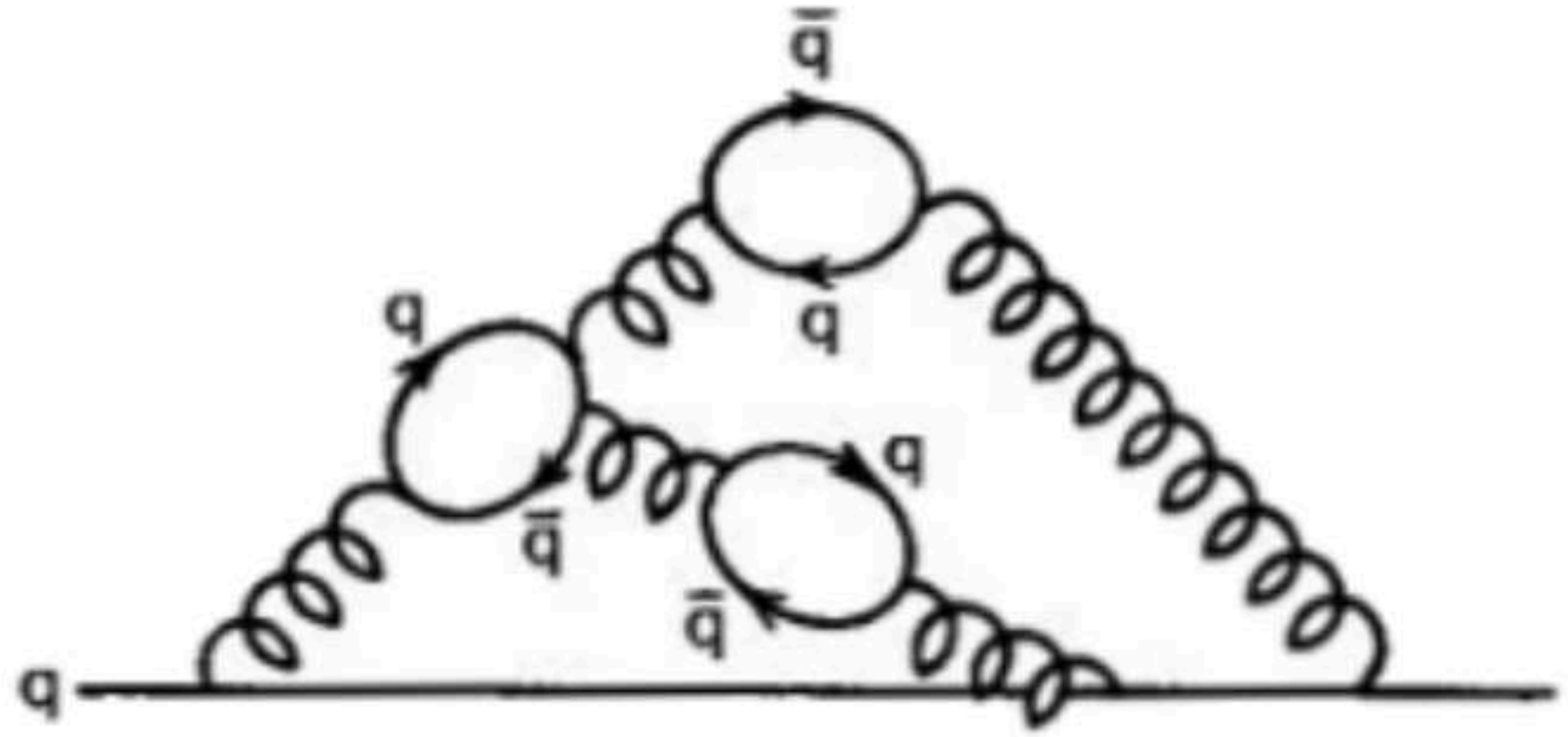
correlations length

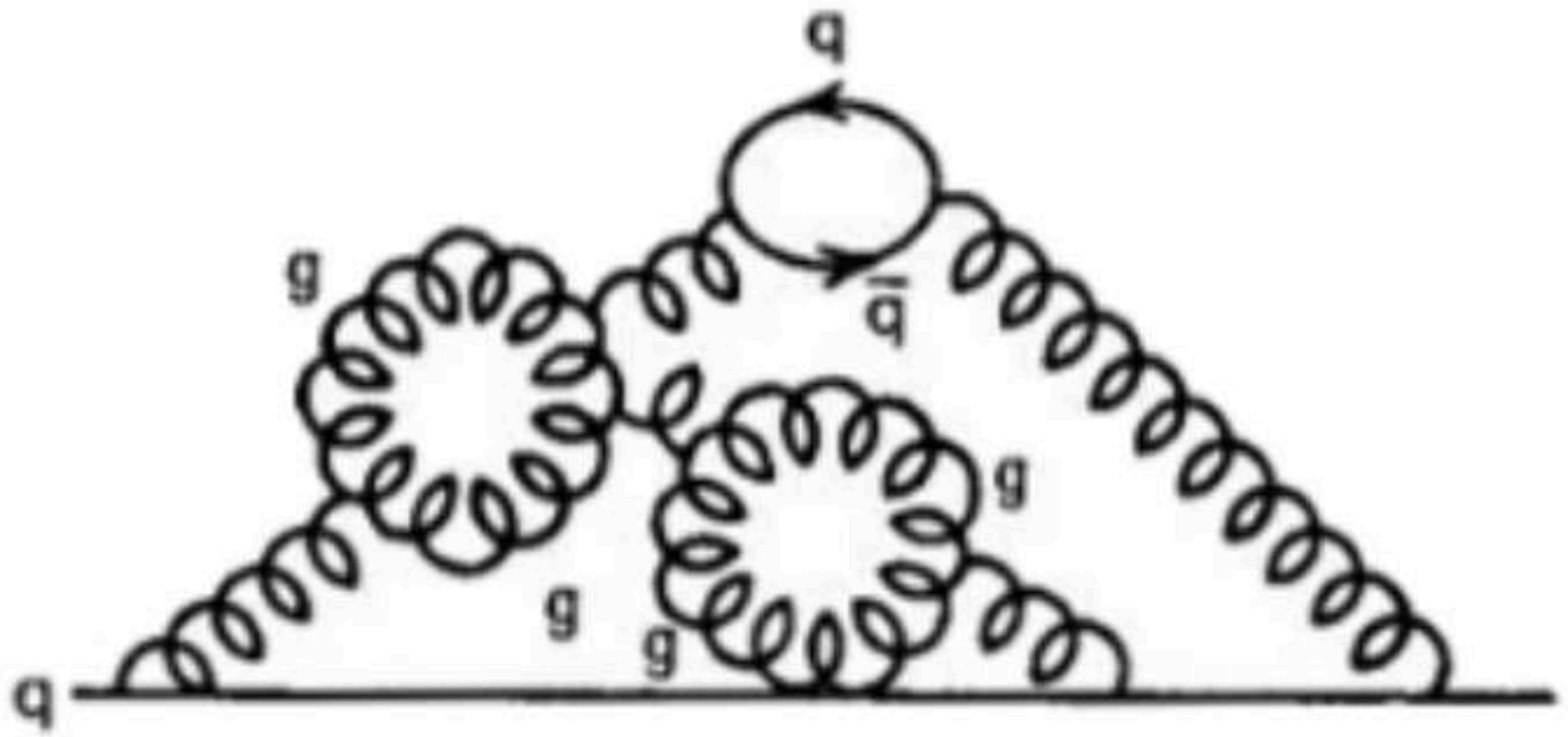


- droplets inside droplets of droplets
- <u>correlation at all lengths</u>
- <u>scale invariance, onset of self-similarity</u>
- intermittency (NA49/NA61/ALICE/STAR)

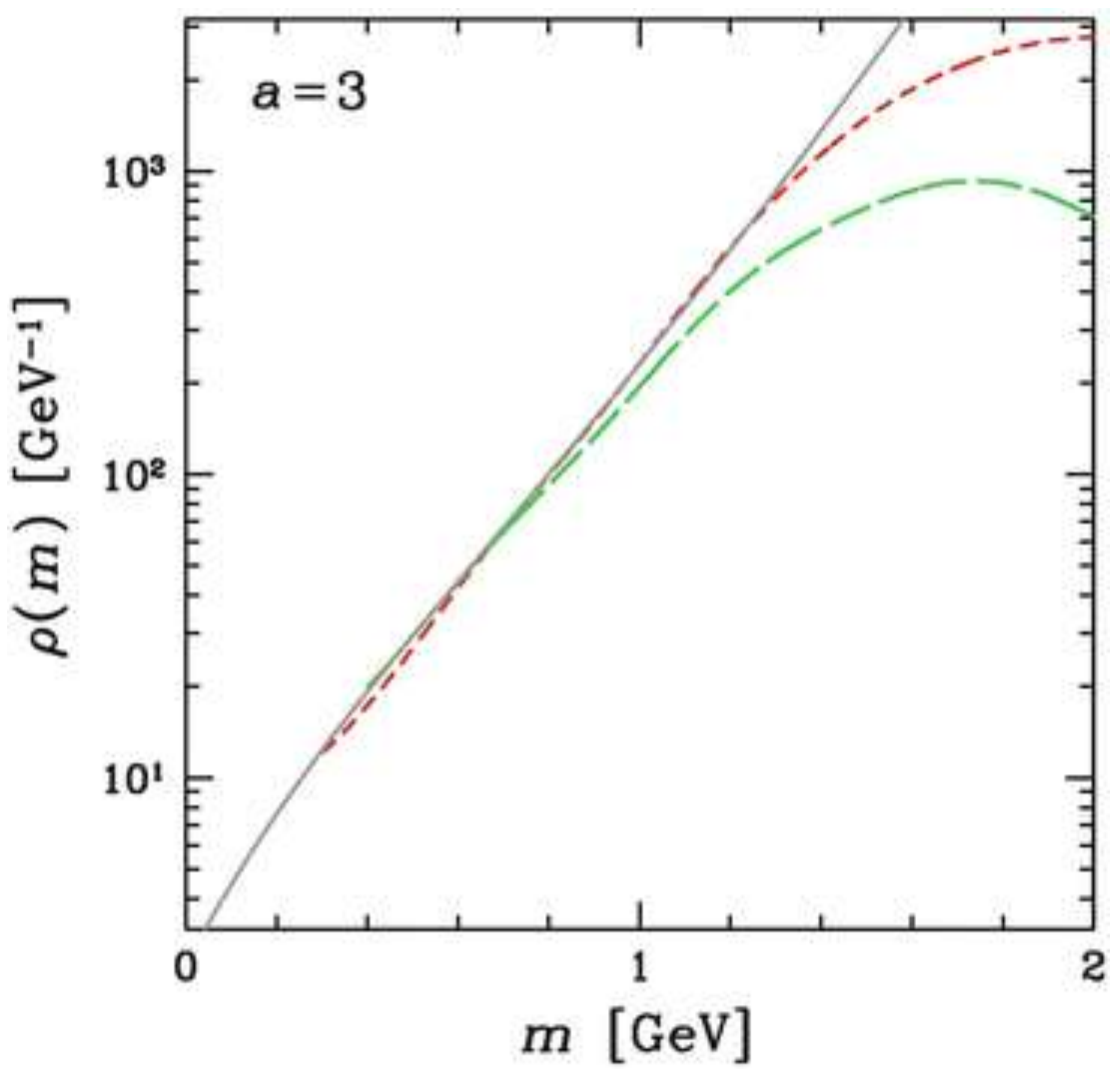
Strongly interacting matter

- Ultimate temperature
- QGP-hadron gas phase transition
- Predicted signals for critical phenomena





Ultimate temperature, strongly interacting matter



statistical bootstrap model (1965)

$$\rho(m) = c \times m^{-3} exp[m/T_H]$$

$T_H \approx$ **160 MeV (Hagedorn, 1965)**

Partition function for a resonance gas in equilibrium (Boltzmann limit)

$$lnZ(T,V) \sim \frac{VT}{2\pi^2}\int dm\rho(m)K_2(m/T)m^2$$

$K_2(m/T) \approx (T/m)^{1/2} exp[-m/T]$, for $m/T \gg 1$ (hadronic matter)

$$lnZ(T,V) \approx V\left(\frac{T}{2\pi}\right)^{3/2}\int \frac{dm}{m^{3/2}} exp\left[\frac{m}{T_H} - \frac{m}{T}\right]$$

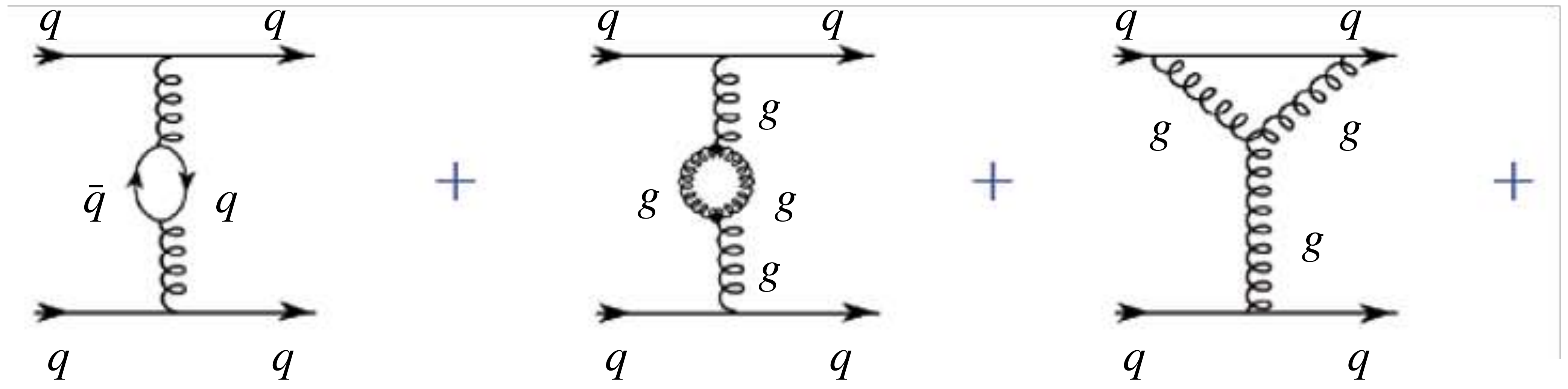
for $T > T_H$ partition function diverges!

Ultimate temperature

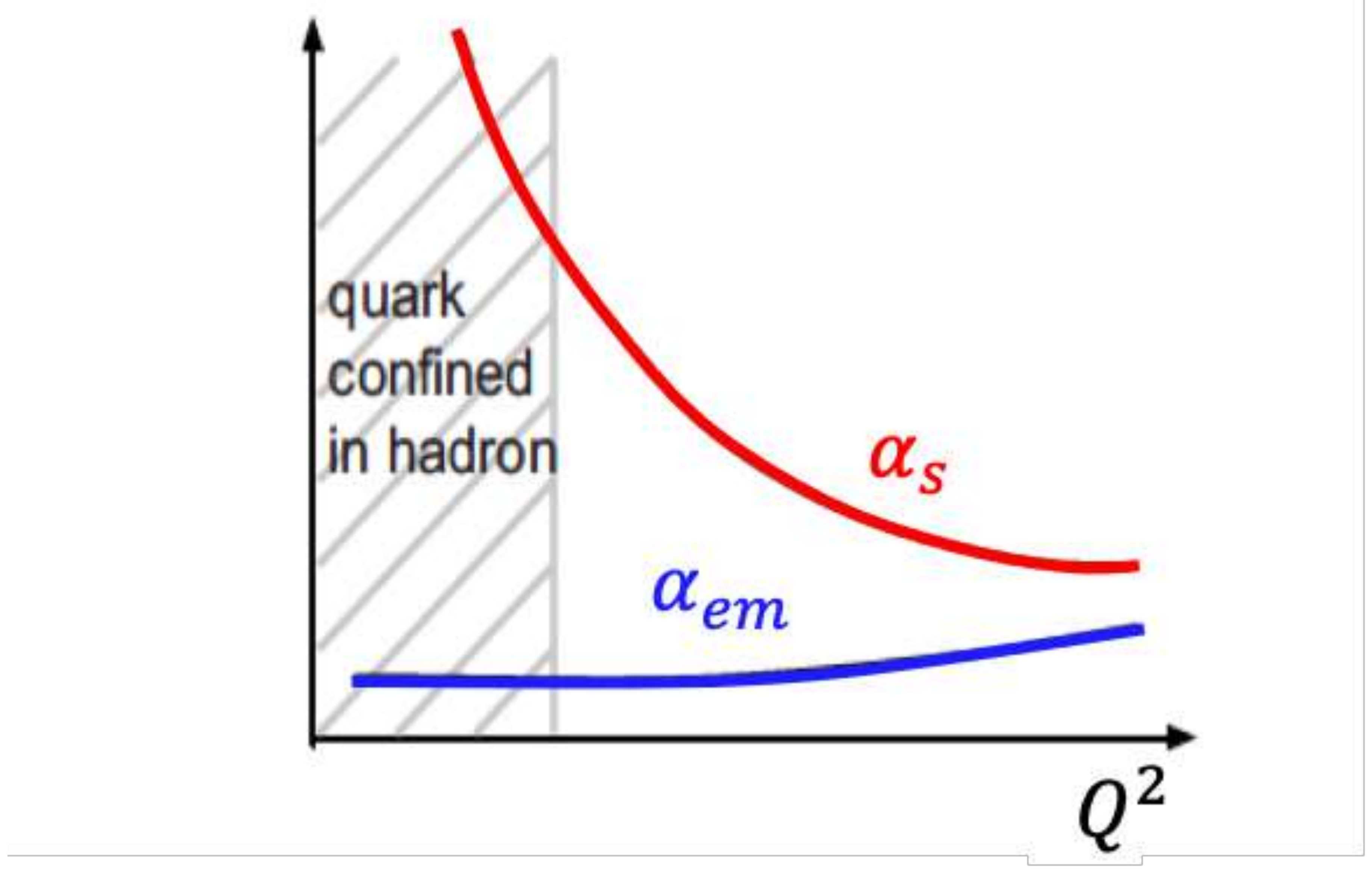
- T_H **is the highest possible temperature of hadronic matter (boiling point)**
  - **signals the transition from hadronic matter to QGP!**

QCD vs. QED

Effective (running) coupling constant

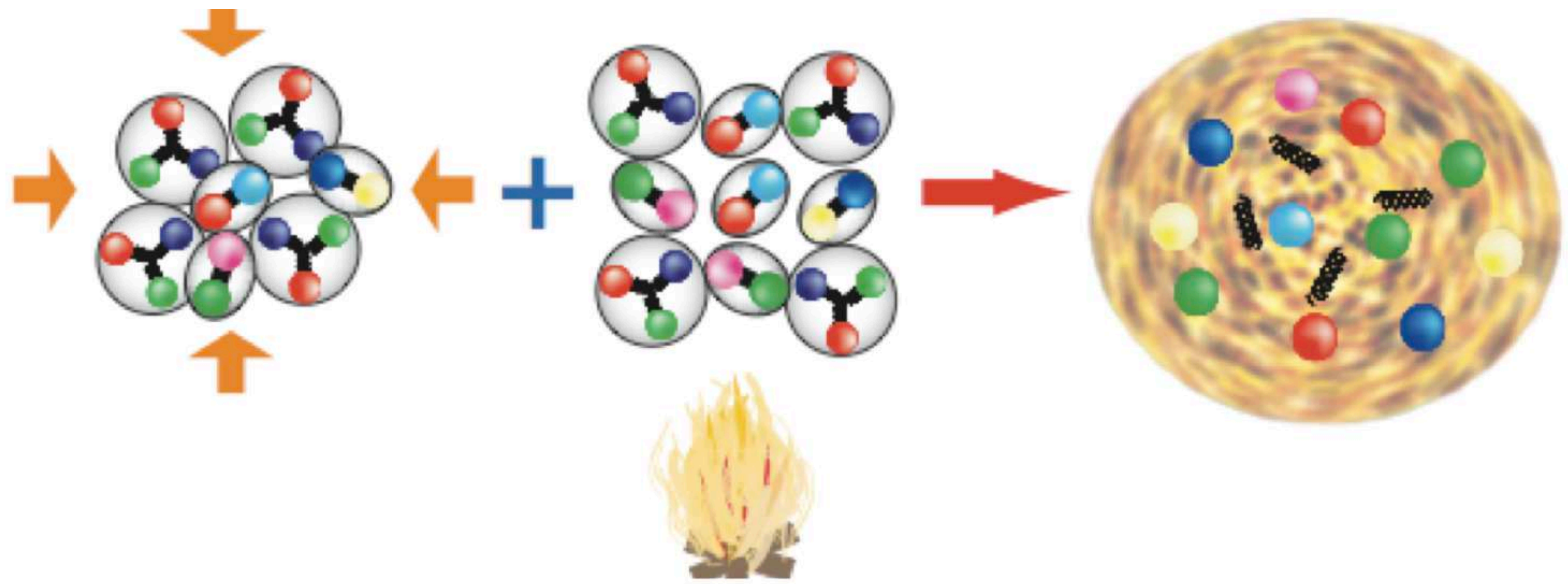


Like in QED — New contributions in QCD



$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1+\beta_0\alpha(\mu^2)ln(Q^2/\mu^2)}$$

recipe for creating the QGP in laboratories



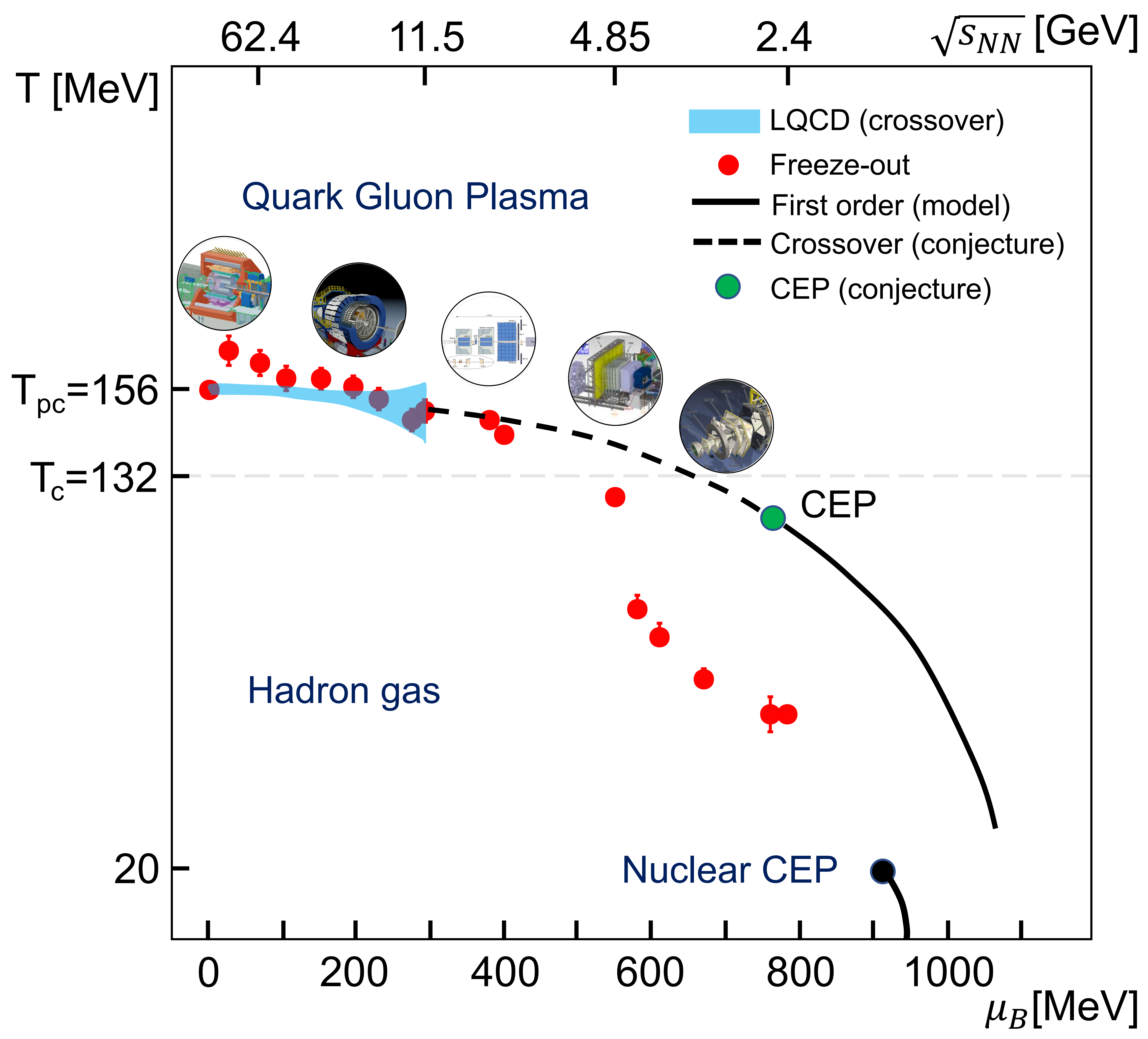
QED: $\beta_0 = -\frac{1}{3\pi} < 0$ - **electric charge screening**

QCD: $\beta_0 = -\frac{2n_f}{12\pi} + \frac{11N_c}{12\pi} > 0$ - **colour charge anti-screening**

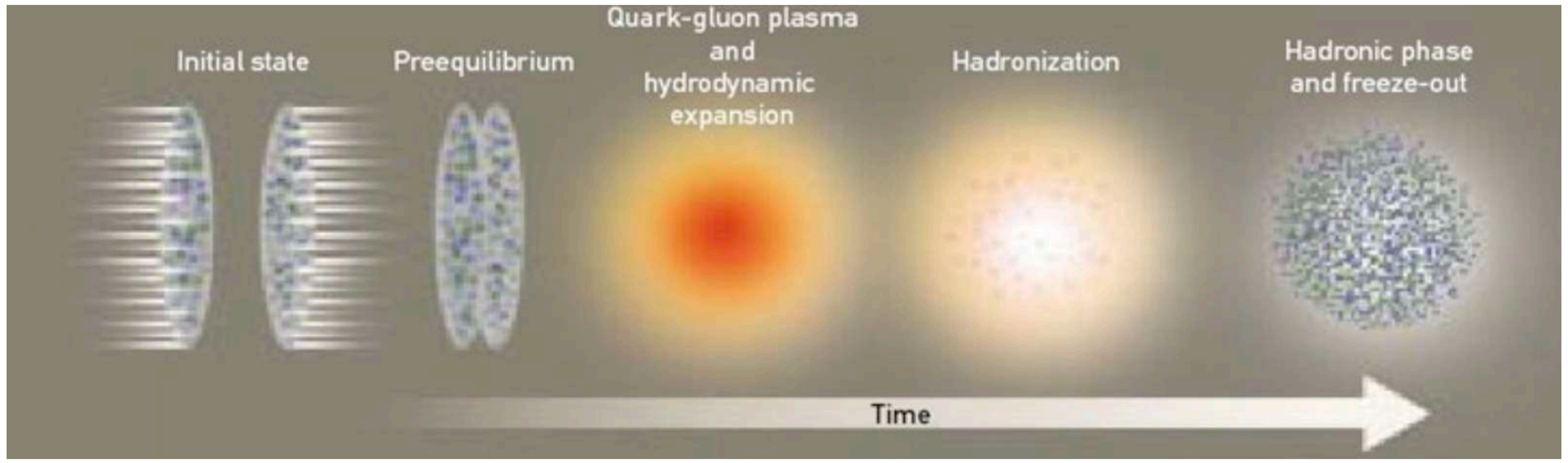
$N_c = 3, n_f = 6$

Asymptotic freedom (1973)
(Nobel Prize 2004)

