Progress in lattice QCD at non-zero temperature and density (part II)

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Recap from last lecture: for physical quark masses deconfinement transition cannot be defined, chiral transition is related to the *O(4)* universality of the chiral phase transition

- Microscopic origin of universality of chiral transition
- Equation of state at zero density
- Taylor expansion: fluctuations and correlations of conserved charges
- Taylor expansion: chiral transition and equation of state
- Heavy quark diffusion coefficient from lattice QCD
- Complex heavy quark potential at $T>0$

comparison with weak coupling calculations (HTL, EQCD)

Comparison with Hadron Resonance Gas (HRG) model

63. Cracow School of Theoretical Physics, Nuclear Matter at Extremes Densities and High Temperatures, September 17-23, 2023, Zakopane, Poland

Chiral transition and spectrum of Dirac eigenvalues

Macroscopic

Microscopic

$$\not{D}[\mathcal{U}]v_k = i\lambda_k v_k$$

$$\bar{\psi}\psi(m) \equiv 2\mathrm{Tr}(\not{D}[\mathcal{U}] + m)^{-1} \qquad = 2\sum_j (i\lambda_j + m)^{-1}$$

$$P_{\mathcal{U}}(\lambda; m) = \frac{4m\rho_{\mathcal{U}}(\lambda)}{\lambda^2 + m^2}\,, \text{ and } \rho_{\mathcal{U}}(\lambda) = \sum_j \delta(\lambda - \lambda_j)$$

$$\mathbb{K}_1(\bar{\psi}\psi) = \frac{T}{V}\langle(\bar{\psi}\psi)\rangle \qquad = \int_0^\infty K_1[P_{\mathcal{U}}(\lambda; m_l)]\ \mathrm{d}\lambda = \frac{T}{V}\int_0^\infty \mathrm{d}\lambda \frac{4m_l \langle\rho_{\mathcal{U}}(\lambda)\rangle}{\lambda^2 + m_l^2} = \int_0^\infty P_1(\lambda)d\lambda$$

$$\mathbb{K}_2(\bar{\psi}\psi) = \frac{T}{V}\langle[(\bar{\psi}\psi) - \langle\bar{\psi}\psi\rangle]^2\rangle \qquad = \int_0^\infty K_1[P_{\mathcal{U}}(\lambda_1; m_l), P_{\mathcal{U}}(\lambda_2; m_l)]\ \mathrm{d}\lambda_1 \mathrm{d}\lambda_2$$

$$= \frac{T}{V}\int_0^\infty \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \frac{(4m_l)^2}{(\lambda_1^2 + m_l^2)(\lambda_2^2 + m_l^2)} \times$$

$$[\langle\rho_{\mathcal{U}}(\lambda_1)\rho_{\mathcal{U}}(\lambda_2)\rangle - \langle\rho_{\mathcal{U}}(\lambda_1)\rangle\langle\rho_{\mathcal{U}}(\lambda_2)\rangle] = \int_0^\infty P_2(\lambda)d\lambda$$

Chiral susceptibility

$$\mathbb{K}_3(\bar{\psi}\psi) = \frac{T}{V}\langle[(\bar{\psi}\psi) - \langle\bar{\psi}\psi\rangle]^3\rangle \qquad = \int_0^\infty P_3(\lambda)d\lambda$$

Binder cumulant

H.T. Ding, W.-P. Huang, S. Mukherjee, PP, arXiv:2305.10916, to be published in PRL

Approaching the chiral limit

$m_l \to 0$: $\frac{m}{\lambda^2+m^2} \to \pi\delta(\lambda)$

$$\lim_{m_l\to 0} \mathbb{K}_1(\bar\psi\psi) = \lim_{m_l\to 0} \frac{T}{V}\langle(\bar\psi\psi)\rangle = 2\pi\mathbb{K}_1[\rho_{\mathcal{U}}(0)] = 2\pi\langle\rho_{\mathcal{U}}(0)\rangle$$

Banks-Casher relation

$$\lim_{m_l\to 0} \mathbb{K}_n(\bar\psi\psi) = \lim_{m_l\to 0} = (2\pi)^n\mathbb{K}_n[\rho_{\mathcal{U}}(0)]$$

Universal *O(N)* scaling of the cumulants of the chiral condensate:

$$\mathbb{K}_n[\bar\psi\psi] = \int_0^\infty P_n(\lambda)d\lambda \sim m_l^{1/\delta-n+1} f_n(z)$$

$f_1(z) = f_G(z)$

$f_2(z) = f_\chi(z)$

$$z \propto z_0 m_l^{-1/\beta\delta}(T-T_c)/T_c$$

Universal *O(N)* scaling functions

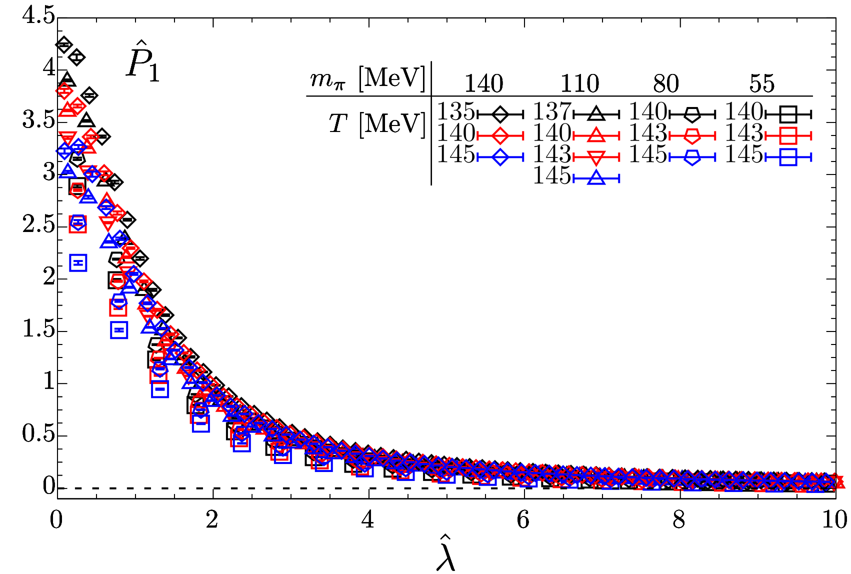
<u>Conjecture:</u> $P_n(\lambda) = m_l^{1/\delta-n+1} f_n(z) g_n(\lambda/m_l)$

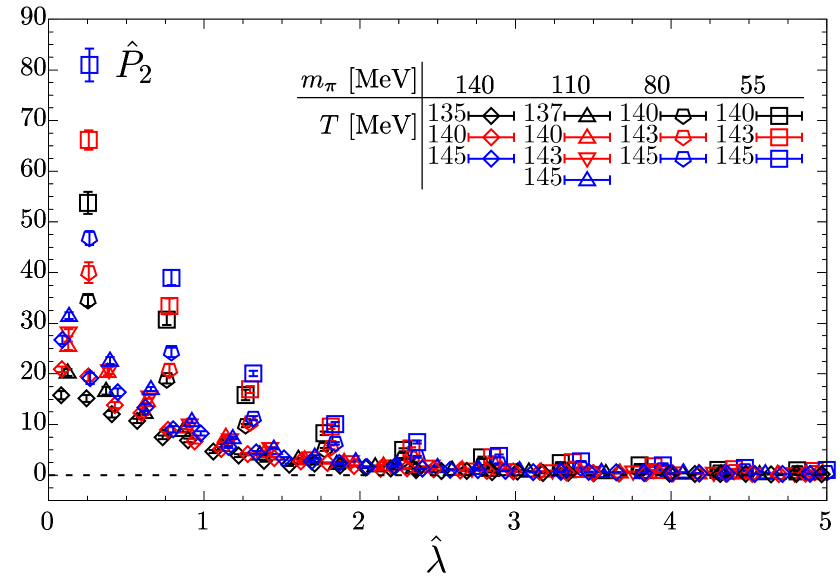
Scaling arises from the deep infrared behavior of $P_n(\lambda)$

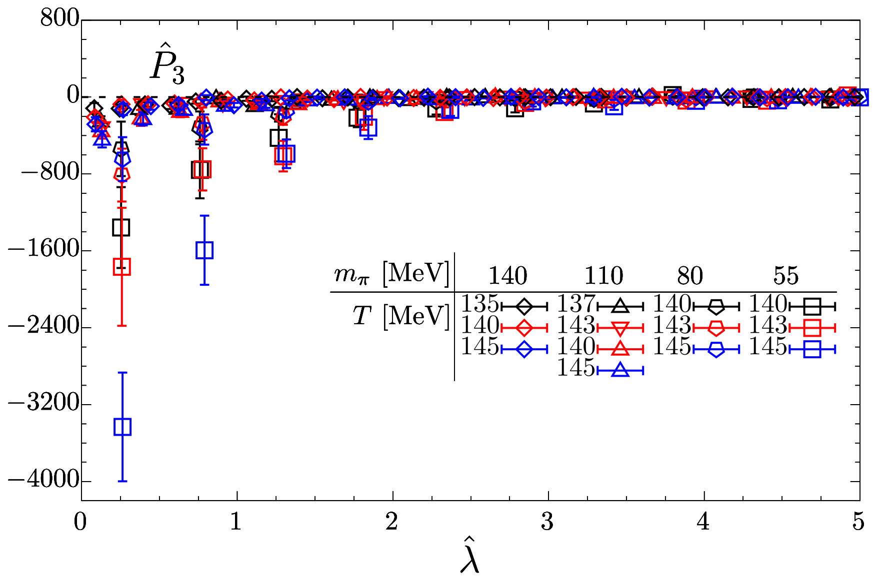
QCD specific dependence on the ratio of the eigenvalues and the quark mass

H.T. Ding, W.-P. Huang, S. Mukherjee, PP, arXiv:2305.10916, to be published in PRL

Chiral observables and spectrum of Dirac eigenvalues







HISQ action, $N_\sigma^3 \times N_\tau$ lattices

$N_\tau = 8,\ N_\sigma = 32 - 56$

$m_l/m_s = 1/27,\ 1/40,\ 1/80,\ 1/160$

$m_\pi = 140,\ 110,\ 80,\ 55$ MeV

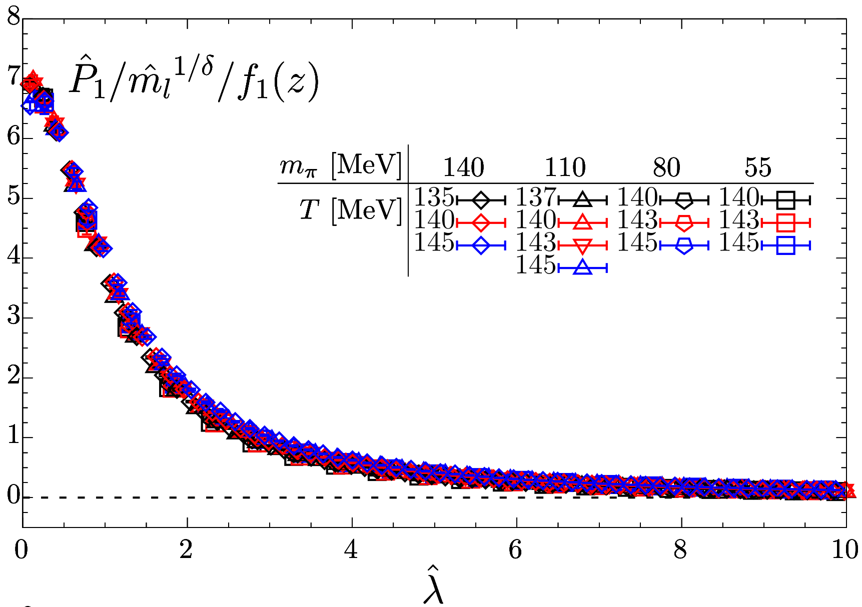
$T = 135 - 176$ MeV

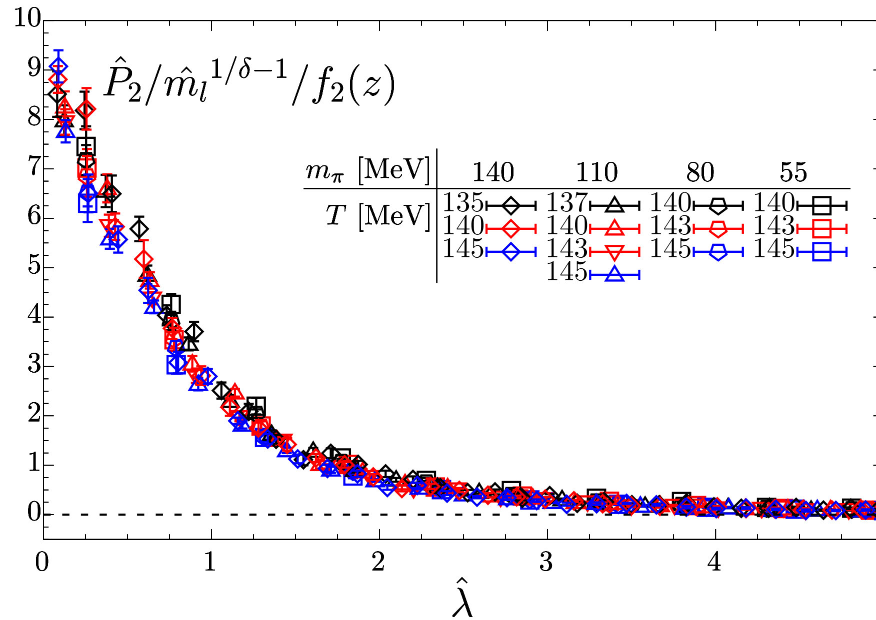
$$\hat{P}_n(\hat{\lambda}) = m_s^2(m_l/m_s)P_n(\lambda)/T_c^4 \qquad \hat{\lambda} = \lambda/m_l$$

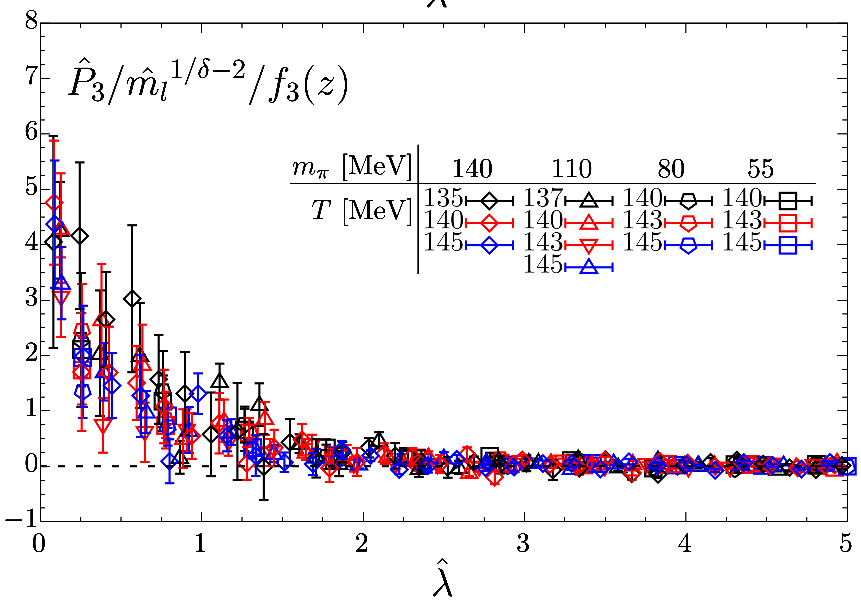
Strong quark mass and temperature dependence

