The QCD phase diagrams, nuclear matter & neutron stars at large Nc

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63. Cracow School of Theoretical Physics

Nuclear Matter at Extreme Densities and High Temperatures

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Outline

- QCD: brief review
- Large-Nc: what is that
- Basic properties
- Large-Nc at nonzero temperature
- QCD Phase-diagram at large Nc
- Nuclear matter at large-Nc
- Neutrons stars at large-Nc
- Conclusions

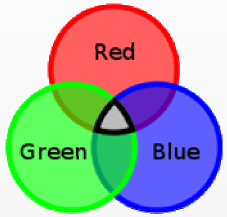
Symmetries of QCD

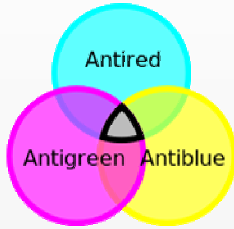
	Giuseppe Lodovico Lagrangia
Born	25 January 1736
	Turin
Died	10 April 1813 (aged 77)
	Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



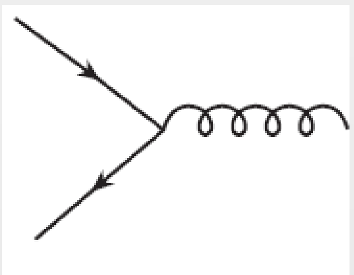


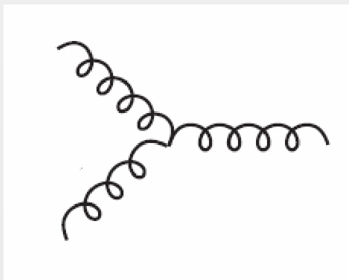
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \; i = u,d,s ,...$$

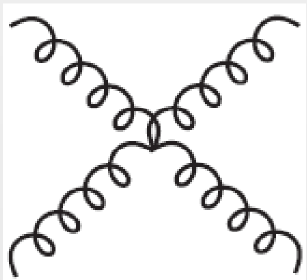
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

8 type of gluons (R$\bar{G}$,B$\bar{G}$,…)

