STATISTICAL HADRONIZATION MODEL FOR Au-Au COLLISIONS AT SIS18 ENERGIES

Szymon Harabasz (TU Darmstadt) in collaboration with:

Wojciech Florkowski
Jagiellonian University
Tetyana Galatyuk
TU Darmstadt / GSI
Malgorzata Gumberidze
GSI
Jędrzej Kołaś
Warsaw University of Technology
Radoslaw Ryblewski
IFJ PAN
Piotr Salabura
Jagiellonian University
Joachim Stroth
Goethe-University Frankfurt / GSI
Hanna Paulina Zbroszczyk
Warsaw University of Technology

Based on:
PRC **102** (2020) 5, 054903,
arXiv: 2003.12992 [nucl-th]

PRC **107** (2023) 3, 034917
arXiv: 2210.07694 [nucl-th],

https://github.com/therminator2/therminator2
Pull Requests, “Issue" reports very welcome

QCD STUDY WITH HEAVY-ION COLLISIONS

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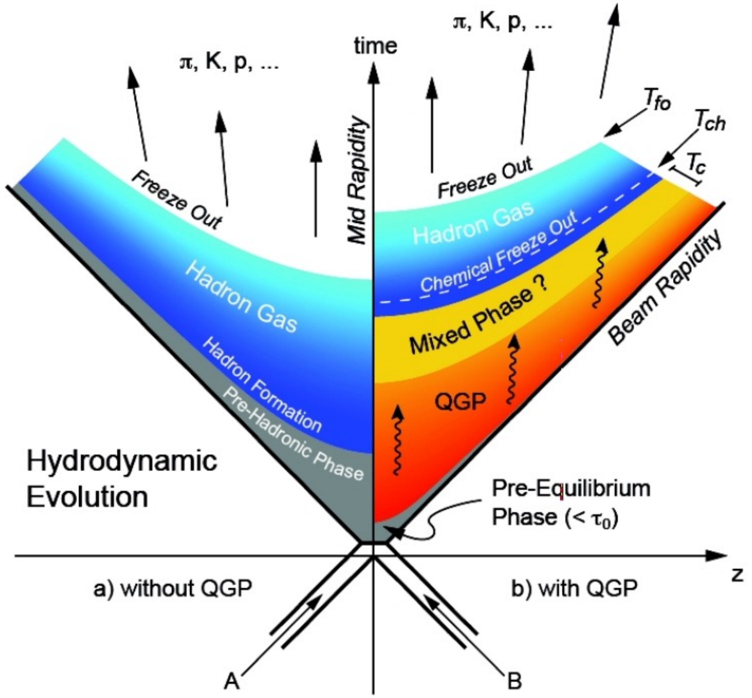
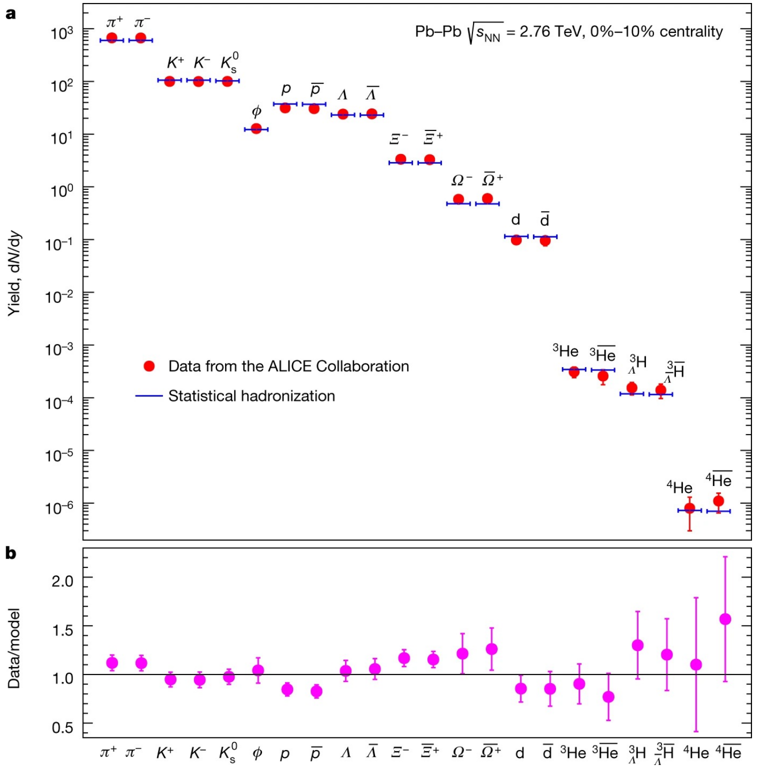


- In chemical equilibrium density of particle *i* can be written as:

$$n_i = \frac{g_s}{2\pi^2} \Upsilon T m^2 K_2\left(\frac{m}{T}\right)$$

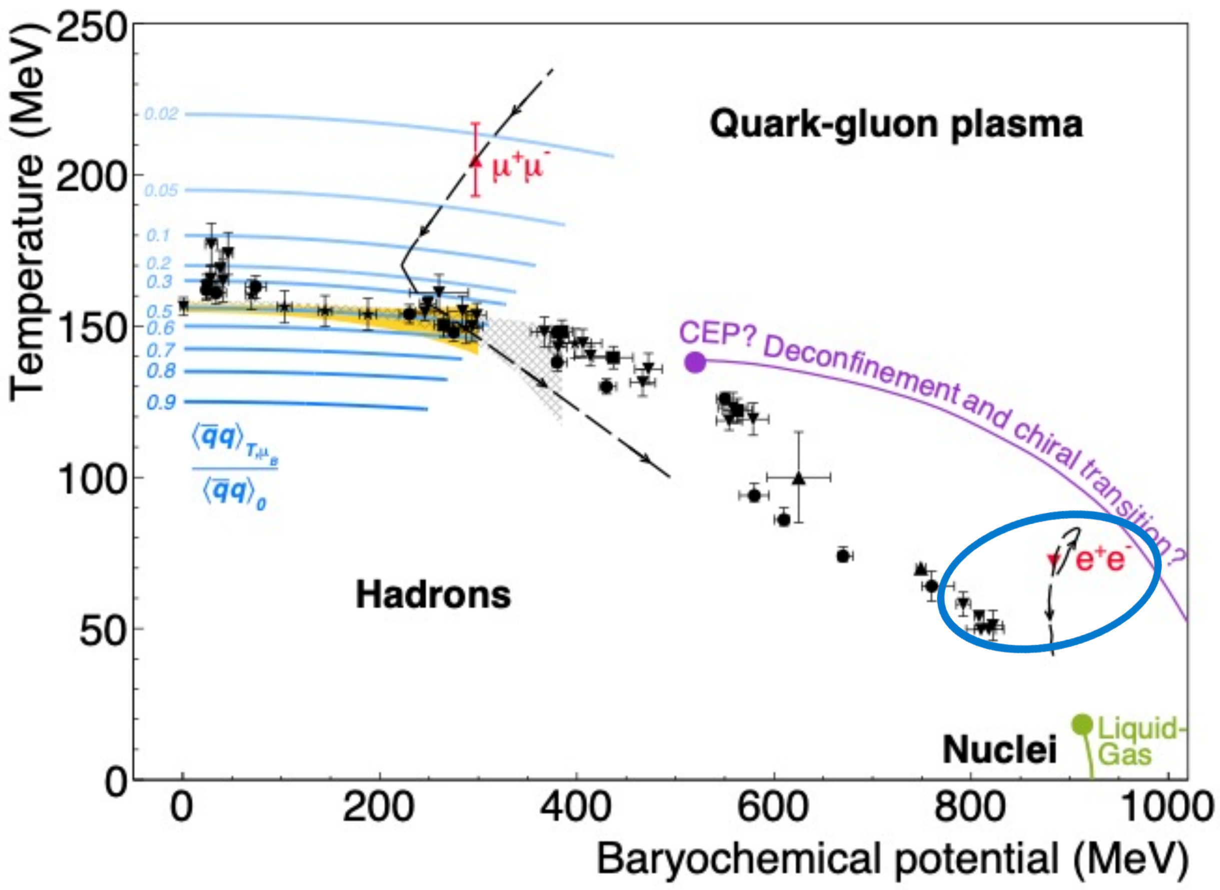
Statistical Hadronization Model (SHM)