Probing the QCD phases via fluctuations



P. Braun-Munzinger, A.R., J. Stachel, arXiv:2211.08819
F. Gross et al., arXiv:2212.11107



Different phases are defined their by EoS

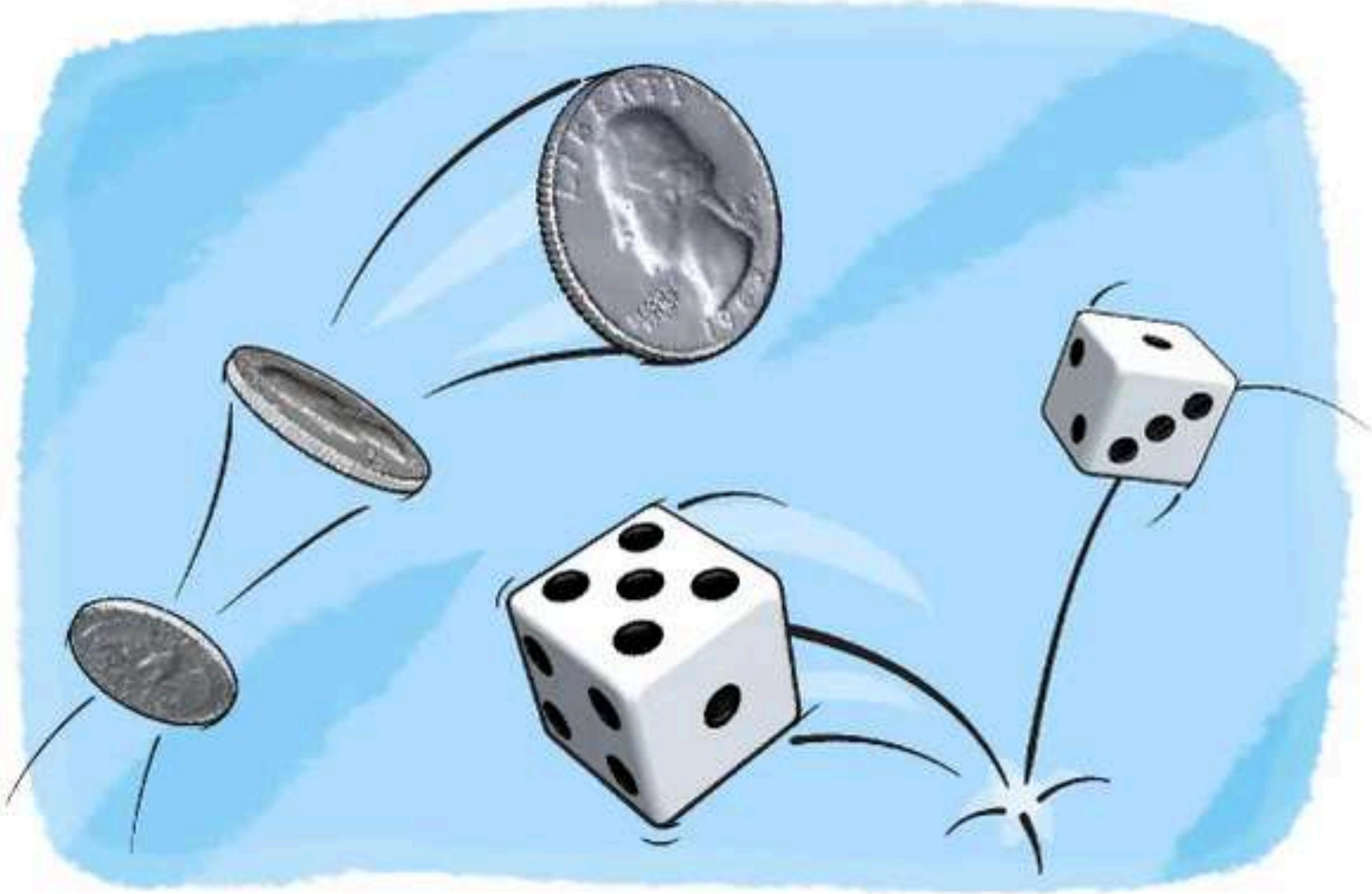
direct link to EoS

$$\hat{\chi}_n^B \equiv \frac{\partial^n \hat{P}}{\partial(\mu_B/T)} = \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} = \frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3}$$

for a thermal system of fixed volume V and temperature T

$\kappa_n(N_B - N_{\bar{B}})$ **- cumulants (measurable in experiment)**

$\hat{\chi}_n^B$ **- susceptibilities (e.g. from lQCD)**



Notations and Definitions

- Moments and cumulants
- Correlation functions

Some probability distributions

- Poisson distribution
- Skellam distribution

Fluctuations in Ideal Gas

Moments, Cumulants

Given a discrete random variable X, with its probability distribution $P(X)$

- **r^{th} order raw moments (moments about origin)**

$$\langle X^r \rangle = \sum_X X^r P(X)$$

- **r^{th} order central moments**

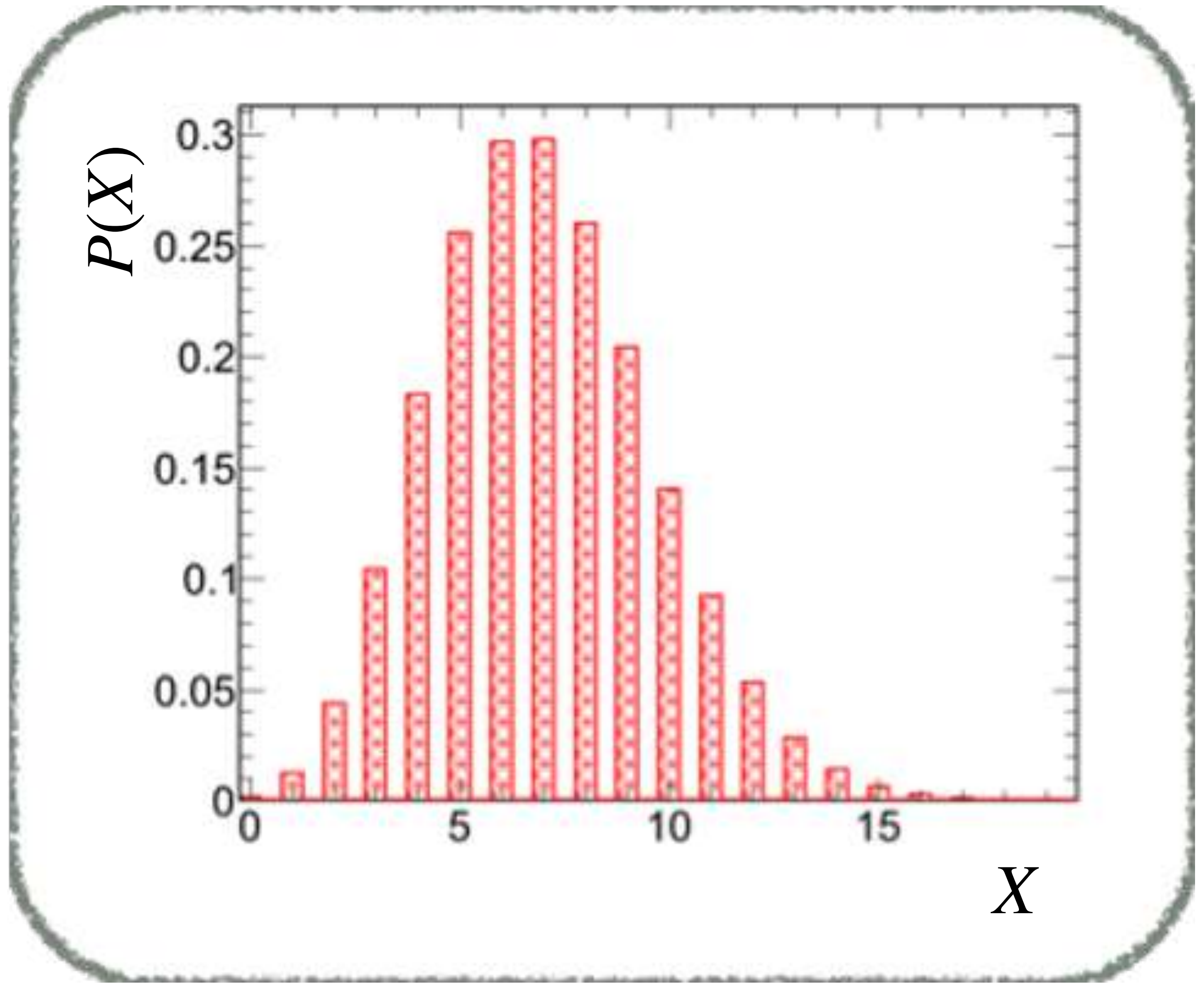
$$\mu_r \equiv \langle \left(X - \langle X \rangle\right)^r \rangle = \sum_X \left(X - \langle X \rangle\right)^r P(X)$$

- **moment generating function**

$$M_X(t) = \langle e^{tX} \rangle = \sum_X e^{tX} P(X) \qquad \langle X^r \rangle = \left. \frac{\partial^r M_X(t)}{\partial t^r} \right|_{t=0}$$

- **cumulant generating function**

$$K_X(t) = ln[M_X(t)] \qquad \kappa_r(X) = \left. \frac{\partial^r K_X(t)}{\partial t^r} \right|_{t=0}$$



Relations between cumulants and moments

$$\kappa_1 = \langle X \rangle, \;\; \kappa_2 = \mu_2, \quad \kappa_3 = \mu_3, \;\; \kappa_4 = \mu_4 - 3\mu_2^2 \; \ldots$$

Poisson distribution

Probability mass function (discrete probability density)

$$P(n;\lambda) = e^{-\lambda}\frac{\lambda^n}{n!}$$

Moment generating function

$$M(t) = \sum_{n=0}^{\infty} e^{tn}e^{-\lambda}\frac{\lambda^n}{n!} = e^{-\lambda}\sum_{n=0}^{\infty}\frac{(e^t\lambda)^n}{n!} = e^{-\lambda}e^{e^t\lambda}$$

$$\sum_{n=0}^{\infty}\frac{a^n}{n!} = e^a$$

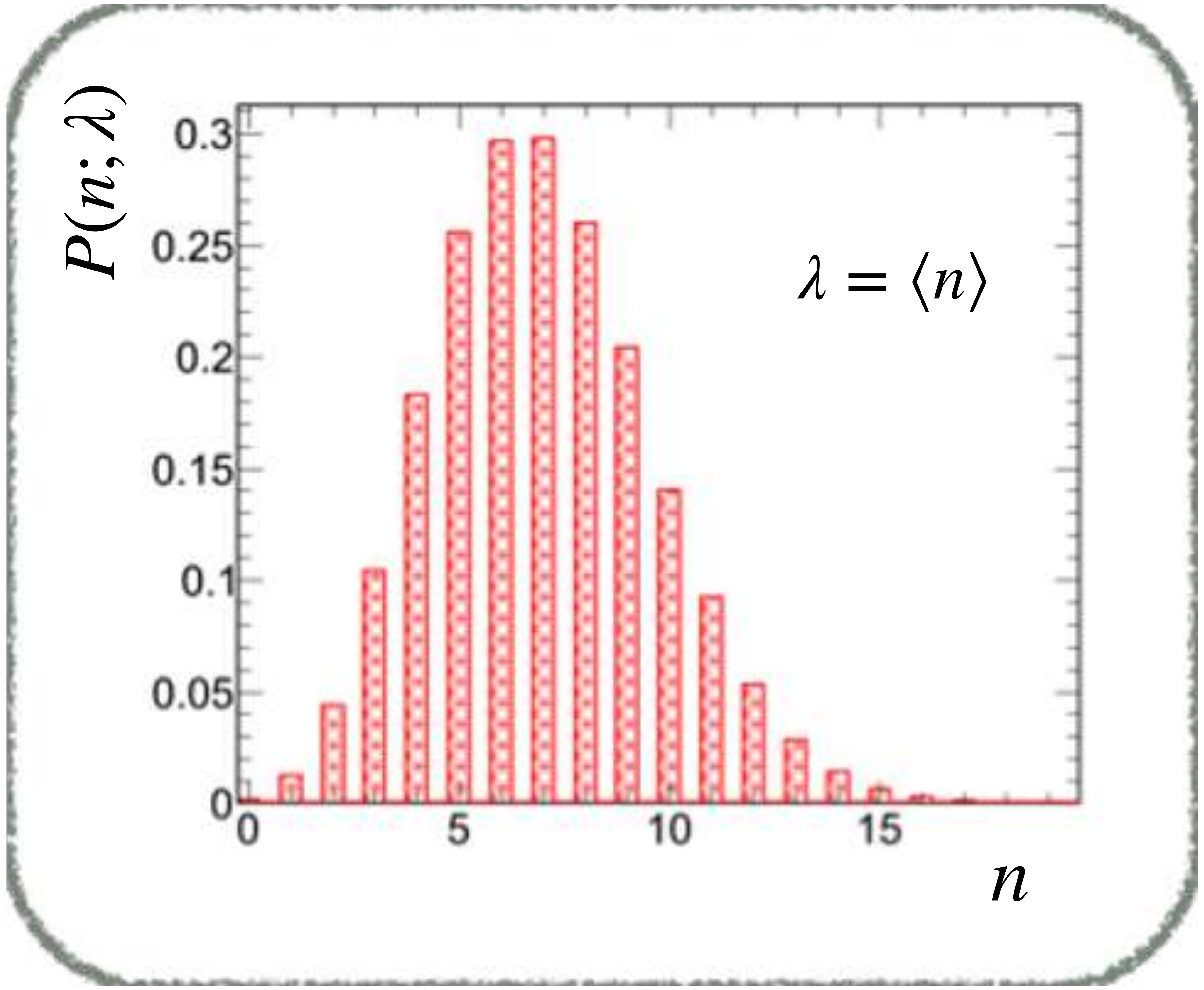
Cumulant generating function

$$K(t) = ln[M(t)] = -\lambda + e^t\lambda$$

Cumulants

$$\kappa_1 = \left.\frac{\partial K(t)}{\partial t}\right|_{t=0} = \lambda, \;\; \kappa_2 = \left.\frac{\partial^2 K(t)}{\partial t^2}\right|_{t=0} = \lambda$$

$$\kappa_3 = \left.\frac{\partial^3 K(t)}{\partial t^3}\right|_{t=0} = \lambda, \;\ldots,\; \kappa_n = \left.\frac{\partial^n K(t)}{\partial t^n}\right|_{t=0} = \lambda$$



all cumulants are equal mean, $\kappa_n = \langle n \rangle$

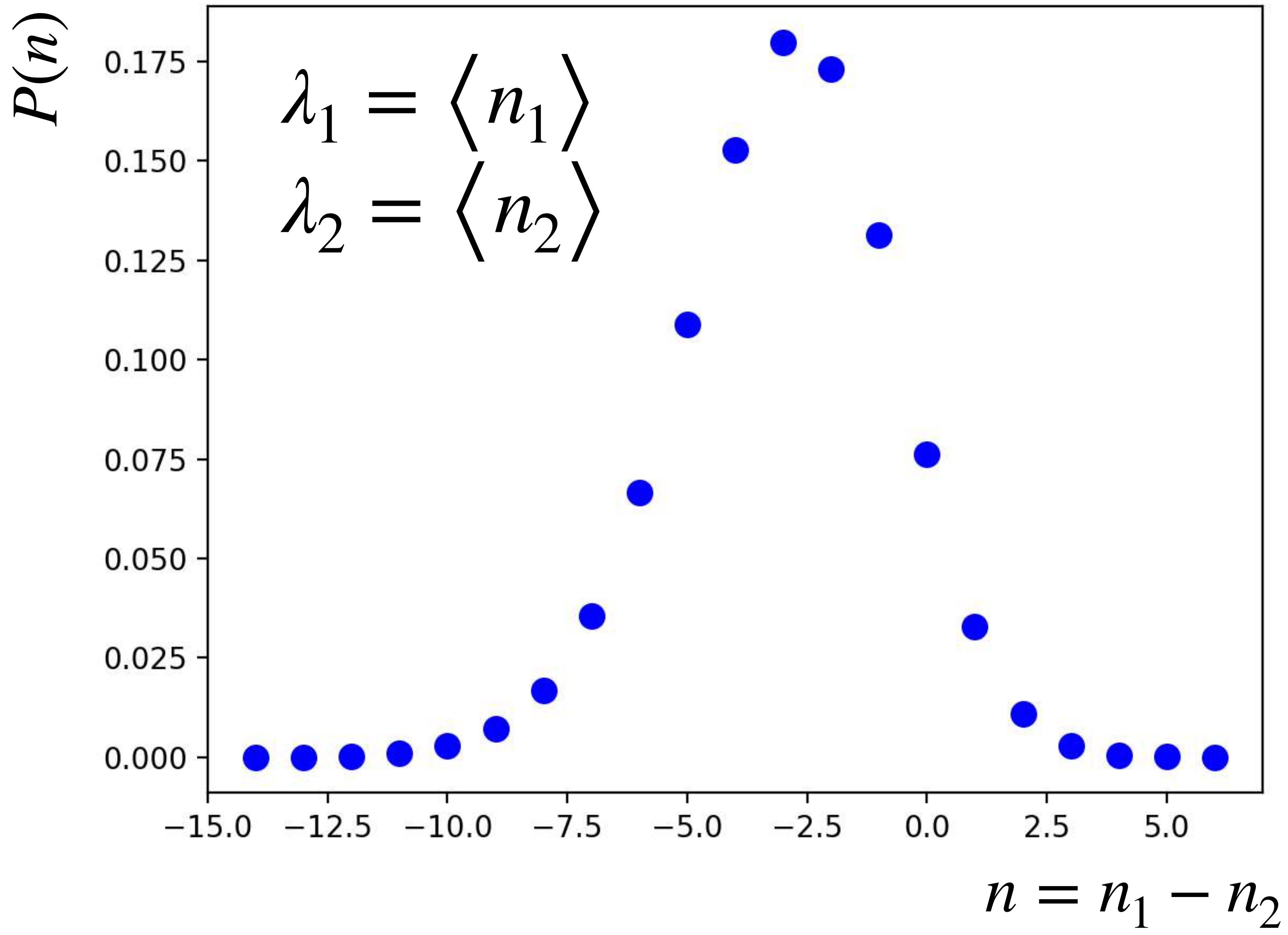
Skellam distribution

the probability distributions of the difference $n = n_1 - n_2$ of two random variables each generated from statistically independent Poisson distributions with mean values λ_1 and λ_2

Probability mass function (discrete probability density)

$$P(n;\lambda_1,\lambda_2) = e^{-(\lambda_1+\lambda_2)}\left(\frac{\lambda_1}{\lambda_2}\right)^{n/2} I_n\left(2\sqrt{\lambda_1\lambda_2}\right)$$

I_n - modified Bessel function of the first kind



Moment generating function

$$M_n(t) = e^{-(\lambda_1+\lambda_2)+\lambda_1 e^t+\lambda_2 e^{-t}}$$

Cumulant generating function

$$K_n(t) = (-\lambda_1 + e^t\lambda_1) + (-\lambda_2 + e^{-t}\lambda_2)$$

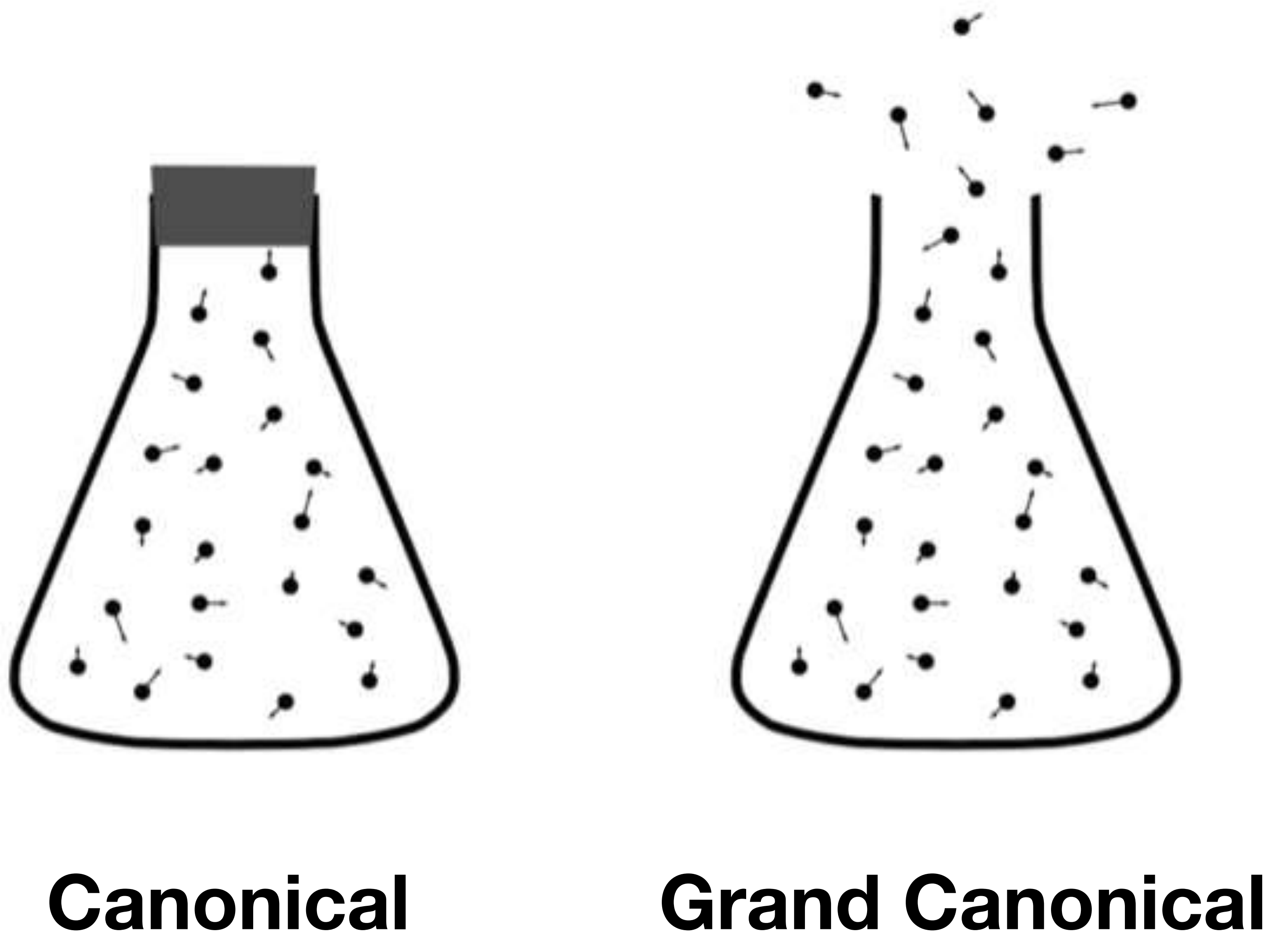
Cumulants

$$\kappa_1 = \lambda_1 - \lambda_2, \quad \kappa_2 = \lambda_1 + \lambda_2, \quad \kappa_3 = \lambda_1 - \lambda_2, \ \ldots$$

$$\kappa_n = \lambda_1 + (-1)^n\lambda_2$$

$$\frac{\kappa_{2k}}{\kappa_{2m}} = \frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2} = 1 \quad \frac{\kappa_{2k+1}}{\kappa_{2m}} = \frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2} \neq 1$$

Event-by-Event particle number fluctuations

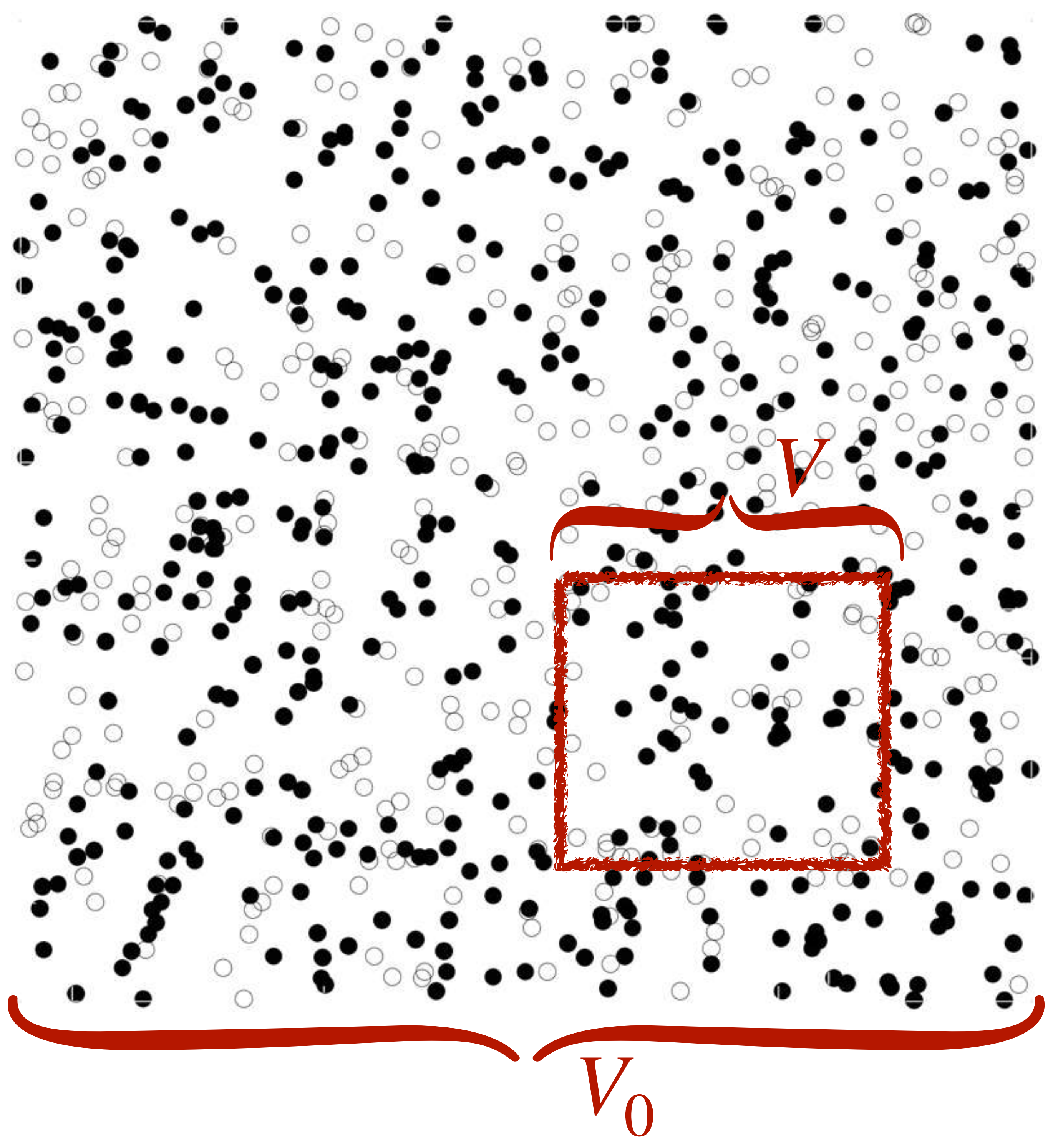


Statistical approach

Ensemble is an idealization consisting of a large number of mental copies of a system, considered all at once, each represents a possible state of the real system!

mean multiplicity

$$\langle N \rangle = \sum_j N_j p(N_j)$$



$V_0 \gg V$

$V \to \infty$

$V_0 \to \infty$

Probability of having a system in a state with E_i and N_i

$$p_j = \frac{exp\left[-\frac{E_j - \mu N_j}{T}\right]}{Z_{GCE}}$$

$$Z_{GCE}(V, T, \mu) = \sum_{state=j} exp\left[-\frac{E_j - \mu N_j}{T}\right]$$

partition function

$$\frac{\partial}{\partial \mu} lnZ_{GCE} = \frac{1}{Z_{GCE}} \frac{\partial Z_{GCE}}{\partial \mu} = \frac{\sum_j exp\left[-\frac{E_j - \mu N_j}{T}\right] \frac{N_j}{T}}{Z_{CGE}} = \frac{1}{T} \sum_j N_j p_j = \frac{1}{T} \langle N \rangle$$

$$\kappa_1(N) = \langle N \rangle = T \frac{\partial lnZ_{GCE}}{\partial \mu} = \frac{\partial lnZ_{GCE}}{\partial (\mu/T)}$$

$$\kappa_2(N) = \langle N^2 \rangle - \langle N \rangle^2 = \frac{\partial^2 lnZ_{GCE}}{\partial (\mu/T)^2}$$

lnZ_{GCE} **- cumulant generating function!**

Fluctuation for the ideal gas EoS

Particle number fluctuations

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{T\chi_k}{V} \qquad \chi_k = -\frac{1}{V\left(\frac{\partial P}{\partial V}\right)_T}$$

Ideal Gas Equation of State

$$PV = \langle N\rangle T \qquad \frac{\partial P}{\partial V} = -\frac{1}{V^2}\langle N\rangle T$$

$$\chi_k = \frac{V}{\langle N\rangle T} \qquad \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{1}{\langle N\rangle}$$

$$\left\langle N^2\right\rangle - \langle N\rangle^2 = < N > \qquad \sigma^2 = \kappa_2(N) = \langle N\rangle$$

Poisson distribution

Minimal baseline

For single particles (Poisson)

$$\kappa_n(\text{Poisson}) = \langle n\rangle$$

$$\frac{\kappa_m}{\langle n\rangle} = 1, \qquad \frac{\kappa_m}{\kappa_n} = 1$$

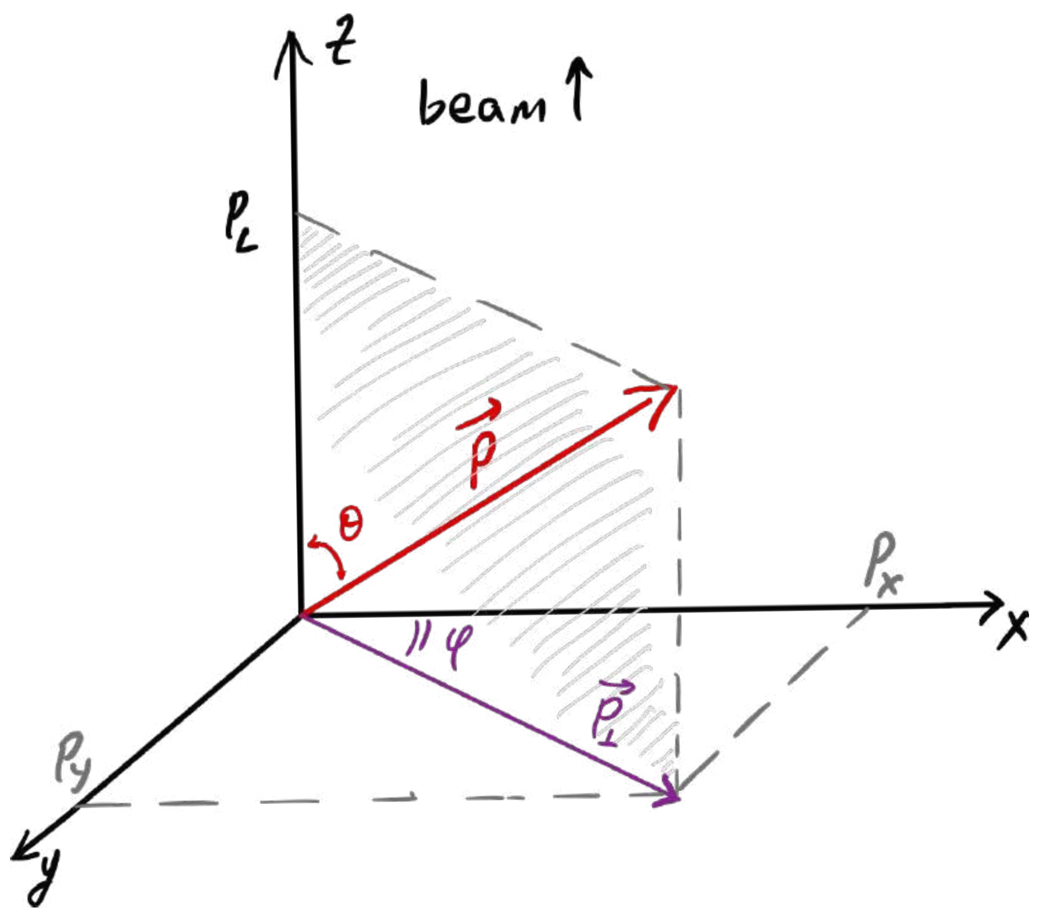
For net-particles (Skellam)