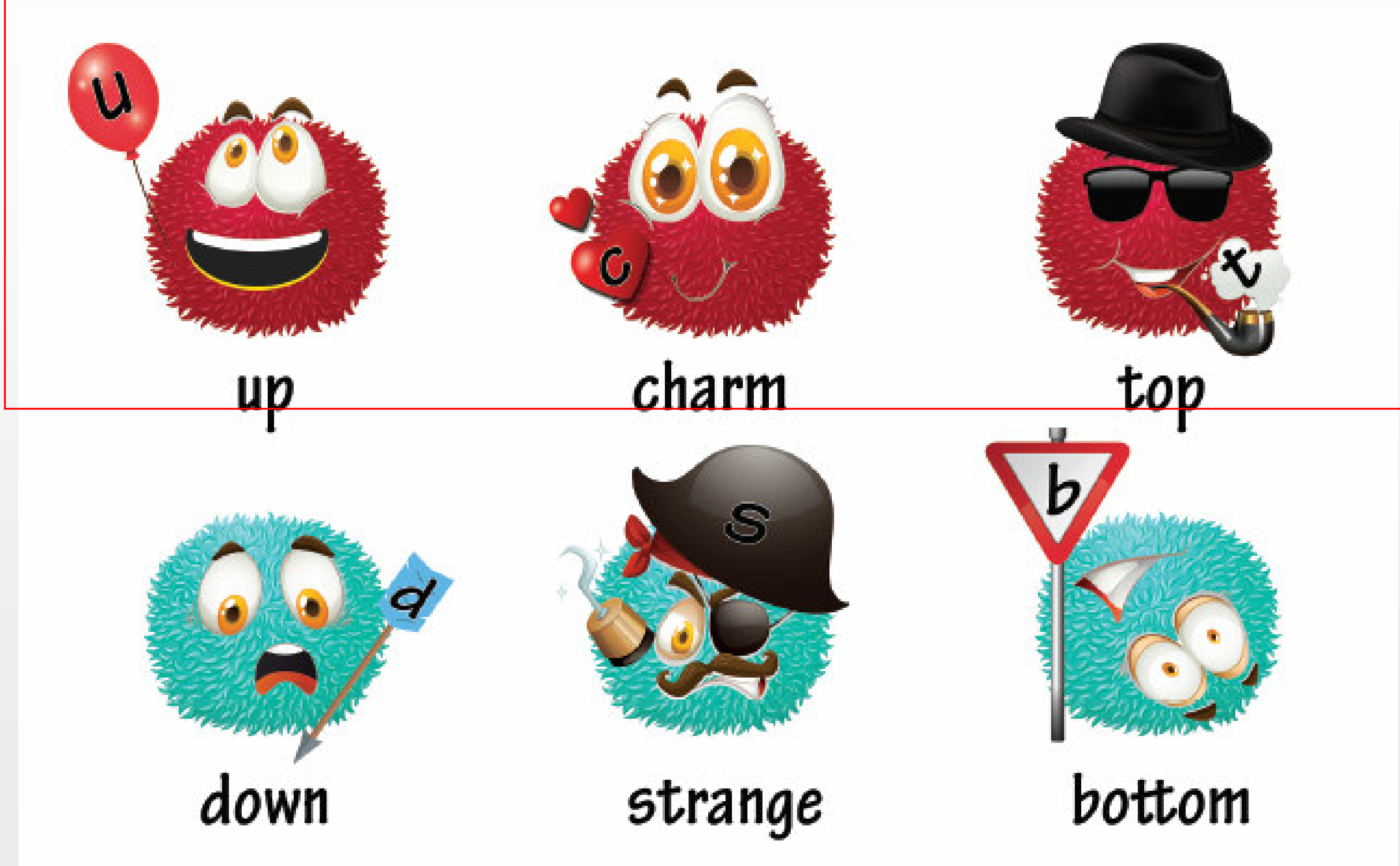
$$A_\mu^a \; ; \; a = 1,...,8$$







Confinement: quarks never ‘seen’ directly.
How they might look like ☺



Picture by Pawel Piotrowski

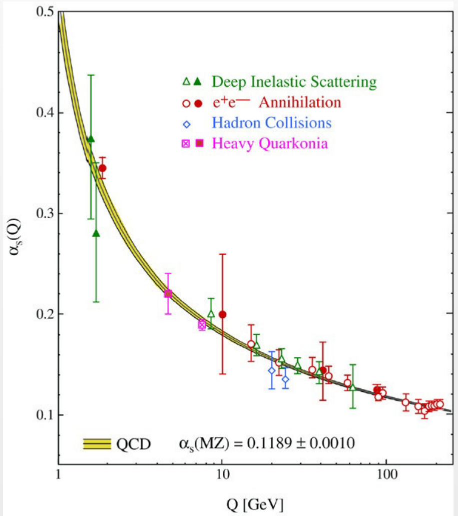
Trace anomaly:
the emergence of a dimension

Chiral limit: $m_{.} = 0$

$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250 \text{ MeV}$

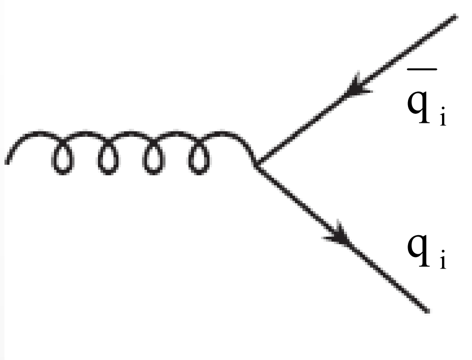
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \rightarrow m^*_{gluon} \approx 500 - 800 \text{ MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Flavor symmetry



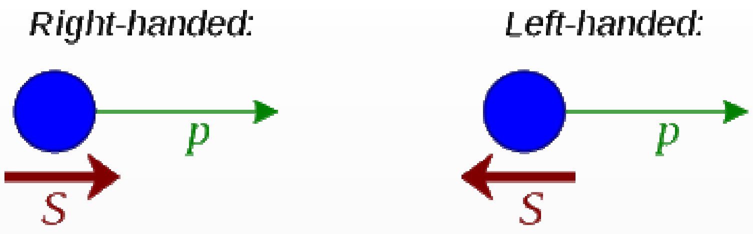
Gluon-quark-antiquark vertex

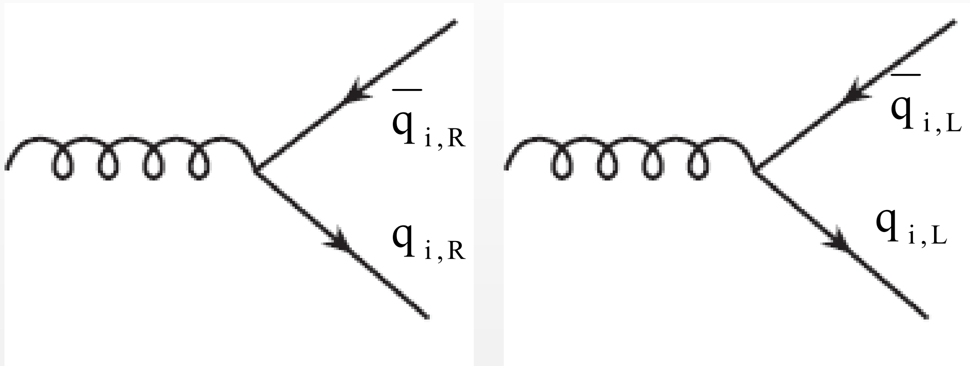
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$U \in U(3)_V \rightarrow U^+ U = 1$$