- One can fit the ratios of measured particle yields and extract free parameters
  - Location in the phase diagram



A. Andronic et al., Nature **561** (2018) 7723

MAPPING THE PHASE DIAGRAM WITH THE SHM



HADES, Nature Phys. **15** (2019) 10, 1040-1045
A. Andronic *et al.*, Nature **561** (2018) no.7723
LQCD: S. Borsanyi *et al.* [Wuppertal-Budapest], JHEP **1009** (2010) 073
LQCD: A. Bazavov *et al.*, PLB **795** (2019) 15-21

- Is it valid at all to use equilibrium methods at low energies?
  - Particles with strange quarks produced deep below the NN threshold
  - Low number of newly produced particles in the interaction zone: ~40 in central events (mainly pions)
- On the other hand:
  - Original nucleons stopped in the interaction zone (~300 particles in central events)
  - Longer life-time of the system (~15 fm/*c*): enough to thermalize

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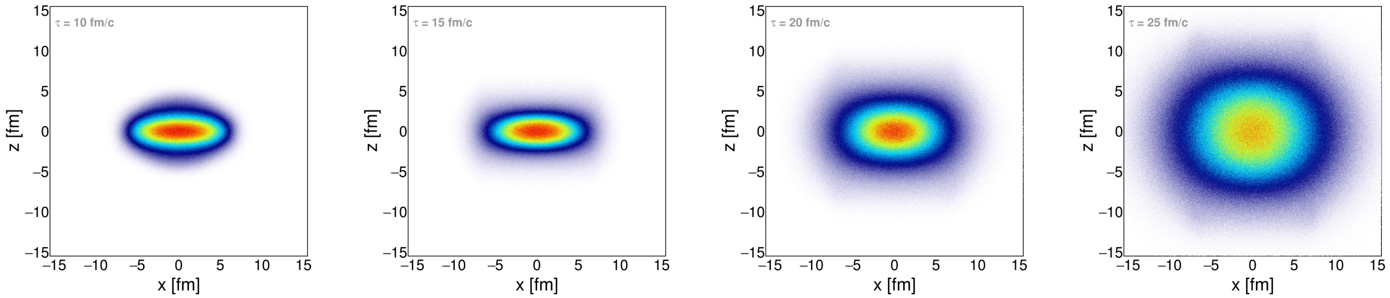
HYDRO-INSPIRED MODELS
OF PARTICLE PRODUCTION AT THE FREEZE-OUT

- **First idea:**
  P. J. Siemens and J. O. Rasmussen, PRL **42** (1979) 880
  — Used for Ne+NaF at E_{kin}/A = 0.8 GeV!
  — Thermal source of spherical geometry and spherically symmetric expansion
  — Constant radial velocity (non-physical for $r = 0$?)

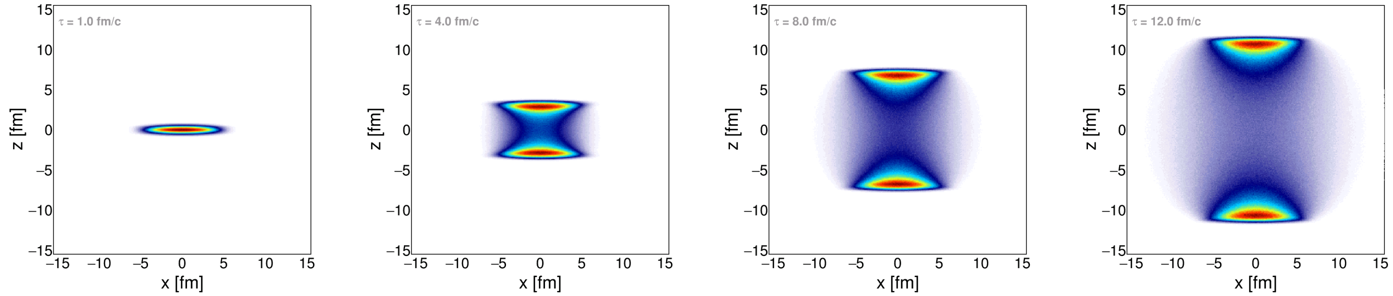
- **Modification:**
  E. Schnedermann, J. Sollfrank, U. W. Heinz, PRC **48** (1993) 2462
  — Appropriate for higher-energy collisions (originally S+S at E_{kin}/A = 200 GeV)
  — Cylindrically-symmetric geometry and expansion
  — Boost invariance in Z direction – "Bjorken scaling"
  — Velocity profile: $\beta(r) = \beta_{\max}(r/\mathrm{r}_{\max})^n$