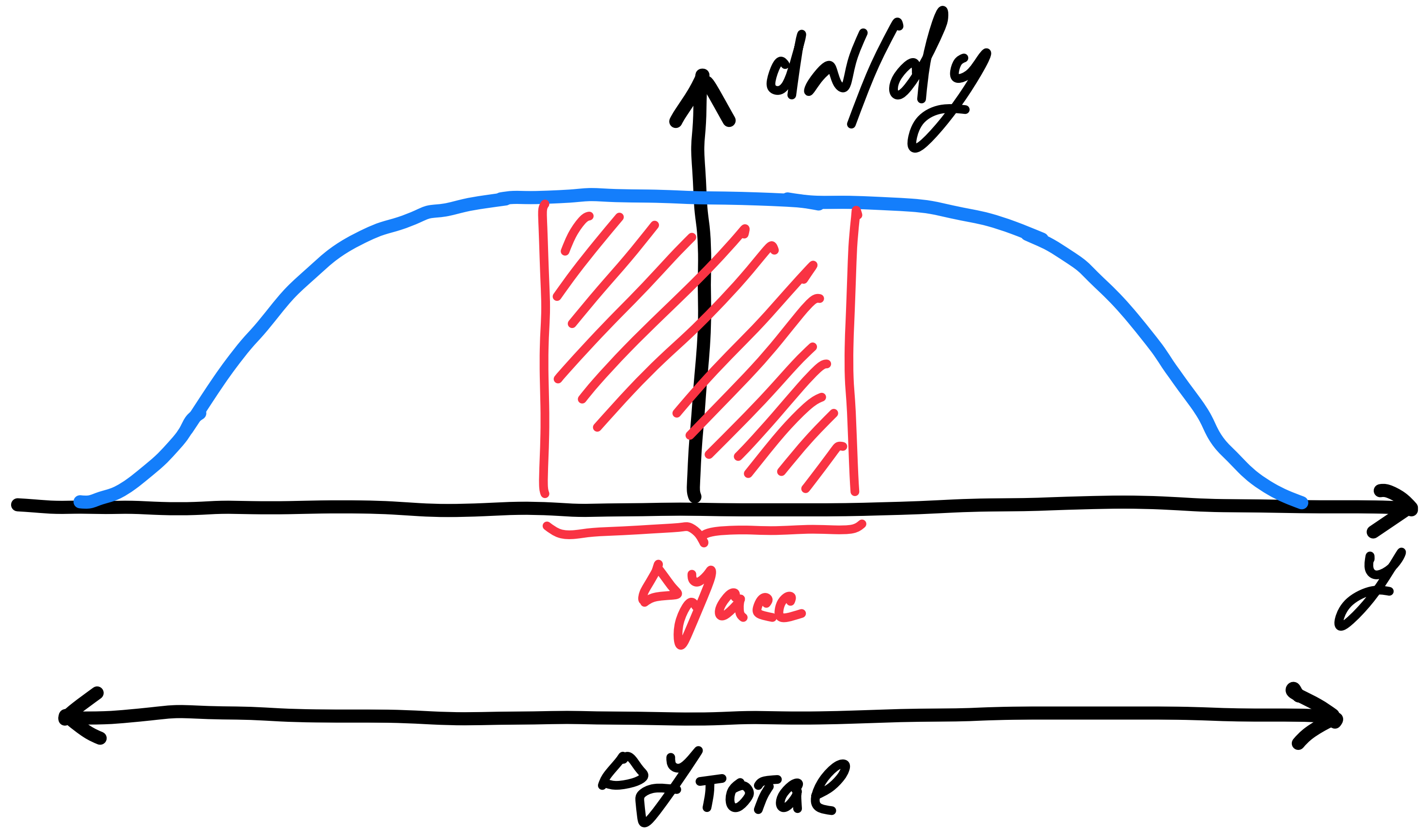
$$\kappa_n(\text{Skellam}) = < n > + (-1)^n < \bar{n} >$$

$$\frac{\kappa_{2m}}{\kappa_{2n}} = \frac{\langle n\rangle + \langle\bar{n}\rangle}{\langle n\rangle + \langle\bar{n}\rangle} = 1$$

$$\frac{\kappa_{2k+1}}{\kappa_{2m}} = \frac{\langle n\rangle - \langle\bar{n}\rangle}{\langle n\rangle + \langle\bar{n}\rangle} \neq 1$$

Experimental conditions for GCE





rapidity: generalisation of longitudinal velocity

$$y = atanh(\beta_L) = \frac{1}{2}ln\frac{1+\beta_L}{1-\beta_L} = \frac{1}{2}ln\frac{E+p_L}{E-p_L}$$

- $\beta_L = p_L/E$
- $p_L = |\vec{p}|cos(\theta)$
- $|\vec{p_T}| = |\vec{p}|sin(\theta)$

$$y = \frac{1}{2}ln\frac{E+|\vec{p}|cos(\theta)}{E-|\vec{p}|cos(\theta)}$$

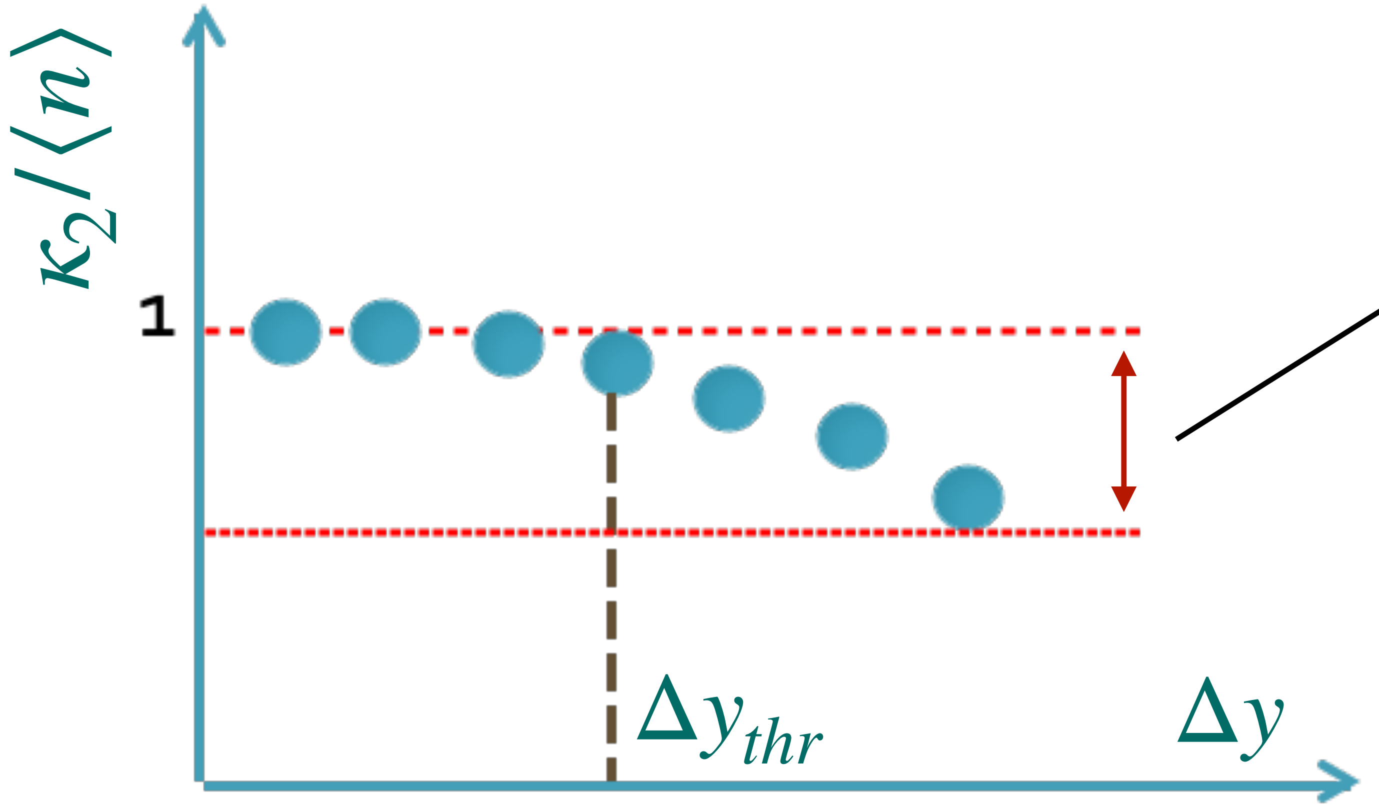
pseudo rapidity

$$|\vec{p}| \gg m, y \to \eta$$

$$\eta = -ln\left(tan\frac{\theta}{2}\right)$$

to achieve requirements of GCE in experiments

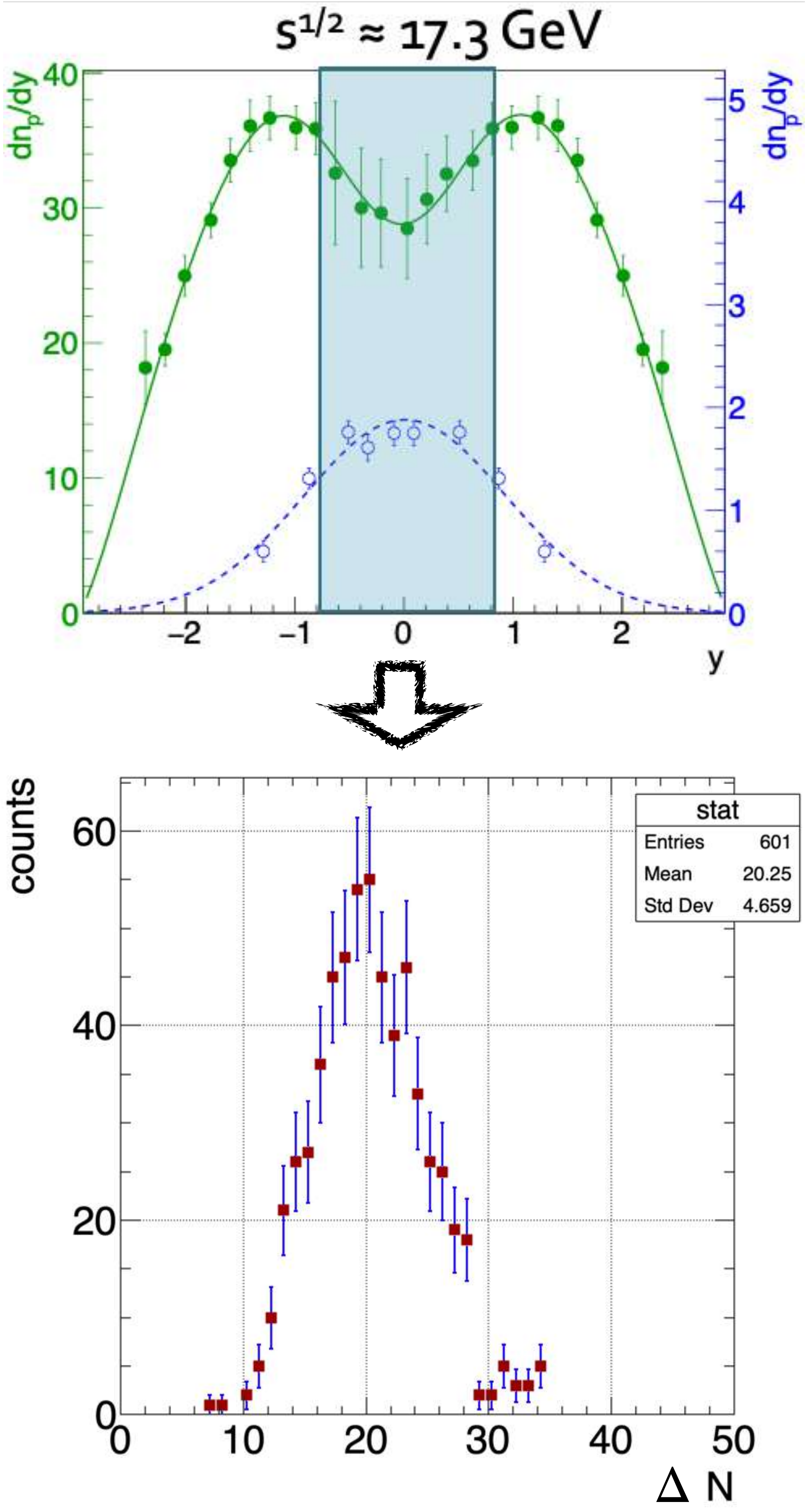
- cuts on p_T, y or η are imposed
  - $\Delta y_{acc} < \Delta y_{thr}$ - dynamical fluctuations disappear (small number Poisson limit)



Deviation from unity has to be observed

- Critical signal or non-critical contributions?
  - Conservation laws
  - Participant fluctuations
  - ...

Measuring Particle number fluctuations



- $\Delta N = N_B - N_{\bar{B}}$ occurs with probability $p(\Delta N)$ (measured)
- r^{th} order central moment: $\mu_r = \sum_{\Delta N} (\Delta N - \langle \Delta N \rangle)^r p(\Delta N)$
- first 4 cumulants: $\kappa_1 = \langle \Delta N \rangle, \ \kappa_2 = \mu_2, \ \ \kappa_3 = \mu_3, \ \kappa_4 = \mu_4 - 3\mu_2^2$
- **advantage:** sensitive to small (critical) signals
- **disadvantage:** sensitive to any non-critical contributions

Compare to minimal baseline: Ideal Gas EoS + GCE

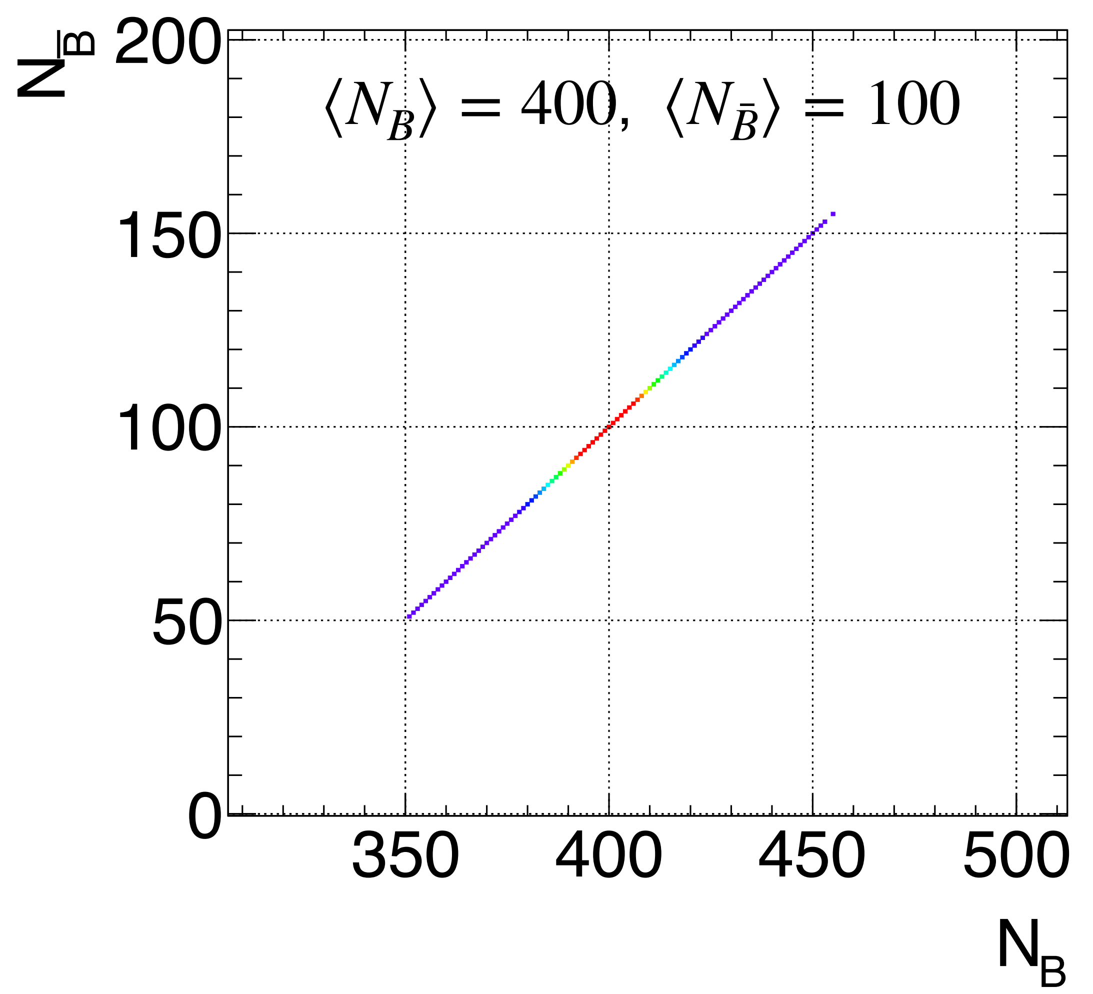
particles (Poisson)

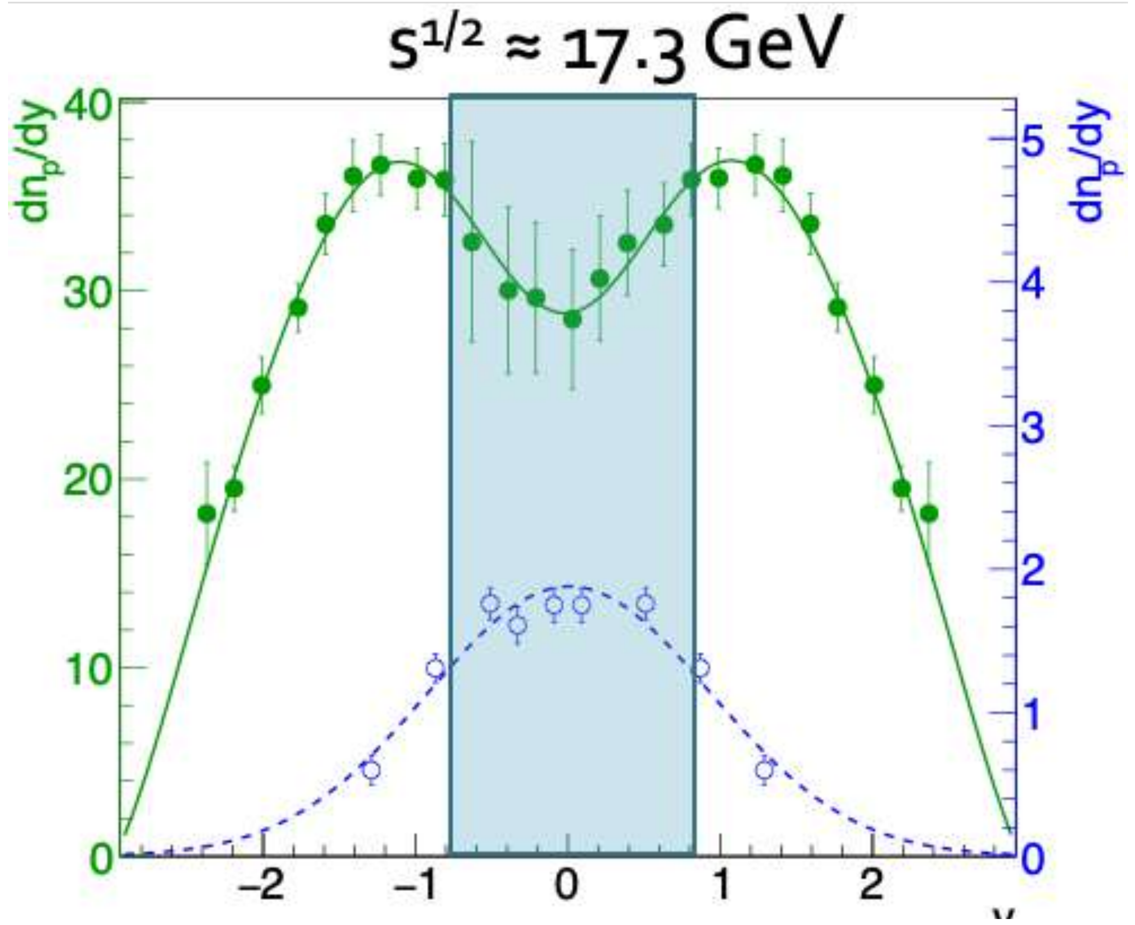
$$\frac{\kappa_m}{\kappa_n} = 1$$

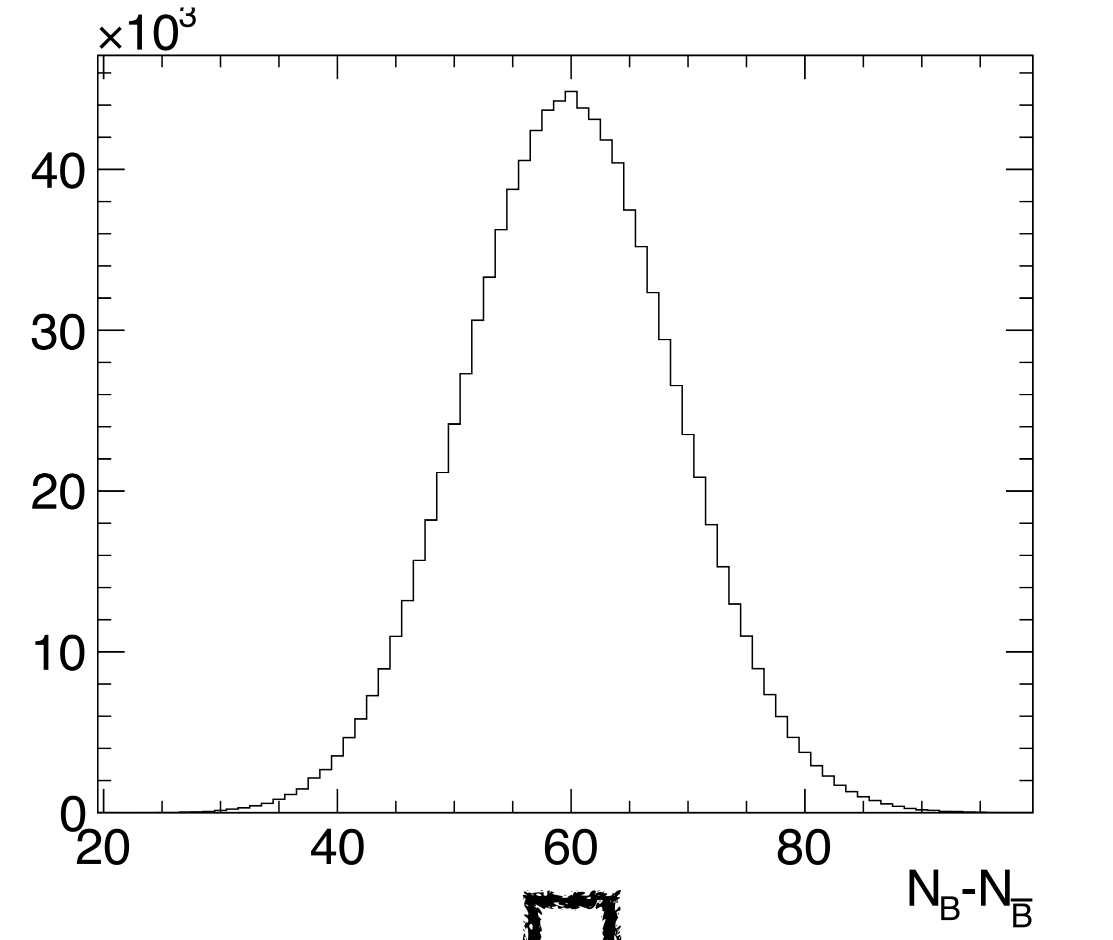
net-particles (Skellam)

$$\frac{\kappa_{2m}}{\kappa_{2n}} = \frac{\langle N \rangle + \langle \bar{N} \rangle}{\langle N \rangle + \langle \bar{N} \rangle} = 1, \qquad \frac{\kappa_{2m}}{\kappa_{2n+1}} = \frac{\langle N \rangle + \langle \bar{N} \rangle}{\langle N \rangle - \langle \bar{N} \rangle}$$

Ideal gas baseline, in the canonical Ensemble







P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel , NPA 1008 (2021) 122141

P. Braun-Munzinger, A.R., J. Stachel, NPA 982 (2019) 307-310

A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

$$\frac{\kappa_2(B-\bar{B})}{\langle n_B + n_{\bar{B}} \rangle} = 1 - \frac{\alpha_B \langle n_B \rangle + \alpha_{\bar{B}} \langle n_{\bar{B}} \rangle}{\langle n_B + n_{\bar{B}} \rangle} + \left(z^2 - \langle N_B \rangle \langle N_{\bar{B}} \rangle\right) \frac{\left(\alpha_B - \alpha_{\bar{B}}\right)^2}{\langle n_B + n_{\bar{B}} \rangle}$$

$\langle N_B \rangle, \langle N_{\bar{B}} \rangle$ - in 4π

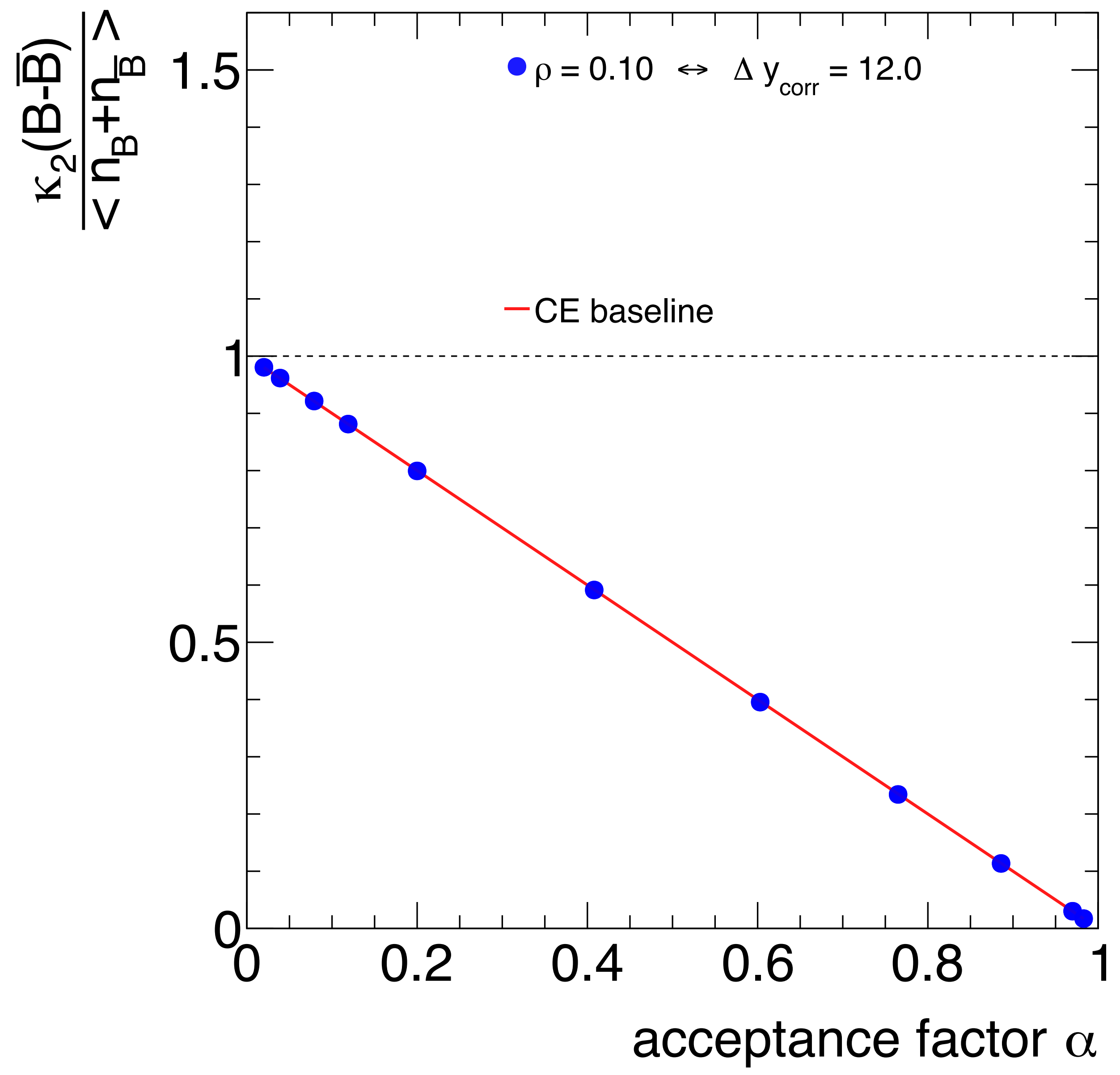
$\langle n_B \rangle, \langle n_{\bar{B}} \rangle$ - inside acceptance

$\alpha_B = \langle n_B \rangle / \langle N_B \rangle$ - acceptance for B

$\alpha_{\bar{B}} = \langle n_{\bar{B}} \rangle / \langle N_{\bar{B}} \rangle$ - acceptance for $\bar{B}$

z - single baryon partition function

- **deviations from unity**
  - **induced by conservation laws**



Canonical Ensemble Calculator

P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel , NPA 1008 (2021) 122141

Cumulants in the canonical thermodynamics

NB : 370

NBar : 20

pB : 0.068

pBar : 0.106

cumulant order: 2

- [x] print analytic formulas
- [ ] Generate .cc file

calculate

NB: number of baryons in 4pi

NBar: number of anti-baryons in 4pi

pB: accepted protons

pBar: accepted anti-protons

Recalculated value of z

z = 86.13349566

Numerical values

kappa_1 = 23.04
kappa_2 = 25.3718

Analytic formulas:

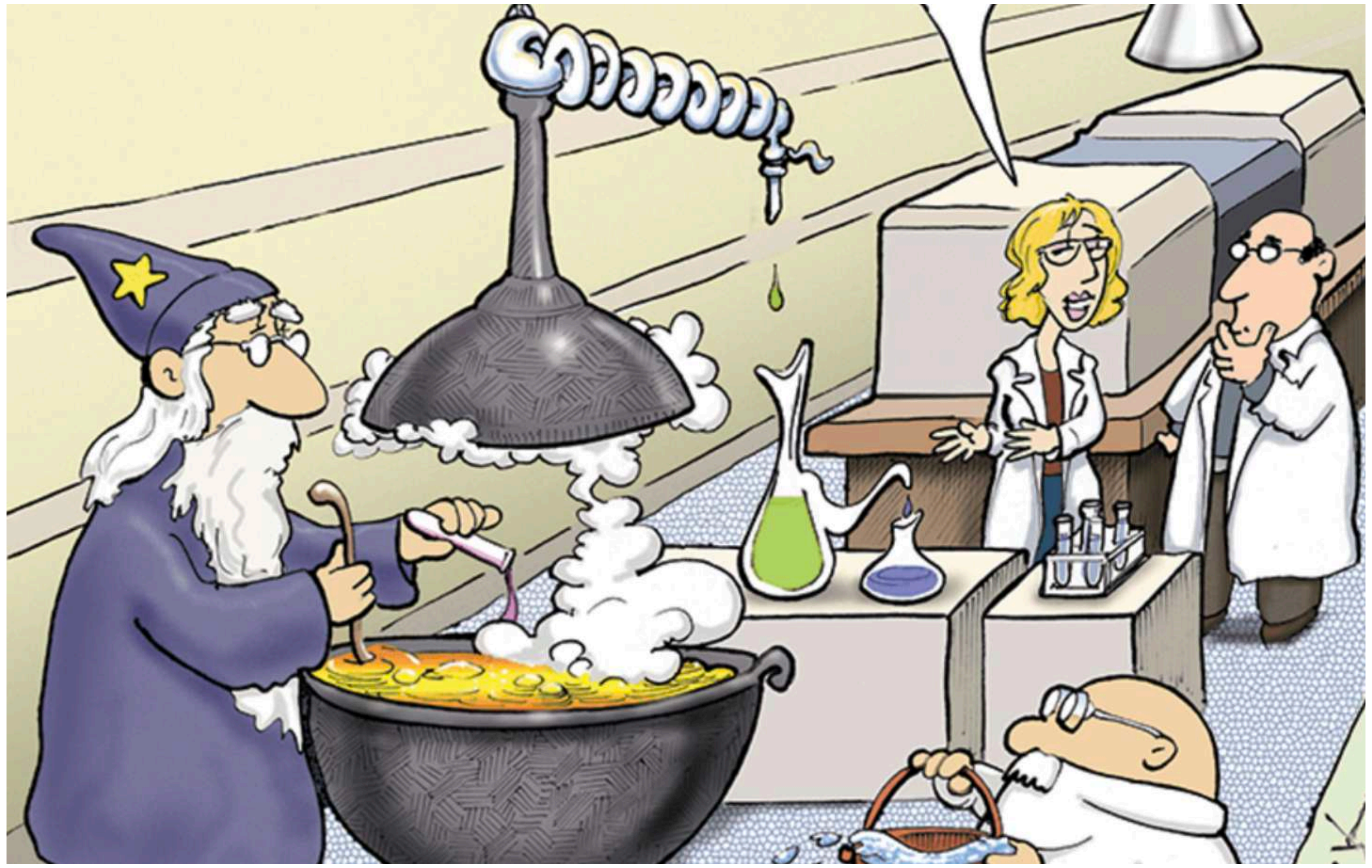
kappa_1 = ((1.0/2.0)*NB - 1.0/2.0*NBar)*(pB + pBar) + (NB + NBar)*((1.0/2.0)*pB - 1.0/2.0*pBar)

kappa_2 = ((1.0/2.0)*NB - 1.0/2.0*NBar)*(-pB*(pB - 1) + pBar*(pBar - 1)) + (NB + NBar)*(-1.0/4.0*pow(pB, 2) - 1.0/2.0*pB*pBar + (1.0/2.0)*pB - 1.0/4.0*pow(pBar, 2) + (1.0/2.0)*pBar) + pow((1.0/2.0)*pB - 1.0/2.0*pBar, 2)*(-4*NB*NBar - NB - NBar + 4*pow(z, 2))