H.T. Ding, W.-P. Huang, S. Mukherjee, PP, arXiv:2305.10916, to be published in PRL

Chiral observables and spectrum of Dirac eigenvalues







Scaling works for $T \leq 145$ MeV $\sim T_c^0(N_\tau = 8)$

Dirac eigenvalues (energy levels of quarks) know about universality class of the QCD chiral transition

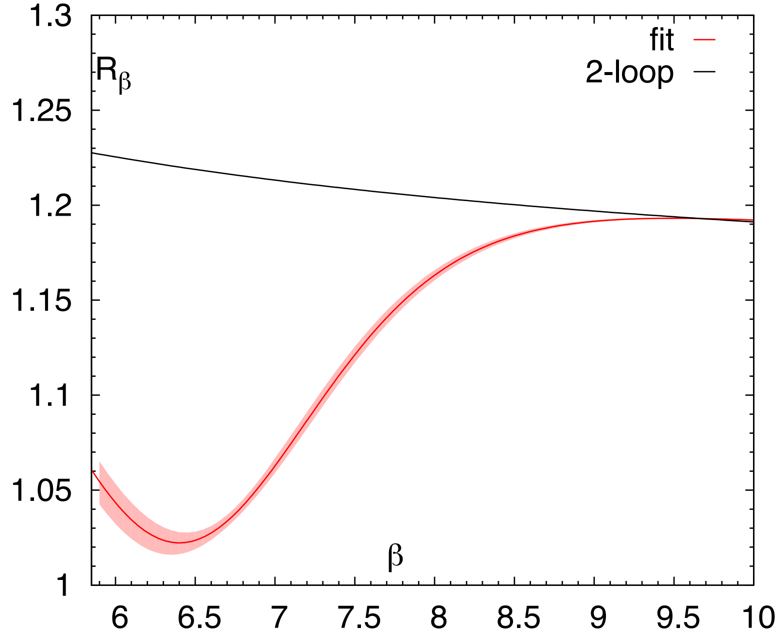
H.T. Ding, W.-P. Huang, S. Mukherjee, PP, arXiv:2305.10916, to be published in PRL

QCD trace anomaly and the integral method

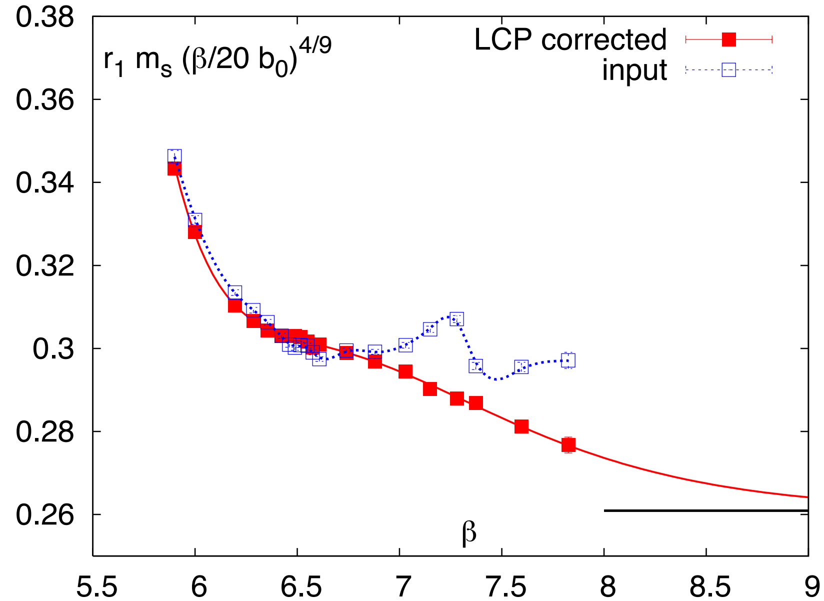
$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) \Rightarrow \frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = R_\beta \{\langle S_G\rangle_0 - \langle S_G\rangle_T\} - R_\beta R_m \{2m_l(\langle\bar{q}q\rangle_0 - \langle\bar{q}q\rangle) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)\}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da},\ R_m = \frac{1}{m_q(\beta)}\frac{dm_q(\beta)}{d\beta},\ \beta = 10/g^2$$



Bazavov, PP, Weber, PRD97 (2018) 014510

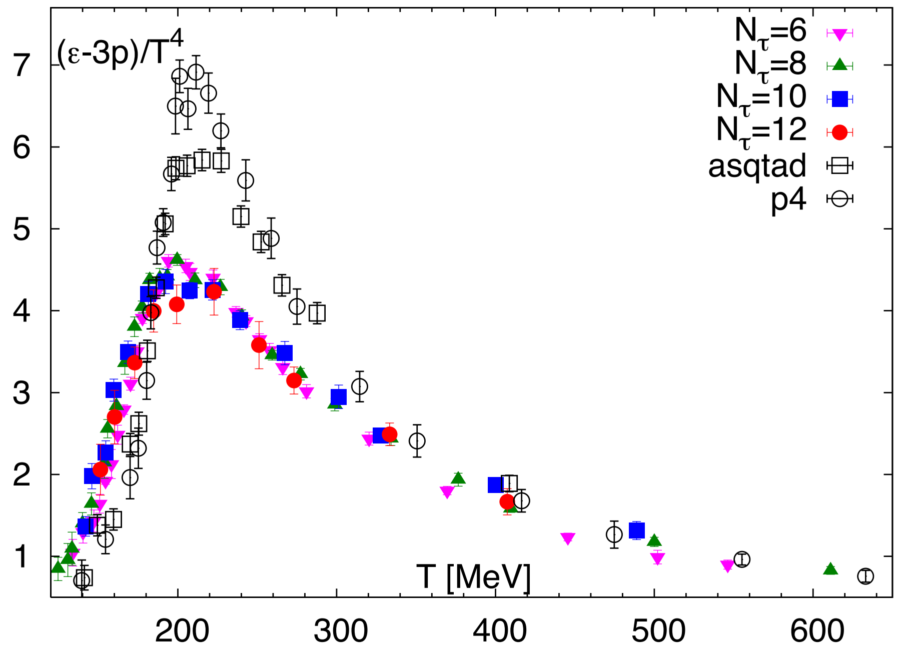


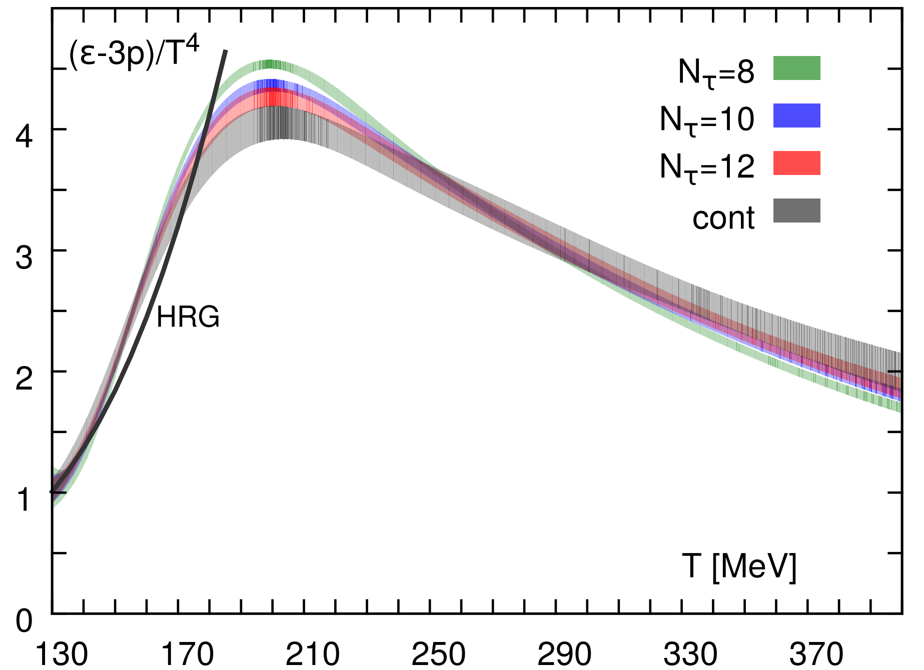
Bazavov et al, PRD 90 (2014) 094503

QCD results on the trace anomaly

2+1 flavor QCD calculations with almost physical light and strange quark masses

Bazavov et al, PRD 90 (2014) 094503





The peak height is much reduced compared to the asqtad and p4 $N_\tau=8$ calculations

Agreement with p4 and asqtad calculations for $T>350$ MeV

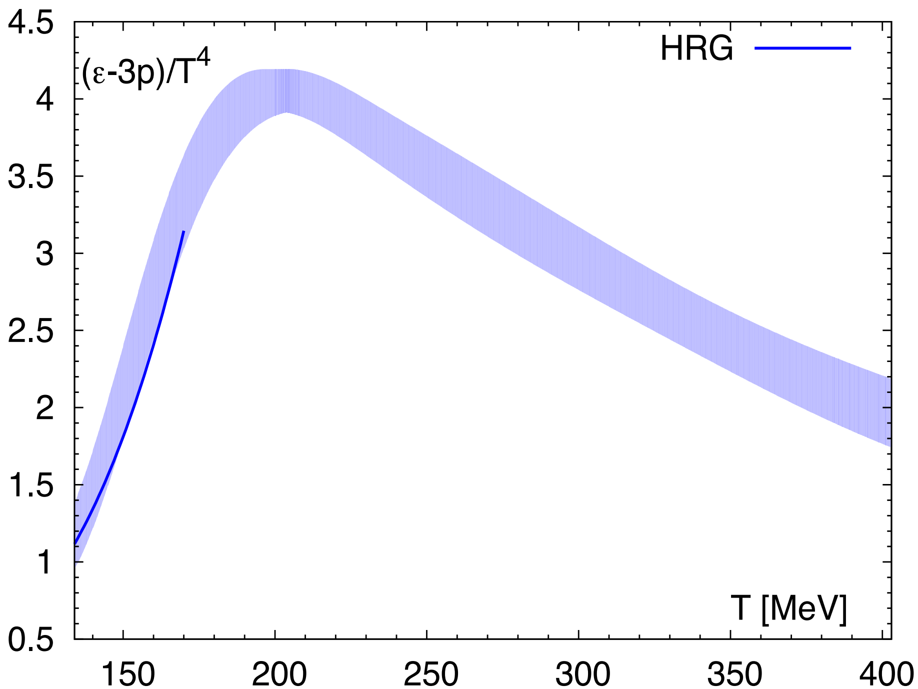
Small cutoff effects for HISQ except for $N_\tau=6$

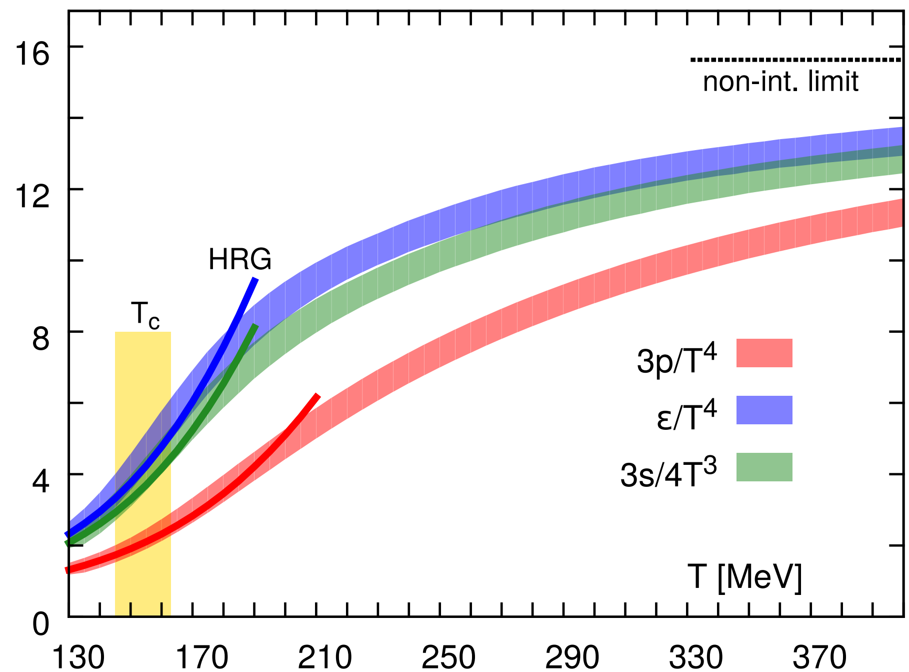
Perform spline interpolation of all the $N_\tau>6$ data with spline coefficients having $a+b/N_\tau^2$ *form* ,
stabilize the spline demanding that ε-$3p$ is given by HRG at $T=130$ MeV

QCD thermodynamics in the continuum limit

Set the lower integration limit to $T_0=130$ MeV and take $p_0=p^{HRG}(T=130$ MeV$)$ → $p(T)$

Bazavov et al, PRD 90 (2014) 094503



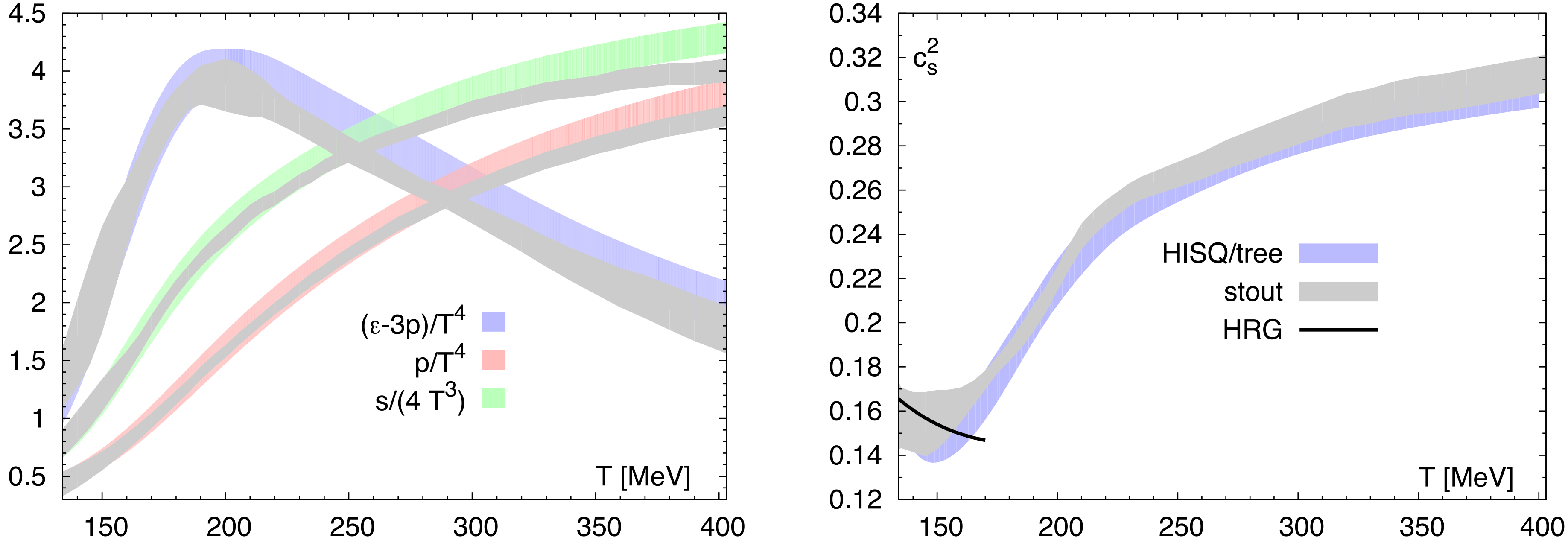


$$T_c = 156 \pm 1.5 \text{ MeV} \qquad \epsilon_{nucl} \simeq 150 \text{MeV}/\text{fm}^3$$

$$\epsilon_c = 420(60) \text{ MeV}/\text{fm}^3 \qquad \epsilon_{proton} \simeq 450 \text{MeV}/\text{fm}^3$$