Guidance from dynamic models T. Galatyuk *et al.*, EPJA **52** (2016) 5, 131

Au+Au at 1.23*A* GeV



In+In at 158*A* GeV



Spherical symmetry at 1-2*A* GeV is clearly more realistic than boost invariance

SINGLE FREEZE-OUT SCENARIO AT RHIC ENERGIES

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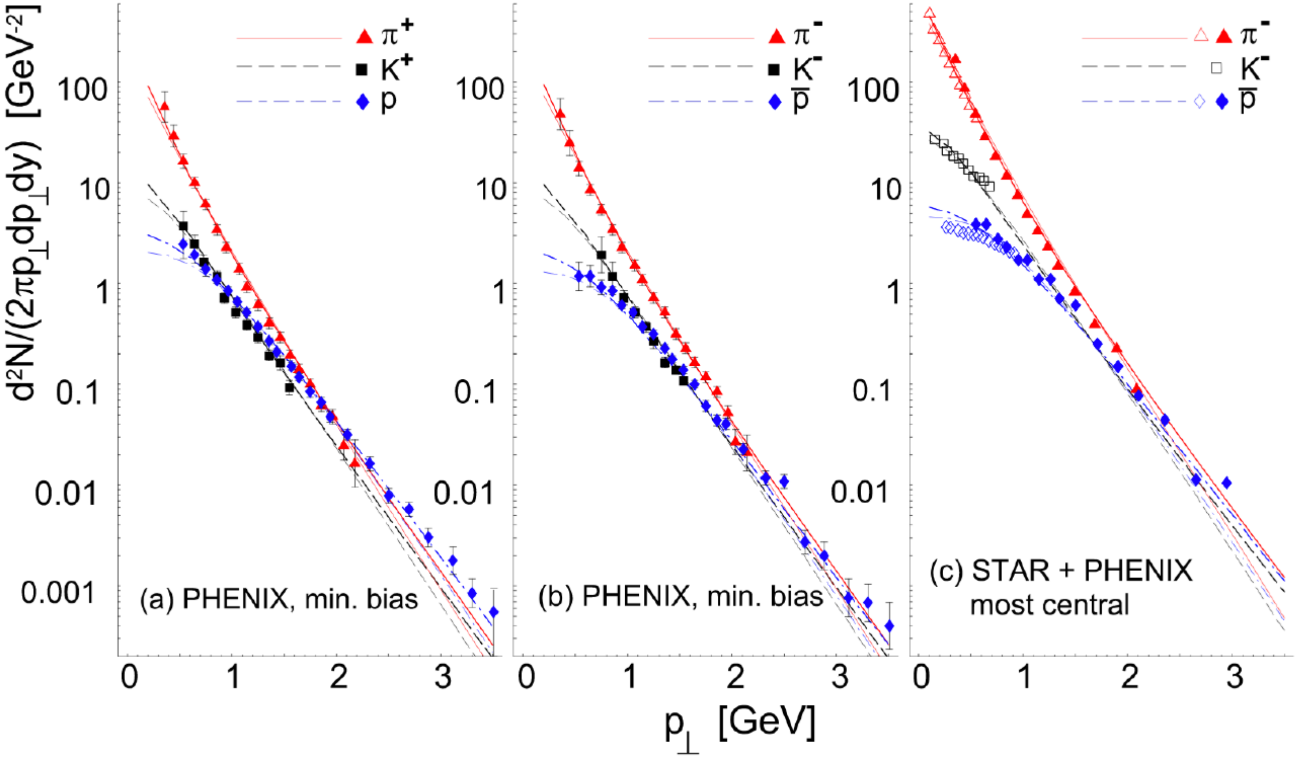
W. Broniowski and W. Florkowski, PRL **87** (2001) 272302

- Chemical freeze-out coincides with kinetic freeze-out
- Hadron yields are given by the integrals of hadron spectra
- Feed-down from resonance decays included
- Successful at RHIC, does it work at SIS18 energies?
- Idea is implemented in the Thermal Event Generator (Therminator 2)



THERMAL EVENT GENERATOR (THERMINATOR 2)

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M. Chojnacki *et al.*, Comput. Phys. Comm. 103 (2012) 746-773
SH, W. Florkowski, T. Galatyuk *et al.*, PRC 102 (2020) 5, 054903
SH, W. Florkowski, T. Galatyuk *et al.*, PRC 107 (2023) 3, 034917

Ingredients of the method:

- Single (chem. & kin.) freeze-out on a **spheroid-symmetric hypersurface**
- Δ spectral function from πN phase shift
- Fix parameters with particle multiplicity ratios:
  - Six equations for six parameters:

protons (incl. those bound in light nuclei): 124.1

π^+:	**9.3**
π^-:	**17.1**
K^+:	**0.0598**
K^-:	**0.00056**
Λ:	**0.0822**

HADES data:
M. Szala, Proc. of SQM 2019
EPJA 56 (2020) 10, 259
PLB 778 (2018) 403-407
PLB 793 (2019) 457-463