Chiral symmetry





$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\partial^\mu(\bar{q}^i \gamma_\mu \gamma_5 q^i) = \frac{3g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}(G_{\mu\nu} G_{\rho\sigma})$$

Spontaneous breaking of chiral symmetry: chiral condensate and constituent mass

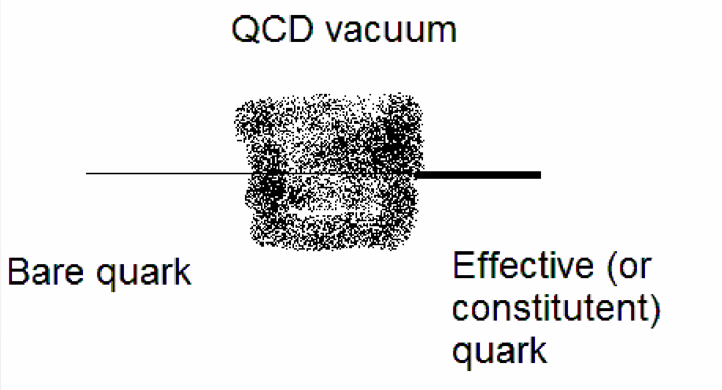
$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$\mathrm{SSB:}\ \mathrm{SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L}}$$

Chiral symmetry → Flavor symmetry

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

$$\mathrm{m \approx m_u \approx m_d \approx 5\ MeV \rightarrow m^* \approx 300\ MeV}$$



$$\mathrm{m_{\rho-meson} \approx 2m^*}$$

$$\mathrm{m_{proton} \approx 3m^*}$$

At nonzero T the chiral condensate decreases

Symmetries of QCD and breakings

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**scale anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**chiral anomaly**)

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

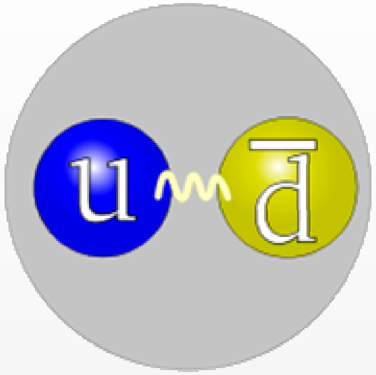
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + \ldots\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

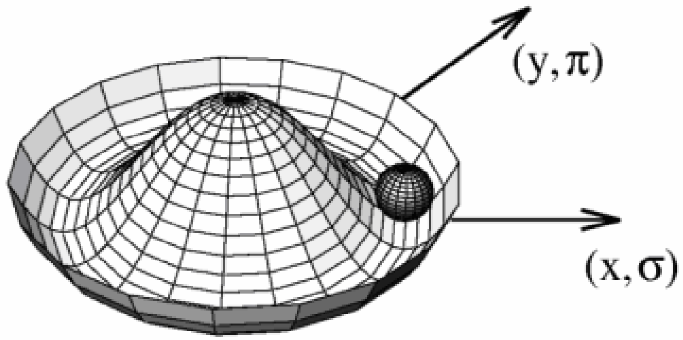
Rho-meson $m_{\rho^+} = 775 \text{ MeV}$

Pion $m_{\pi^+} = 139 \text{ MeV}$

$$m_u + m_d \approx 7 \text{ MeV}$$

Mass generation in QCD is a nonpert. penomenon based on SSB (mentioned previusly).