HRG: all resonances from PDG treated as stable (zero width) particles in an ideal gas

Comparison of different continuum limit

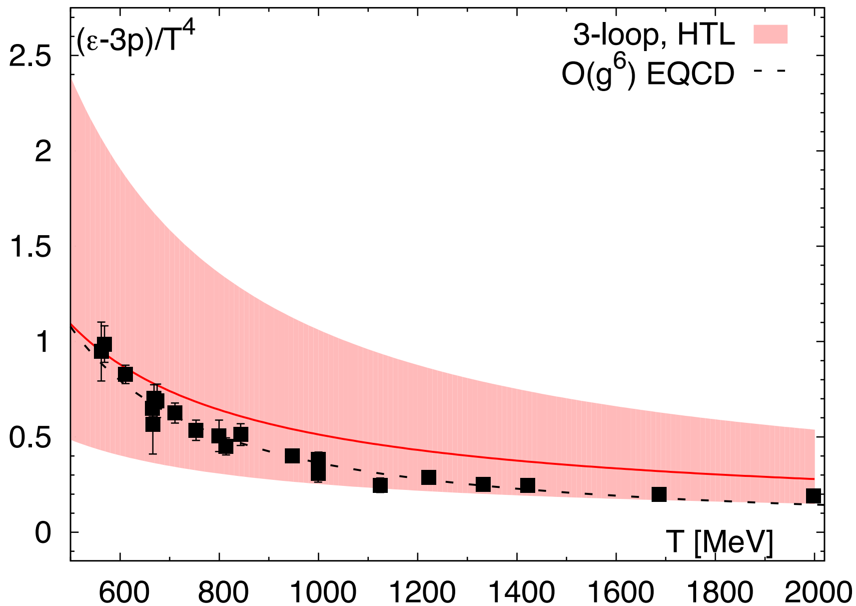


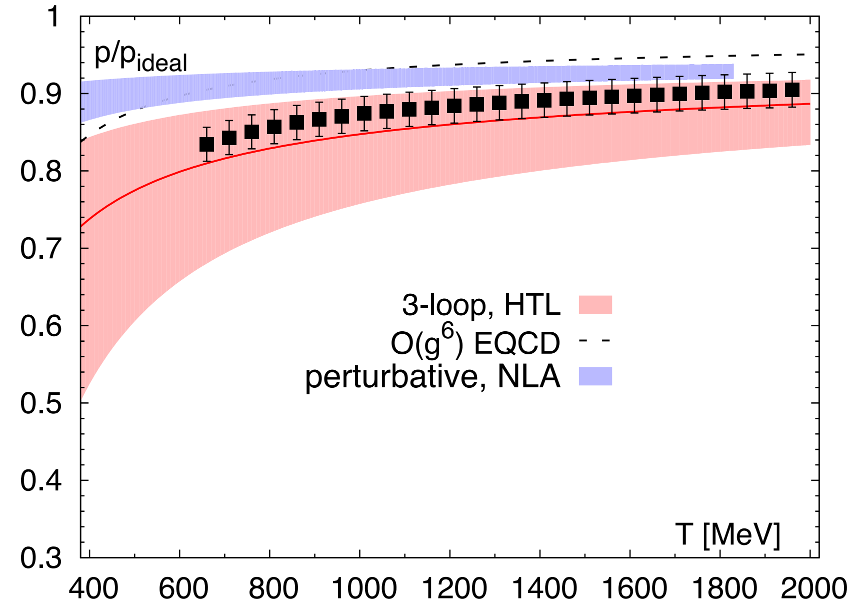
Continuum results obtained with stout and HISQ action agree reasonably well given their errors (some tension for the entropy density)

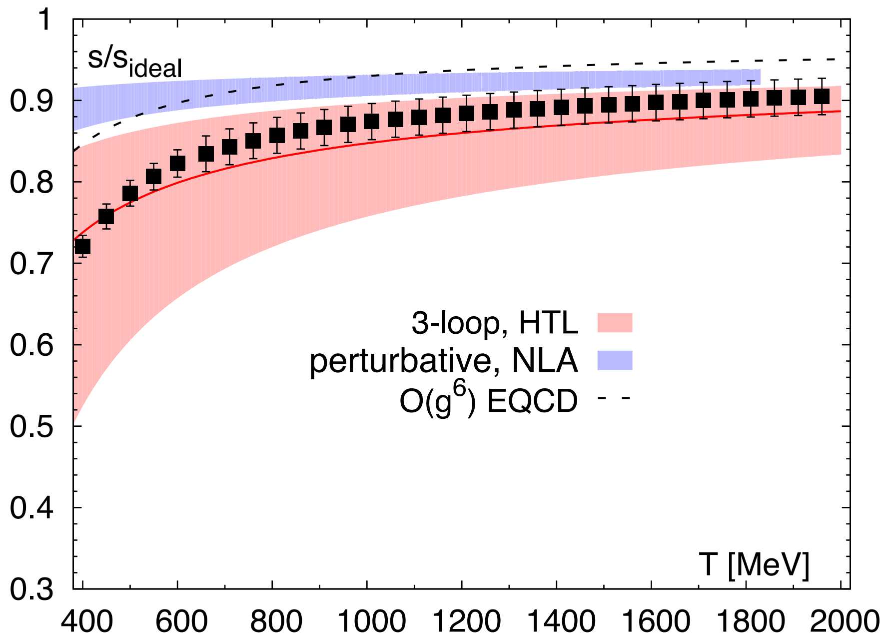
Even in the transition region the speed of sound is not much smaller than the HRG speed of sound (the EoS is never really soft)

HISQ: Bazavov et al, PRD 90 (2014) 094503
stout: Borsányi et al, PLB730 (2014) 99

Comparison of EoS with weak coupling results







Reasonably good agreement
Between the lattice and the weak
coupling calculations
for *T>400* MeV

Bazavov, PP, Weber, PRD97 (2018) 014510

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{udsc} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l$$ quark

$$\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial\mu_b^i} \frac{\partial^j}{\partial\mu_b^j} \frac{\partial^k}{\partial\mu_c^i} \frac{\partial^l}{\partial\mu_d^l} \ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d) \,\big|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the susceptibilities, i.e. the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}\left(\langle X^2\rangle - \langle X\rangle^2\right) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}\left(\langle XY\rangle - \langle X\rangle\langle Y\rangle\right)$$

information about carriers of the conserved charges (hadrons or quarks)

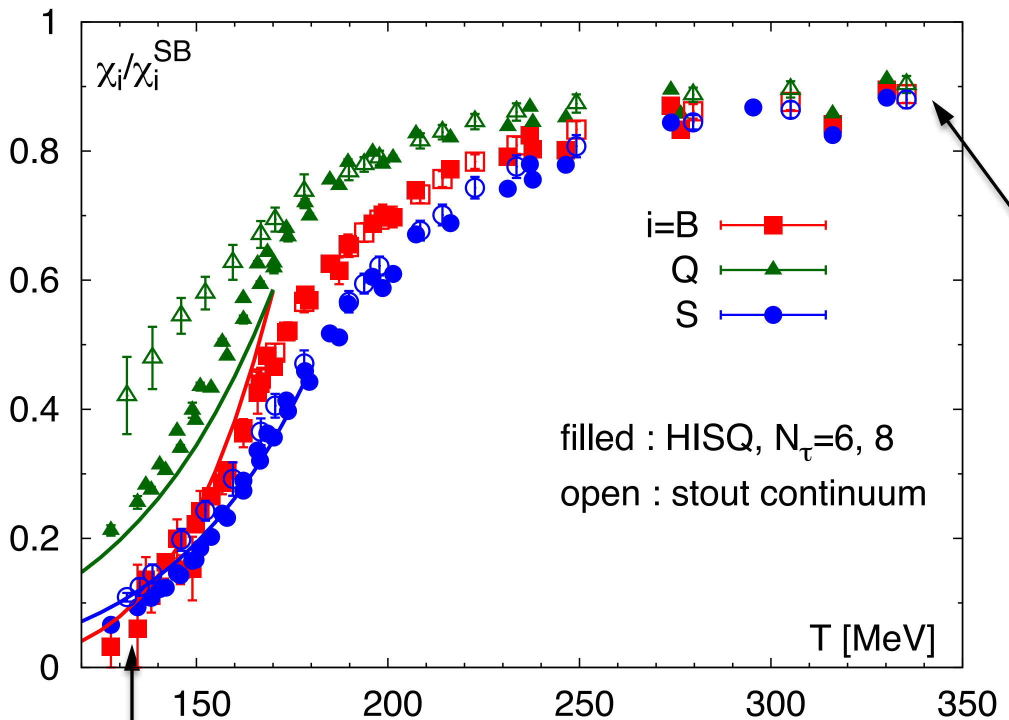
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

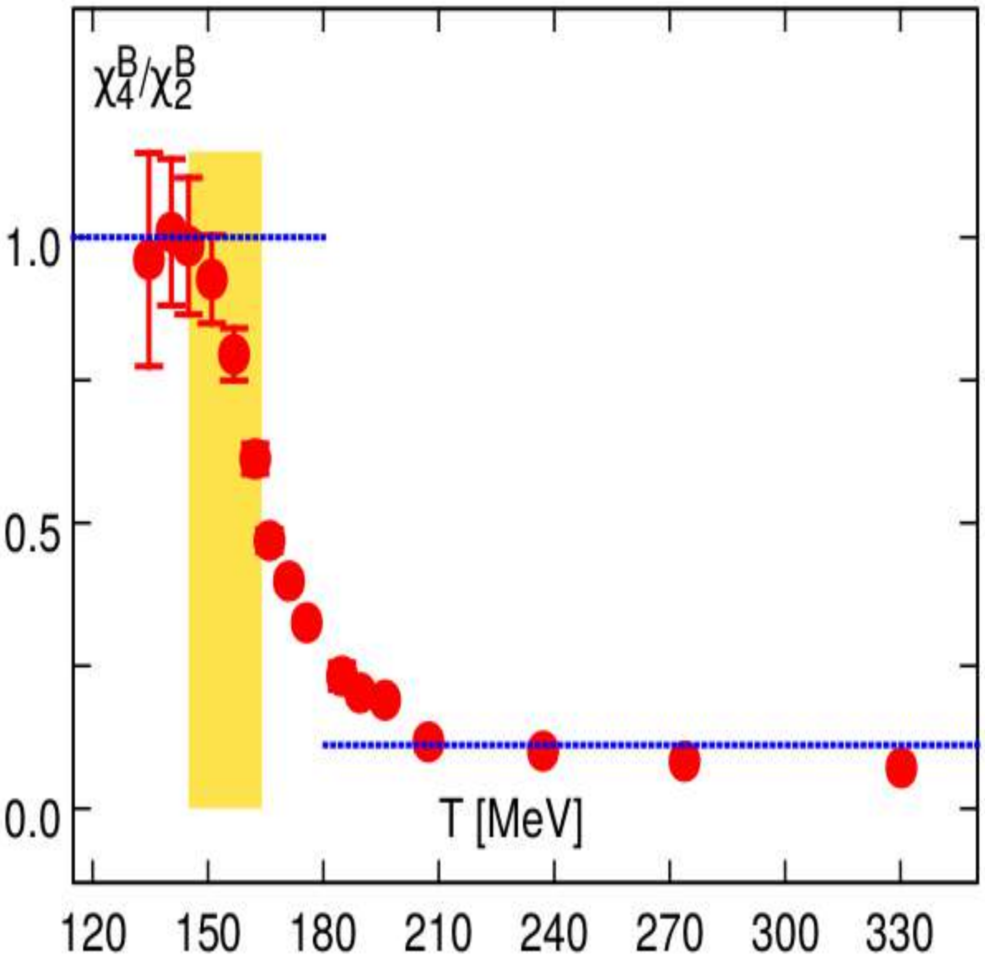
conserved charges carried by light quarks

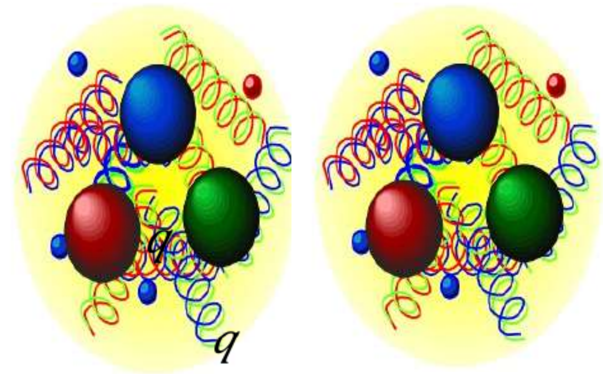
conserved charges are carried by massive hadrons

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

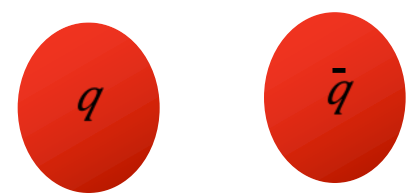
Deconfinement : fluctuations of conserved charges





B=1, -1

$$\chi_2^B = \langle B^2 \rangle = 1, \ \chi_4^B = \langle B^4 \rangle = 1$$



B=1/3, -1/3

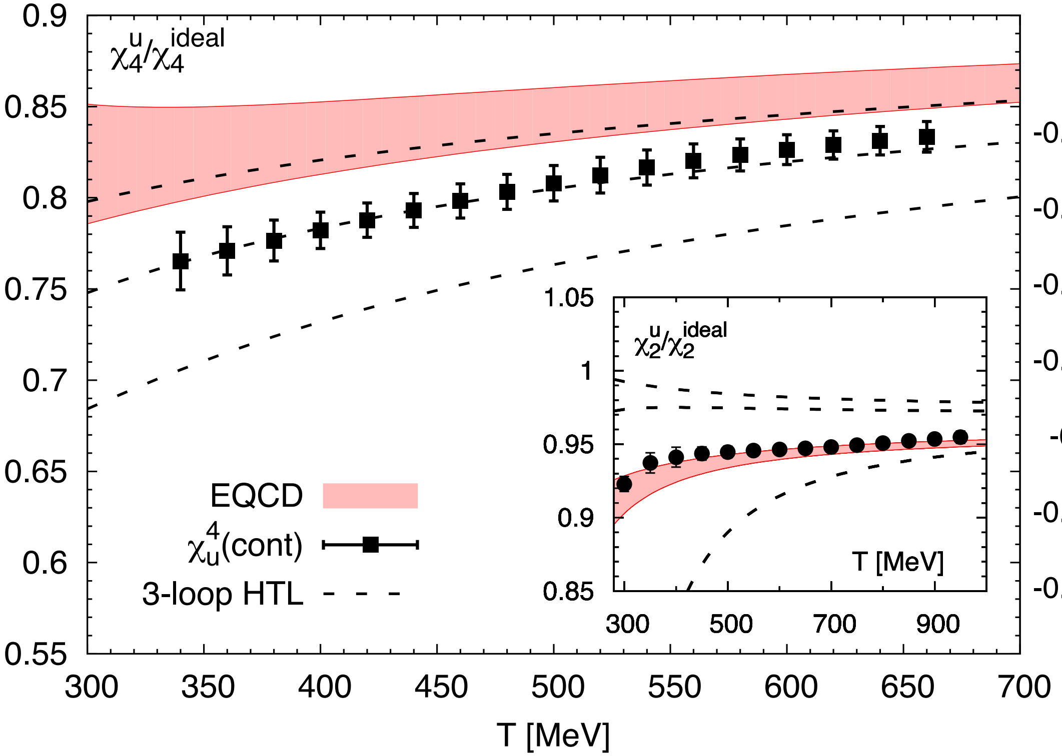
$$\chi_2^B = \langle B^2 \rangle = \frac{1}{9}, \ \chi_4^B = \langle B^4 \rangle = \frac{1}{81}$$

Degrees of freedom for 150 MeV < T< 200 MeV ?