→ $T \quad \mu_B \quad \mu_{I3} \quad \mu_S \quad \gamma_S \quad R$

- m_t spectra of p and π in five rapidity bins

→ ***H*** in the radial expansion velocity profile $v = \tanh(Hr)$

δ - eccentricity parameter for the spheroid in the *momentum* space

- Femtoscopic radii

→ **ε** - eccentricity parameter for the spheroid in the *position* space

APPROACHES TO THERMAL PARAMETERS

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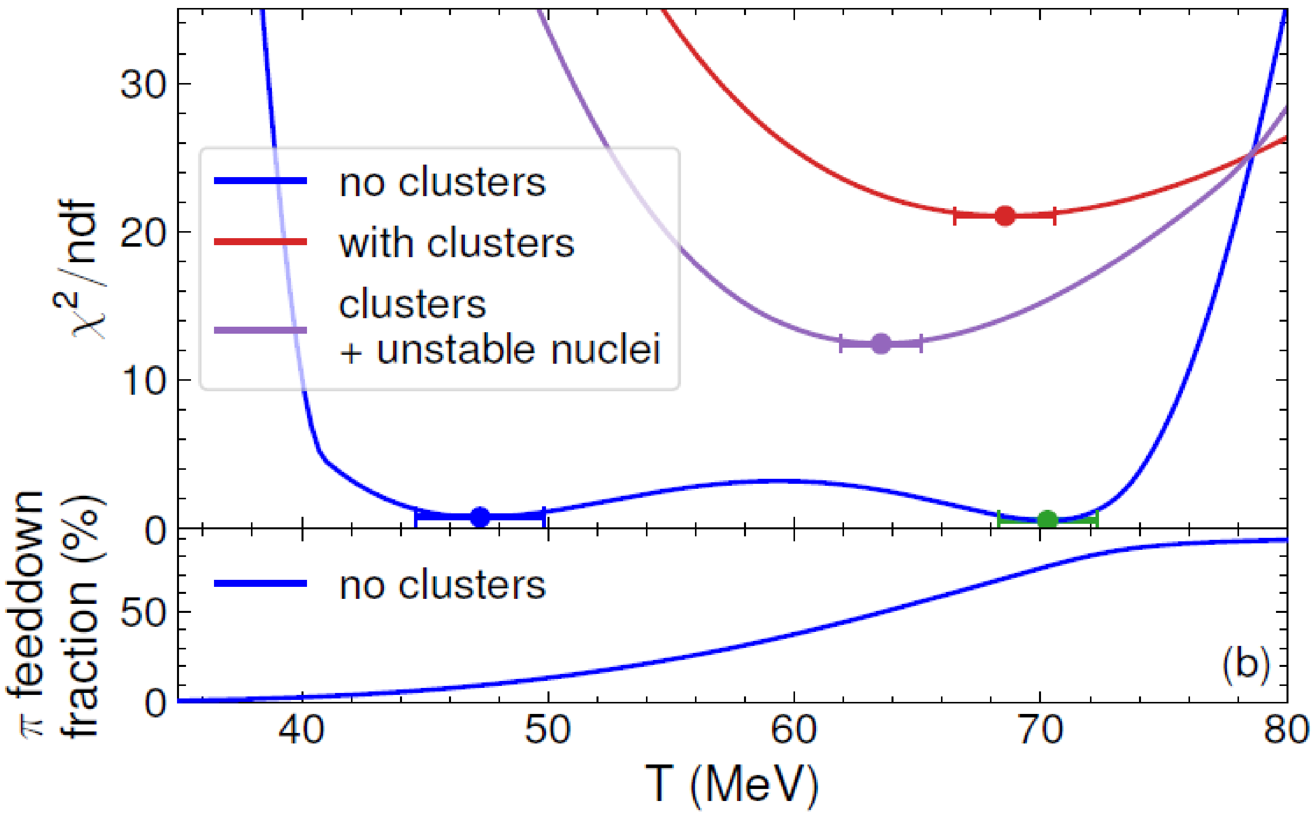
A. Motornenko *et al.*, PLB **822** (2021) 136703



Parameter	Harabasz *et al.* [1]	no clusters low T minimum	no clusters high T minimum	with clusters	with clusters + unstable nuclei
T (MeV)	49.6 ± 1.1	47.2 ± 2.6	70.3 ± 2.0	68.6 ± 2.0	63.5 ± 1.6
R (fm)	16.0	18.9 ± 2.2	6.8 ± 0.9	9.0 ± 0.4	10.4 ± 0.3
μ_B (MeV)	776 ± 3	780.1 ± 3.8	872.1 ± 24.3	786.7 ± 2.9	781.1 ± 3.3
γ_S	0.16 ± 0.02	0.19 ± 0.07	0.05 ± 0.01	0.03 ± 0.01	0.04 ± 0.01
χ^2/N_{df}	$N_{\mathrm{df}} = 0$	1.58/2	1.13/2	105.30/5	62.30/5

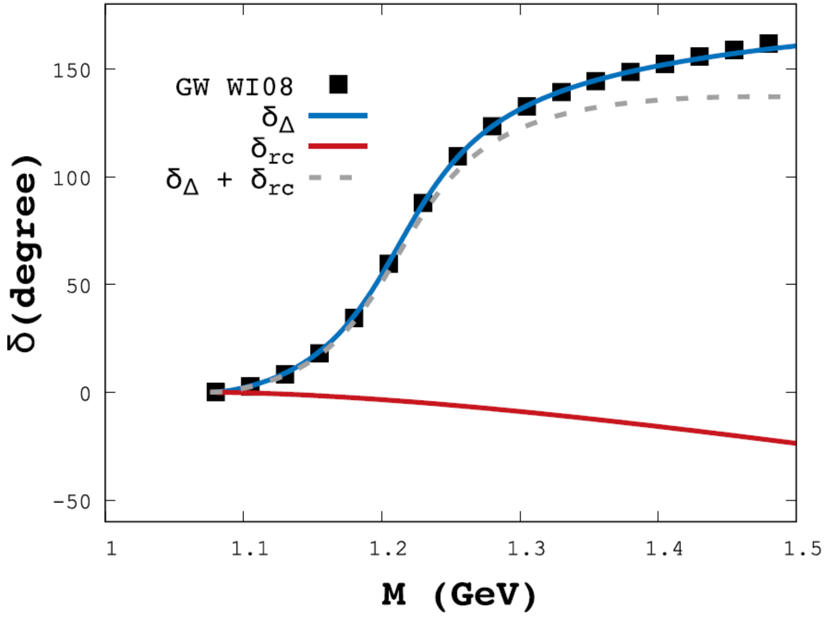
- There Q/B = 0.4 and total S = 0 are kept as constraints
- We recover parameters needed to run Therminator: μ_{I3} μ_S
- We fix the Hubble constant H and readjust R

RESONANCE TREATMENT

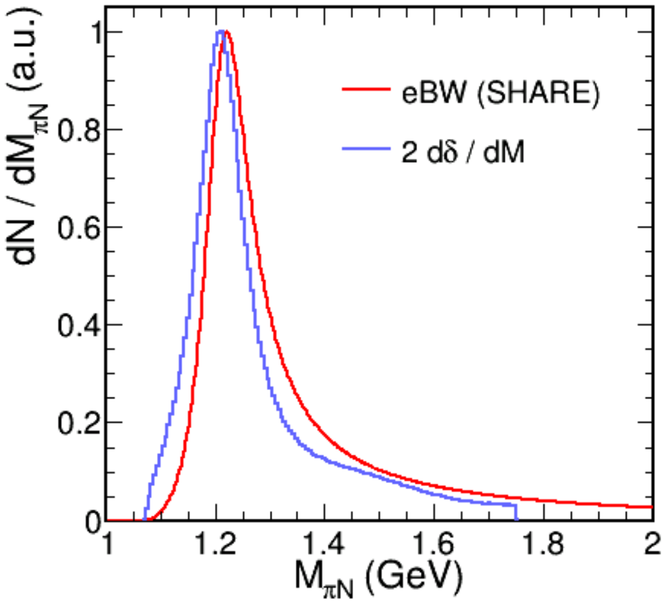
R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. **187** (1969) 345 (1969)
R.Venugopalan, and M. Prakash, NPA **546** (1992) 718
W. Weinhold, and B. Friman, PLB **433** (1998) 236
Pok Man Lo, EPJC **77** (2017) no.8, 533