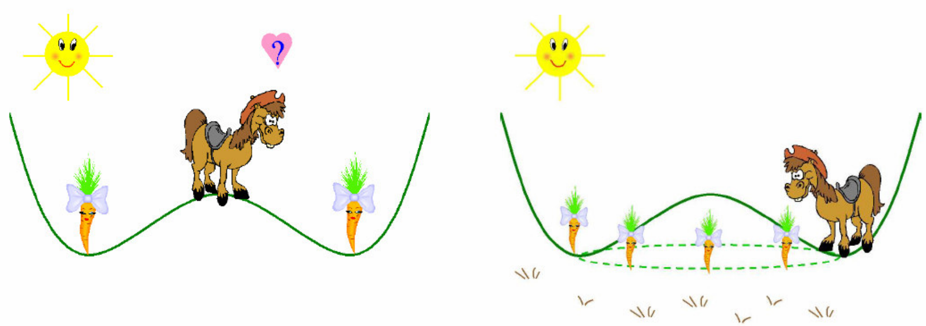
SSB and the donkey of Buridan: hadronic approaches



$$\sigma_N \to \sigma_N + \phi$$

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)

Spontaneous Symmetry Breaking



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d},\ \bar{u}d,\ \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s},\ d\bar{s};\ \bar{d}s,\ -\bar{u}s$	I = 0 f'	I = 0 f	θ_{quad} [°]	θ_{lin} [°]
$1\,^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$	−11.3	−24.5
$1\,^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	39.2	36.5
$1\,^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^{\dagger}$	$h_1(1380)$	$\boldsymbol{h_1(1170)}$		
$1\,^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
$1\,^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^{\dagger}$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
$1\,^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^{\dagger}$	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$		
$1\,^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$		$\boldsymbol{\omega(1650)}$		
$1\,^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$				
$1\,^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	31.8	30.8
$1\,^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$		
$2\,^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$		
$3\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1800)}$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d},\ d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s},\ d\bar{s}$ $s\bar{d},\ s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

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From well-known tensor mesons to yet unknown axial-tensor mesons

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While the ground-state tensor ($J^{PC} = 2^{++}$) mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ are well known experimentally and form an almost ideal nonet of quark-antiquark states, their chiral partners, the ground-states axial-tensor ($J^{PC} = 2^{--}$) mesons are poorly settled: only the kaonic member $K_2(1820)$ of the nonet has been experimentally found, whereas the isovector state ρ_2 and two isoscalar states ω_2 and ϕ_2 are still missing. Here, we study masses, strong, and radiative decays of tensor and axial-tensor mesons within a chiral model that links them: the established tensor mesons are used to test the model and to determine its parameters, and subsequently various predictions for their chiral partners, the axial-tensor mesons, are obtained. The results are compared to current lattice QCD outcomes as well as to other theoretical approaches and show that the ground-state axial-tensor mesons are expected to be quite broad, the vector-pseudoscalar mode being the most prominent decay mode followed by the tensor-pseudoscalar one. Nonetheless, their experimental finding seems to be possible in ongoing and/or future experiments.

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TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

J^{PC}, $^{2S+1}L_J$	$I=1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$ $I=1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$ $I=0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**}$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_\mathrm{L} \times SU(3)_\mathrm{R} \times \times U(1)_\mathrm{A}$
0^{-+}, $^1S_0$	π K $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ $(\Phi^{ij} = \bar{q}^j_\mathrm{R} q^i_\mathrm{L})$	$\Phi \to e^{-2i\alpha} U_\mathrm{L} \Phi U_\mathrm{R}^\dagger$
0^{++}, $^3P_0$	$a_0(1450)$ $K_0^*(1430)$ $f_0(1370), f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, $^1S_1$	$\rho(770)$ $K^*(892)$ $\omega(782), \phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_\mathrm{L} \gamma_\mu q^i_\mathrm{L})$	$L_\mu \to U_\mathrm{L} L_\mu U_\mathrm{L}^\dagger$
1^{++}, $^3P_1$	$a_1(1260)$ $K_{1,A}$ $f_1(1285), f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_\mathrm{R} \gamma_\mu q^i_\mathrm{R})$	$R_\mu \to U_\mathrm{R} R_\mu U_\mathrm{R}^\dagger$
1^{+-}, $^1P_1$	$b_1(1235)$ $K_{1,B}$ $h_1(1170), h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + iP_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_\mathrm{R} i\overleftrightarrow{D}_\mu q^i_\mathrm{L})$	$\Phi_\mu \to e^{-2i\alpha} U_\mathrm{L} \Phi_\mu U_\mathrm{R}^\dagger$
1^{--}, $^3D_1$	$\rho(1700)$ $K^*(1680)$ $\omega(1650), \phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j i\overleftrightarrow{D}_\mu q^i$		
2^{++}, $^3P_2$	$a_2(1320)$ $K_2^*(1430)$ $f_2(1270), f_2'(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{L}(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i_\mathrm{L})$	$L_{\mu\nu} \to U_\mathrm{L} L_{\mu\nu} U_\mathrm{L}^\dagger$
2^{--}, $^3D_2$	$\rho_2(?)$ $K_2(1820)$ $\omega_2(?), \phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{R}(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_\mathrm{R})$	$R_{\mu\nu} \to U_\mathrm{R} R_{\mu\nu} U_\mathrm{R}^\dagger$
2^{-+}, $^1D_2$	$\pi_2(1670)$ $K_2(1770)$ $\eta_2(1645), \eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(i\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{R}(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_\mathrm{L})$	$\Phi_{\mu\nu} \to e^{-2i\alpha} U_\mathrm{L} \Phi_{\mu\nu} U_\mathrm{R}^\dagger$
2^{++}, $^3F_2$	$a_2(?)$ $K_2^*(?)$ $f_2(?), f_2'(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$		
3^{--}, $^3D_3$	$\rho_3(1690)$ $K_3^*(1780)$ $\omega_3(1670), \phi_3(1850)$	$\vdots$	$\vdots$	$\vdots$