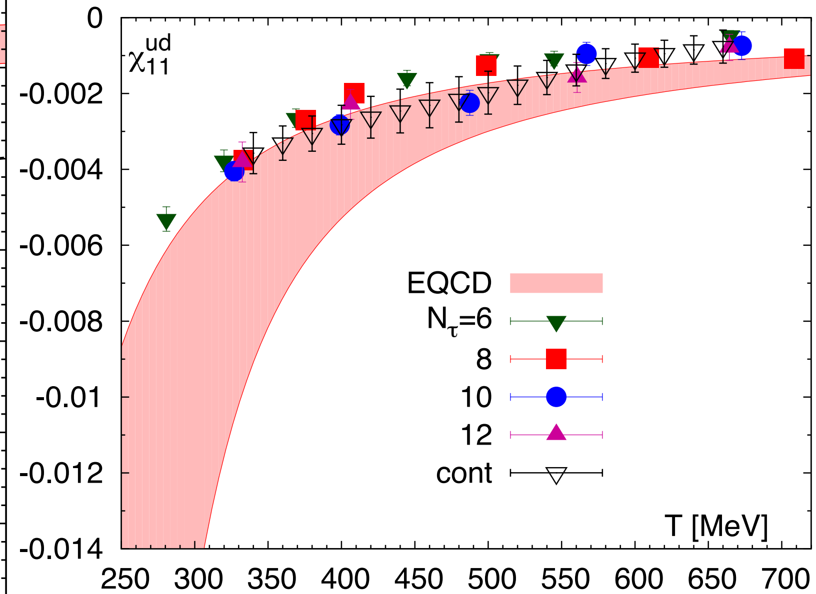
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations

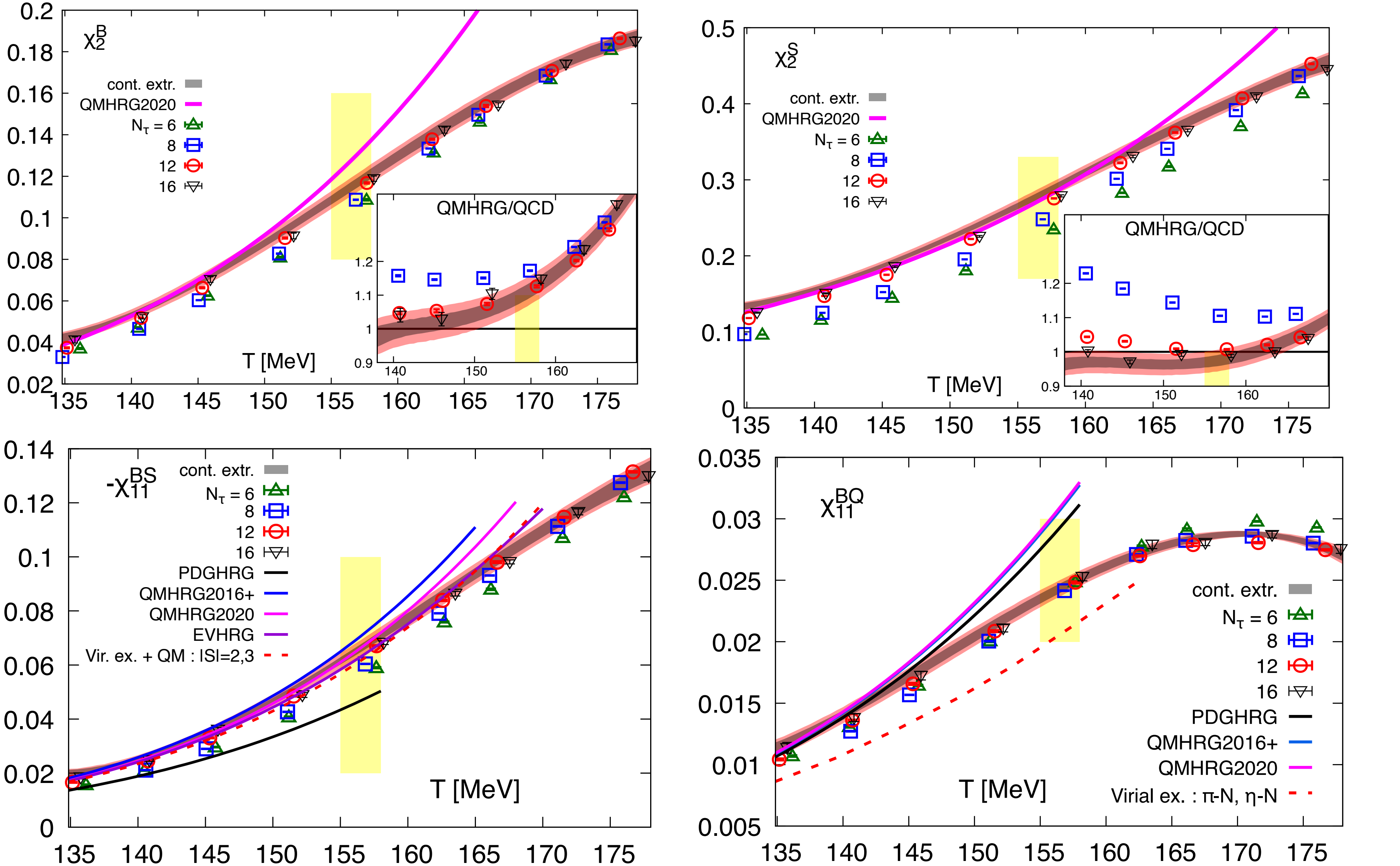


- Good agreement between continuum extrapolated lattice results and the weak coupling approach
- Quark number correlations vanish at any loop order but can be calculated in EQCD and the EQCD calculations agree with the continuum extrapolated lattice results

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

Second order Taylor expansion coefficients and HRG

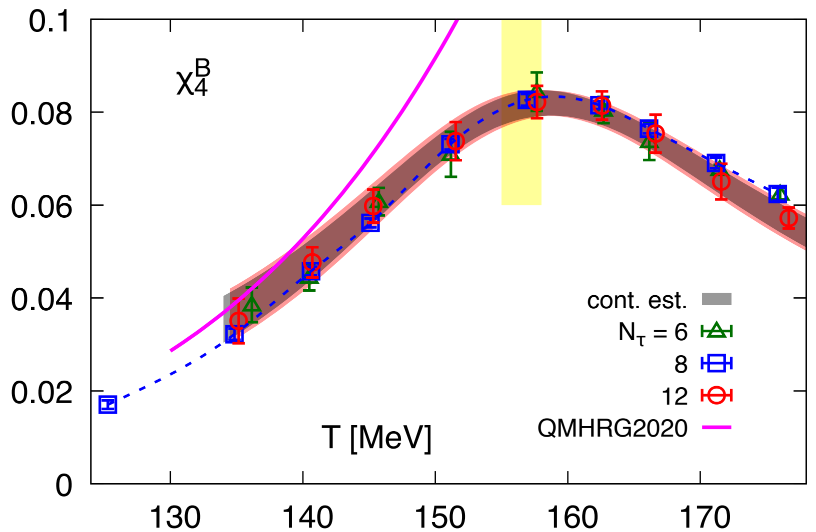
HISQ, m_π^{phys}, $a = 1/(TN_\tau)$

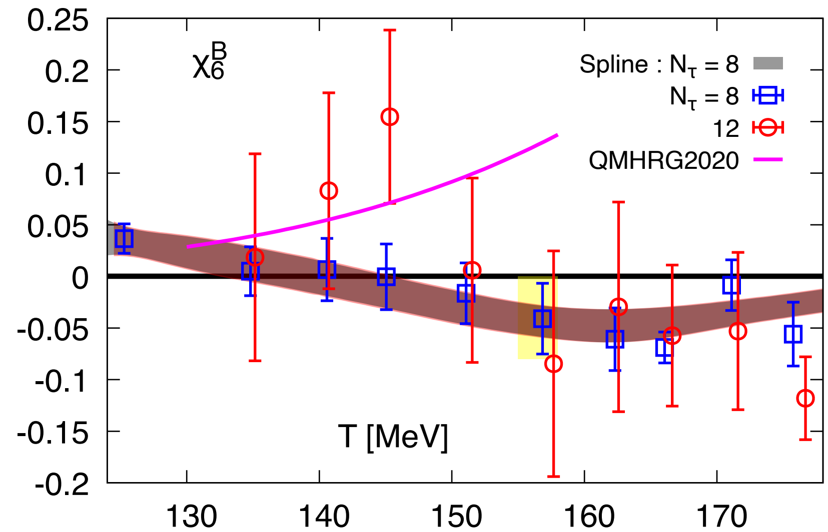


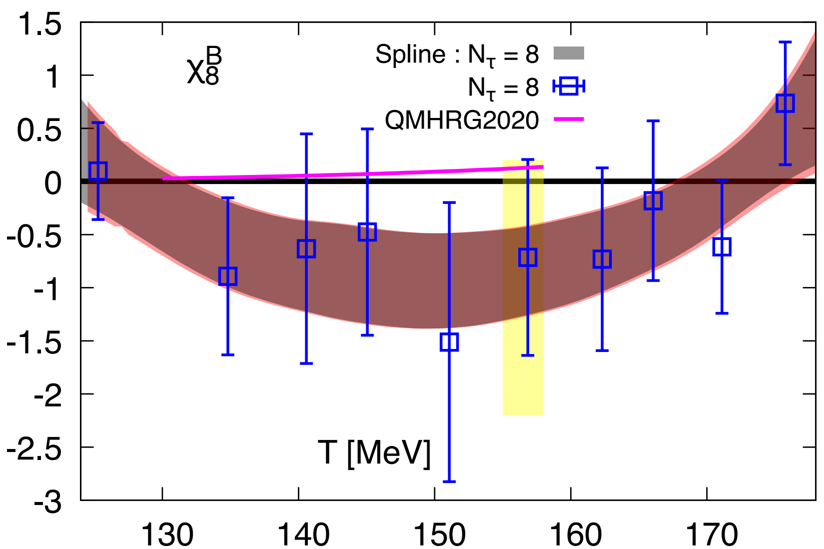
HRG works up to temperatures ≃145-150 MeV

Higher order Taylor expansion coefficients and HRG

HISQ, m_π^{phys}, $a = 1/(TN_\tau)$





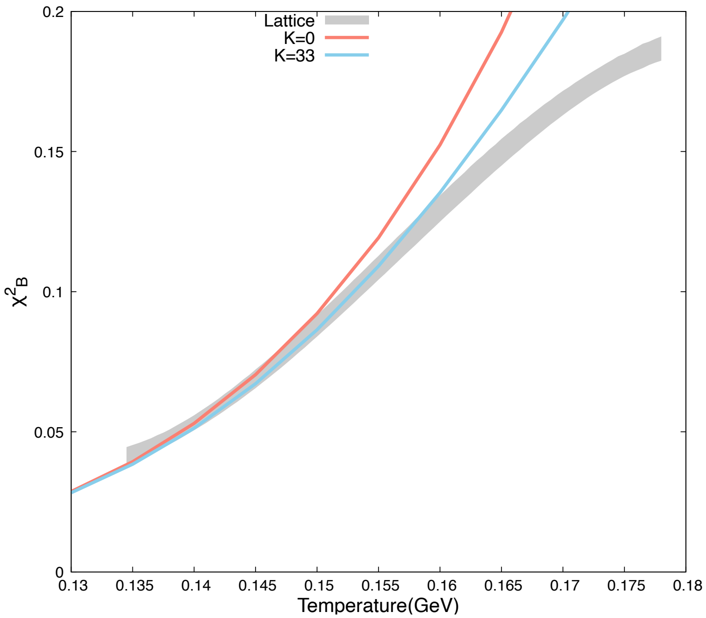


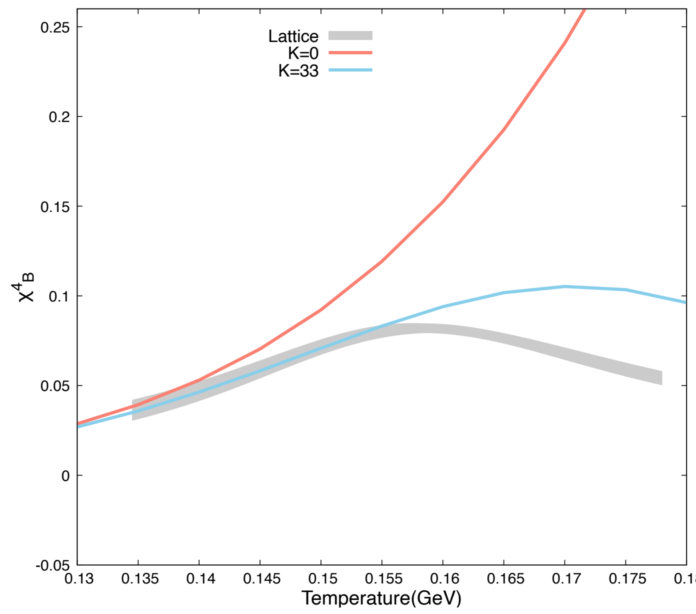
For 4[th] order expansion coefficient HRG may work only for $T<140$ MeV

For 6[th] and 8[th] order expansion coefficients turn negative around T_c HRG, only works for $T<135$ MeV

Possibly no singularity for real values of baryon chemical potential.