Authors: B. Friman, A. Rustamov

a Python package for calculating both analytic formulas and numerical values for net-baryon cumulants of any order in the finite acceptance is available for download

git clone https://github.com/e-by-e/Cumulants-CE.git



Challenges in measurements

- Particle misidentification issues
- Contributions from non-critical fluctuations

Particle Identification strategy

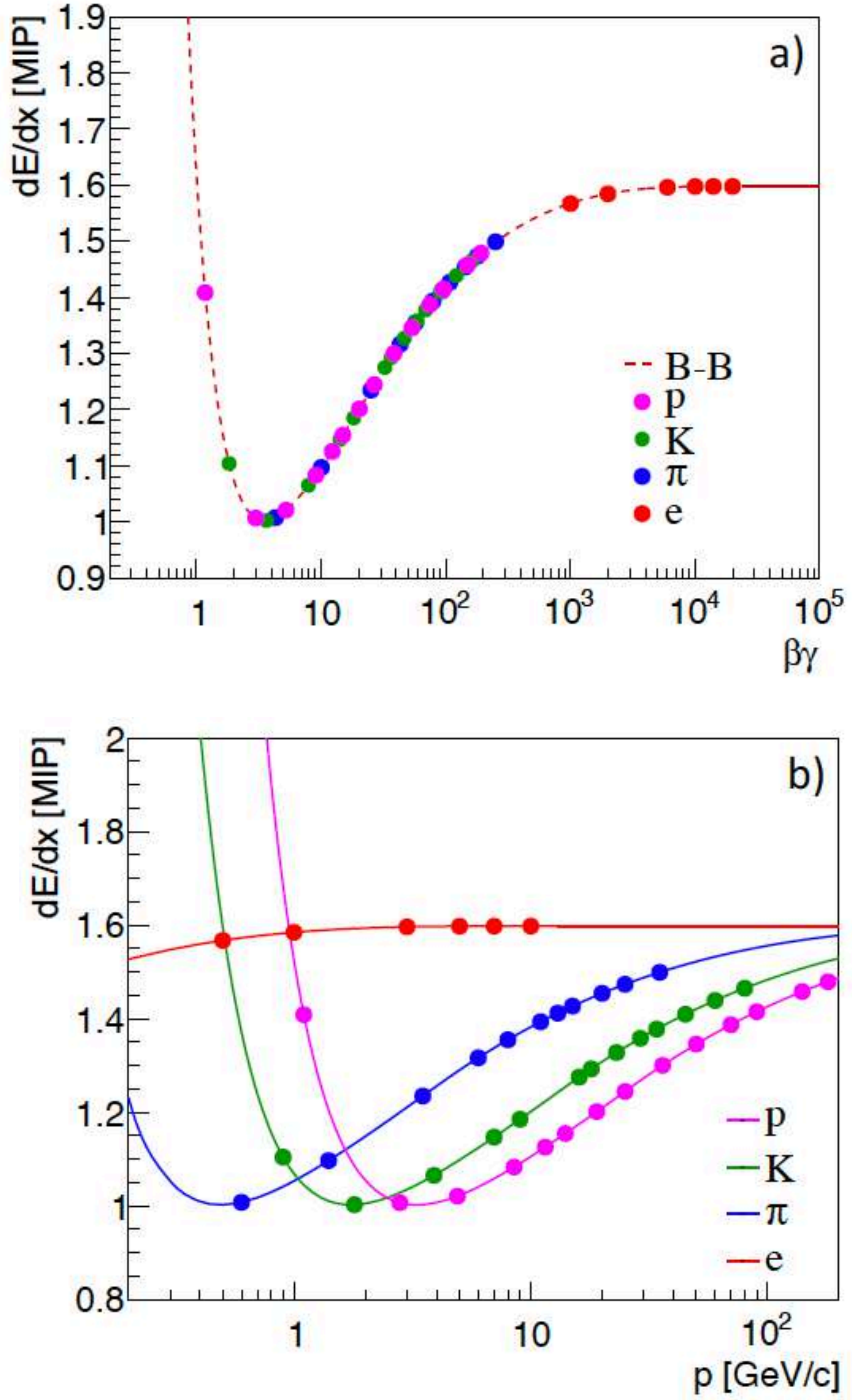
Based on the definition of relativistic momentum

$$\vec{p} = \frac{m\vec{\beta}c}{\sqrt{1-\beta^2}}$$

- **To identify the particle at least two independent measurements are needed, e. g., momentum $\vec{p}$ and velocity $\vec{v}$**

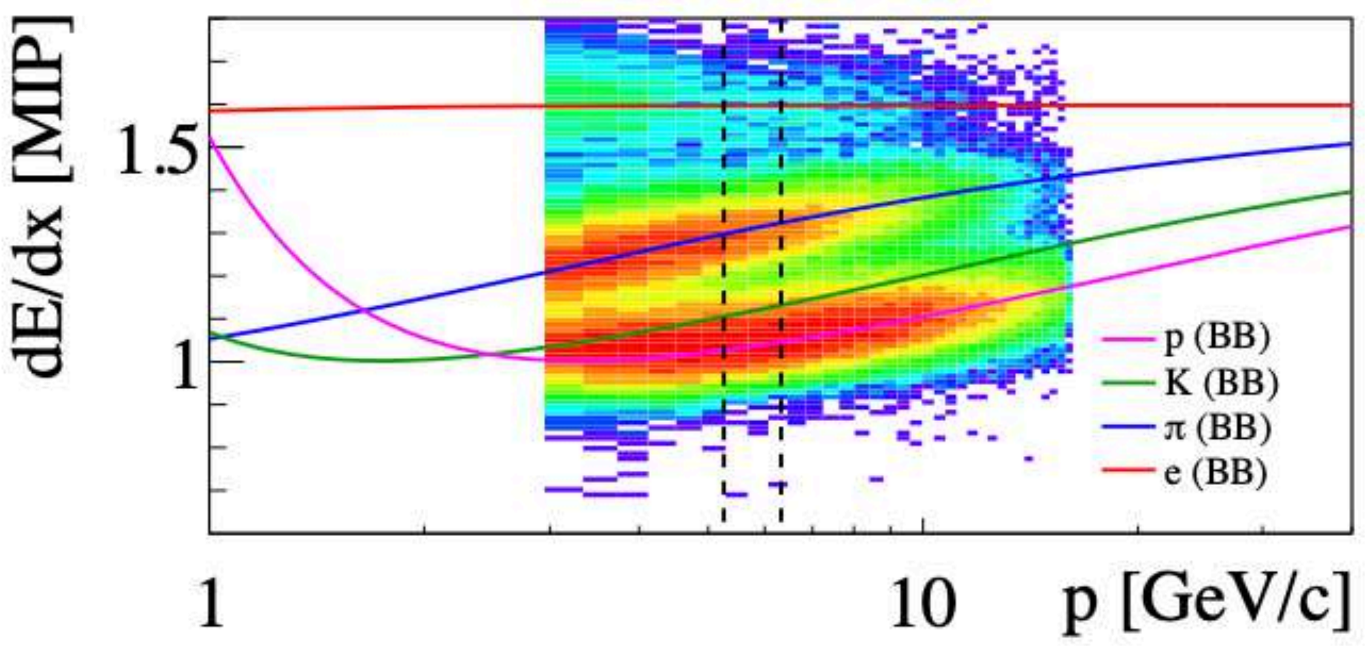
$$-\left\langle \frac{dE}{dx} \right\rangle (\beta\gamma) \sim \frac{z^2}{\beta^2} ln(\alpha\beta\gamma)$$

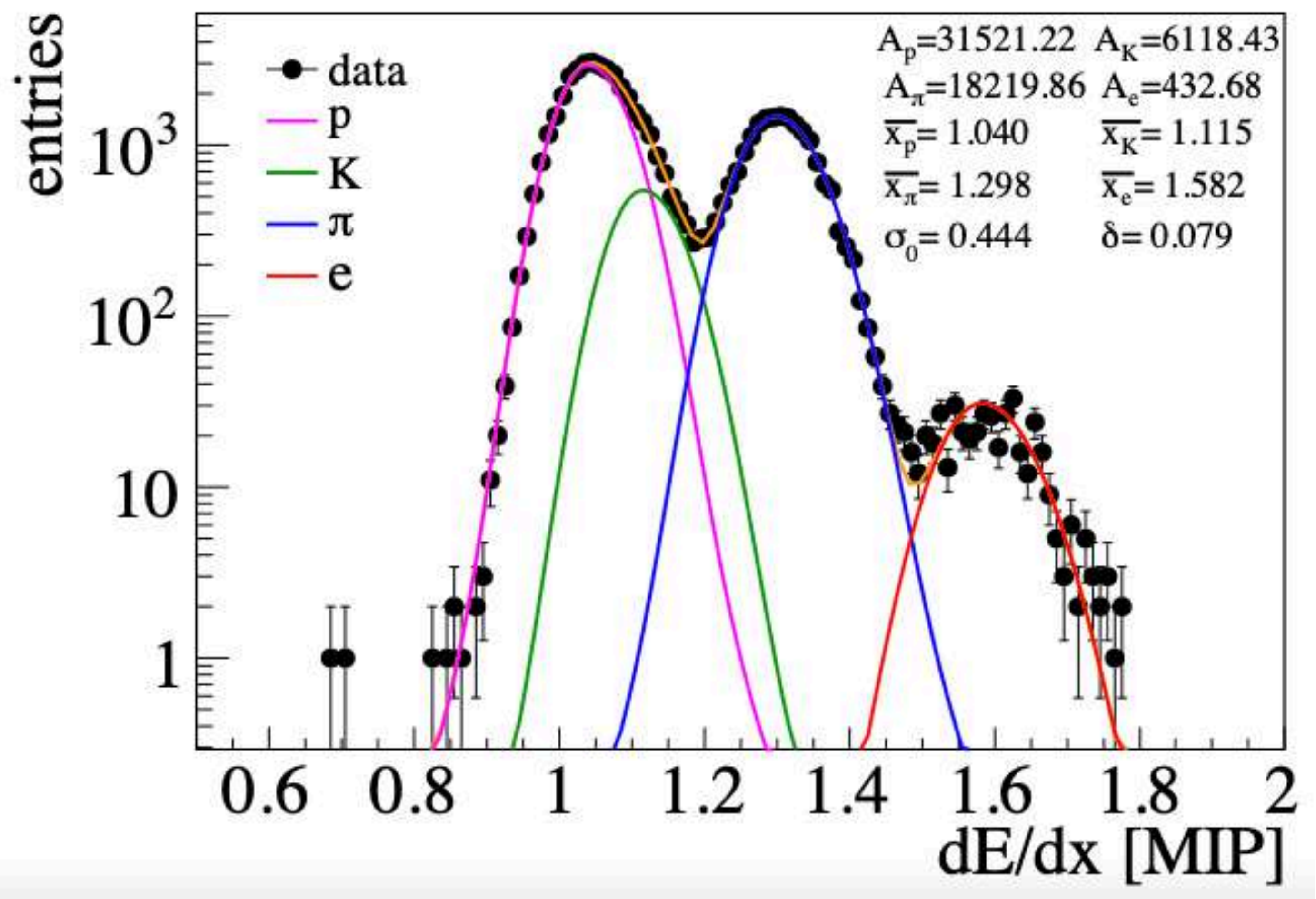
- **momentum is obtained by solving equations of motion of a particle inside the magnetic field**



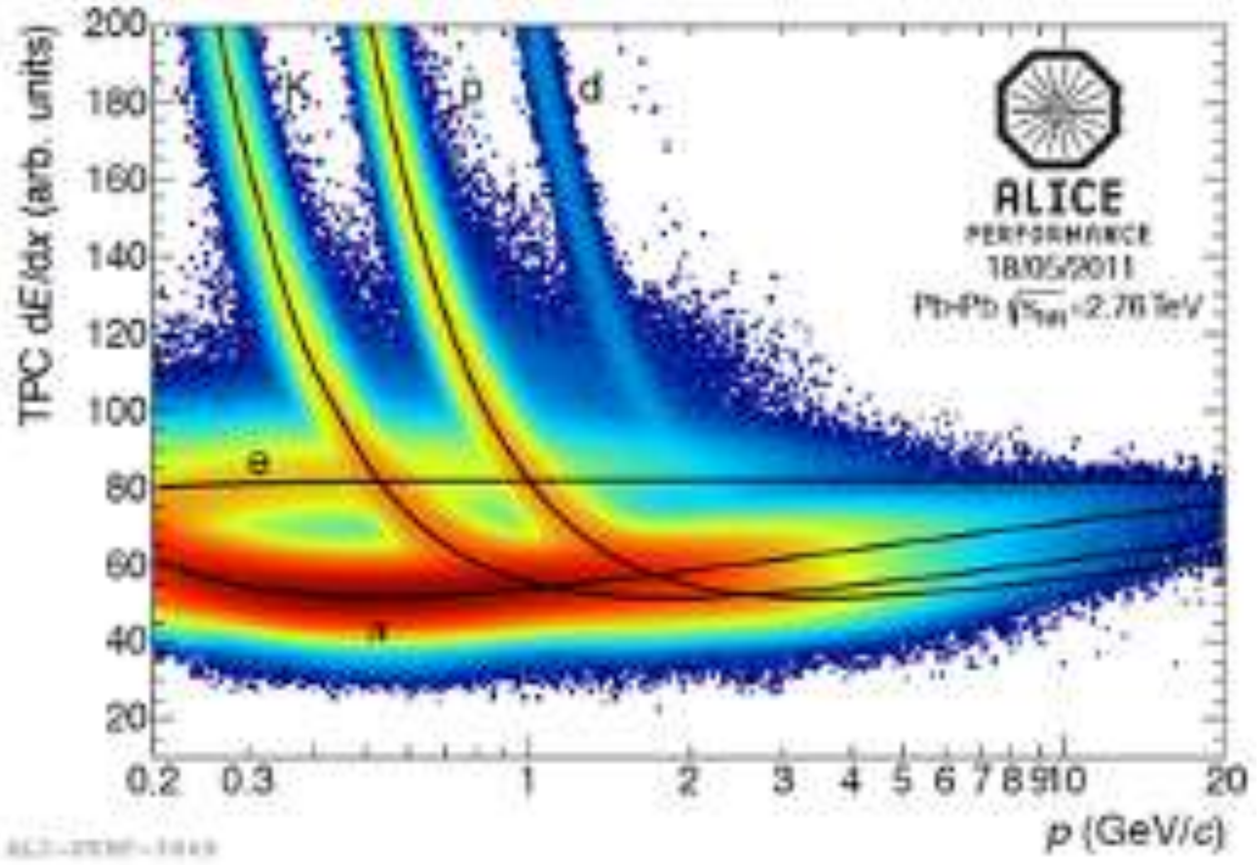
Particle identification

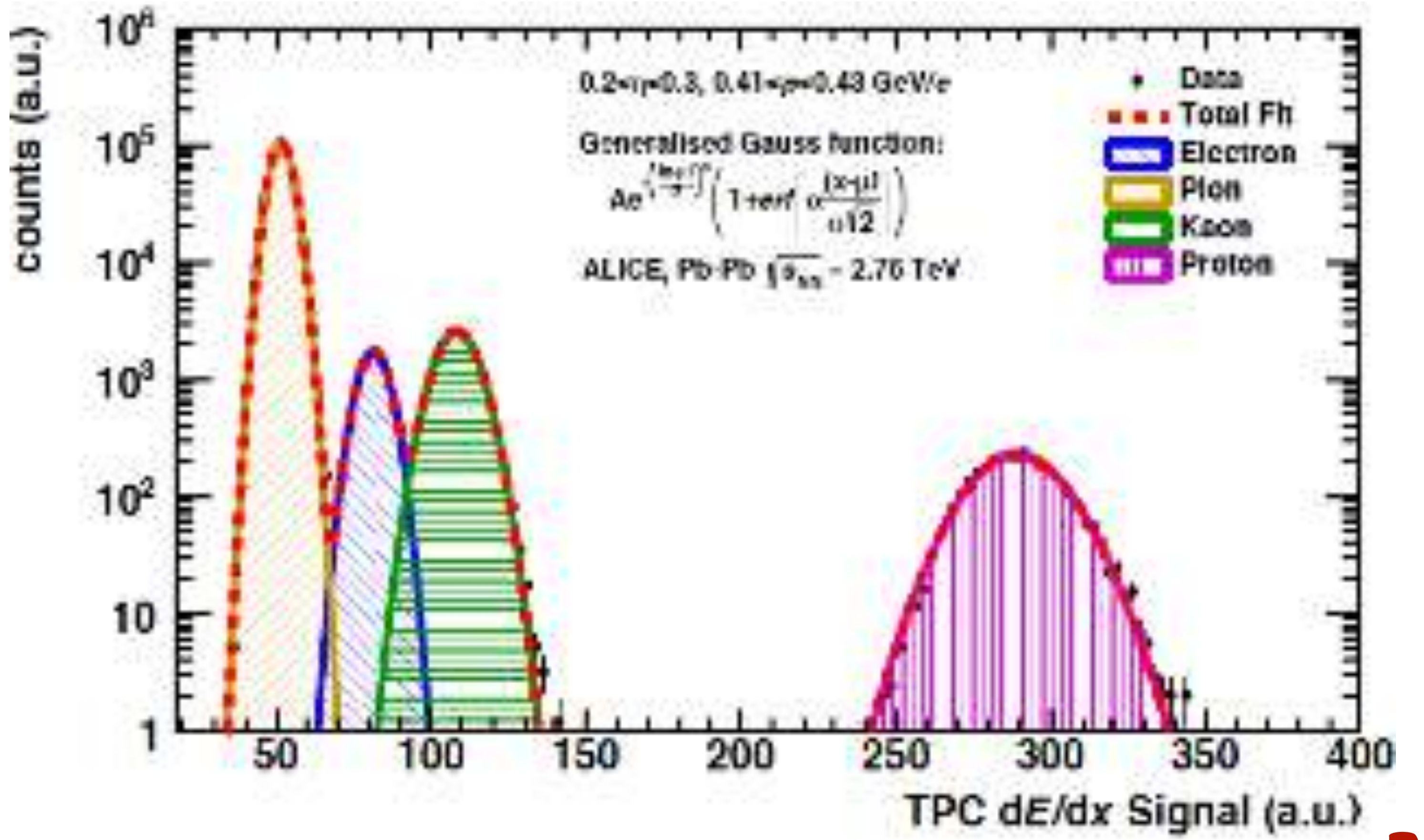
NA49, Pb-Pb@7.6GeV



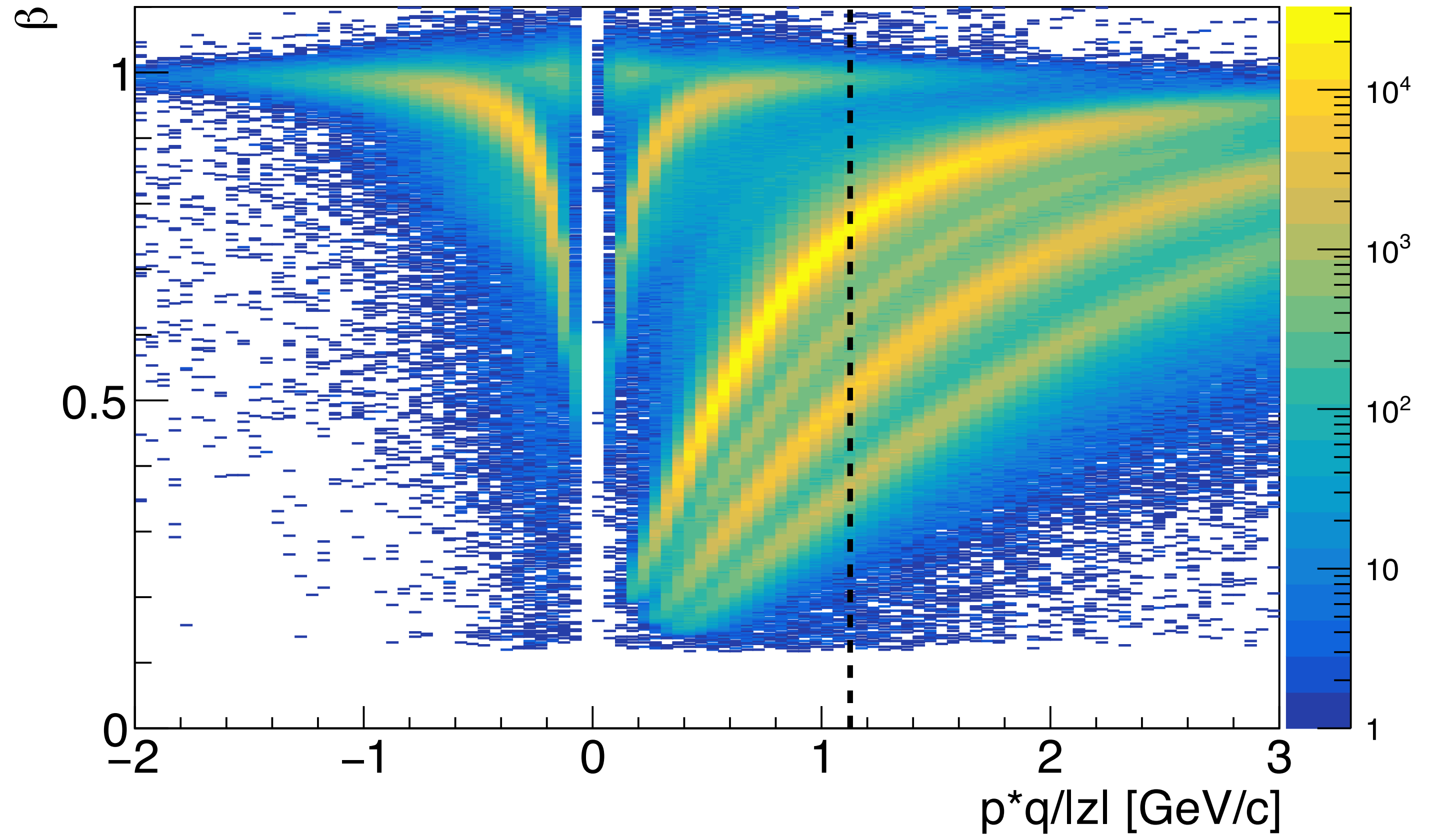


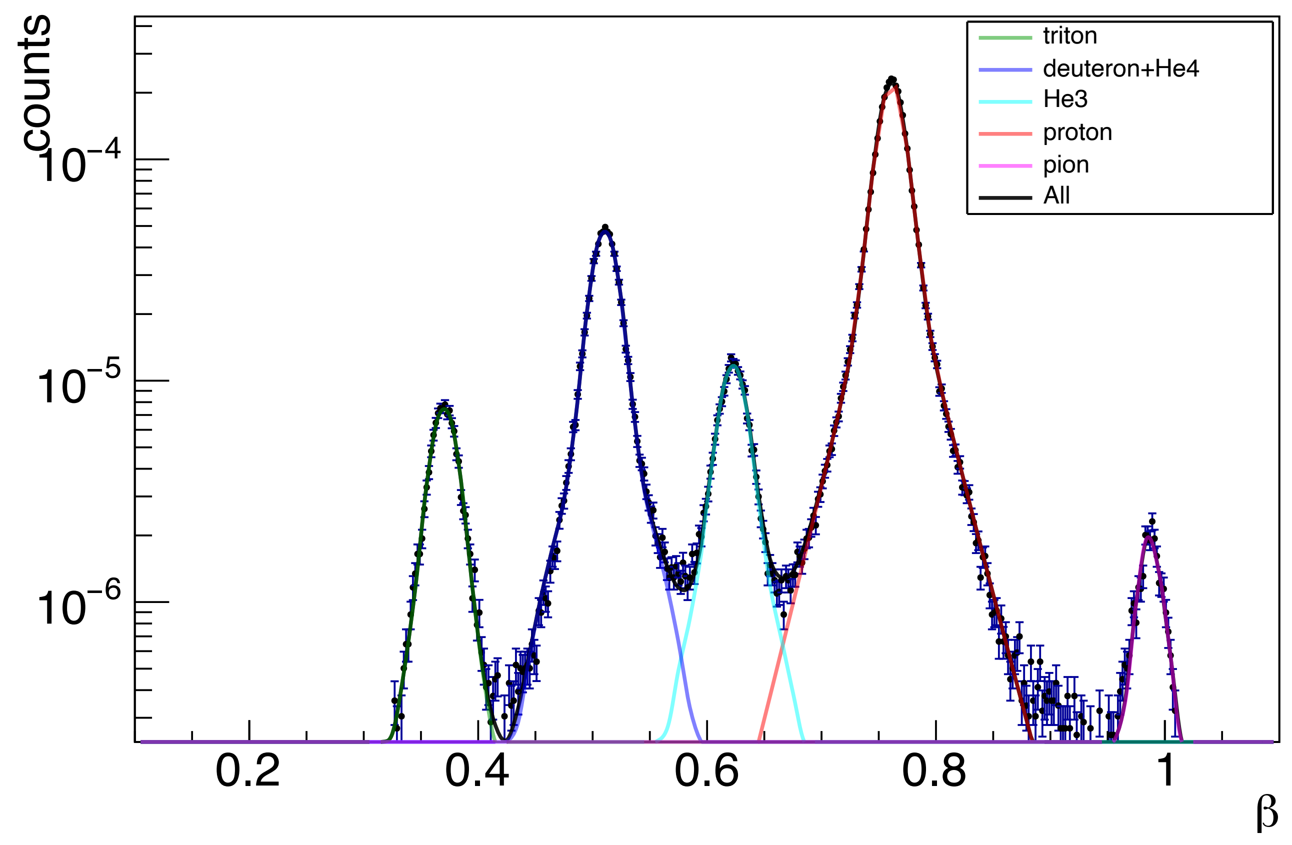
ALICE, Pb-Pb@2.76 TeV





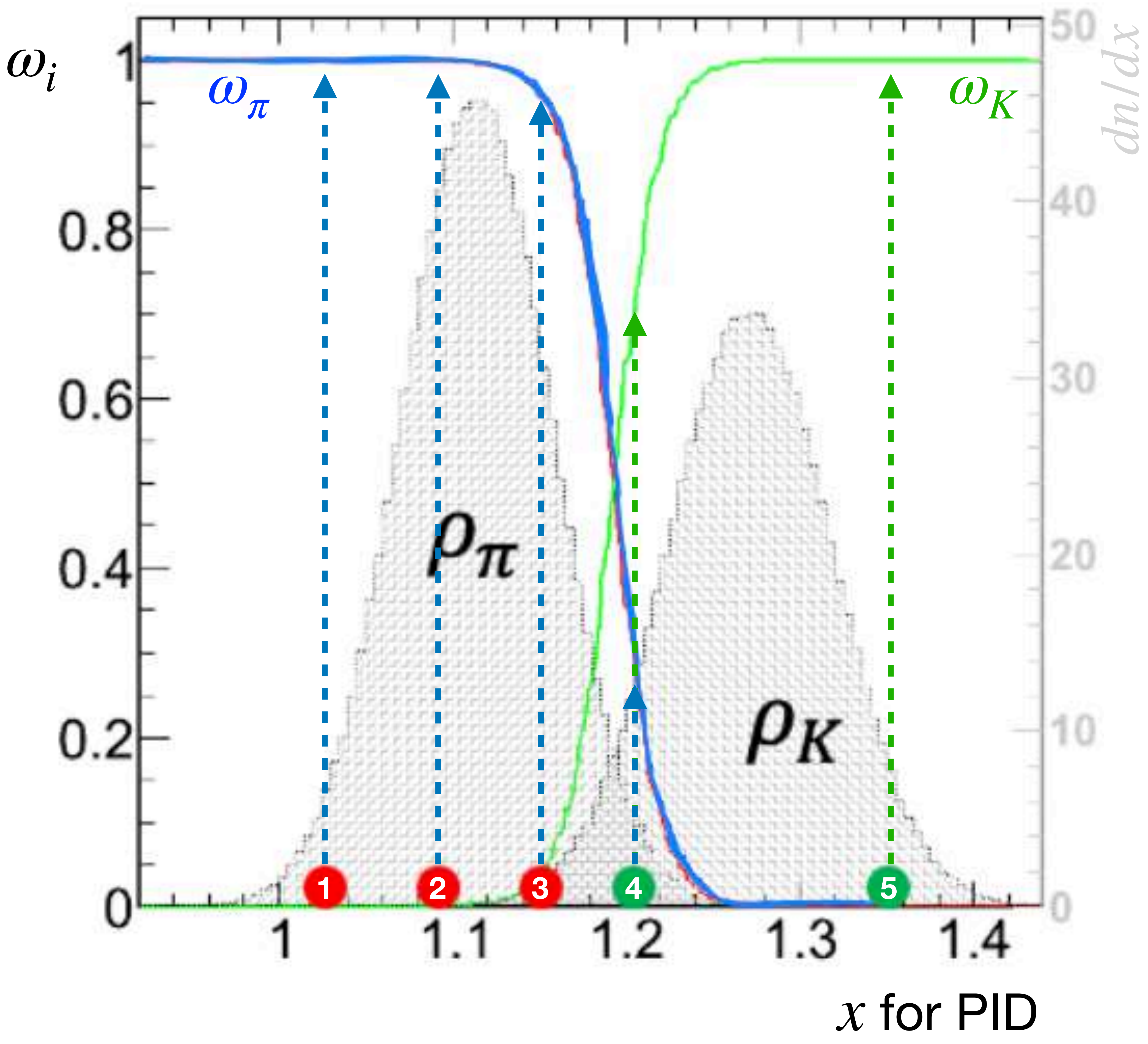
HADES, Au-Au@2.4 GeV





TPCs

drift chambers+TOF



- Input for this event: 3 pions ❶ ❷ ❸, 2 kaons ❹ ❺
- Probabilities that a given measurement x_i is pion or Kaon

$$\omega_\pi^i = \frac{\rho_\pi(x_i)}{\rho_\pi(x_i) + \rho_K(x_i)} \qquad \omega_K^i = \frac{\rho_K(x_i)}{\rho_\pi(x_i) + \rho_K(x_i)}$$

