Semi-Classical Yang-Mills with $n_f = 12$ at $T = 0$

Seth Grable

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We want to solve Yang-Mills... What's Stopping us?

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- But wait...

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- Methods used were derived from Euler/Heisenburg-1936 and Swinger-1951 & 1954

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- AdS/CFT has stolen the spotlight for the last 30 years,

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- $A^a_\mu \to B^a_\mu + a^a_\mu$

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- Dropping terms linear in a_μ^a, the gauge fixed Lagrangian is

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\begin{aligned}
L =& \frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}a_\mu^a\big[-(D^2)^{ac}g_{\mu\nu} - 2f^{abc}F_{\mu\nu}^b\big]a_\nu^c \\
&+ \bar{\psi}\big[\not{D}^2\big]\psi + \bar{c}^a\big[(D^2)^{ac}\big]c^c.
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$$Z = \int dB_\mu^a e^{-[\int_x (\frac{1}{4}\bar{F}_{\mu\nu}^a)^2] - \frac{1}{2}\ln\det(\theta_{\text{Glue}}) + \frac{n_f}{2}\ln\det(\theta_{\text{Dirac}}) + \ln\det(\theta_{\text{Ghosts}})} \tag{4}$$

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- These θ's are just the operators found in (3)

Solving Methods

- I use

$$\ln \det(\theta) = -\frac{d}{ds}\zeta_\theta\Big|_{s=0} \tag{5}$$

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- and K_θ is the heat kernels of the operator (Bertlmann Anomalies in QFT ch. 5)

$$K_\theta = \mathrm{Tr}[e^{-\tau\theta}], \tag{7}$$

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Heat Kernels

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- HW: find the heat kernels need for (3) for SU(3).

Results

- Using all of this machinery the λ_8 abelian sub-sector of $SU(3)$ gives

$$\frac{1}{\Gamma(s)}\int_0^\infty d\tau\tau^{s-1}\Big(-\frac{1}{2}K_{\theta_{\text{Gluon}}}+\frac{n_f}{2}K_{\theta_{\text{Dirac}}}+K_{\theta_{\text{Ghosts}}}\Big)$$
$$=\frac{3\text{Vol}}{16\pi^2}\frac{B^2}{\Gamma(s)}\int_0^\infty d\tau\tau^{s-1}\Bigg[4(1-\frac{d}{2})+B^2\left(\frac{3dn_f}{4}-9d\right) \tag{10}$$
$$+\frac{dn_f}{2}\frac{1}{\sinh^2(B\tau)}+\frac{dn_f}{4}\frac{1}{\sinh^2(\frac{B\tau}{2})}+9\left(1-\frac{d}{2}\right)\frac{1}{\sinh^2(\frac{3B\tau}{2})}\Bigg]$$

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- For $d=4$ and $n_f=12$ this gives

$$-\frac{d}{ds}\zeta_{\theta_{\text{total}}}\Big|_{s=0}=-\frac{B^2\text{Vol}}{16\pi^2}\left(36\log(A)+3\log\left(\frac{\mu^2}{B}\right)-3+\log\left(\frac{27}{4}\right)\right) \tag{11}$$

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$$\langle l \rangle = \lim_{\beta\to\infty} \mathrm{Tr} \frac{1}{NZ} \int d\bar{B} e^{-\int_x S[\bar{B}]} \mathcal{P} e^{i\int_0^\beta \ell \bar{B}_0^a t^a} \tag{12}$$

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- recall $SU(3)$ is three copies of $SU(2)$ embedded which overlap (in the adjoint representation) via λ_3 and λ_8. These two diagonal generators can give all eigenvalues of $SU(3)$. Without them we have $SU(2) \times SU(2) \times SU(2)$ which would not give a vanishing Polyakov loop for constant B.

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