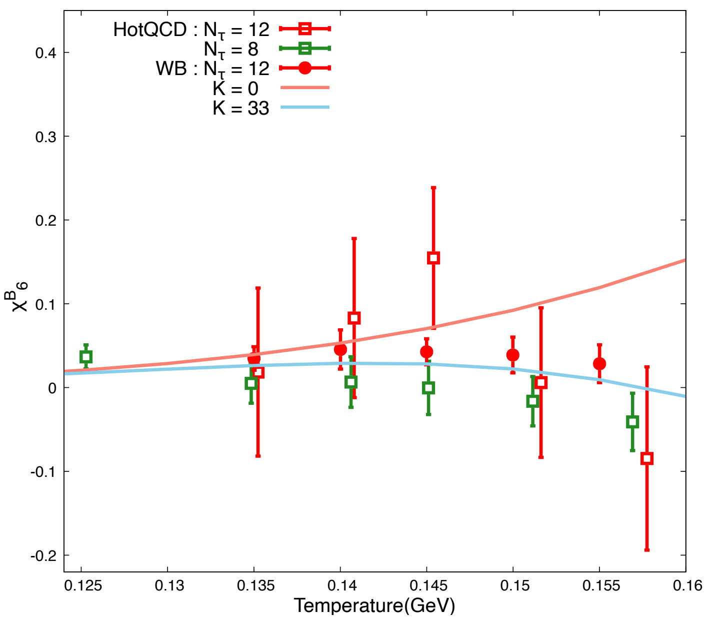
HRG with repulsive mean field

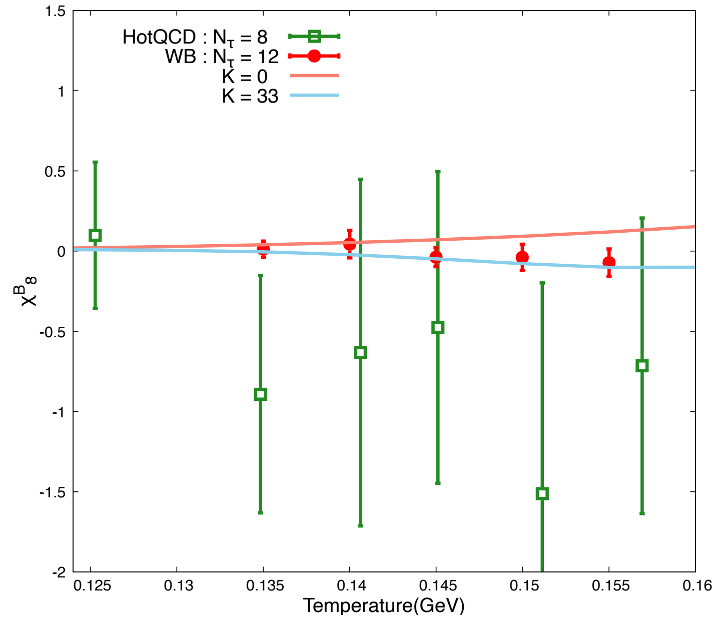




D. Biswas, PP,
S. Sharma,
work in progress

Improved agreement between lattice and HRG.





Pade approximation and radius of convergence

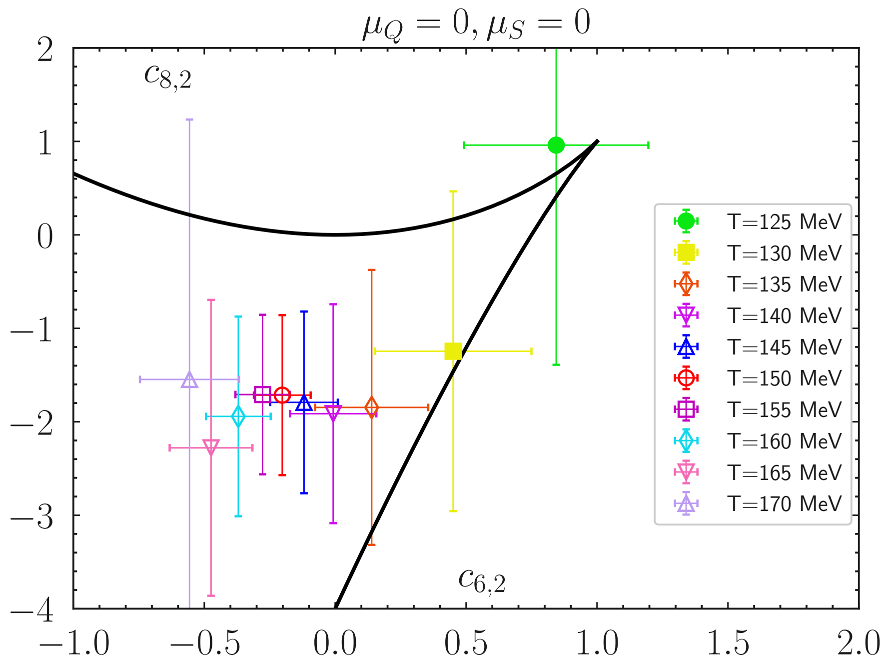
$$\Delta P(T,\mu_B) = P(T,\mu_B) - P(T,0) = \sum_{k=1}^{\infty} P_{2k}\mu_B^{2k} \qquad \bar{x} = (\mu_B/T)\cdot\sqrt{P_4/P_2}$$

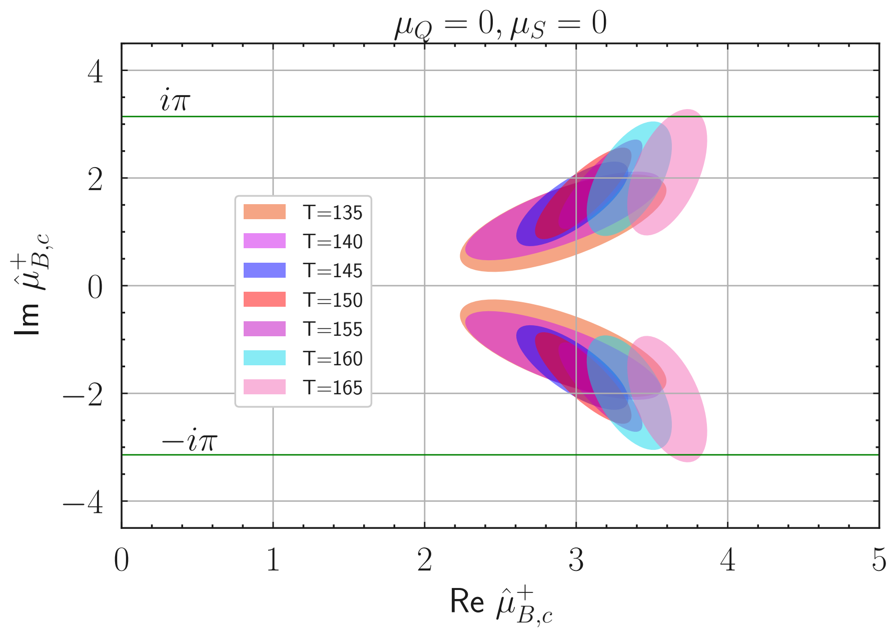
$$\frac{\Delta P(T,\mu_B)}{T^4} = \frac{P_2^2}{P_4}\sum_{k=1}^{\infty} c_{2k,2}\bar{x}^{2k} = \frac{P_2^2}{P_4}(\bar{x}^2 + \bar{x}^4 + c_{6,2}\bar{x}^6 + c_{8,2}\bar{x}^8 + \ldots) \to \frac{P_2^2}{P_4}P_{[4,4]}$$

$$c_{6,2} = \frac{P_6 P_2}{P_4^2} = \frac{2}{5}\frac{\chi_6^B\chi_2^B}{(\chi_4^B)^2}, \qquad c_{8,2} = \frac{P_8 P_2^2}{P_4^3} = \frac{3}{35}\frac{\chi_8^B(\chi_2^B)^2}{(\chi_4^B)^3}.$$

$$P_{[4,4]} = \frac{(1-c_{6,2})\bar{x}^2 + (1-2c_{6,2}+c_{8,2})\,\bar{x}^4}{(1-c_{6,2}) + (c_{8,2}-c_{6,2})\bar{x}^2 + (c_{6,2}^2 - c_{8,2})\bar{x}^4}.$$

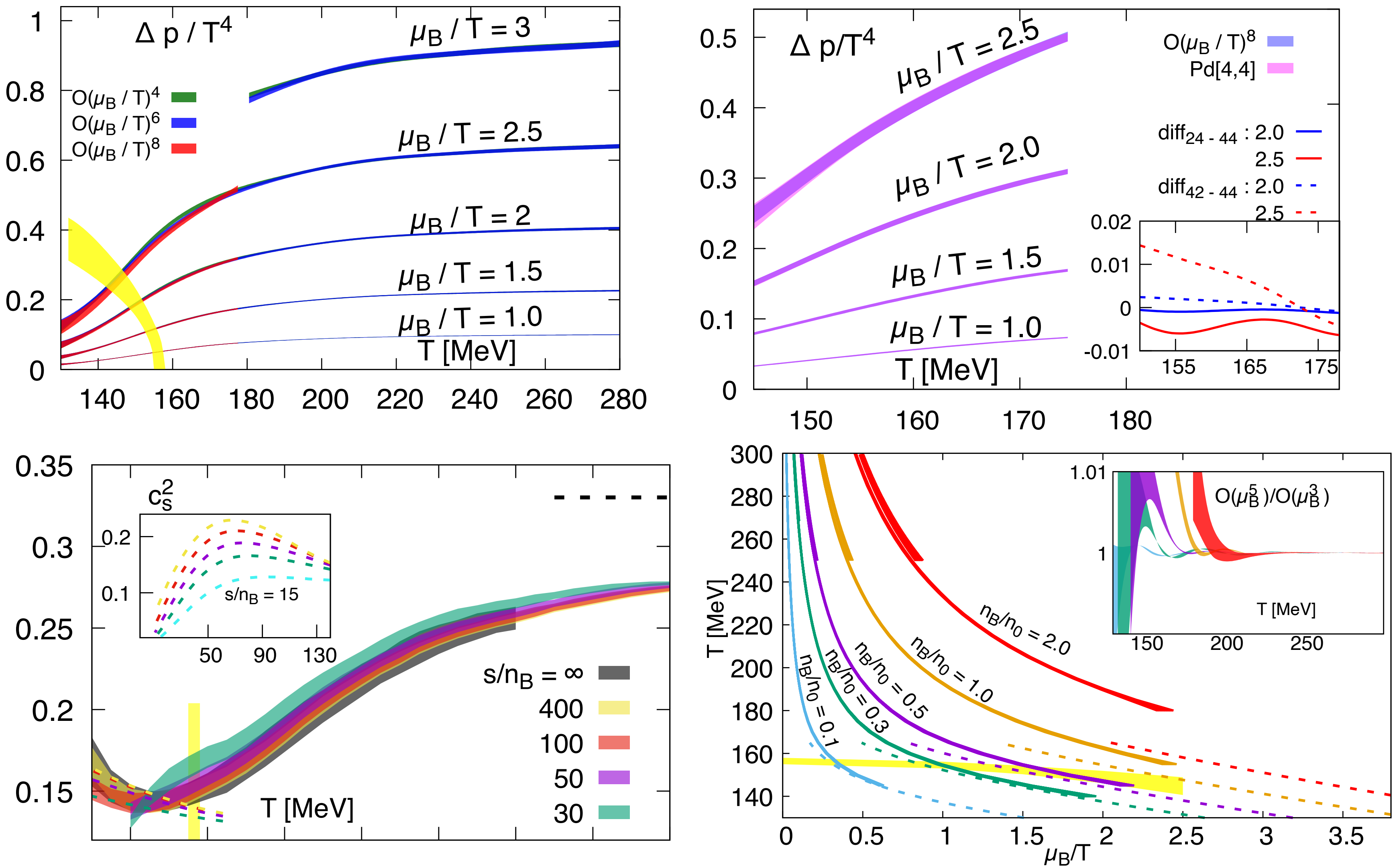
Padé poles: Mercer-Roberts estimators of radius of convergence





For 135 MeV $< T <$ 165 MeV only complex poles for $|\mu_B/T| > 2.5$

Equation of State at non-zero baryon density



$$\epsilon(T_{\rm pc}(\mu_B)) = \begin{cases} 370(40)(30)\ \mathrm{MeV/fm^3}\ ,\ \mu_B/T = 0 \\ 330(28)(53)\ \mathrm{MeV/fm^3}\ ,\ \mu_B/T = 2.5 \end{cases}$$

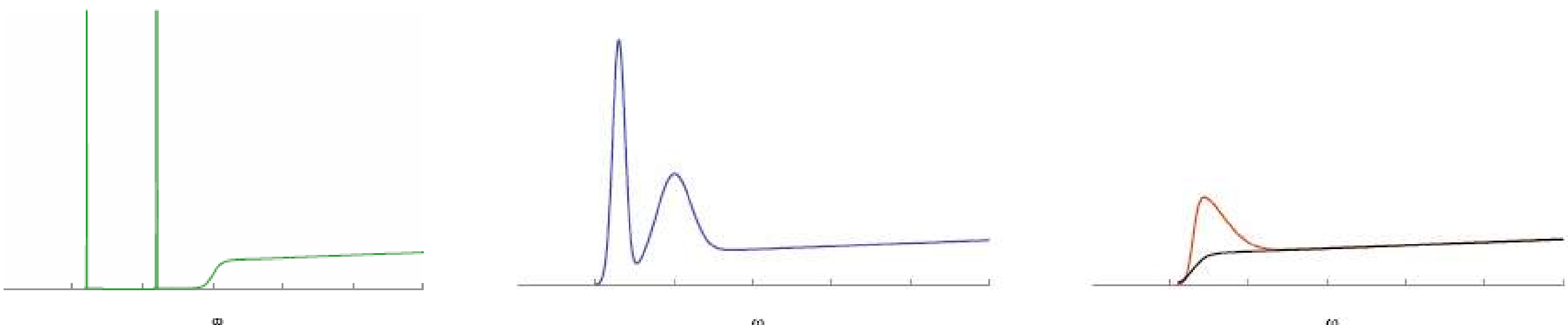
Meson correlators and spectral functions

Vacuum and in-medium properties as well as dissolution of mesons are encoded in the spectral functions:

$$\rho(\omega, p, T) = \frac{1}{2\pi} \text{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [O(x,t), O(0,0)] \rangle_T, \quad O(x,t) \sim \bar{Q}(x,t) \Gamma Q(x,t)$$

Melting is seen as progressive broadening and disappearance of the bound state peaks

Modifications of quarkonium yields in heavy ion collisions   Matsui and Satz, PLB 178 (1986) 416



$$C(\tau, T) = \sum_x \langle O(x,\tau) O(0,0) \rangle_T \quad \longleftrightarrow \quad C(\tau, T) = \int_{-\infty}^{+\infty} d\omega \rho(\omega, T) e^{-\tau\omega}$$

Consider large τ behavior of $C(\tau, T=0)$:

$$C(\tau, T) \sim \sum_n |\langle 0|O|n\rangle|^2 e^{-M_n \tau} \simeq f_1 e^{-M_1 \tau} + f_2 e^{-M_2 \tau} + \ldots$$