- New Idea: Introducing proxies for particle numbers

$$W_\pi = \sum_{i=1}^{i=5} \omega_\pi(x_i) \qquad W_K = \sum_{i=1}^{i=5} \omega_K(x_i)$$

❶ ❷ ❸ $\omega_\pi = 1$

❹ $\omega_\pi \approx 0.3$

❺ $\omega_\pi = 0$

$n_\pi = 3$: true event multiplicity

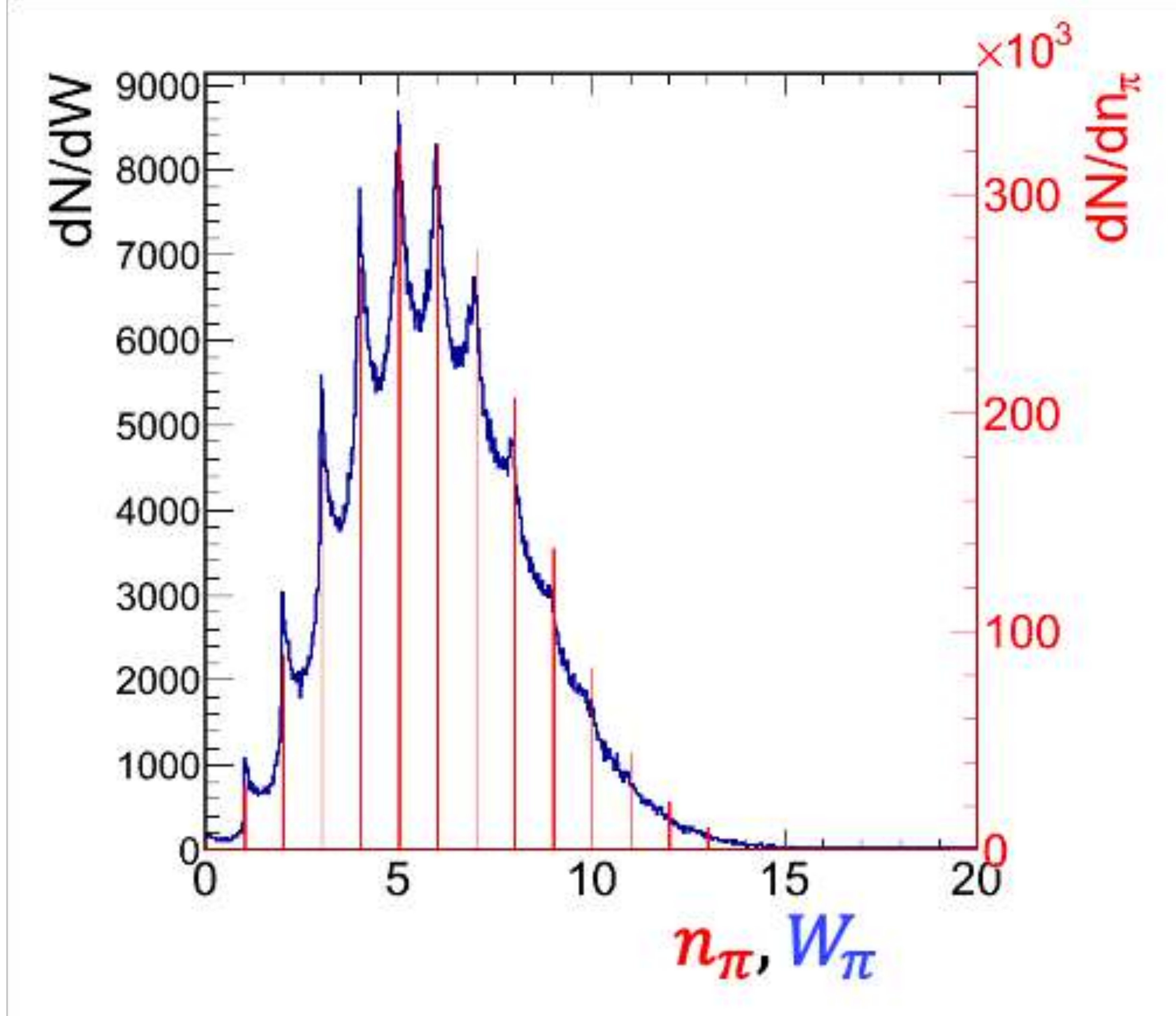
$W_\pi = 3.3$: proxy for event multiplicity

W_π and W_K can be measured in each event

$n_\pi, n_K \to W_\pi, W_K$: from integer to floating particle numbers

M. Gazdzicki et al., Phys.Rev.C 83 (2011) 054907
M. I. Gorenstein, PRC 84, 024902 (2011)
AR, M. I. Gorenstein, PRC 86, 044906 (2012)
M. Arslandok, AR, NIM A946, 162622 (2019)

Identity Method, used in ALICE, NA61/Shine, NA49, HADES



M. Gazdzicki et al., Phys.Rev.C 83 (2011) 054907
M. I. Gorenstein, PRC 84, 024902 (2011)
AR, M. I. Gorenstein, PRC 86, 044906 (2012)
M. Arslandok, AR, NIM A946, 162622 (2019)

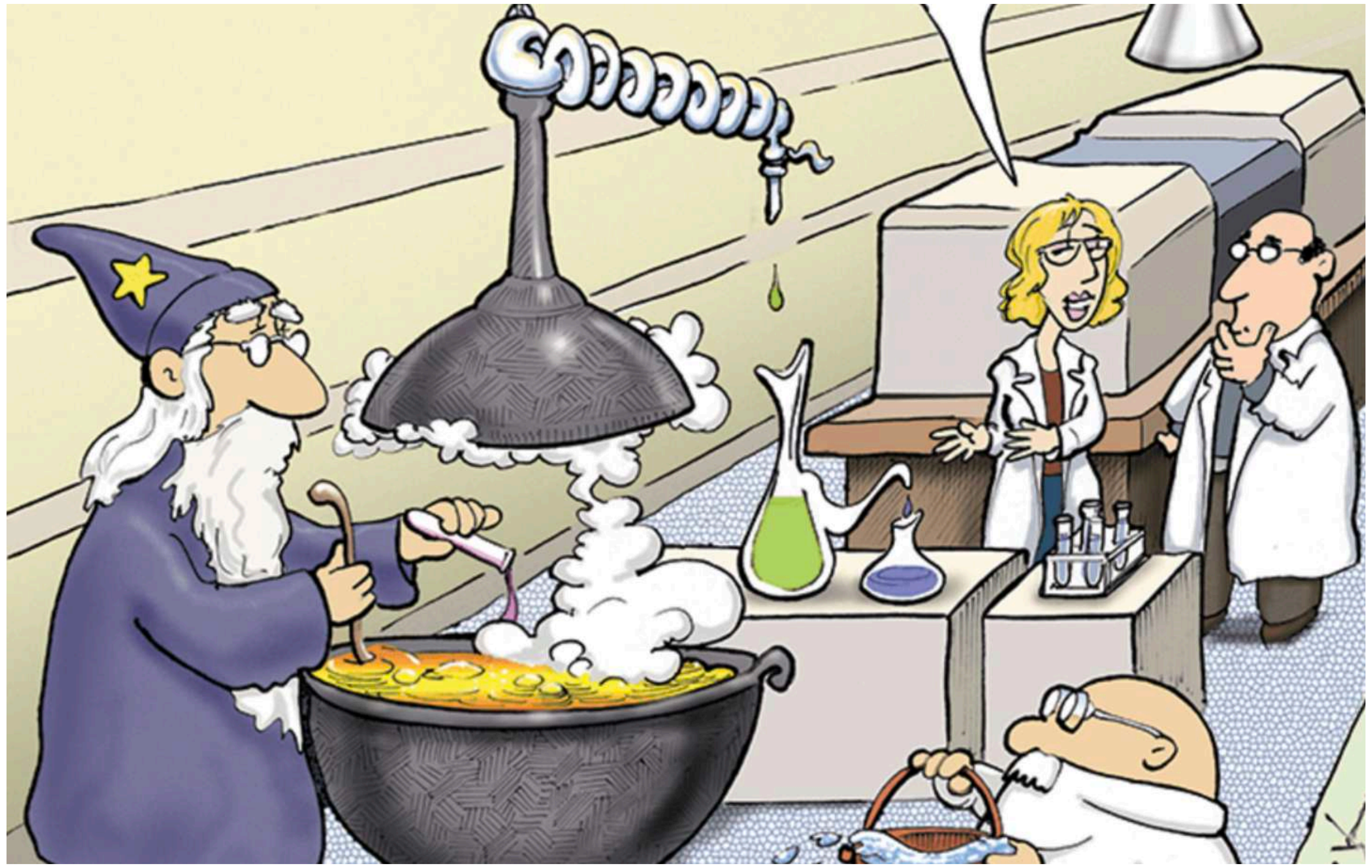
NA49: Phys.Rev.C 89 (2014) 5, 054902
ALICE:*Eur.Phys.J.C* 79 (2019) 3, 236
ALICE: Phys.Lett.B 807 (2020) 135564
ALICE: e-Print: 2206.03343
NA61: Eur.Phys.J.C 81 (2021) 5, 384
HADES: ongoing
STAR: ongoing

M. Gzdzicki, M. Gorenstein,
M. Mackowiak, AR, NPA 1001 (2020) 121915

Identity Method: Moments of W distributions ⇒ moments of multiplicity distributions

$$\begin{pmatrix} \langle N_K^2 \rangle \\ \langle N_\pi^2 \rangle \\ \langle N_\pi N_K \rangle \end{pmatrix} = \begin{pmatrix} A^{-1} \\ \text{defined by} \\ \rho_i(x) \end{pmatrix} \times \begin{pmatrix} \langle W_K^2 \rangle - f_1(\rho_i) \\ \langle W_\pi^2 \rangle - f_2(\rho_i) \\ \langle W_\pi W_K \rangle - f_3(\rho_i) \end{pmatrix}$$

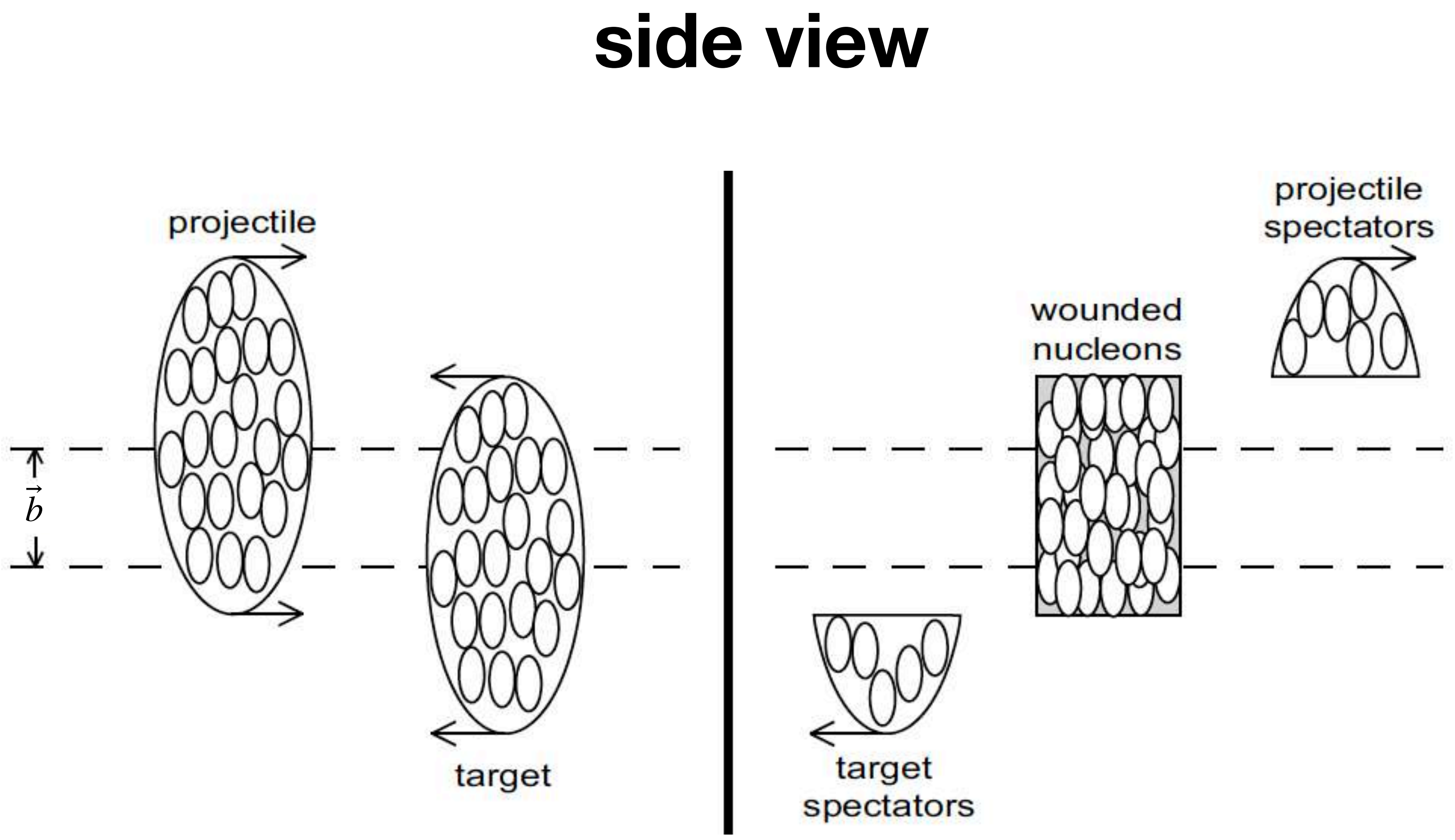
- **provides unique solutions**
- **works for any number of particles**
- **works for higher order pure and mixed moments**

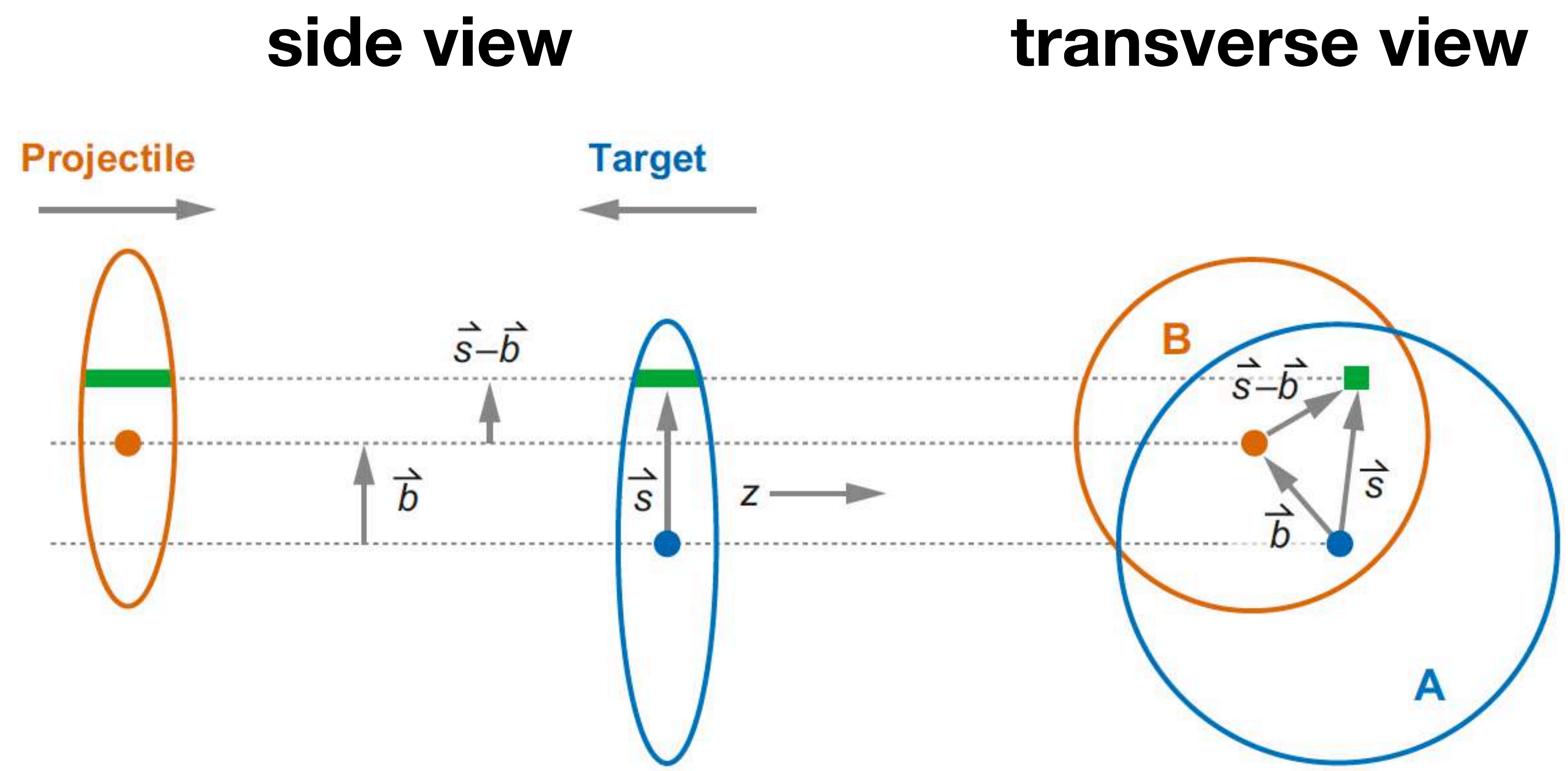


Challenges in measurements

- Particle misidentification issues
- Contributions from participant/volume fluctuations

Collision Geometry





Spectators

- Nucleons which do not meet any other nucleons on their way

Wounded nucleons, N_W

- Nucleons which collided at least once inelastically

Number of collisions, N_{coll}

- All possible binary collisions

Impact parameter, $\vec{b}$

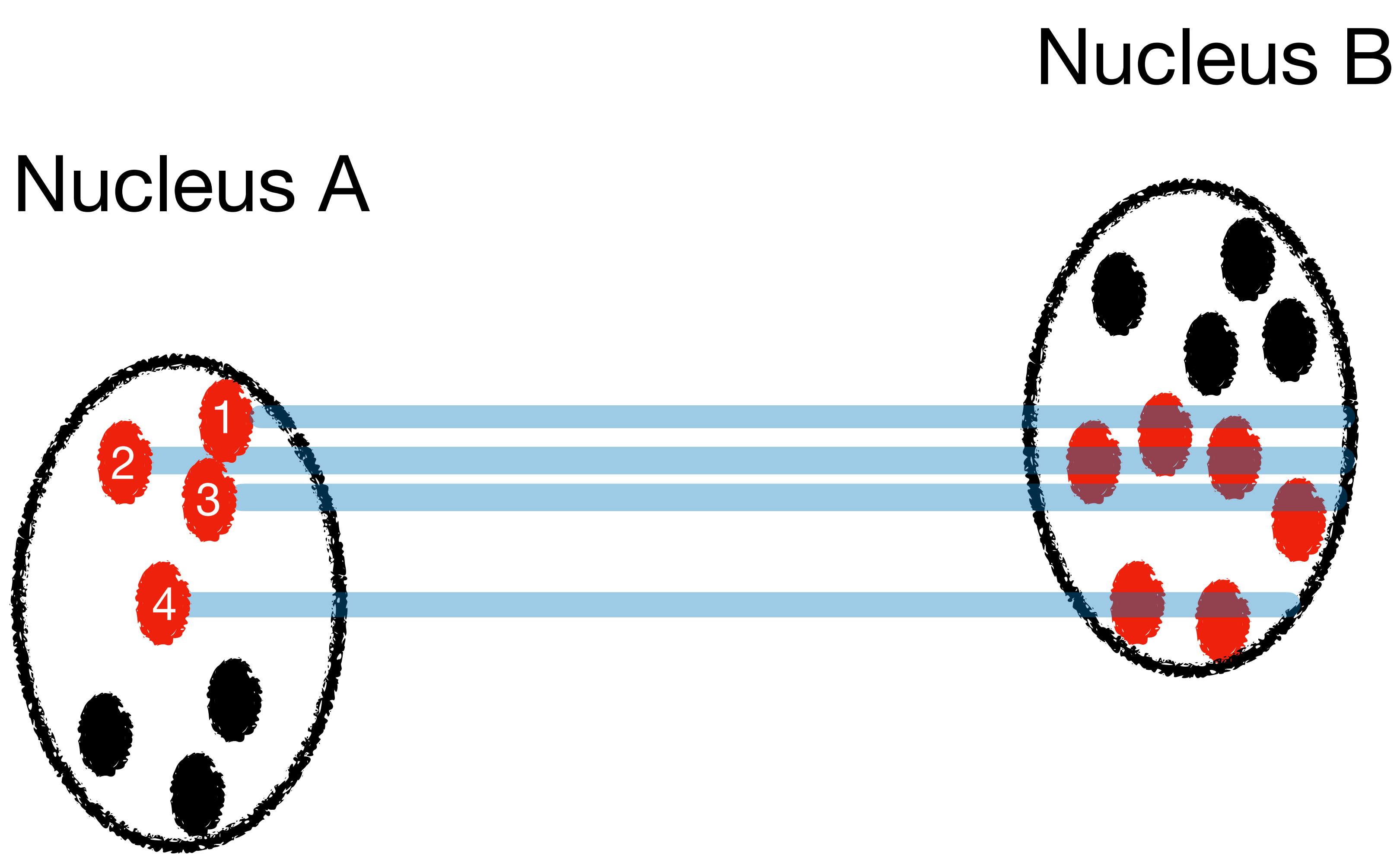
- A 2D vector connecting centres of the colliding nuclei in plane transverse to the nucleon trajectories
- **Central collisions**
  - Characterised by the smallest values of $|\vec{b}|$
- **Minimum bias collisions**
  - Averaged over all different values of $|\vec{b}|$

Wounded nucleon fluctuations

A. Bialas, and M. Bleszynski, W. Czyz, Nucl. Phys. B111 (1976) 461

In this paper we propose to describe the nucleus-nucleus collisions in terms of the number of "wounded" nucleons (w) i.e. the number of nucleons which underwent at least one inelastic collisions in this process.

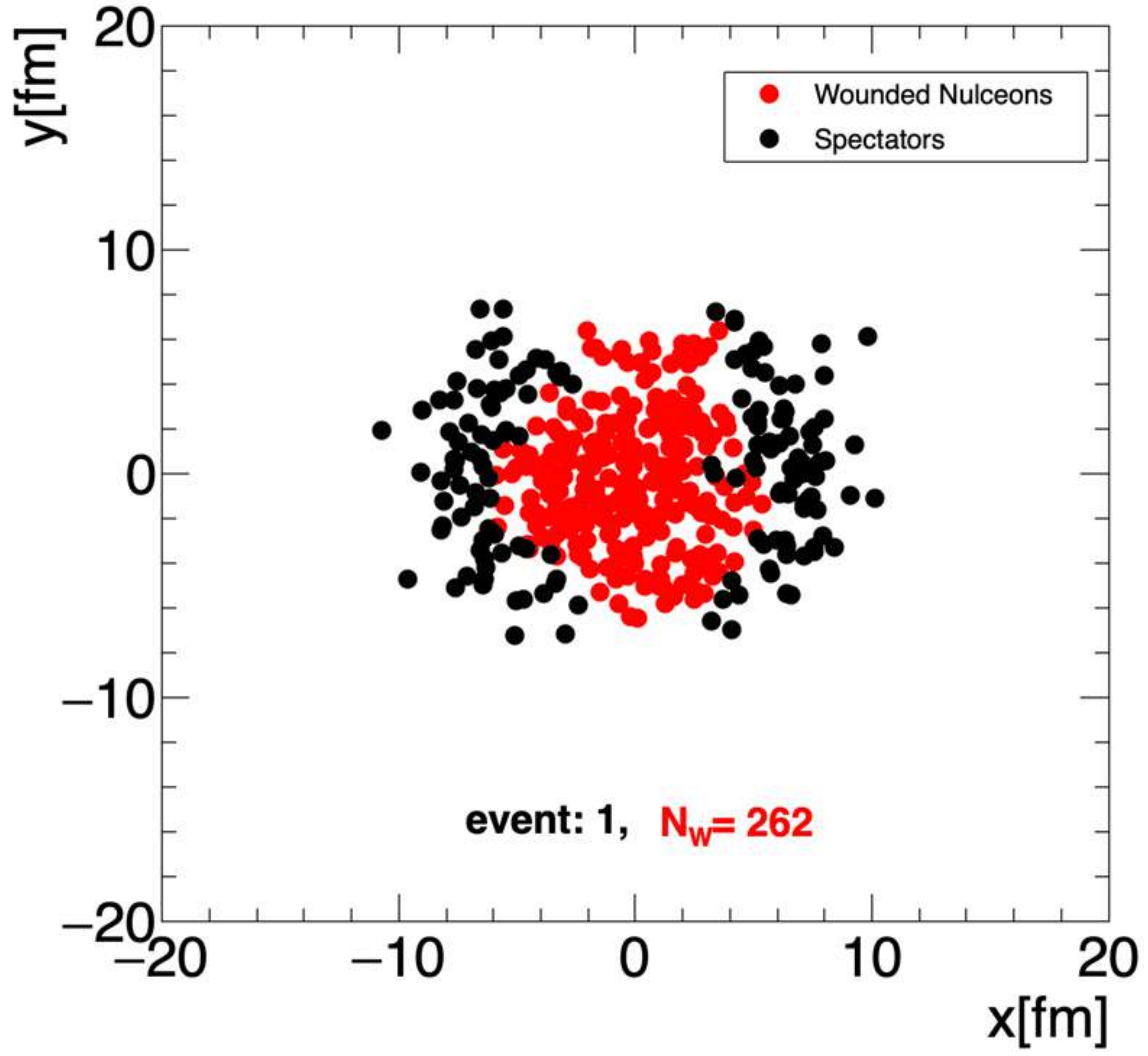
Example



$$N^1_W = 1 + 3,\ N^1_{coll} = 3$$
$$N^2_W = 1 + 0,\ N^2_{coll} = 3$$
$$N^3_W = 1 + 1,\ N^3_{coll} = 3$$
$$N^4_W = 1 + 2,\ N^3_{coll} = 2$$

$$N_W \equiv \sum N^i_W = 10$$
$$N_{coll} \equiv \sum N^i_{coll} = 11$$

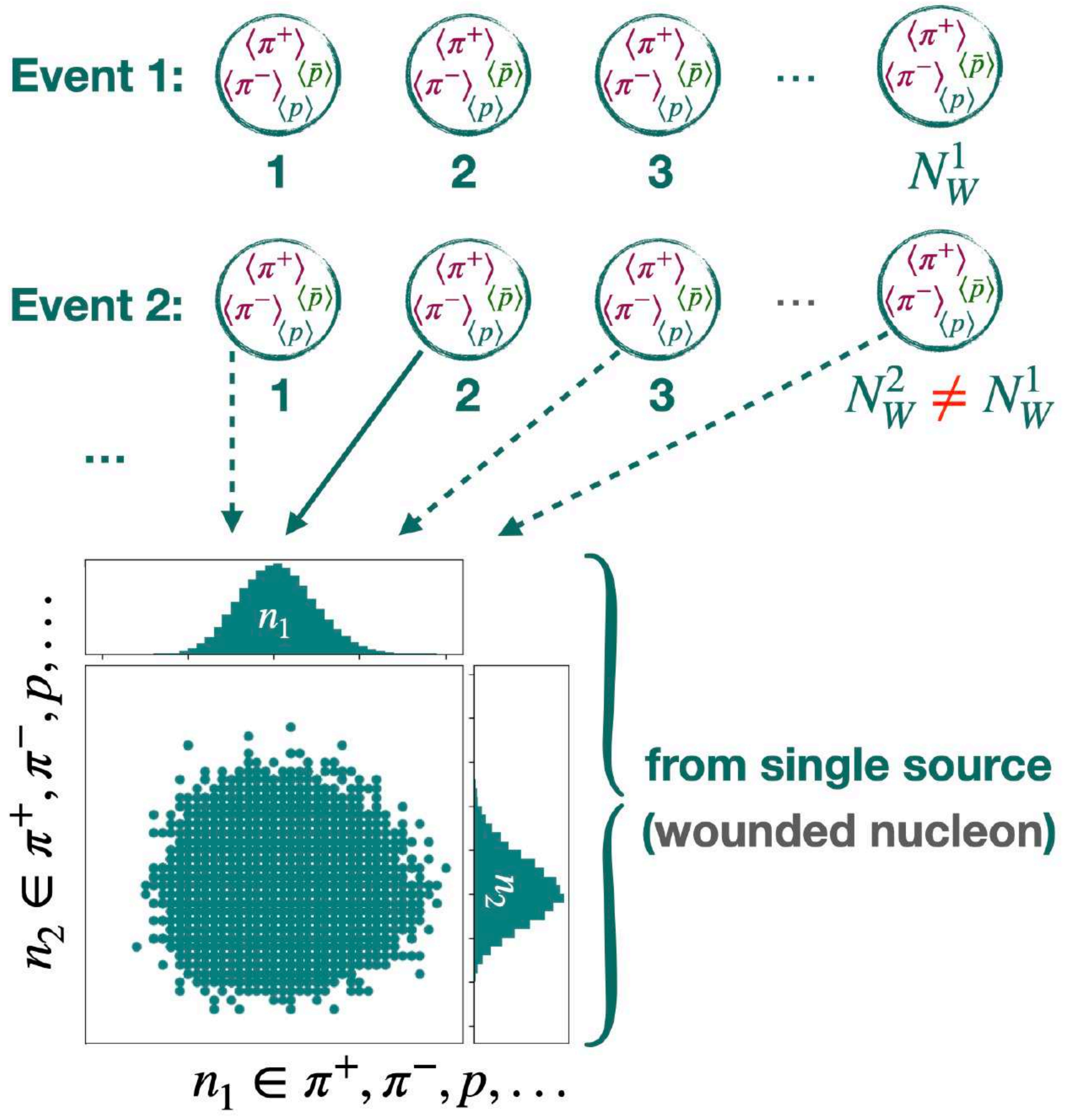
C. Loizides, J. Nagle, P. Steinberg, SoftwareX 1-2 (2015) 13-18 e-Print: 1408.2549 [nucl-ex]