πN phase shift in the P_{33} channel

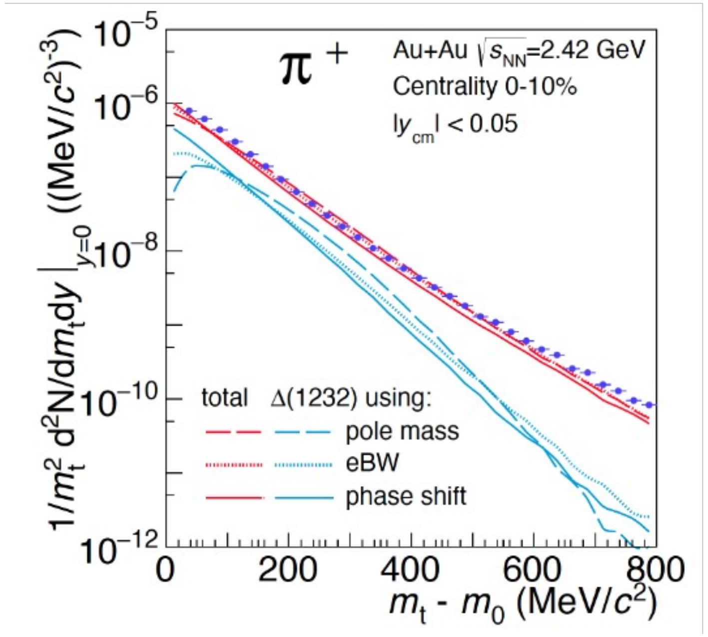
Pok Man Lo *et al.*, PRC **96**, 015207
GW WI08: R.L. Workman *et al.* PRC **86**, 035202



Spectral function: $B_l(M) = 2\frac{d}{dM}\delta_l$



Sensitivity of hadron spectra



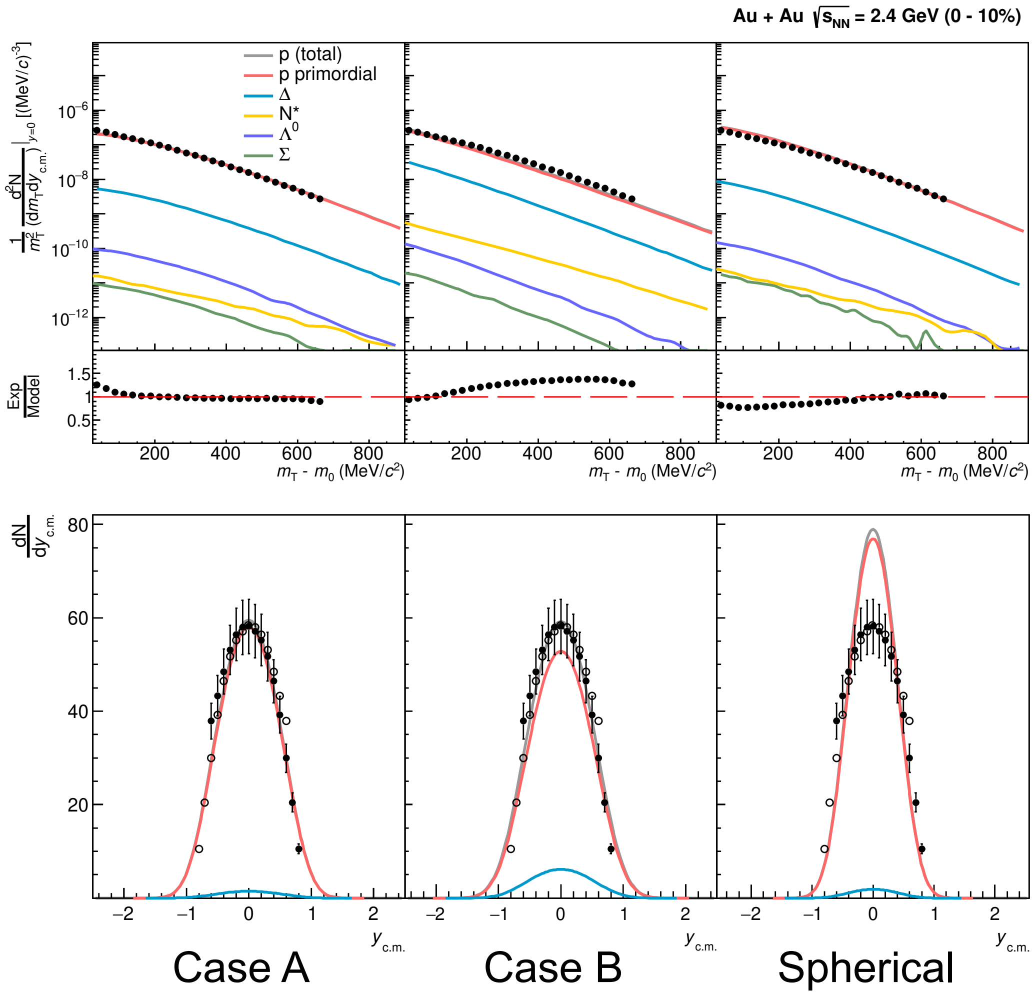
PROTON SPECTRA

HADES data:
M. Szala, Proceedings of SQM 2019
EPJA 56 (2020) 10, 259
PLB 778 (2018) 403-407
PLB 793 (2019) 457-463

Ref. [29]: PRC **102** (2020) 5, 054903
Case A & Case B: arXiv: 2210.07694 [nucl-th]

Parameter	Case A	Case B	Spherical
T (MeV)	49.6	70.3	49.6
R (fm)	15.7	6.06	15.7
μ_B (MeV)	776	876	776
μ_S (MeV)	123.4	198.3	123.4
μ_{I3} (MeV)	-14.1	-21.5	-14.1
γ_S	0.16	0.05	0.16
H (GeV)	0.01	0.0225	0.008
δ	0.2	0.4	0
$\sqrt{Q^2}$	0.238	0.256	0.285

- Spheroid fireball is more realistic than spherical



PION SPECTRA (POSITIVE CHARGE)

HADES data:
M. Szala, Proceedings of SQM 2019
EPJA 56 (2020) 10, 259
PLB 778 (2018) 403-407
PLB 793 (2019) 457-463