$T > 0: \quad \tau < 1/T \Rightarrow \text{reconstruct } \rho(\omega, T)$

Current-current correlators and heavy quark diffusion coefficient

$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$

$$\partial_t p_i = -\eta p_i + f_i(t),$$

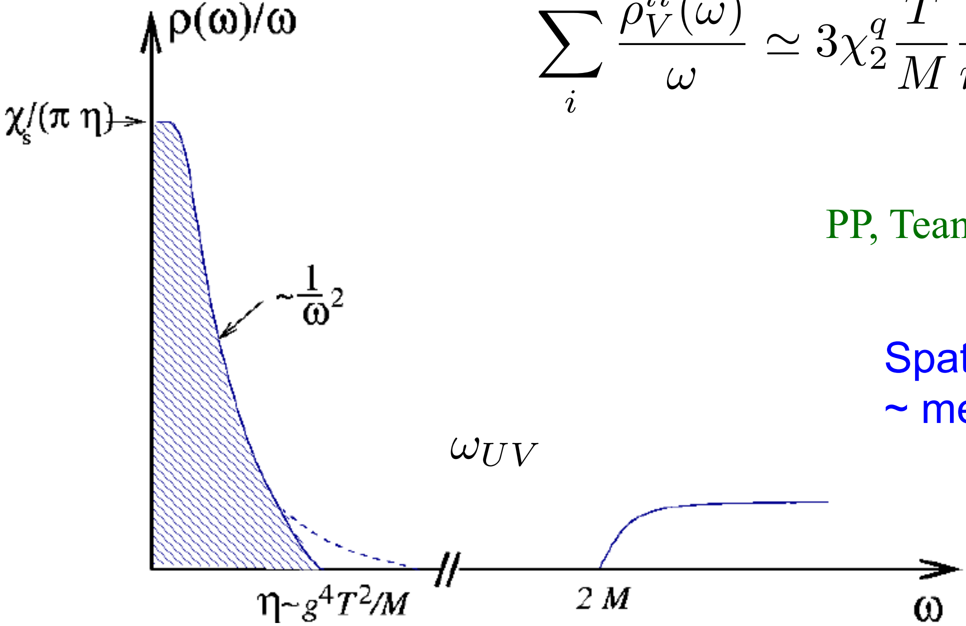
$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t-t')$$

$$\kappa = 2MT\eta = 2T^2/D_s$$

Momentum diffusion coefficient

$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q \frac{T}{M} \frac{\eta}{\eta^2+\omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D_s}$$

drag constant



PP, Teaney, PRD 72 (2006) 014508

Spatial diffusion constant
~ mean free path (weak coupling)

$$D_s \sim \frac{1}{g^4 T}$$

area under the peak ~ $\chi_2^q \frac{T}{M}$

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

Heavy quark diffusion and lattice QCD

Obtain the momentum heavy quark transport coefficient through the force correlator

$$\langle f_i(t) f_j(t)\rangle = \langle E_i(t) E_j(t')\rangle + \frac{1}{3}\langle \mathbf{v}^2\rangle \langle \delta_{ij} B_k(t) B_k(t') - B_i(t') B_j(t)\rangle \qquad \langle \mathbf{v}^2\rangle = \frac{3T}{M}$$

$$t \to i\tau$$

Can be rigorously derived in Heavy Quark Effective Theory

Casalderrey-Solana, Teaney, PRD 74 (2006) 085012; Caron-Huot, Laine, Moore, JHEP 0904 ('09) 053

Bouttefeux, Laine, JHEP 12 (2020) 150

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gE_i(\tau,\vec{0})\, U(\tau,0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle} \qquad \kappa_E = \lim_{\omega\to 0} \frac{2T}{\omega}\rho_E(\omega)$$

$$G_B(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gB_i(\tau,\vec{0})\, U(\tau,0)\, gB_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle} \qquad \kappa_B = \lim_{\omega\to 0} \frac{2T}{\omega}\rho_B(\omega)$$

$$G_{E,B}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{E,B}(\omega) \frac{\cosh\left(\tau - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v}^2\rangle \kappa_B$$

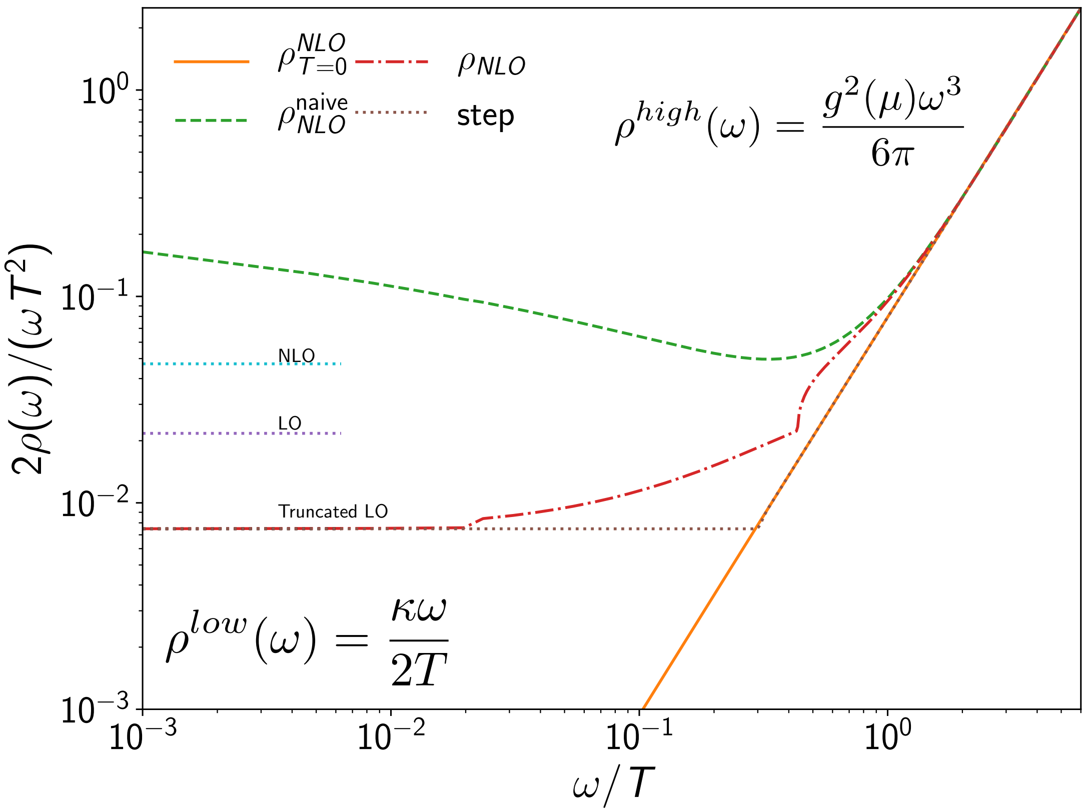
Extracting momentum diffusion coefficient from the lattice

Challenge 1: obtain precise results for chromo-electric and chromo-magnetic (very noisy)

⇒ Noise reduction via multi-level algorithm, applicable to quenched QCD (pure glue plasma)

⇒ Noise reduction by gradient flow method (new development !) , also applicable in full QCD

Challenge 2: reconstruct the spectral function from the Euclidean time lattice correlator



⇒ use known large and small energy behavior of the spectral

Parameterize $\rho(\omega, T)$ as smooth interpolation between $\rho^{low}(\omega, T)$ and $\rho^{high}(\omega)$, and treat κ as well as the additional nuisance parameters of interpolation as fit parameters

Extracting momentum diffusion coefficient from the lattice

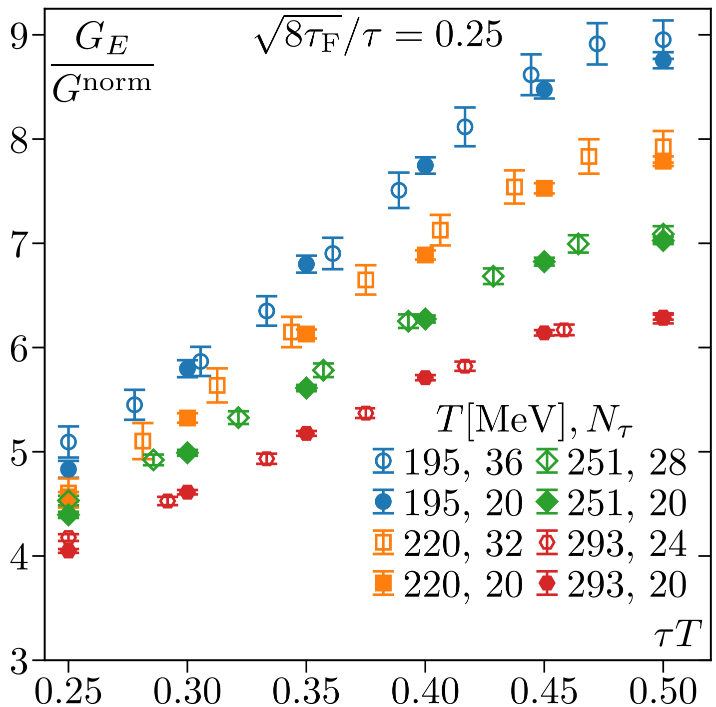
2+1 flavor QCD with $m_l = m_s/5$ ($m_\pi = 320$ MeV), $T = 195 - 354$ MeV, $96^3 \times N_\tau$ lattices with $N_\tau = 36,\ 32,\ 28,\ 24,\ 20$; additional $64^3 \times N_\tau$ lattices with $N_\tau = 20,\ 22,\ 24 \Rightarrow 3$ lattice spacings at each T; Gradient flow for noise reduction

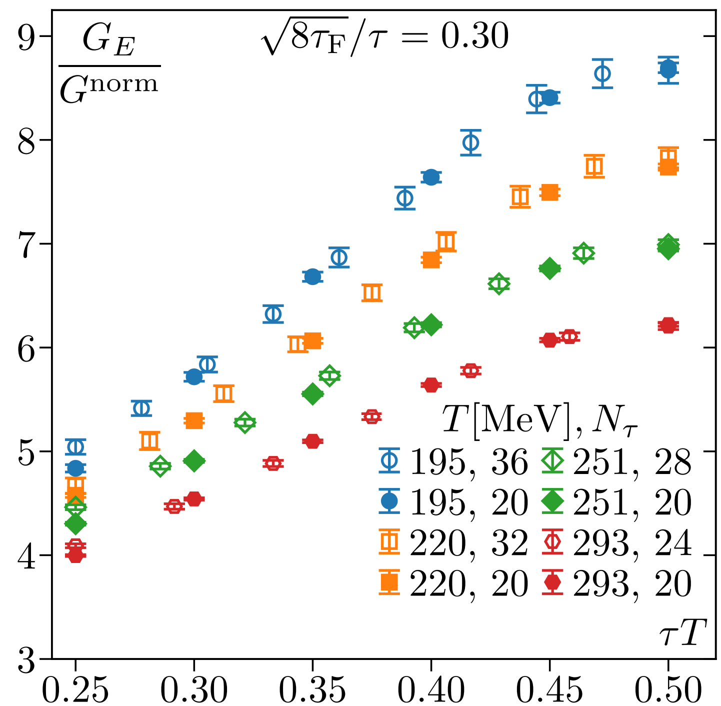
HotQCD, PRL 130 (2023) 231902

$$A_\mu(x) \to B_\mu(\tau_F, x) \qquad B_\mu(0,x) = A_\mu(x) \qquad \partial_{\tau_F} B_\mu(\tau_F, x) = -g_0^2 \frac{\delta S_{\rm YM}[B]}{\delta B_\mu(\tau_F, x)}$$

Symanzik gauge action and Zeuthen flow

Gauge fields are smeared in the radius $\sqrt{8\tau_F}$ $\qquad a < \sqrt{8\tau_F} < \tau/3$





We see small cutoff effects thanks improved actions

Analysis and modeling the chromo-electric correlator

Analysis of the chromo-electric correlator:

- Extrapolate the lattice results on the chromo-electric correlator to the continuum limit
- Perform the zero flow time extrapolation

HotQCD, PRL 130 (2023) 231902

Fits to model spectral function:

$$\rho^{low}(\omega,T)=\frac{\kappa\omega}{2T} \qquad \rho^{high}(\omega)=\rho^{LO,NLO}(\omega)$$

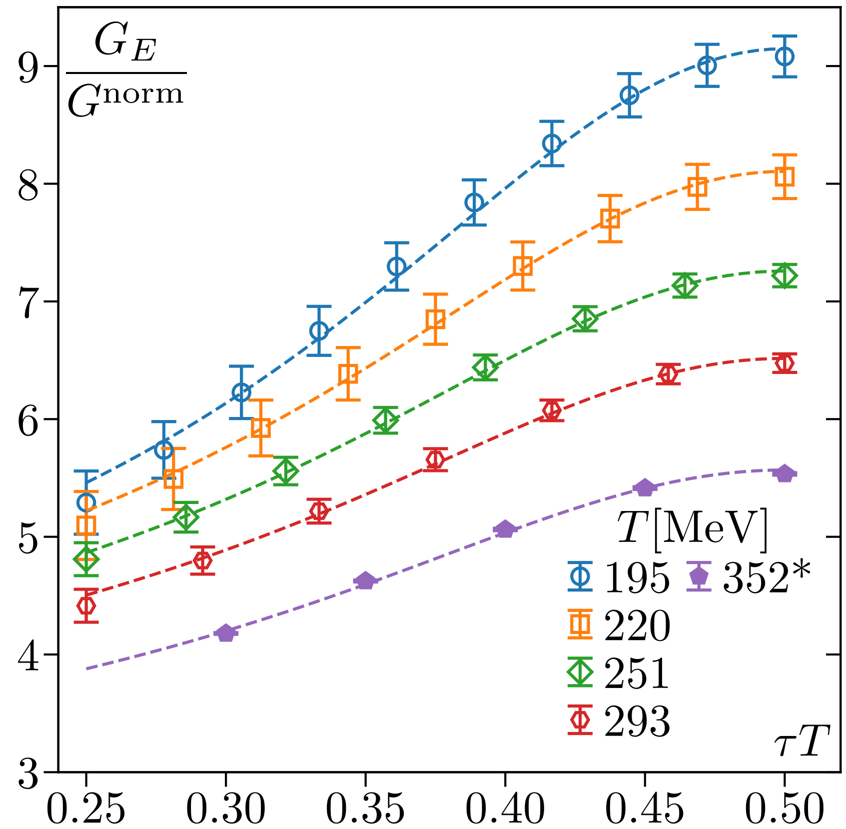
$$\rho^{max}(\omega,T)=max(\rho^{low}(\omega,T),\rho^{high}(\omega))$$