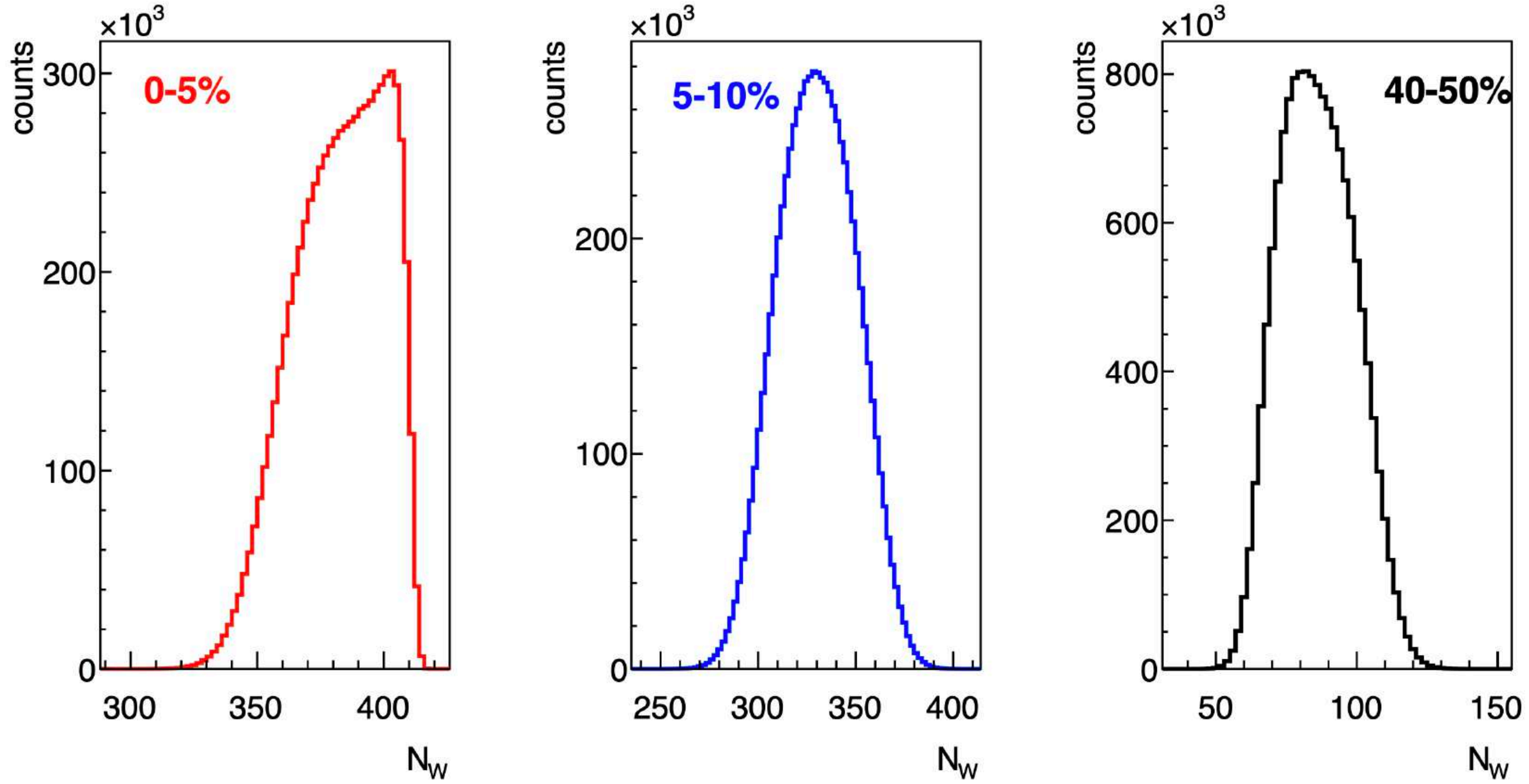
Contributions from participant/volume fluctuations

$$\kappa_n(\Delta N) = VT^3\hat{\chi}_n^B \qquad \frac{\kappa_n(\Delta N)}{\kappa_m(\Delta N)} \neq \frac{\hat{\chi}_n^B}{\hat{\chi}_m^B} \quad \text{with } \Delta N = N_B - N_{\bar{B}}$$

model of independent sources



N_W **distributions from Glauber model (Pb-Pb@2.76 TeV)**



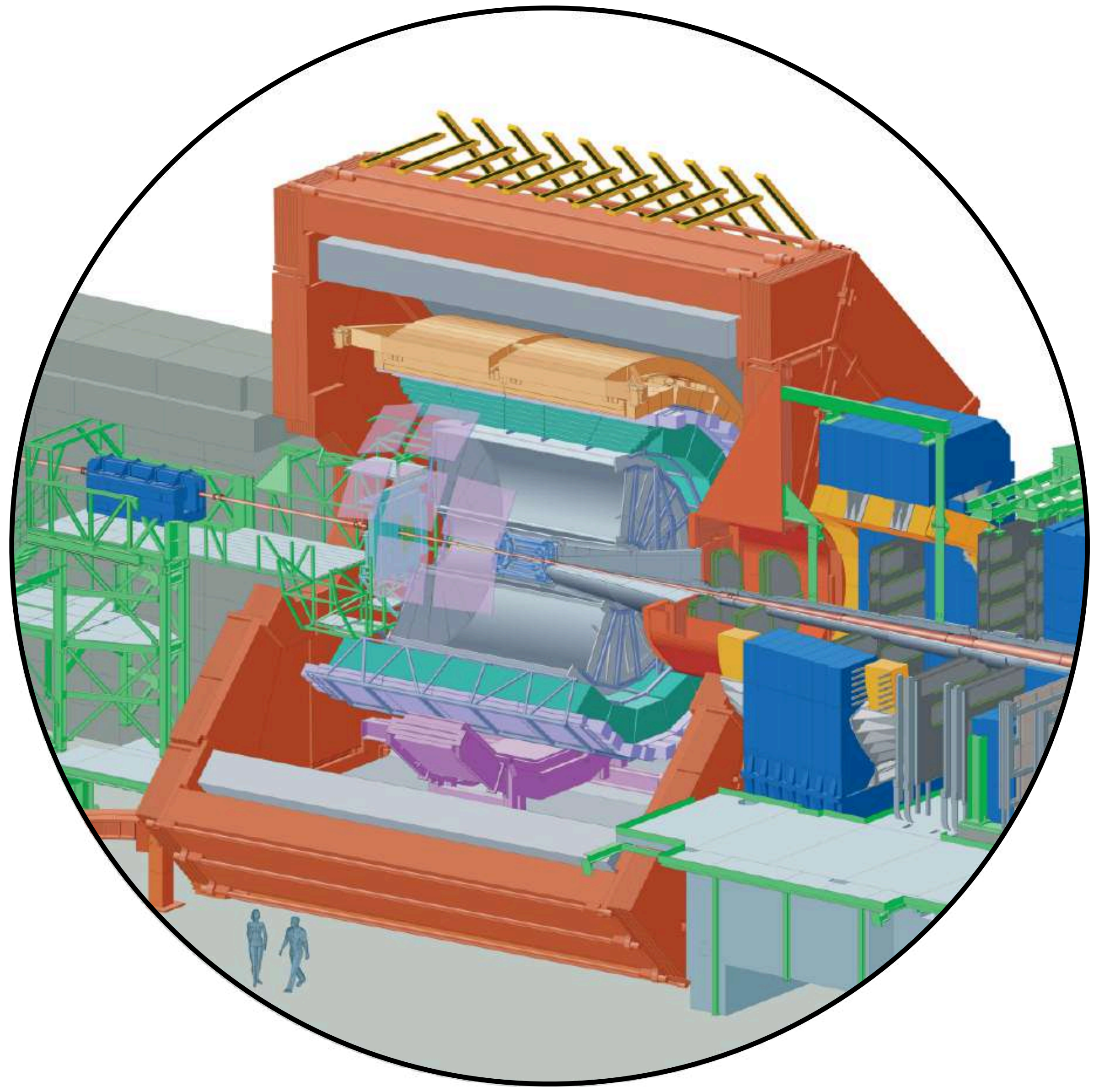
$$\kappa_2(\Delta N) = \langle N_W \rangle\, \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \kappa_2(N_W)$$

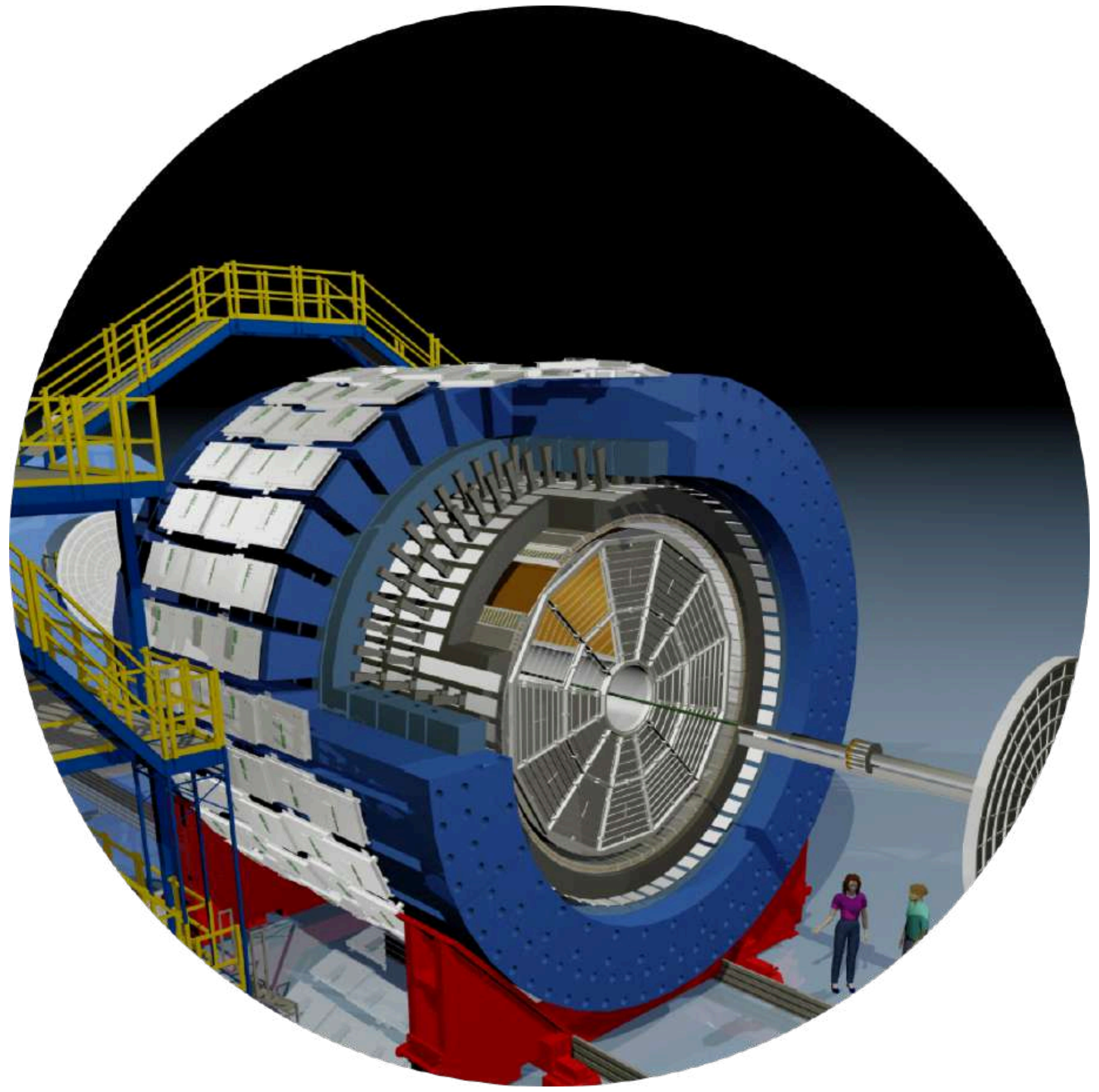
$$\kappa_3(\Delta N) = \langle N_W \rangle\, \kappa_3(\Delta n) + 3\,\langle \Delta n \rangle\, \kappa_2(\Delta n)\kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$$

...

$$\Delta N = N_B - N_{\bar{B}}, \;\; \Delta n = n_B - n_{\bar{B}}$$

P. Braun-Munzinger, A.R., J. Stachel, NPA 960 (2017) 114
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911
A.R., R. Holzmann, J. Stroth, NPA 1034 (2023) 122641
V. Koch, R. Holzmann, A.R., J. Stroth, in preparation

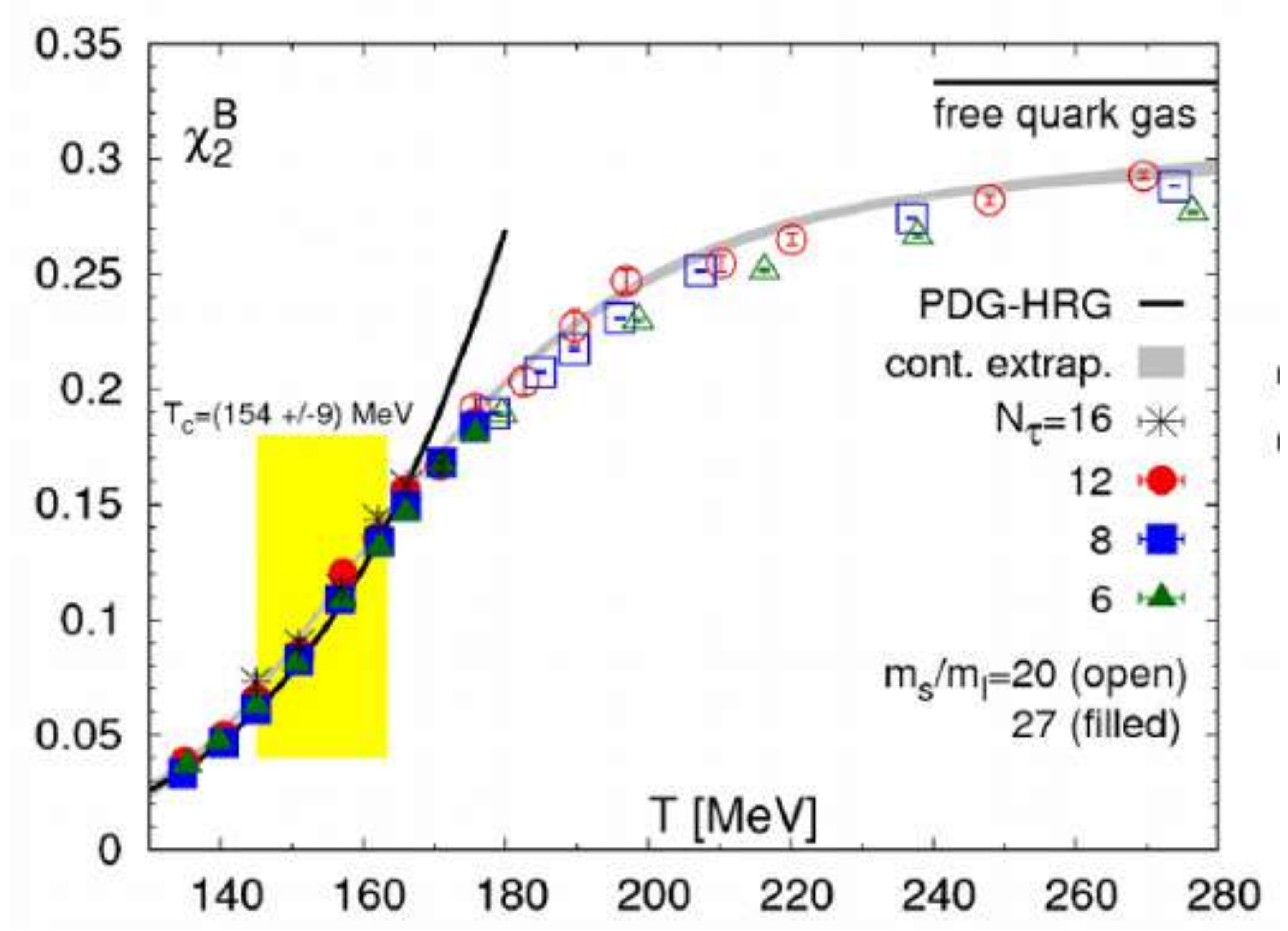


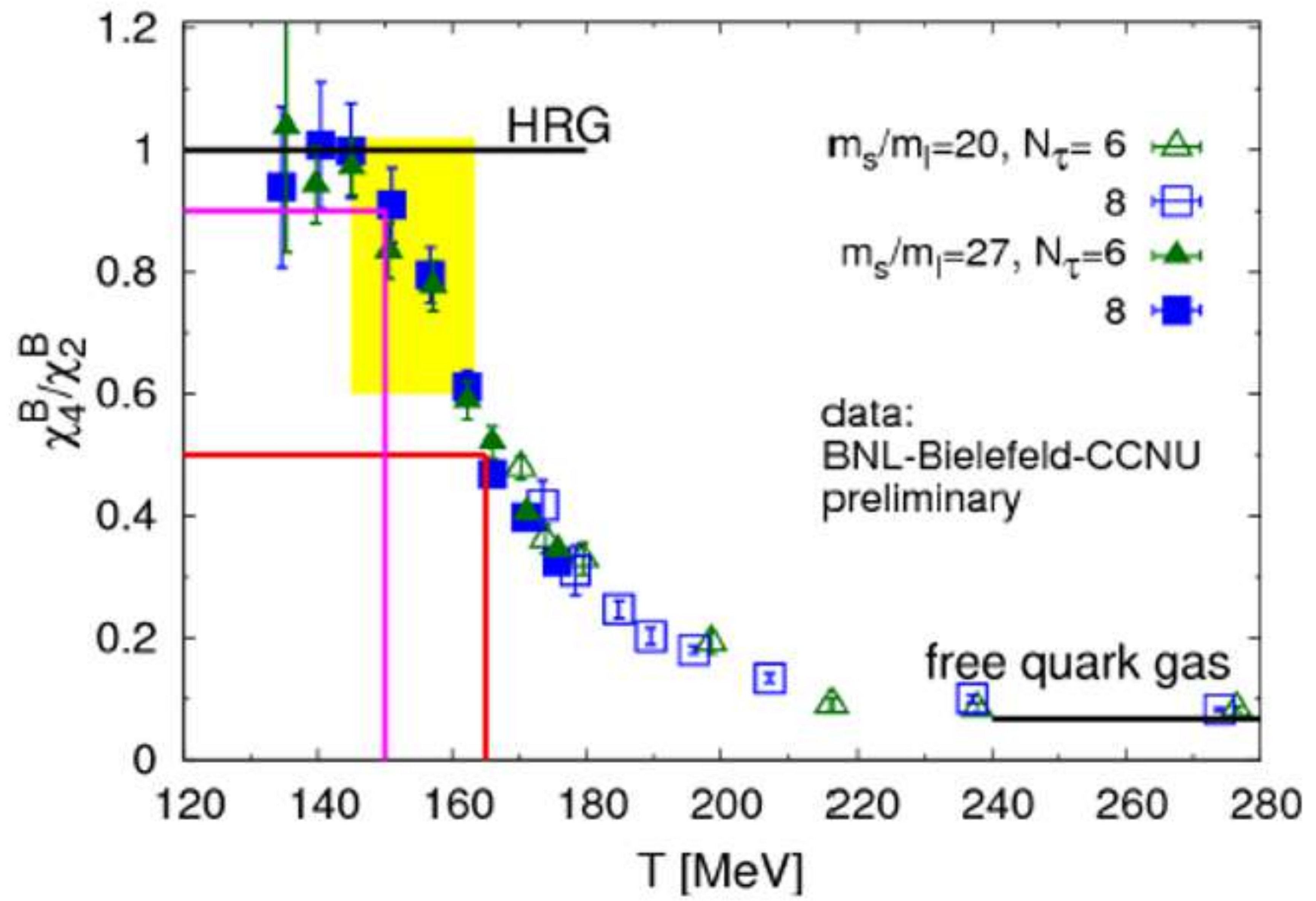


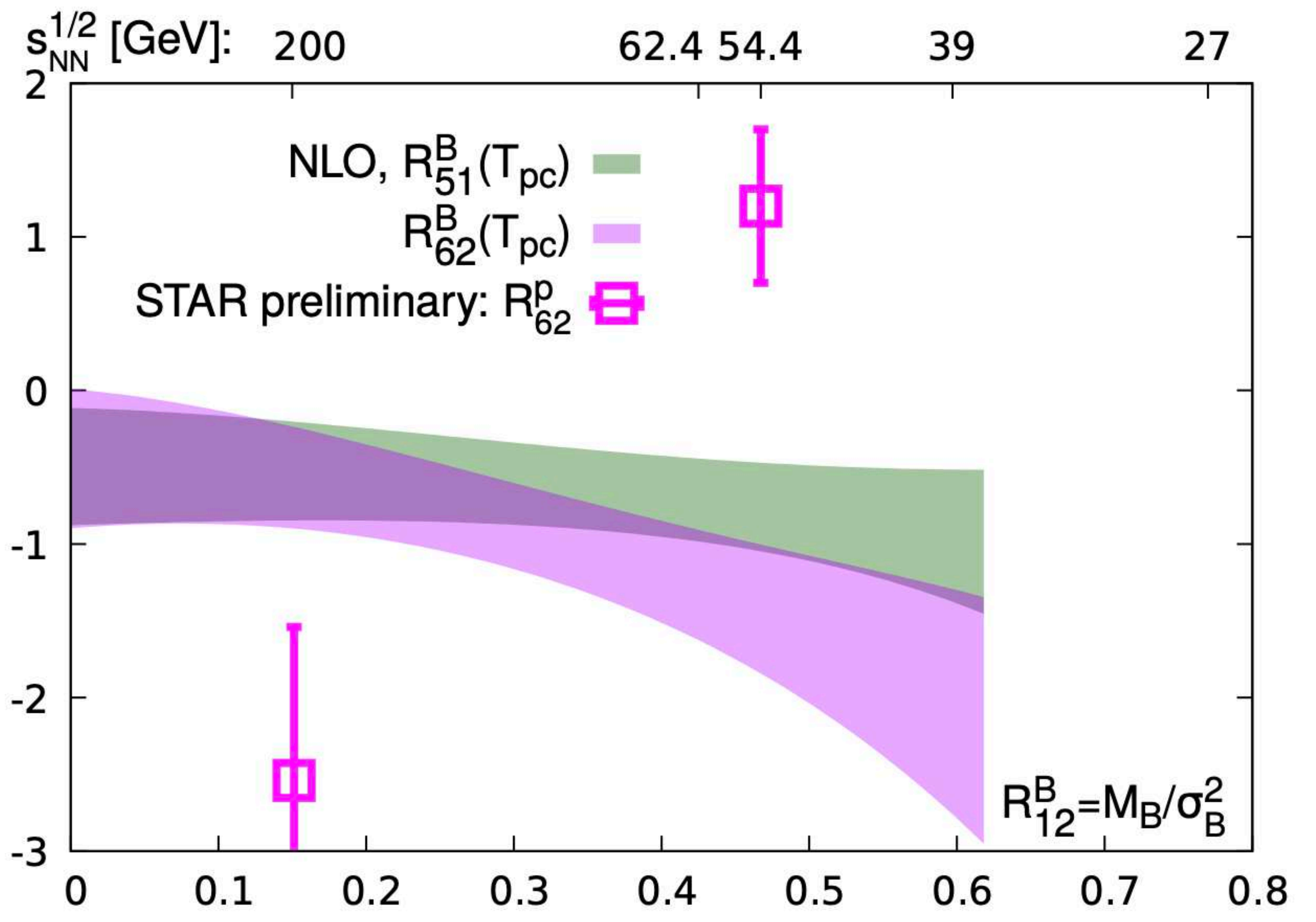
Experimental results

- Search for crossover transition
- Search for critical point and first order phase transition

Predictions from LQCD







A. Bazavov et al [HotQCD], PRD 101 (2020) 074502
A. Bazavov et al., Phys.Rev. D85 (2012) 054503

- χ_2^B : agreement with the HRG in GCE (for T < 165 MeV)
- χ_4^B/χ_2^B : significant reduction compared to HRG in GCE (for T > 150 MeV)
- $\chi_{5(6)}^B/\chi_{1(2)}^B$: (progressively) negative sign towards lower energies, probe for crossover
- hierarchy of cumulant ratios: $\chi_3^B/\chi_1^B > \chi_4^B/\chi_2^B > \chi_5^B/\chi_1^B > \chi_6^B/\chi_2^B$, probe for crossover

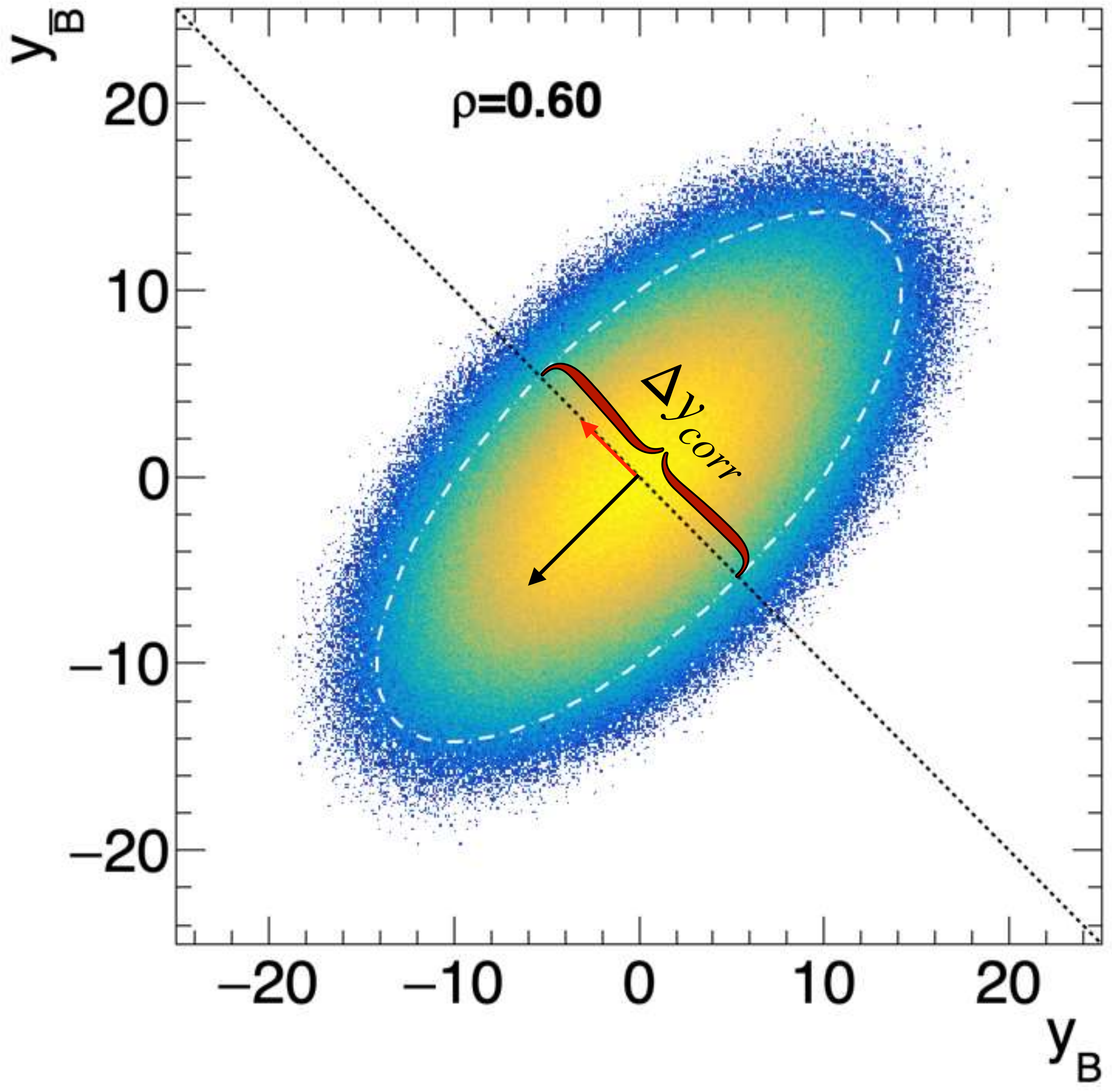
To compare with experiments

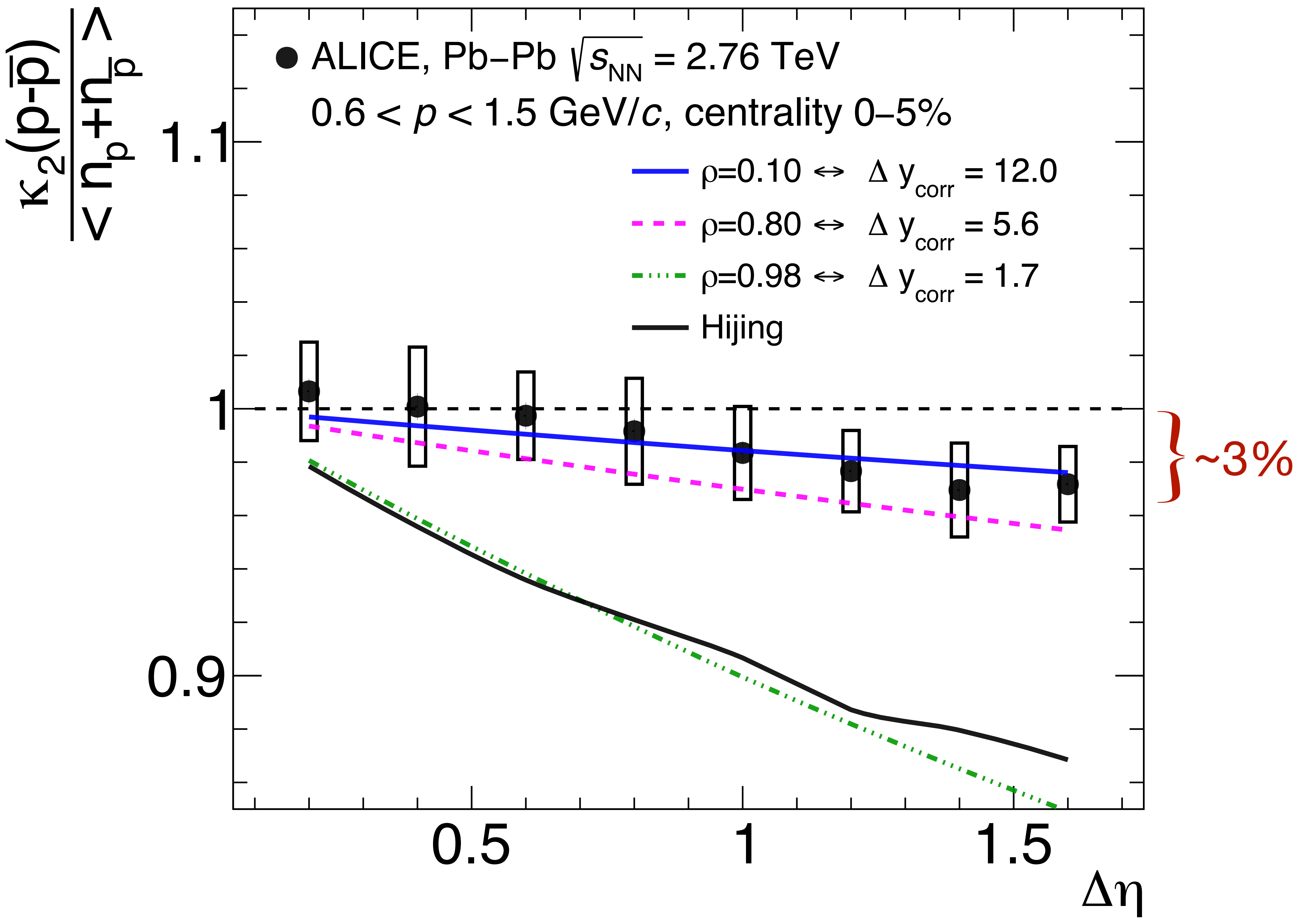
- volume is fixed
- charge conservations are imposed on the averages
- predictions are for net-baryon number