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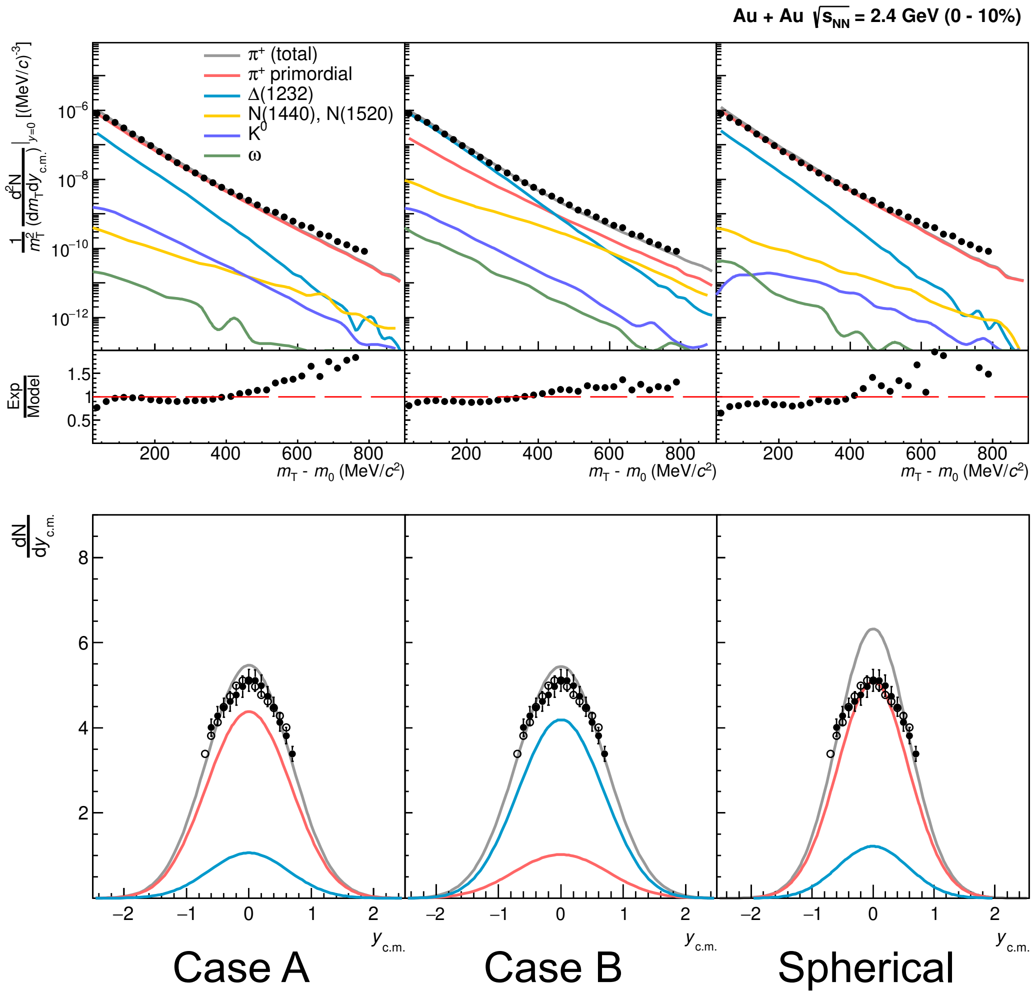
$\sqrt{Q^2}$	0.238	0.256	0.285

- Spheroid fireball is more realistic than spherical
- Single δ works well for both protons and pions
- One can infer the shape of the fireball from data



PION SPECTRA (NEGATIVE CHARGE)

HADES data:
M. Szala, Proceedings of SQM 2019
EPJA 56 (2020) 10, 259
PLB 778 (2018) 403-407
PLB 793 (2019) 457-463

TECHNISCHE UNIVERSITÄT DARMSTADT

Ref. [29]: PRC **102** (2020) 5, 054903
Case A & Case B: arXiv: 2210.07694 [nucl-th]

Parameter	Case A	Case B	Spherical
T (MeV)	49.6	70.3	49.6
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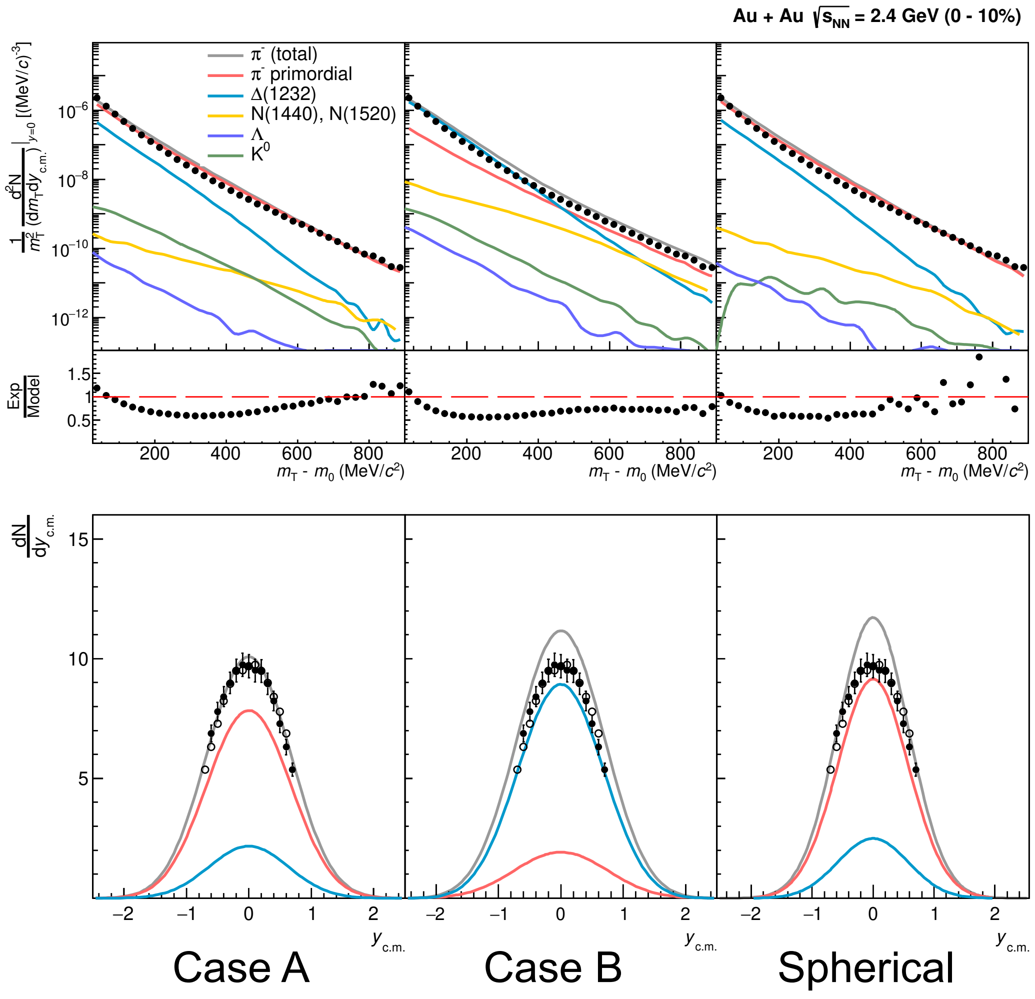
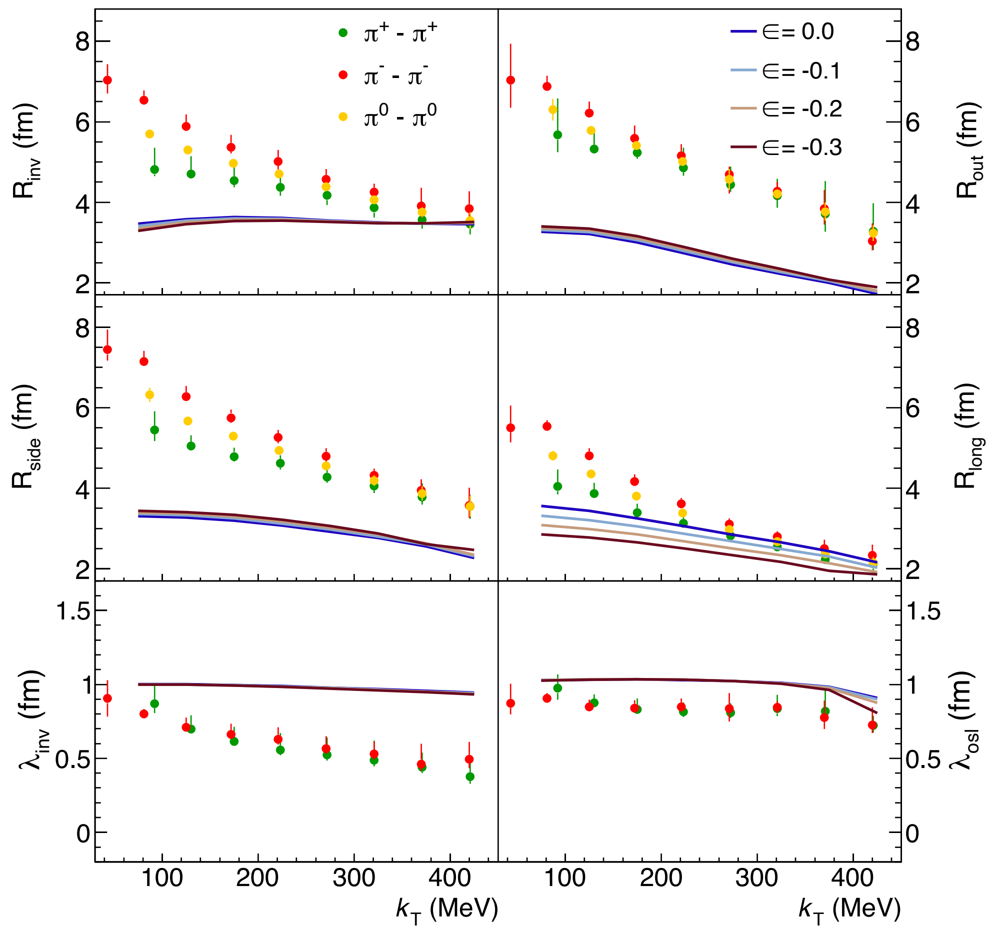