$$\rho^{smax}(\omega,T)=\sqrt{(\rho^{low})^2+(\rho^{high})^2}$$

$$\rho^{pow}(\omega,T)=\rho^{low}(\omega,T),\ \omega\le\omega_{IR}$$

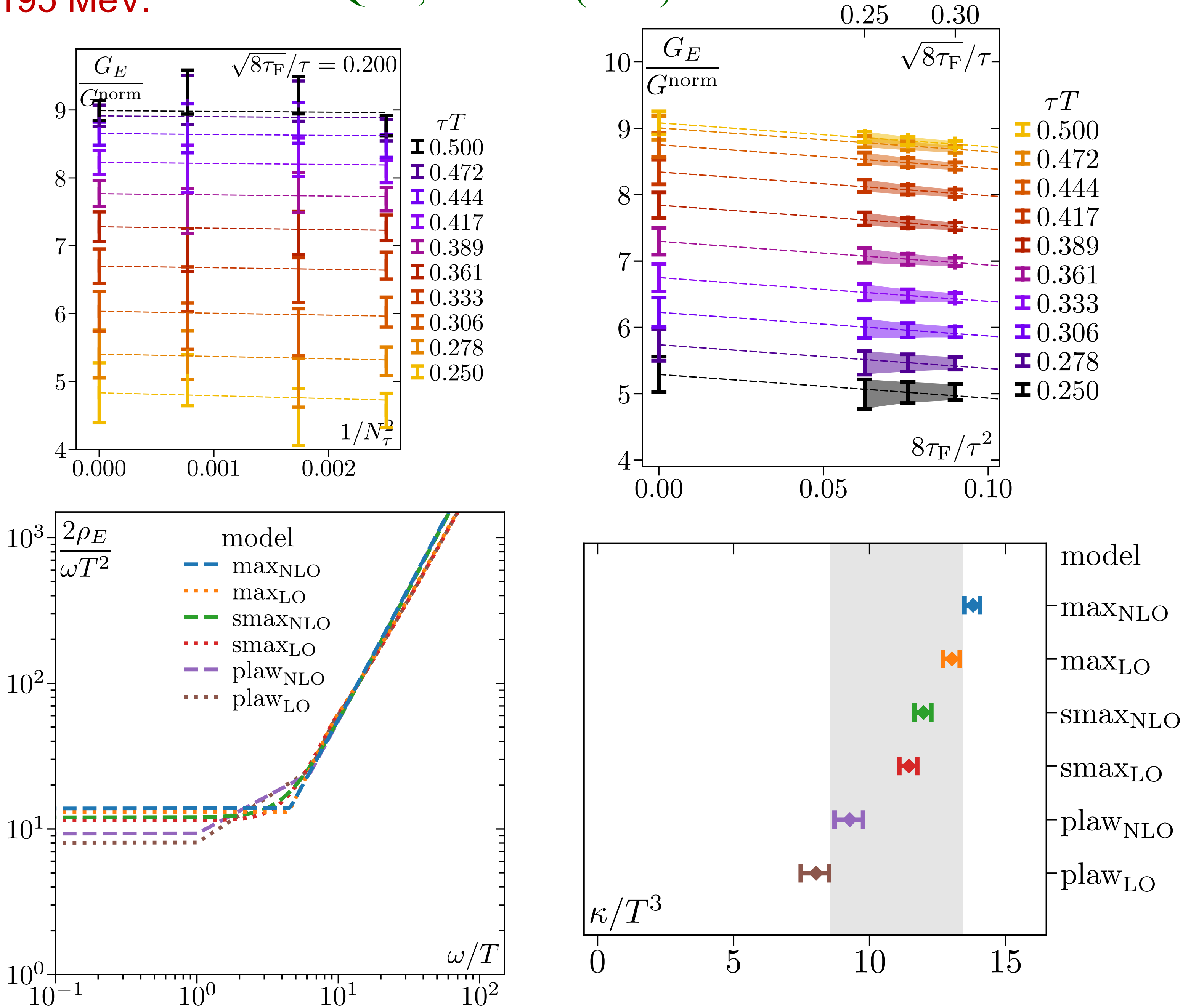
$$\rho^{pow}(\omega,T)=A\omega^{\alpha},\ \omega_{IR}<\omega<\omega_{UV}$$

$$\rho^{pow}(\omega)=\rho^{high}(\omega),\ \omega\ge\omega_{UV}$$



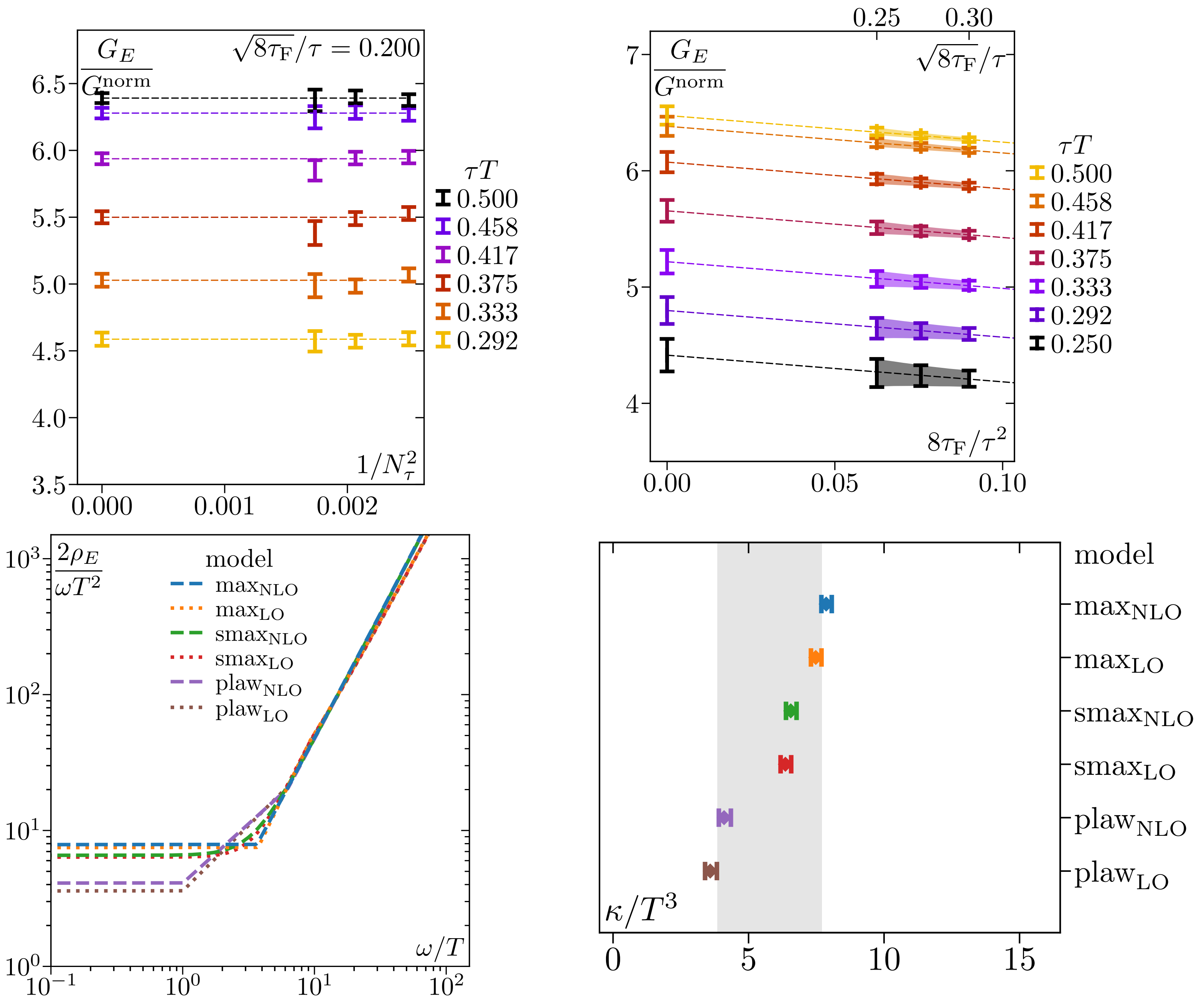
T=195 MeV:

HotQCD, PRL 130 (2023) 231902



T=293 MeV:

HotQCD, PRL 130 (2023) 231902

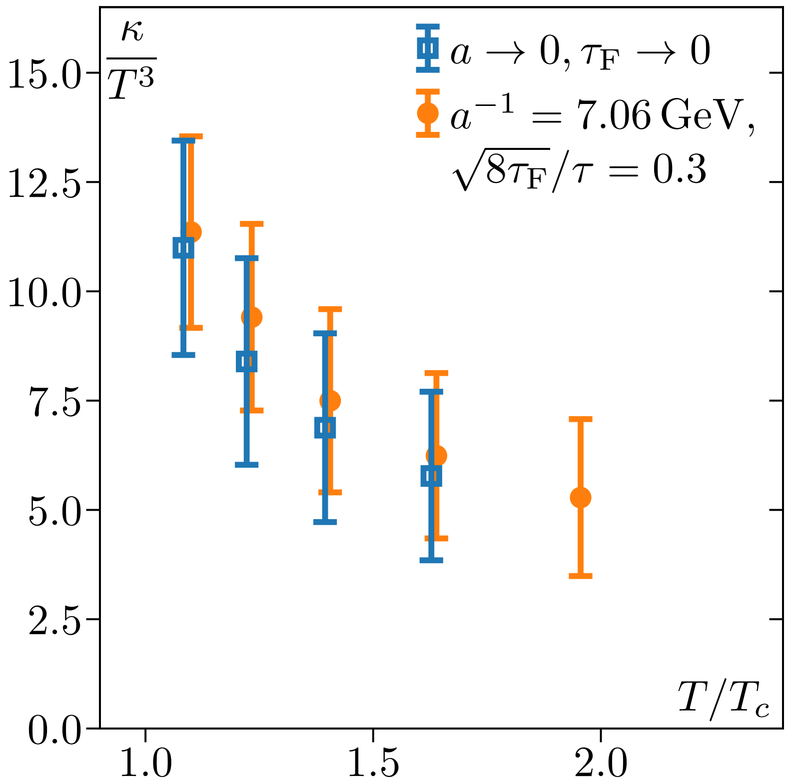


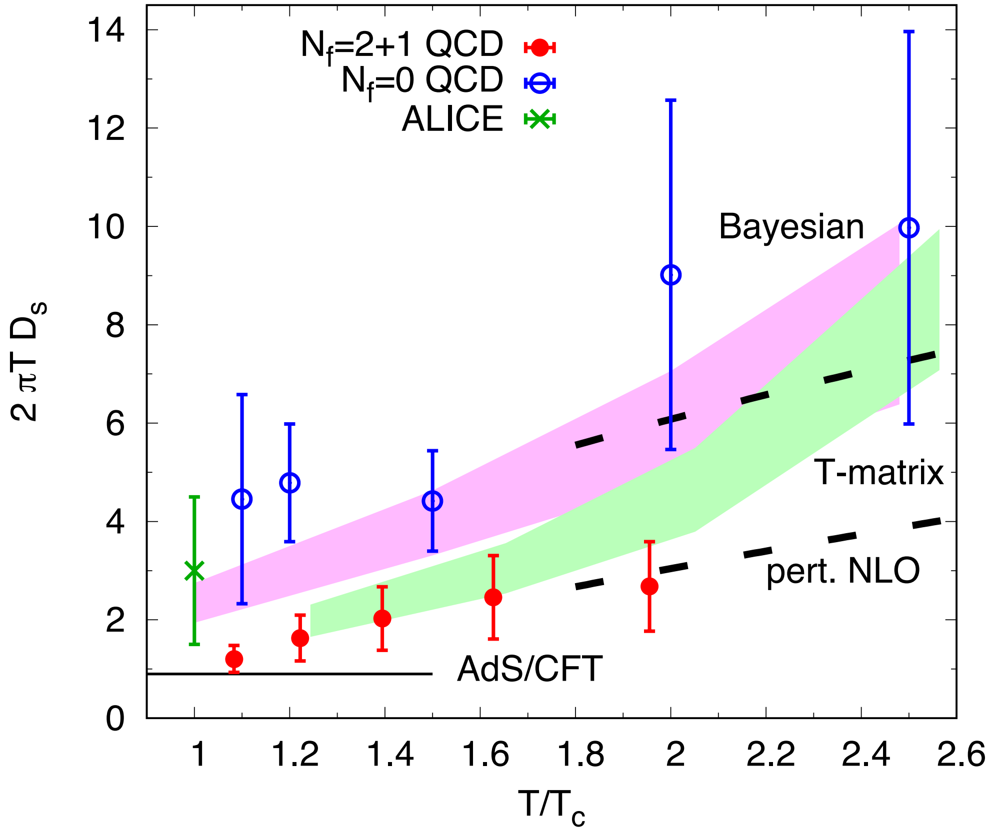
Heavy quark diffusion coefficient in QCD

- κ/T^3 has significant temperature dependence
- D_s is significantly smaller in 2+1 flavor QCD than in quenched QCD and is close to the AdS/CFT limit

HotQCD, PRL 130 (2023) 231902

$$D_s = \frac{2T^2}{\kappa}$$



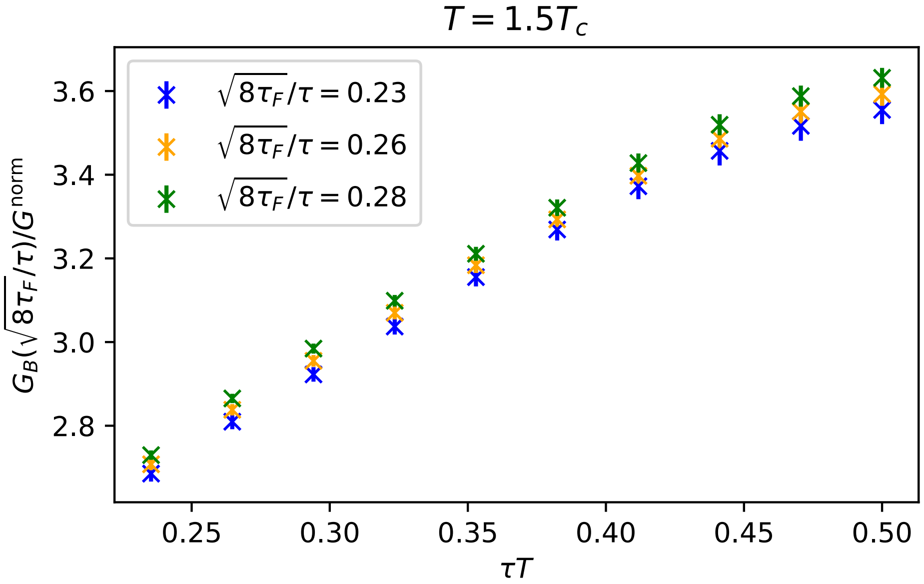


Mass suppressed correction to heavy quark diffusion coefficient

$1/M$ correction to the momentum heavy quark diffusion coefficient $\Rightarrow G_B(\tau, T)$
$G_B(\tau, T)$ has anomalous dimension $\Rightarrow$ additional matching to $\overline{MS}$ is needed

Quenched QCD

Gradient flow + incomplete 1-loop matching



Multi-level algorithm + non-perturbative matching via Schrödinger functional

$1.5T_c : \kappa_B = (1.23 - 2.54)T^3,$

Brambilla, Leino, Mayer-Steudte, PP (TUMQCD), PRD 107 (2023) 054508

$\kappa_B = (1.0 - 2.1)T^3$

Banerjee, Datta, Laine JHEP 08 (2022) 128

$\langle v^2 \rangle$ is taken from PP, EPJC 62 (2009) 85

10-20% correction for bottom quark, ~30% correction for charm quark

Quark anti-quark potential at *T>0*

Conjecture, Matsui and Satz, PLB 178 (86) 416 $-\frac{4}{3}\frac{\alpha_s}{r}+\sigma r \to -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r}, T>T_c$