Probing the matter at the phase boundary (ALICE)

P. Braun-Munzinger, A.R., J. Stachel, under preparation
A.R., P. Braun-Munzinger, J. Stachel, QM 2022
P. Braun-Munzinger, A.R., J. Stachel, NPA 982 (2019) 307-310

A.R., NPA 967 (2017) 453-456 **ALICE:** Phys. Lett. B 807 (2020) 135564
Phys. Lett. B (2022) 137545



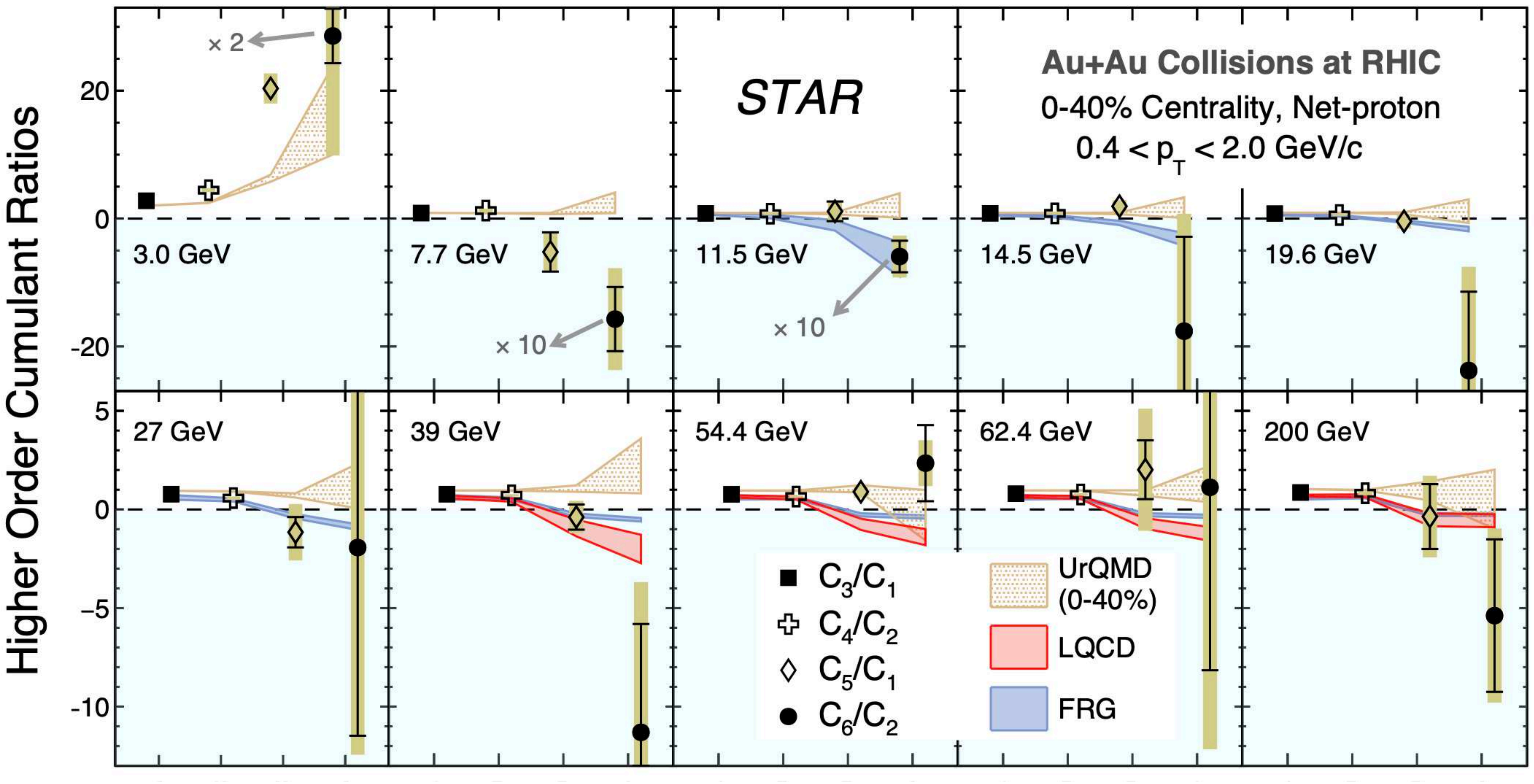


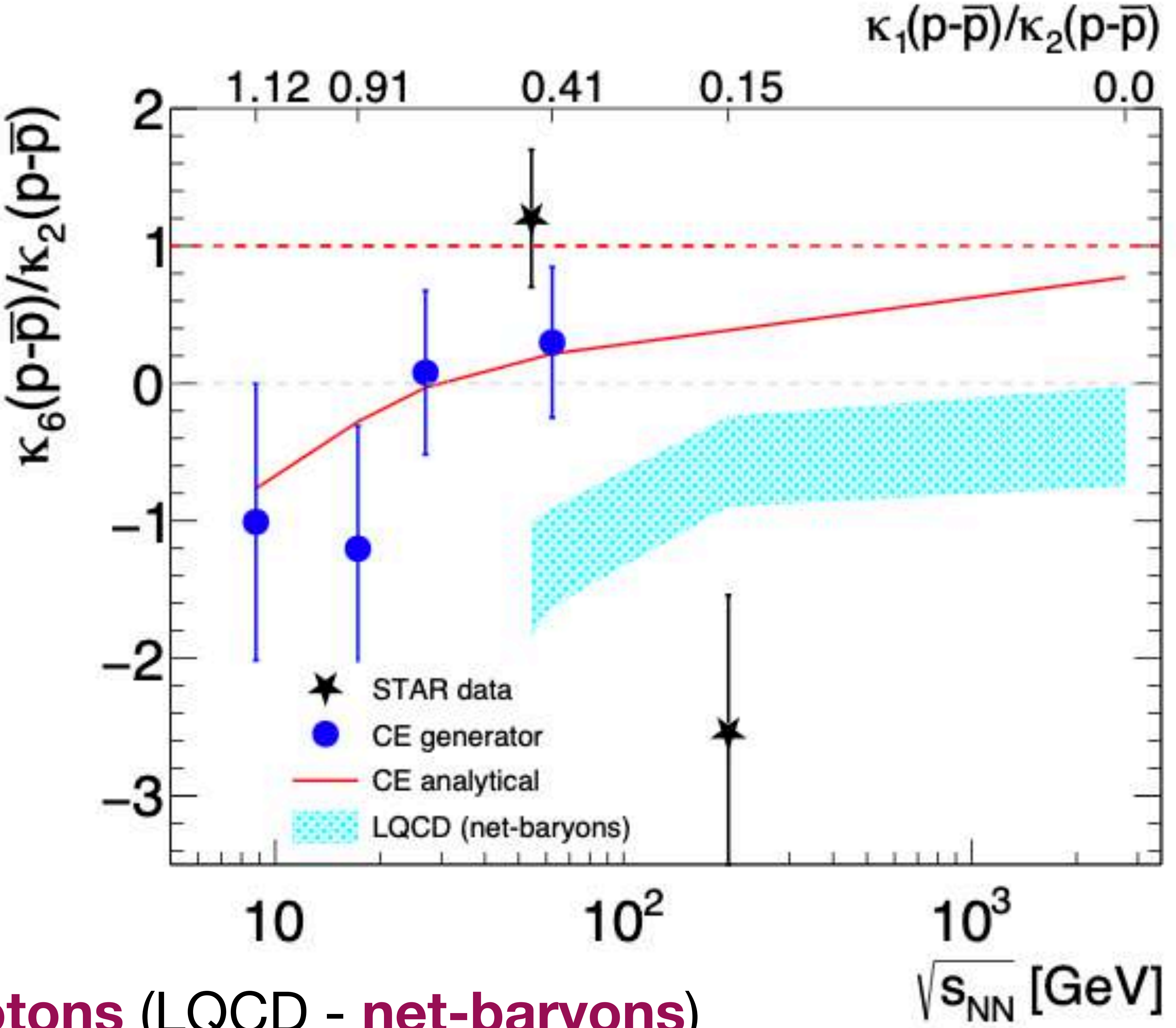
- Alice data: best description with $\rho = 0.1$ ($\Delta y_{corr} = 12$) ↔ **Global baryon number conservation**
- Calls into question baryon production mechanism in Hjing (Lund String Fragmentation)
  - Hijing results suggest $\rho = 0.98$ ($\Delta y_{corr} = 1.7$) ↔ **Strong local correlations**

Experimental search for crossover transition

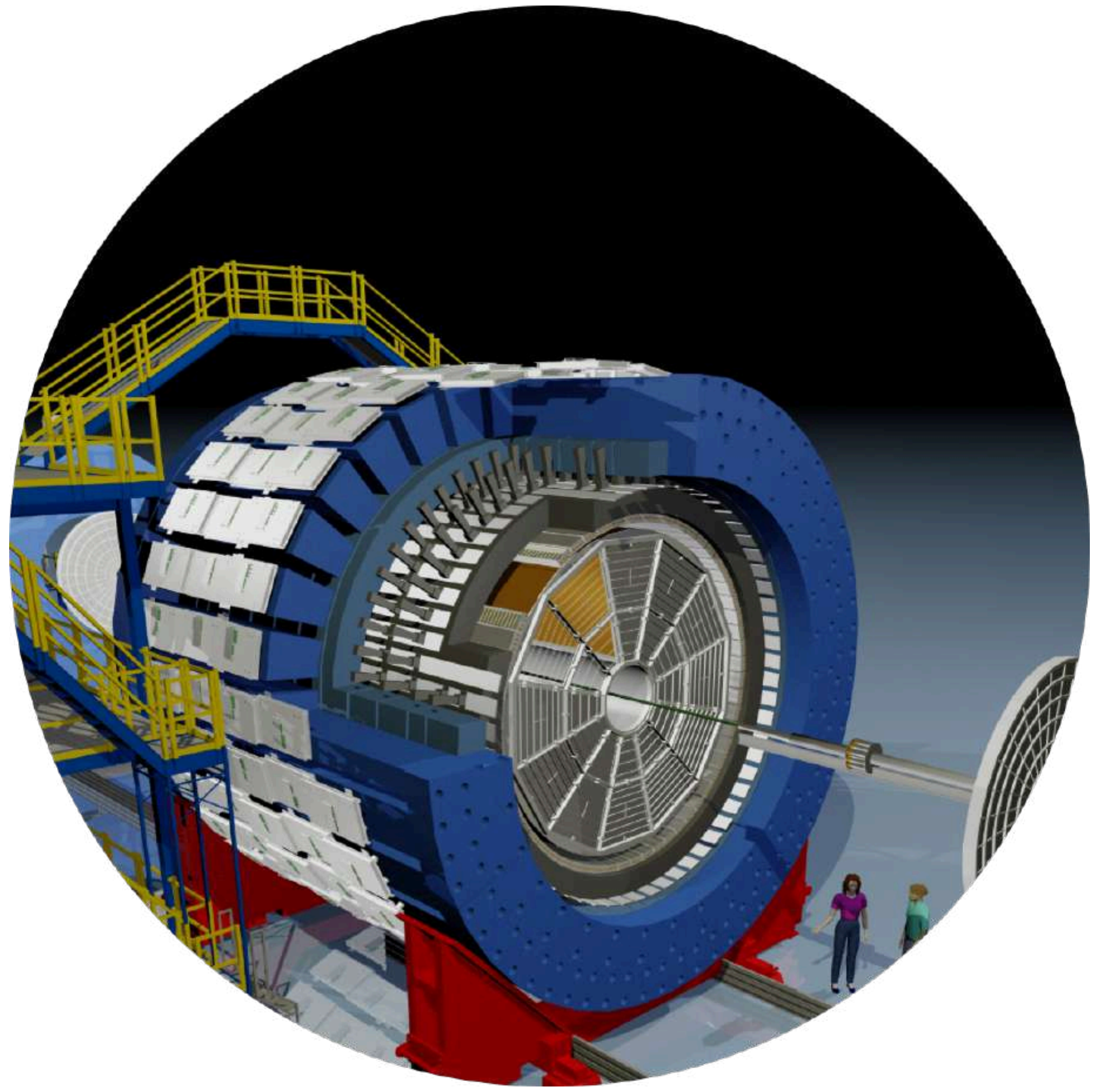
STAR: Phys.Rev.Lett. 127 (2021) 26, 262301
Phys.Rev.Lett. 130 (2023) 8, 082301

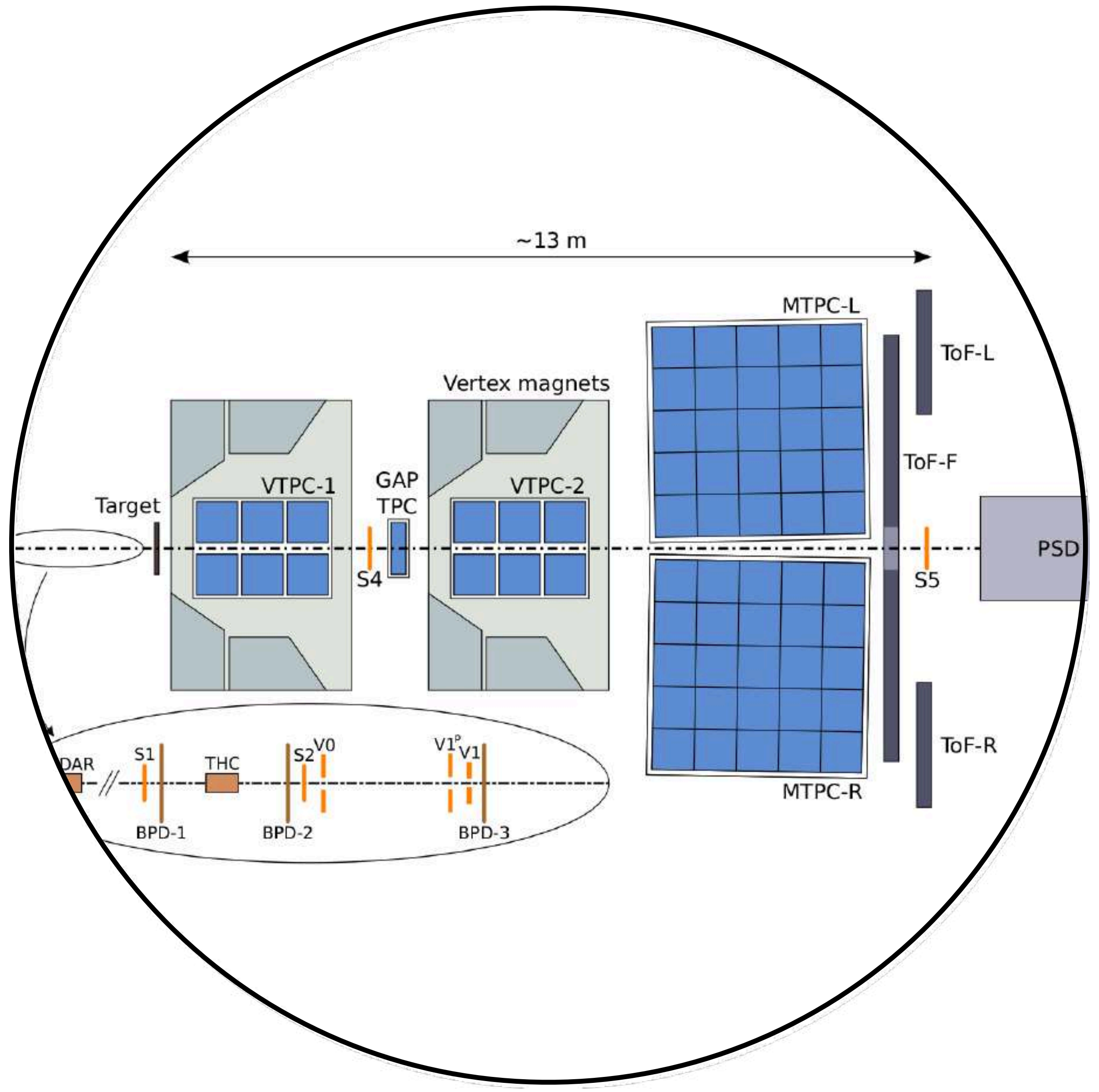
CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

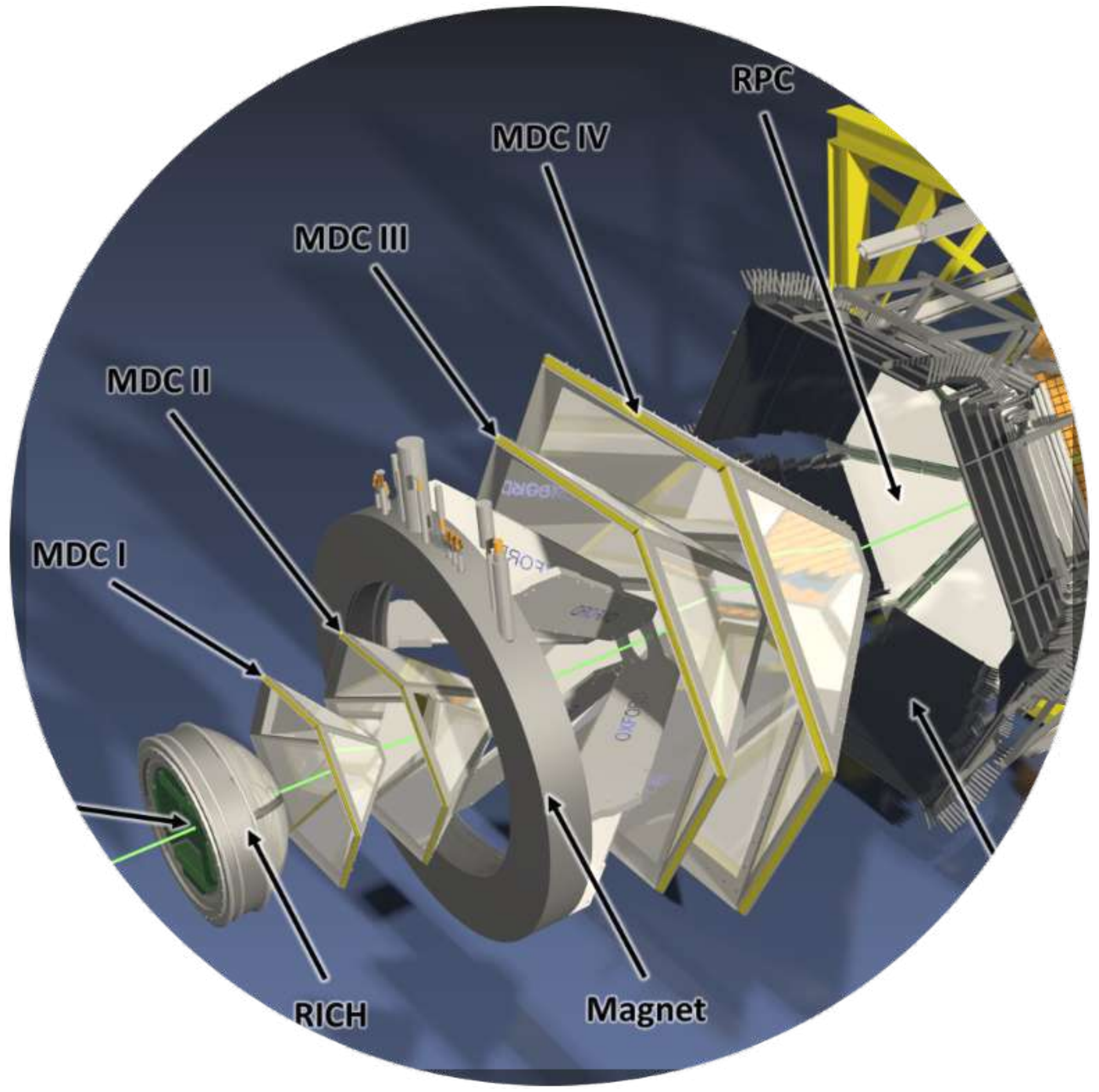




- first try to measure the cross-over signals via fluctuations of **net-protons** (LQCD - **net-baryons**)
  - reverse ordering at 3 GeV (driven by volume fluctuations of genuine physics?)
    - no anticipated ordering at 54.4 GeV !
  - no consistent trend between 54.4 and 200 GeV data (both are negative in LQCD)
- **Experimental verification of crossover signal is not conclusive**





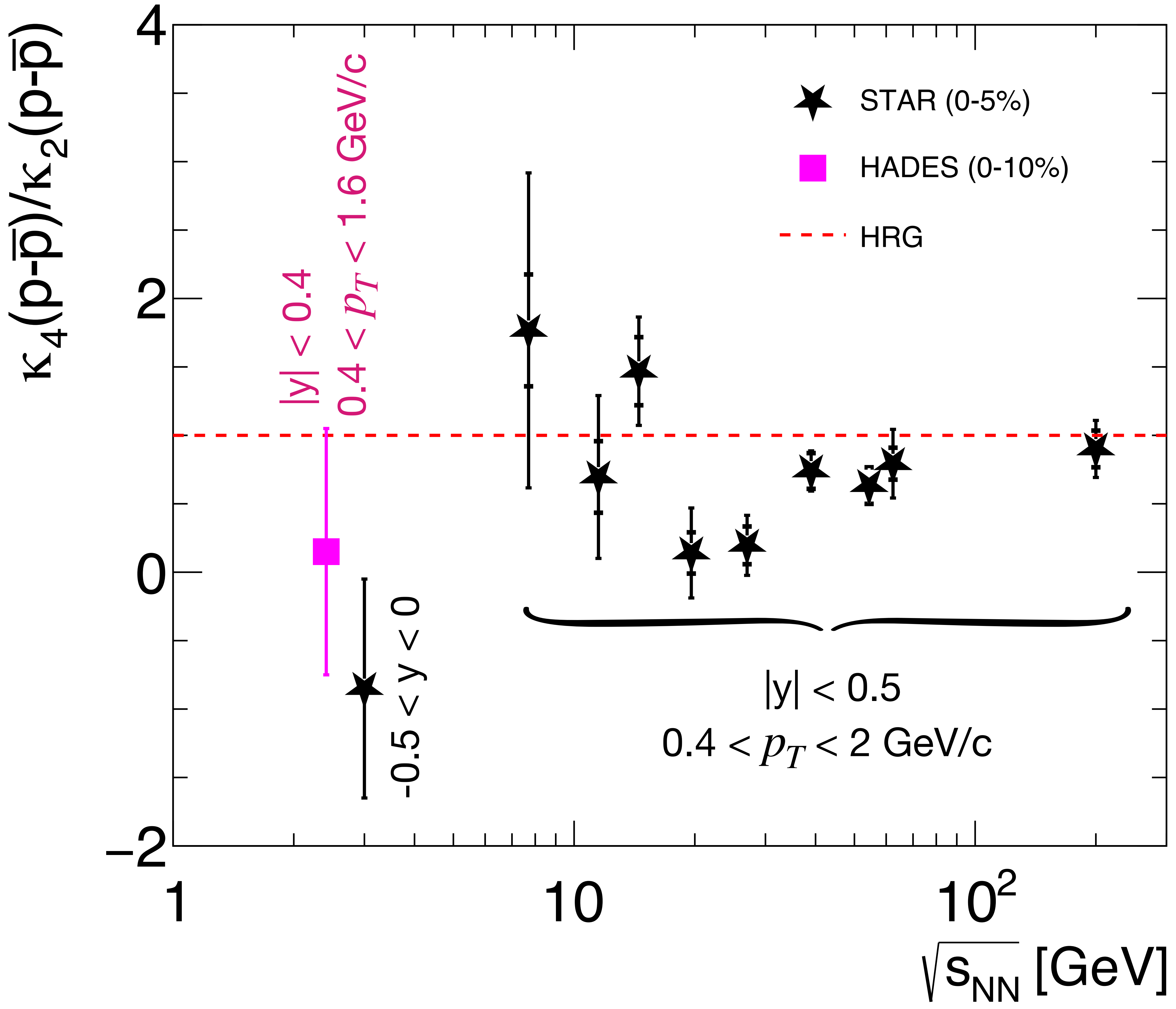


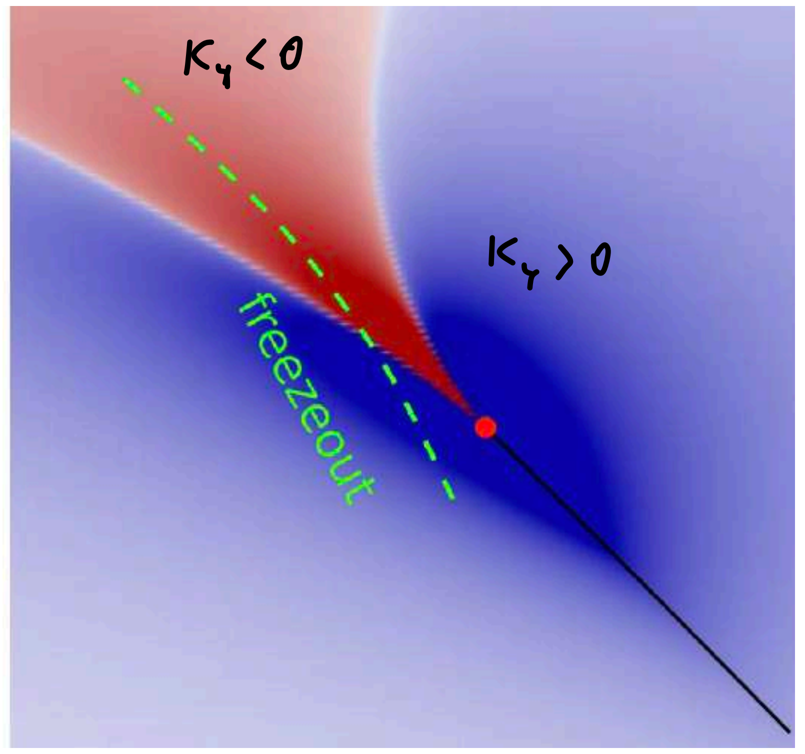
Experimental results

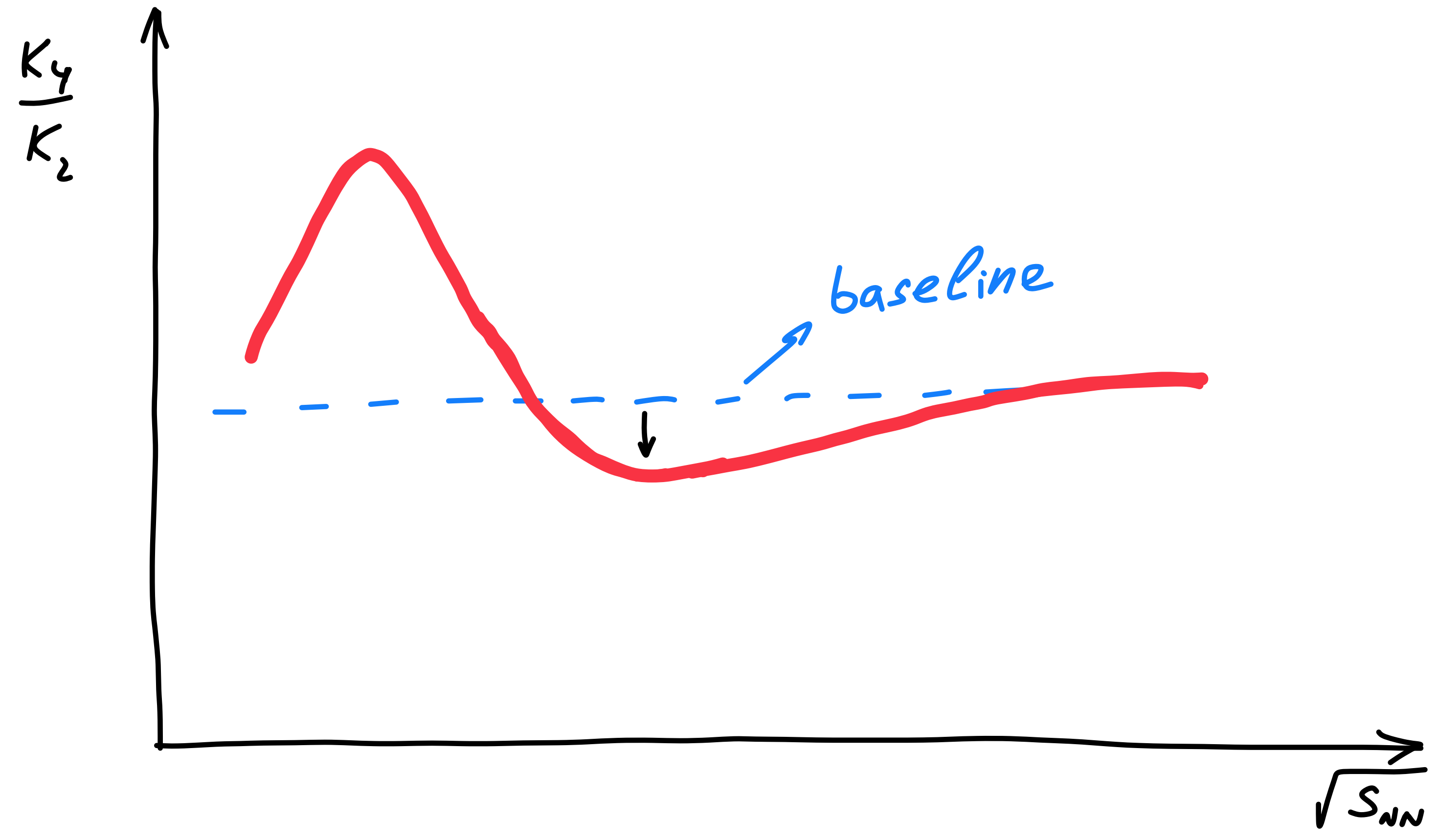
- Search for crossover transition
- Search for critical point and first order phase transition