PION FEMTOSCOPIC RADII

Case B (T = 70.3 MeV, …)

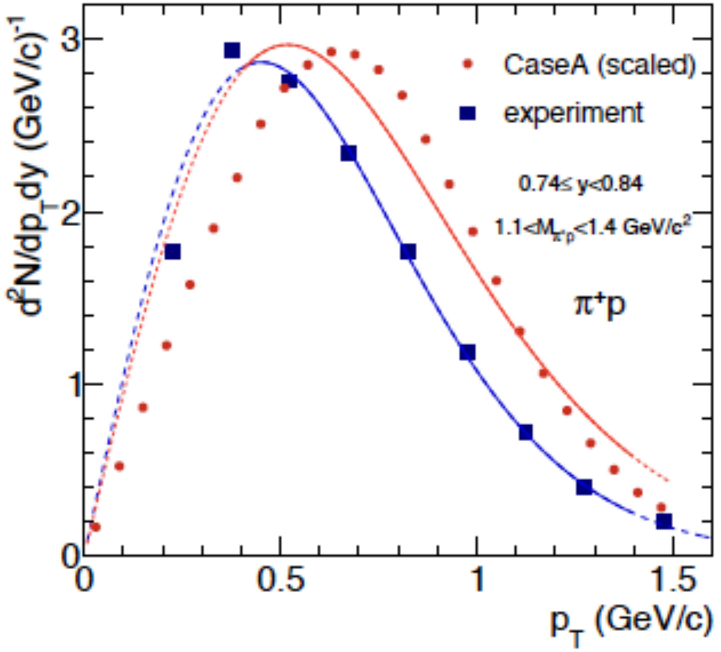


HADES data:
PLB 795 (2019) 446

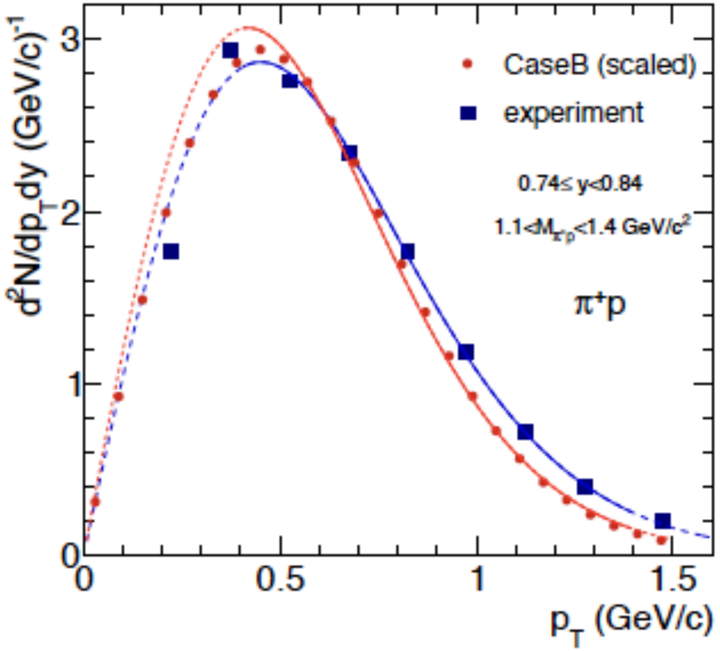
- Trends as a function of transverse momentum and order of magnitude or R's are OK-ish
- Many moving parts in the modeling (pair wave function etc.) not just the freeze-out parametrization

OUTLOOK: PION-PROTON PAIR SPECTRA

Case A (T = 49.6 MeV, …)



Case B (T = 70.3 MeV, …)