Extending pNRQCD to *T>0*: the potential is complex, the real part can have thermal correction but is not necessarily screened, except when $r \sim 1/m_D$

Based on weak coupling

Laine, Philipsen, Romatschke, Tassler, JHEP 03 (06) 054
Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017

Calculate the potential non-perturbatively on the lattice by considering Wislon loops of size *r x* τ at *T>0*

$$W(r,\tau,T)=\int_{-\infty}^{\infty}\rho_r(\omega,T)e^{-\omega\tau}$$

If potential at $T>0$ exists the $\rho_r(\omega,T)$ should have a well defined peak at $\omega \simeq \mathrm{Re}V(r,T)$, and the width of the peak is $\mathrm{Im}V(r,T)$

Rothkopf, Hatsuda, Sasaki, PRL 108 (2012) 162001

Challenge: reconstruct $\rho_r(\omega,T)$

Hybrid potentials,
pairs of static-light mesons ...

$$\rho_r(\omega,T=0)=\delta(\omega-V(r))+\sum_n \delta(\omega-E_n(r))$$

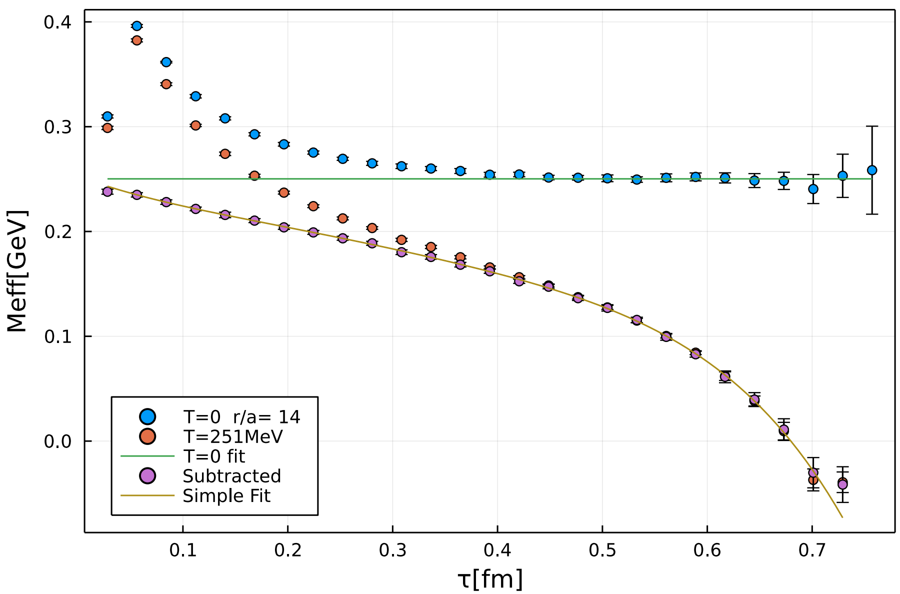
Calculations on fine lattices

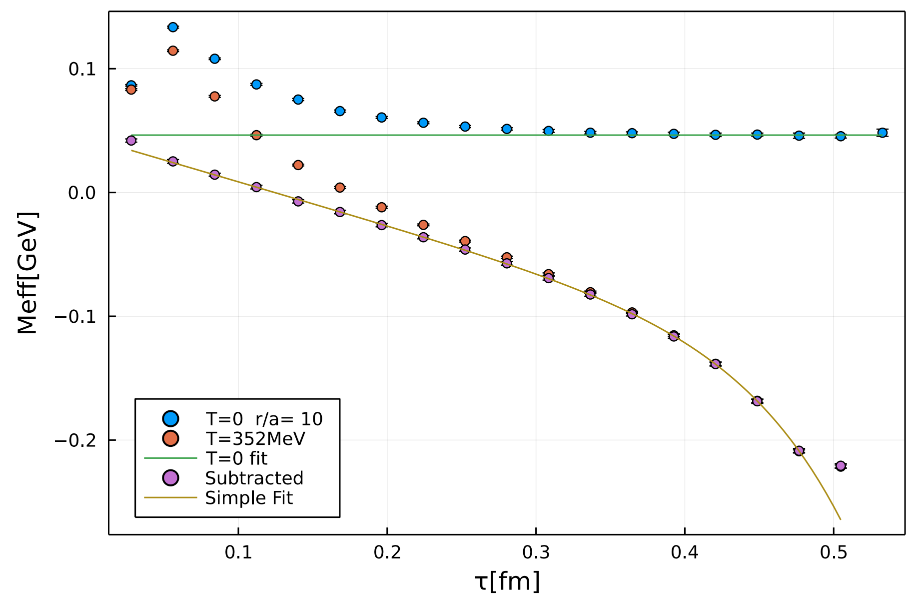
$2+1$ f QCD, $m_\pi = 300$ MeV $\quad T = 126,\ 196,\ 220,\ 252,\ 294,\ 354$ MeV

$a = 0.028$ fm, $96^3 \times N_\tau, N_\tau = 56,\ 36,\ 32,\ 28,\ 24,\ 20$

Gradient flow for noise reduction: $\sqrt{8\tau_F}T = 0.04 - 0.05$

$$m_{eff}(r,\tau,T) = -\partial_\tau \log(W(\tau,r,T)) \simeq \frac{1}{a} \ln \frac{W(r,\tau,T)}{W(r,\tau+a,T)}$$

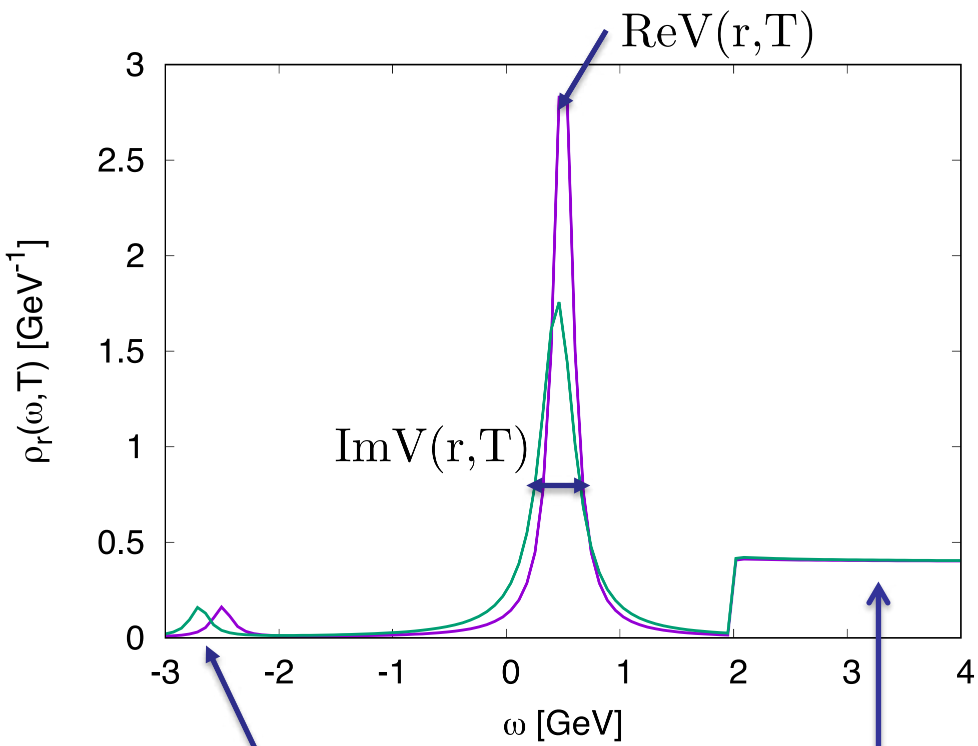




- No plateau at $T > 0$ in m_{eff} at $T > 0$
- Only tiny T-dependence for small τ

HotQCD, arXiv:2308:16587

Spectral function and the subtracted correlators



$$W^{high}(r,\tau) = \int_{-\infty}^{\infty} d\omega \rho_r^{high}(\omega) e^{-\omega\tau}$$

On the lattice:

$$W^{high}(r,\tau) = W(r,\tau,T=0) - A_0 \exp(-V(r)\tau)$$

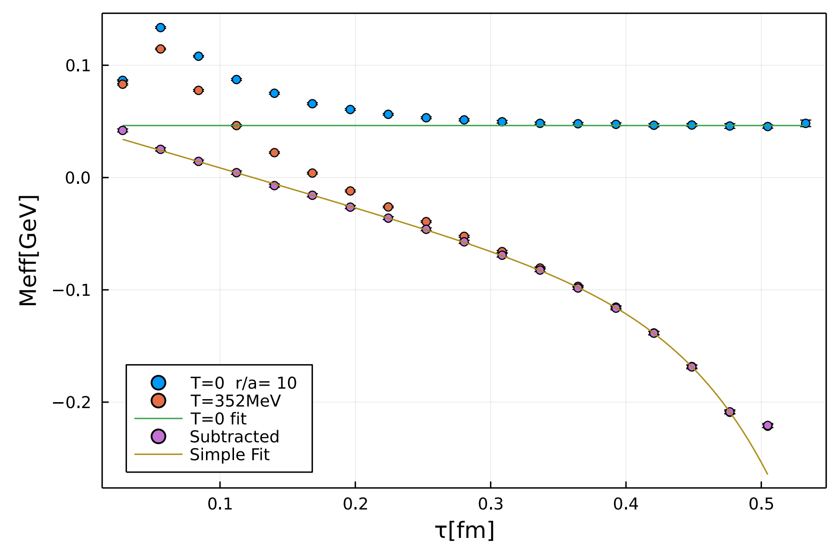
$$W^{sub}(r,\tau,T) = W(r,\tau,T) - W^{high}(r,\tau)$$

$$\rho_r(\omega,T) = \rho_r^{tail}(\omega,T) + \rho_r^{peak}(\omega,T) + \rho_r^{high}(\omega)$$

See, Bala et al (HotQCD), PRD 105 (2022) 054513

Cumulants of $W^{sub}(r,\tau,T)$ carry information about T-dependent part of the spectral function

m_{eff} for the subtracted correlator has milder τ-dependence, which is approximately linear

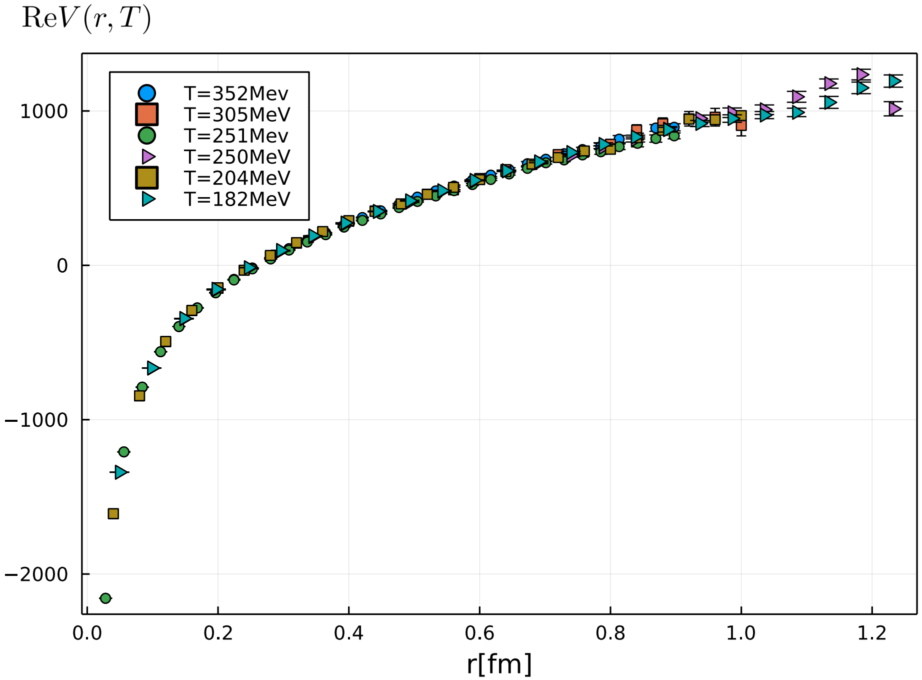


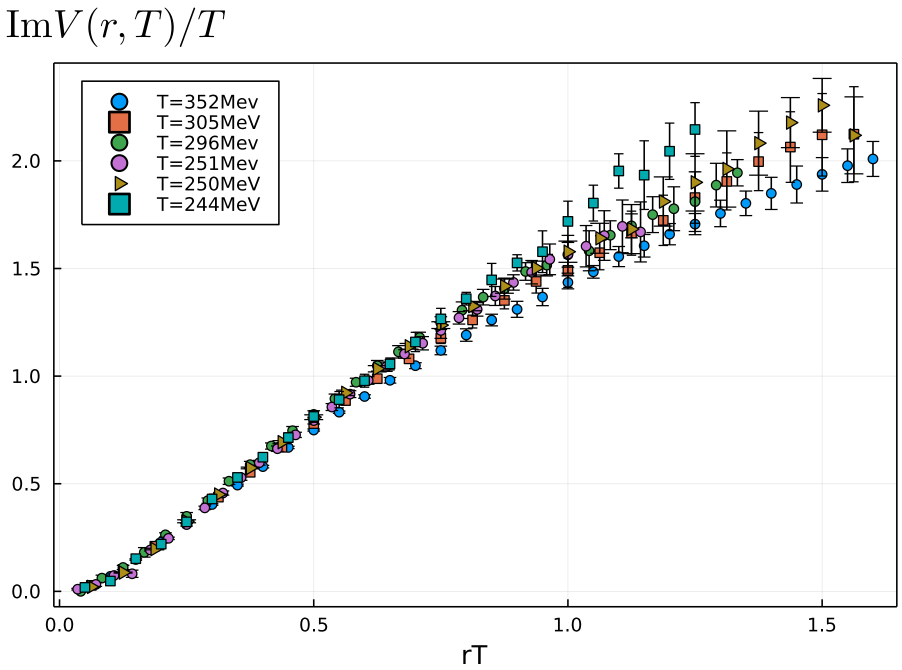
HotQCD, arXiv:2308:16587

Model spectral function and the complex potential

$$\rho_r^{peak}(\omega,T) = \frac{A}{\pi}\frac{\Gamma(\omega,r,T)}{(\omega-\mathrm{Re}V(r,T))^2+\Gamma^2(\omega,r,T)} \quad \Gamma(\omega,r,T) = \begin{cases}\Gamma_0(r,T) & -2\Gamma_0<\omega<2\Gamma_0 \\ 0 & n \text{ otherwise}\end{cases}$$

$$\rho_r^{tail}(\omega,T) = A^{tail}\delta(\omega - E^{tail})$$





$\mathrm{Re}V(r,T)$ shows tiny temperature dependence and no hint of screening

$\mathrm{Im}V(r,T)$ increases with rT and is proportional to T

HotQCD, arXiv:2308:16587