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







a dip in the excitation function is generic

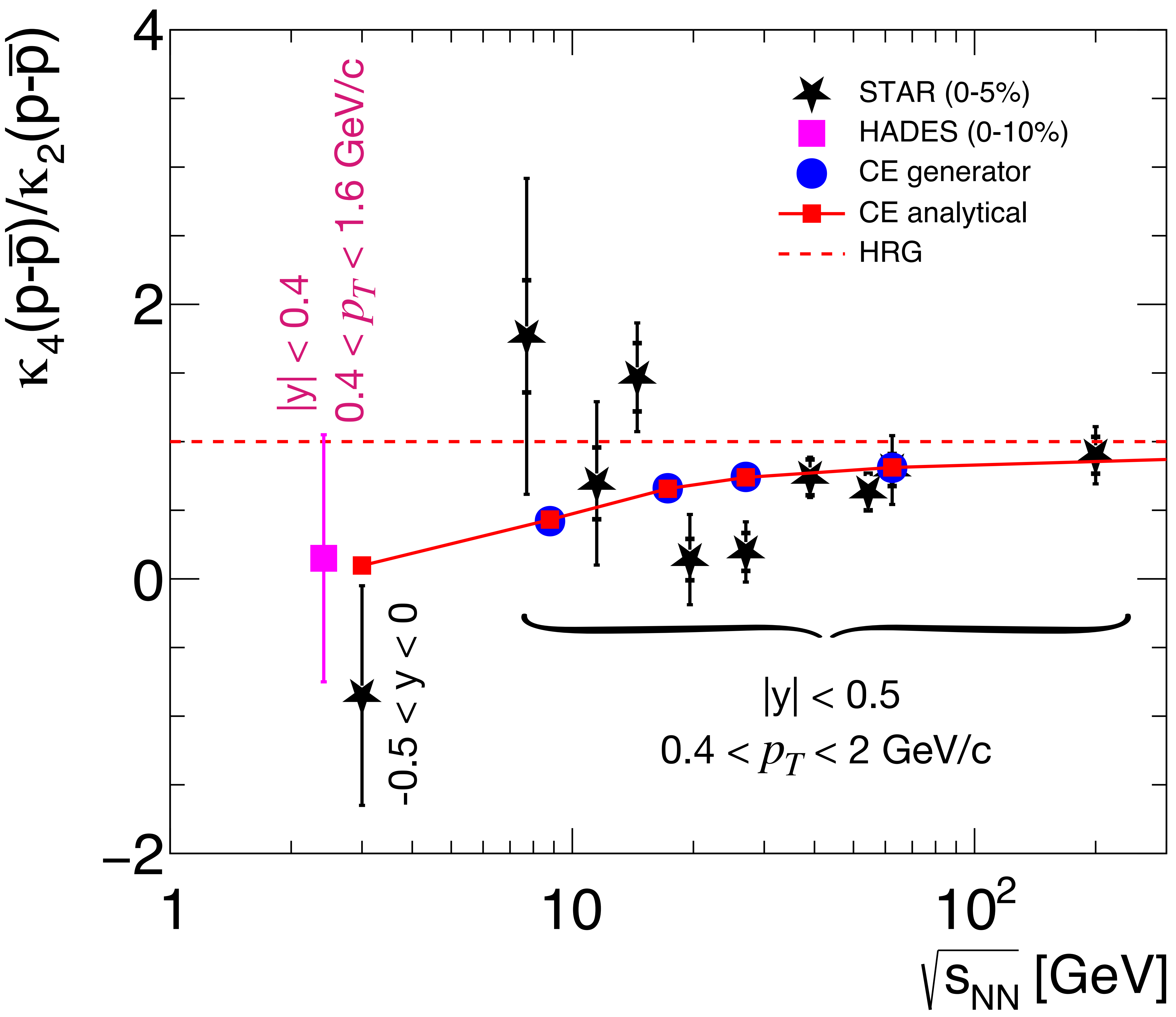
M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

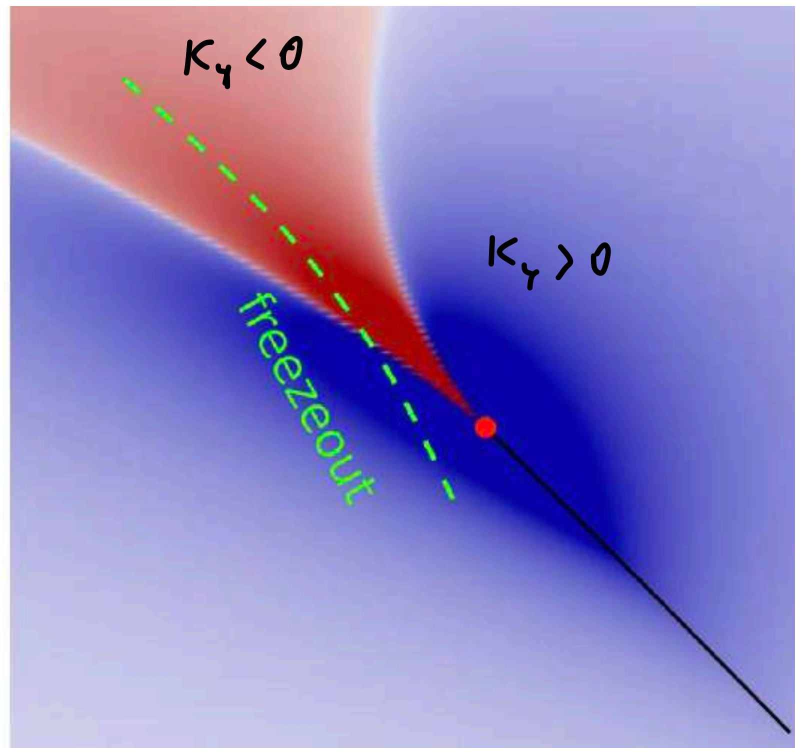
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

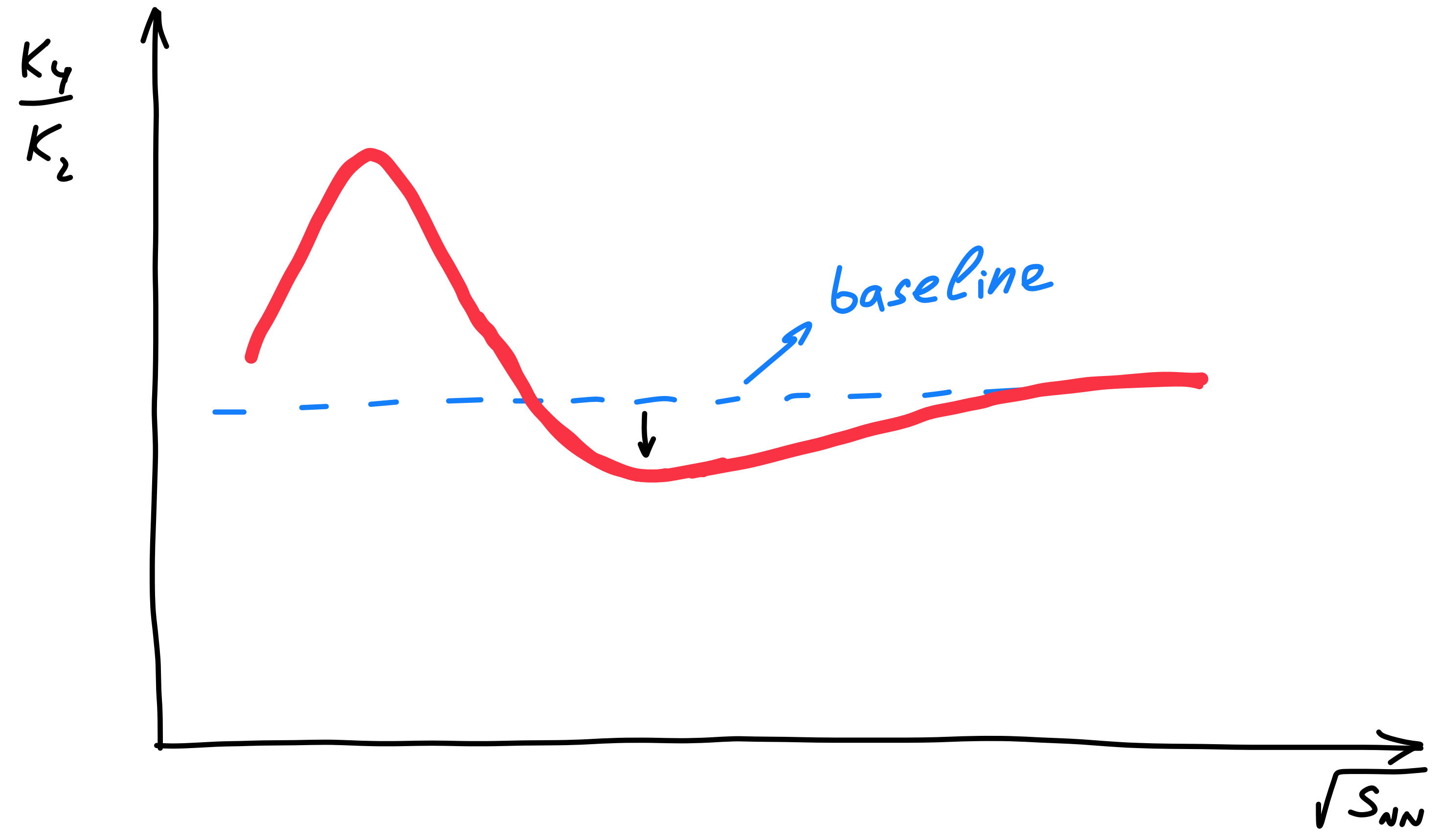
non-monotonic behaviour with a significance of 3.1σ relative to Skellam expectation

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







a dip in the excitation function is generic

M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

non-monotonic behaviour with a significance of 3.1σ relative to Skellam expectation

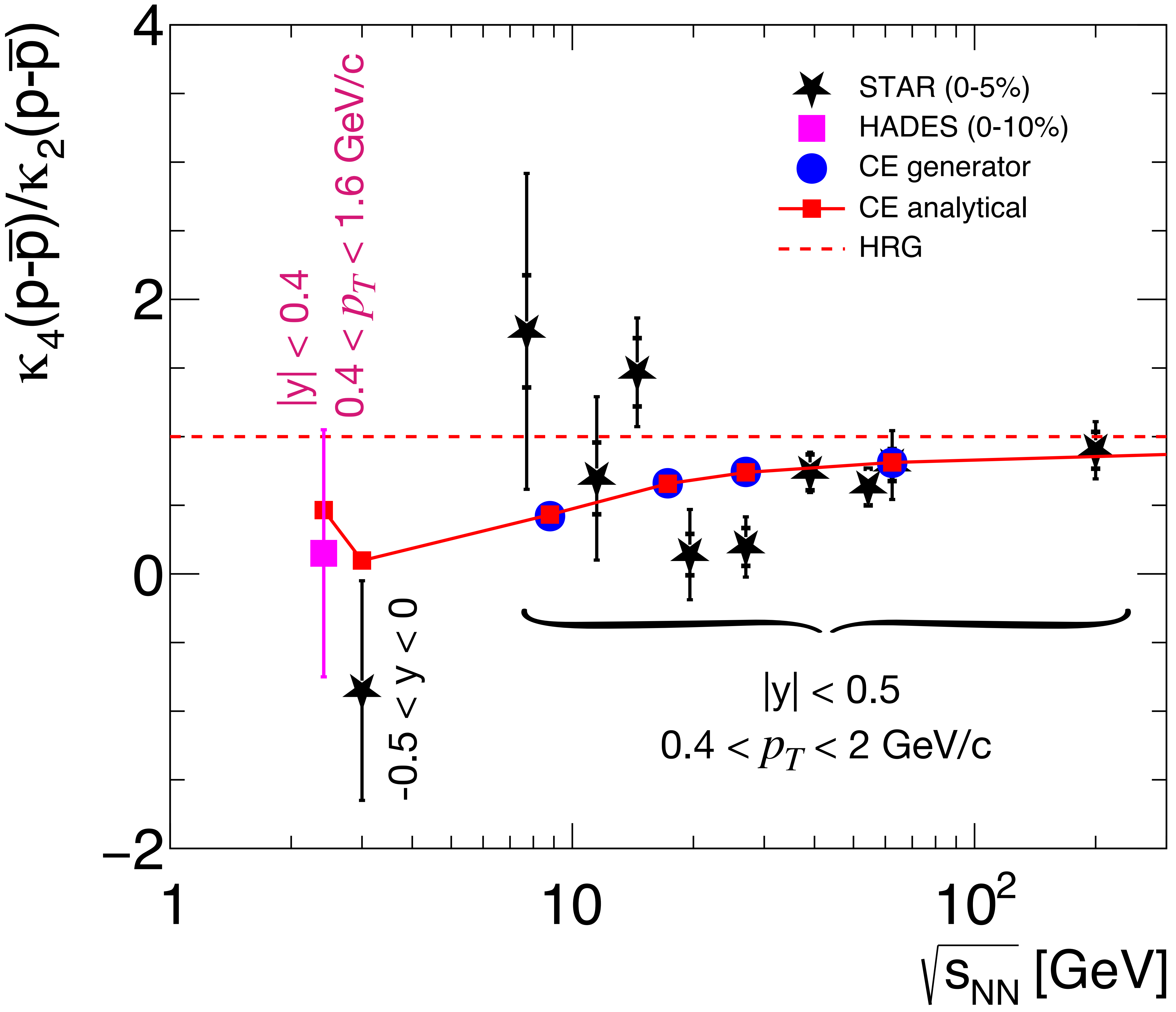
CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

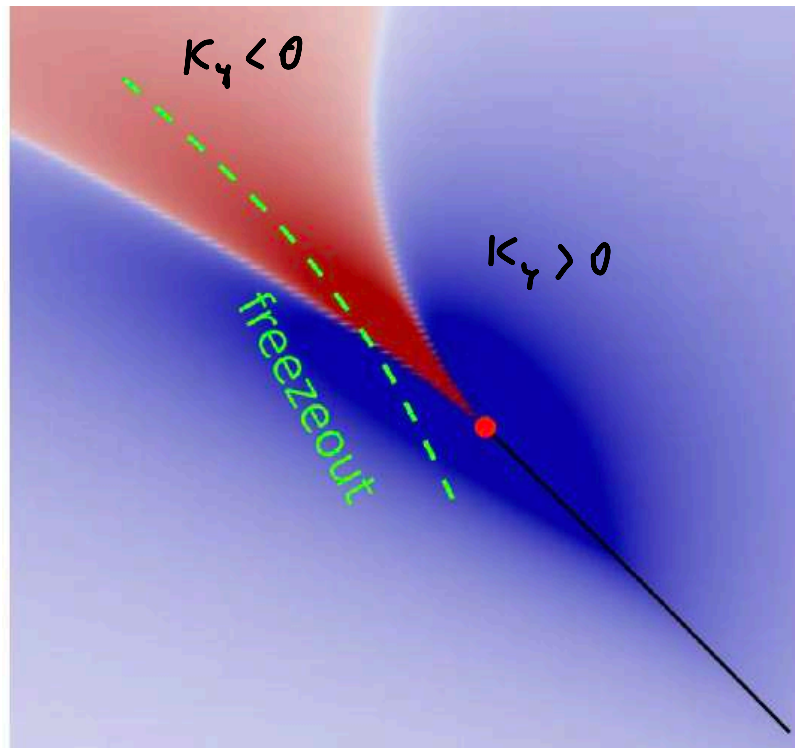
no statistically significant difference between the data and the canonical baseline (KS test: 1.2σ, χ^2 test: 1.5σ)

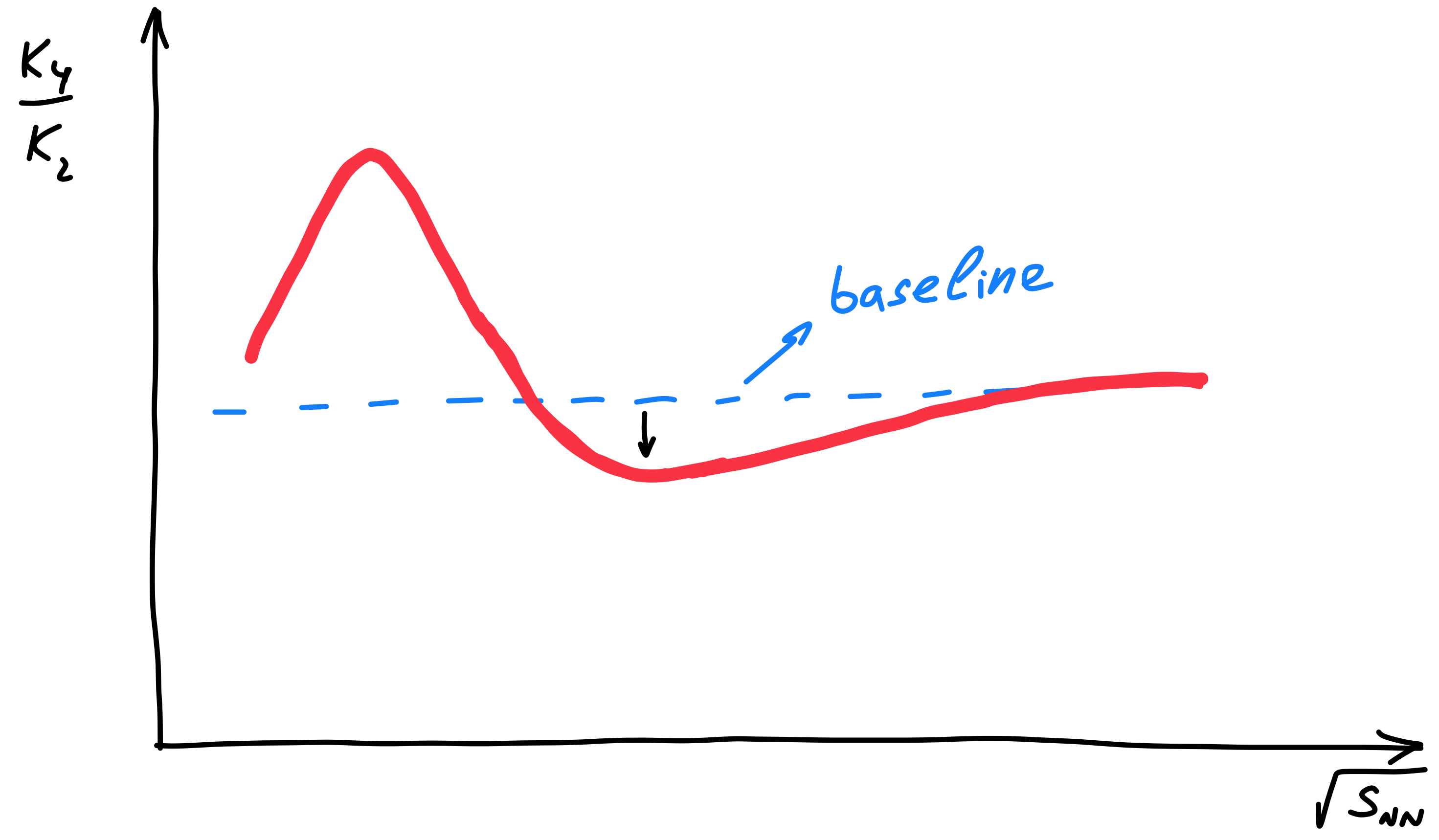
no clear signal for critical point (yet)

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







a dip in the excitation function is generic

M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

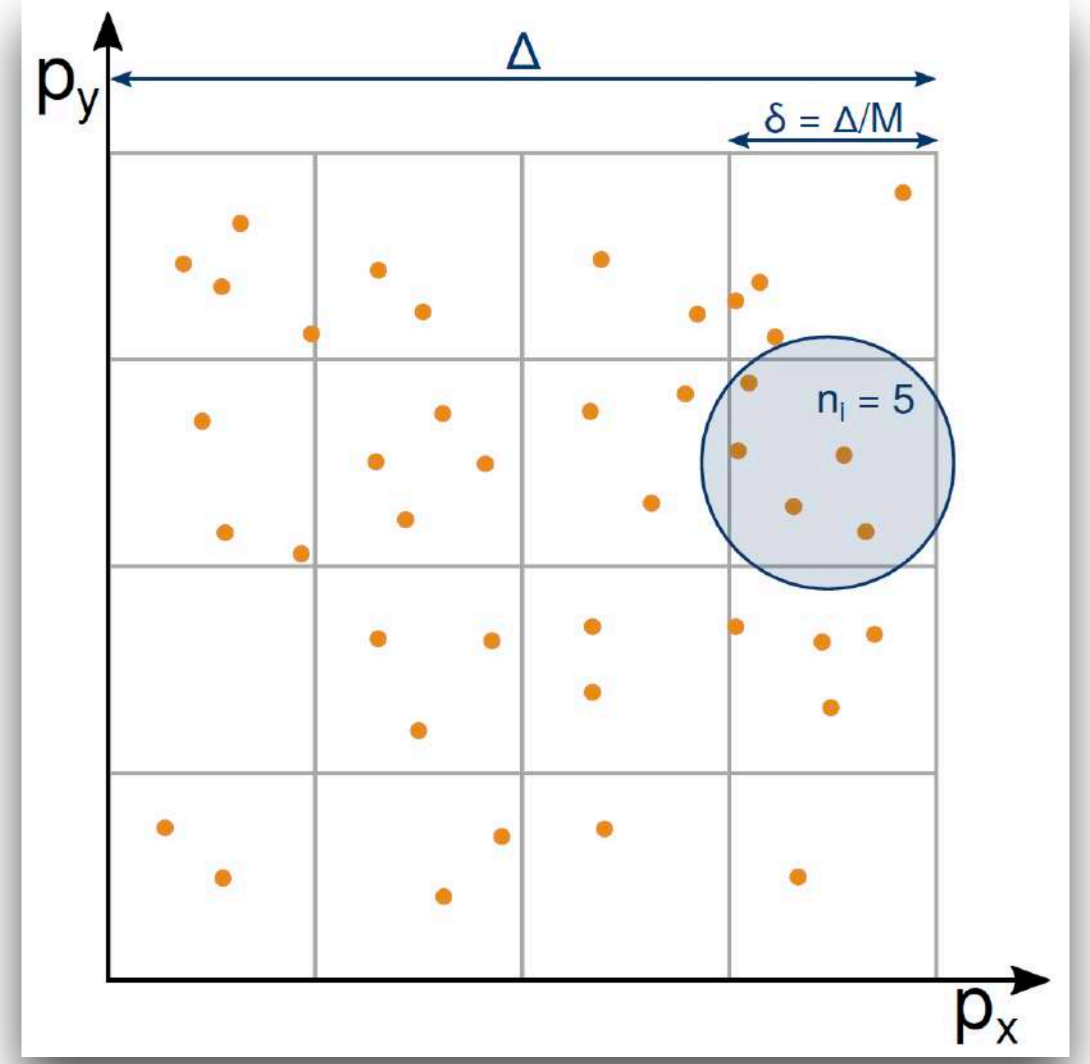
non-monotonic behaviour with a significance of 3.1σ relative to Skellam expectation

CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

no statistically significant difference between the data and the canonical baseline (KS test: 1.2σ, χ^2 test: 1.5σ)

no clear signal for critical point (yet)

Intermittency, direct access to the critical point



The scaled second order factorial moment

$$F_2(M) = \frac{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \rangle}{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \rangle^2}$$

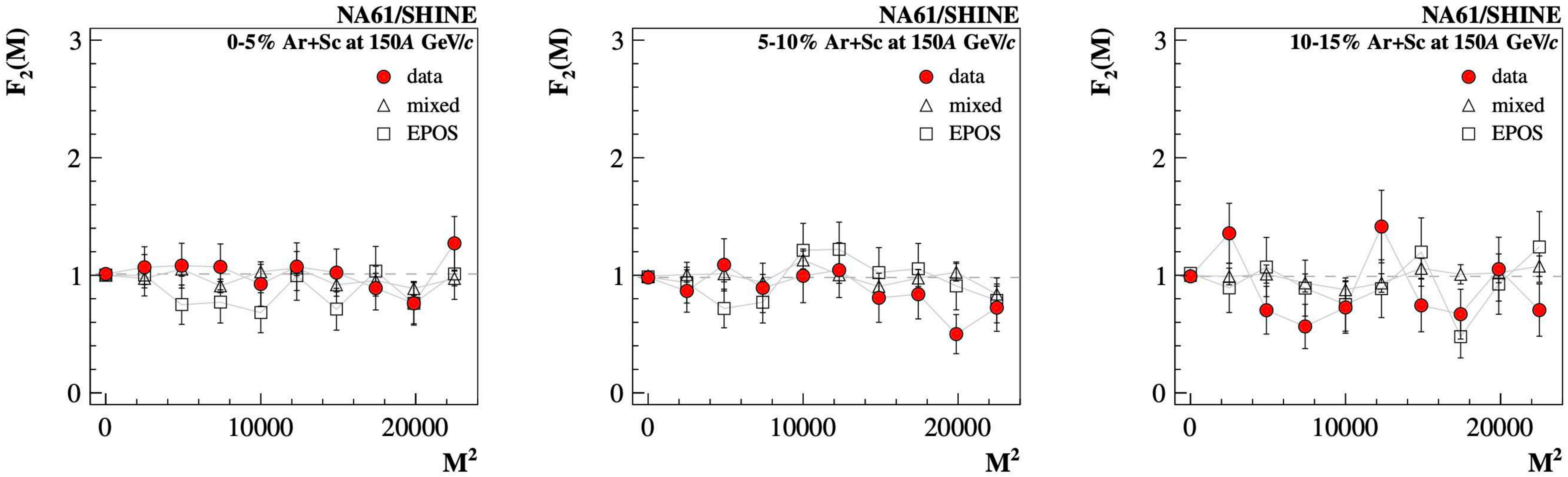
A. Bialas and R. B. Peschanski Nucl. Phys. B273 (1986) 703–718.

M - number of bins in each direction

n_i - number of particles in i^{th} bin

$\langle \ldots \rangle$ - averaging over events

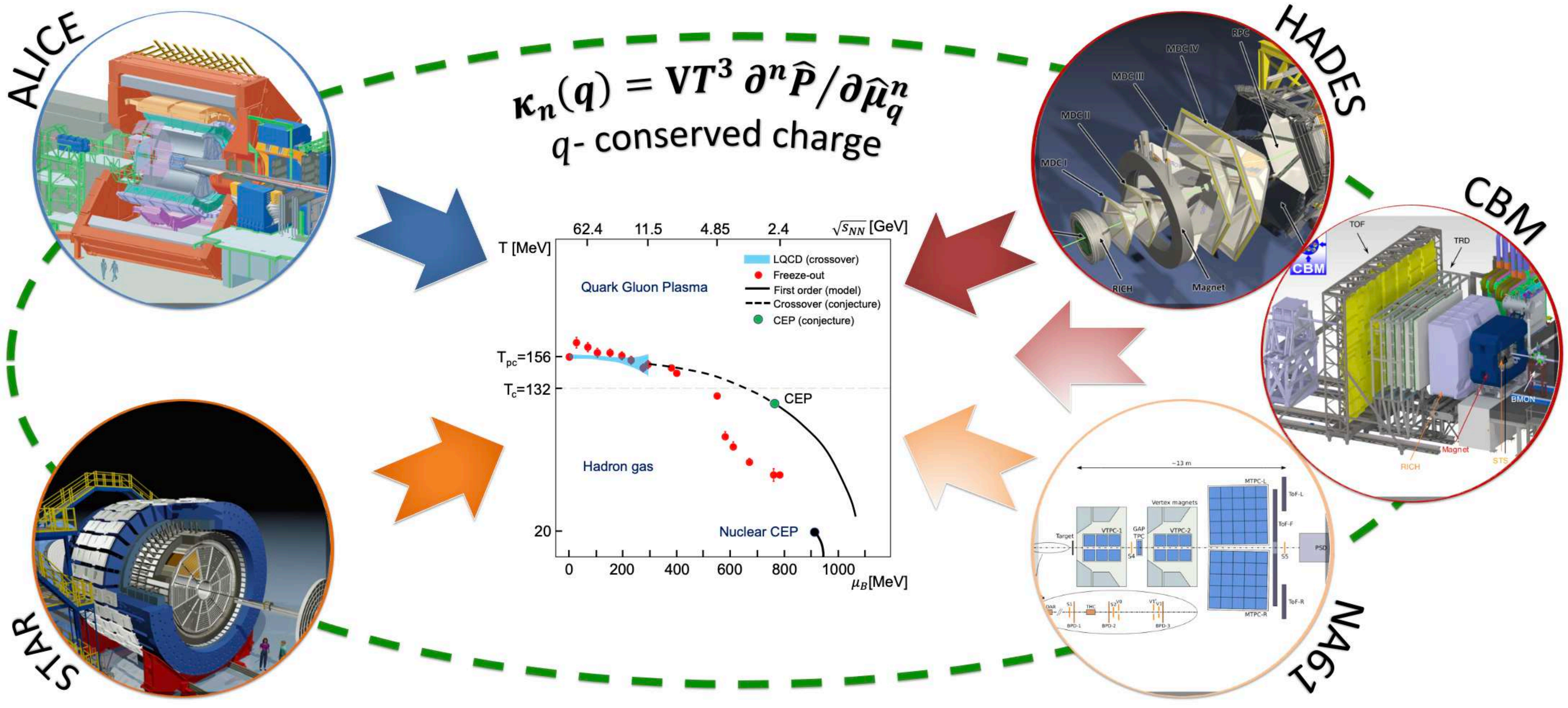
near the critical point (assuming 3D Ising universality class) $F_2(M) \sim (M^2)^{\phi_2}, \quad \phi_2 = 5/6$



no indication of critical point!

NA61: e-Print: 2305.07557 [nucl-ex]

Outlook



Currently available and near future data from ALICE, STAR, HADES/CBM and NA61/SHINE

will be a game changer


THANKS FOR YOUR ATTENTION