M. Kurach, Internship and Training Project Report,
GET INvolved Programm, GSI/FAIR Darmstadt

HADES data:
PLB **819** (2021) 136421

- Extra hint to select the ”right” set of parameters
- Possibility to disentagle effects of the resonance shape, the thermal factor and the kinematic shift
- The event generator allows experimental collaborations to study combinatorial background effects

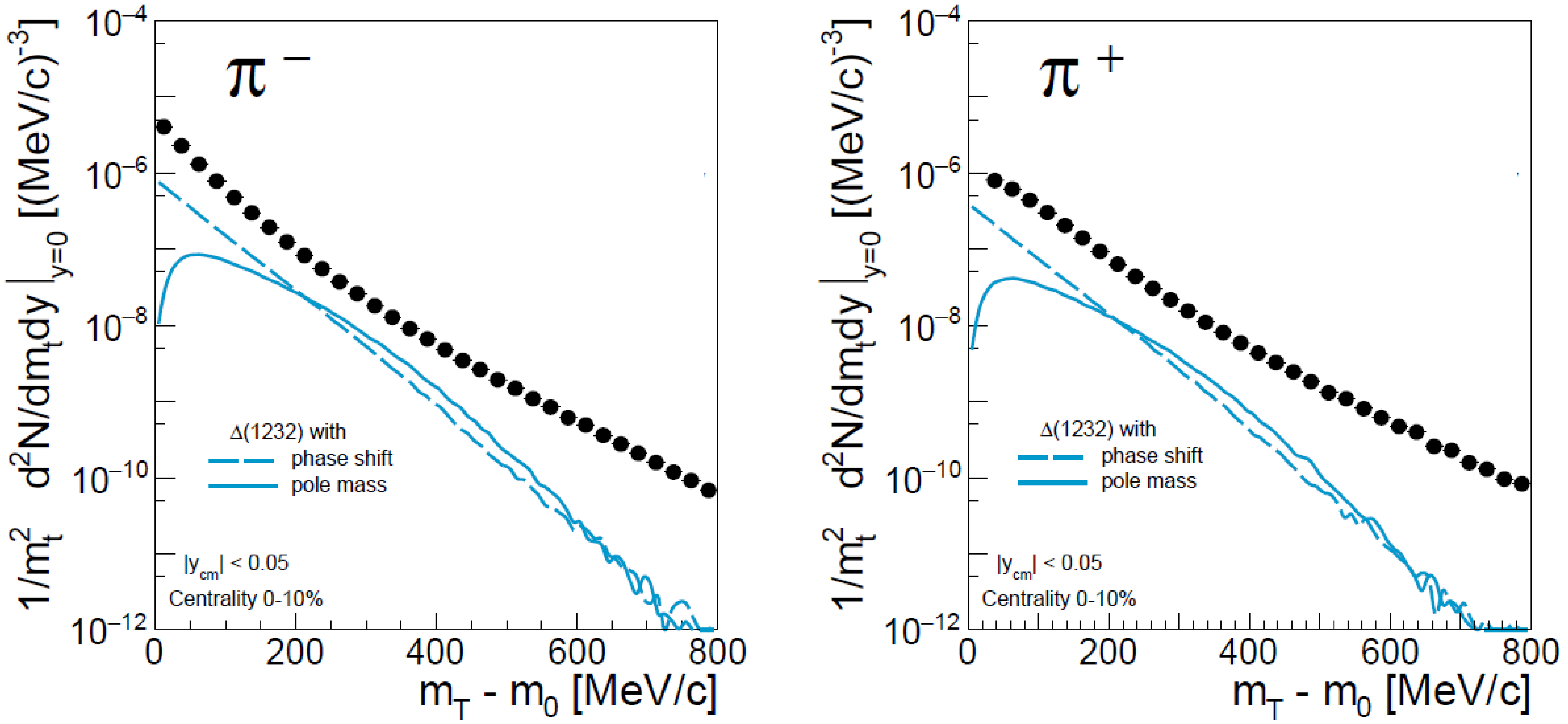
SUMMARY

- Statistical hadronization model can describe not only multiplicities, but also spectra of bulk particles produced in heavy-ion collisions in $\sqrt{s_{NN}}$ of few GeV
- Input:
  - Spheroid fireball shape and expansion
  - Hubble-like velocity profile
  - Instantaneous freeze-out
  - Careful treatment of baryonic resonances
- Output:
  - Thermodynamic conditions and fireball shape at the freezeout
  - Resonance shape vs. thermal factor vs. kinematic shift
  - Convenient, easily tunable event generator for experiment:
    - Efficiency study with a realistic event shape
    - Combinatorial background effects
    - **Future: constraining freeze-out cocktail for dileptons**

Q&A

EXTRA SLIDES

INFLUENCE OF THE Δ DESCRIPTION ON PION SPECTRA



Transverse mass of pions from Δ decay for different spectral functions:

- Δ with fixed mass of 1.232 GeV
- Spectral function from the πN phase shift in the P_{33} channel

Finite Δ width:
$\rightarrow$ populate low m_t pions

COOPER-FRYE FORMULA

F. Cooper and G. Frye, PRD 10 (1974) 186

"Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production"

$$E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x)\, p^\mu\, f(x,p)$$

- Spherically symmetric system:
$$x^\mu = (t(r), r\mathbf{e_r})$$
- Spherical expansion of the "fluid":
$$u^\mu = \frac{1}{\sqrt{1-v^2(r)}}(1, v(r)\mathbf{e_r})$$
- Sudden freeze-out in the "lab" frame ($t = \text{const}(r)$):
$$d^3\Sigma_\mu \equiv \varepsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial \zeta} \frac{\partial x^\beta}{\partial \phi} \frac{\partial x^\gamma}{\partial \theta} d\zeta\, d\phi\, d\theta$$
$$= (r^2 \sin\theta\, d\theta\, d\phi\, dr, 0, 0, 0)$$

Parameter of $\zeta \to (t(\zeta), r(\zeta))$

Local thermodynamic equilibrium

$$f(x,p) = \frac{g_s}{2\pi}\left[\Upsilon^{-1} \exp\left(\frac{p_\mu u^\mu}{T}\right) \pm 1\right]^{-1}$$

Fugacity factor:
$$\Upsilon \equiv \gamma_q^{N_q+N_{\bar{q}}} \gamma_s^{N_s+N_{\bar{s}}} \exp\left(\frac{\mu_B B + \mu_S S + \mu_{I_e} I_3}{T}\right)$$
(in this work we assume $\gamma_q = 1$)

- Integrating over the freeze-out hypersurface and phase-space gives back particle multiplicity
- Right sets of assumptions recover the original Siemens-Rasmussen and Schnedermann-Sollfrank-Heinz formulas
- **But we assume Hubble-like expansion:**
$$\boldsymbol{v(r) = \tanh(Hr)}$$