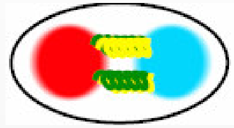
Uniwersytet Jana Kochanowskiego w Kielcach

Table from:

F.G., R. Pisarski,
A. Koenigstein
Phys.Rev.D 97 (2018) 9, 091901
e-Print: 1709.07454

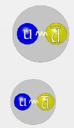
Non-conventional mesons: beyond qq

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

Large-Nc: basics/1

- Instead of 3 colors, Nc colors. Then Nc is taken as a large number.

- Why to do that? Certain simplifications appear! (Yet QCD not solvable also in that limit).

- (Some) mesons become stable and slowly interacting.

- Confinement, symmetry breaking, etc…are believed to hold in large-Nc as well.

Large-Nc: basics/3

Running coupling and the ‘t Hooft limit

$$N_c \longrightarrow \infty, \qquad g^2_{\mathrm{QCD}} N_c \longrightarrow \text{finite.}$$

$$\mu \frac{dg}{d\mu} = -bg^3$$

$$b = \frac{1}{2}\frac{1}{8\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

$$g^2(\mu) = \frac{8\pi^2}{\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)} \frac{1}{\ln \frac{\mu}{\Lambda_{QCD}}}$$

For large-$N_c$we get:

$$g^2(\mu) = \frac{8\pi^2}{\left(\frac{11}{3}N_c\right)} \frac{1}{\ln \frac{\mu}{\Lambda_{QCD}}} \propto \frac{1}{N_c}$$

Large-Nc: consequences

- Constituent quark mass Nc^0

- Masses of conventional quark-antiquark states mesons and glueballs (and hybrids): Nc^0
  (with one notable exception…)

- Decay width of these states decreases with Nc

- Masses of baryons proprtional to Nc; meson-baryon coupling proportional to Nc^(1/2)

Large Nc at nonzero T

PHYSICAL REVIEW D **85**, 056005 (2012)

Restoration of chiral symmetry in the large-N_c limit

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- NJL model
- Sigma model(s)
- Comparison and improvements

Is 3 a large number?

Spoiler: in some cases yes, in some cases no!

Finite T, NJL model

$$\mathcal{L}_{\mathrm{NJL}}(N_c) = \bar{\psi}(\imath\gamma^{\mu}\partial_{\mu} - m_q)\psi + \frac{3G}{N_c}[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\psi)]^2$$

$$1 = \frac{m_q}{m^*} + \frac{3G}{N_c}\left(2N_c + \frac{1}{2}\right)\int_0^{\Lambda}\frac{dkk^2}{\pi^2}\frac{2\tanh\left(\frac{\sqrt{k^2+m^{*2}}}{2T}\right)}{\sqrt{k^2+m^{*2}}},$$

$$T_c(N_c) \simeq \Lambda\sqrt{\frac{3}{\pi^2}}\sqrt{1 - \frac{\pi^2}{6\Lambda^2 G}} \propto N_c^0.$$

FInite T, sigma model

$$\mathcal{L}_{\sigma}(N_c) = \frac{1}{2}(\partial_{\mu}\Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4,$$

$$\Phi^t = (\sigma, \vec{\pi})$$

$$0 = \varphi(T)^2 - \frac{N_c}{3\lambda}\mu^2 + 3\int(G_{\sigma} + G_{\pi}).$$

$$\int G_i = \int_0^{\infty} \frac{dkk^2}{2\pi^2\sqrt{k^2 + M_i^2}}\left[\exp\left(\frac{\sqrt{k^2 + M_i^2}}{T}\right) - 1\right]^{-1},$$

$$T_c(N_c) = \sqrt{2}f_{\pi}\sqrt{\frac{N_c}{3}} \propto N_c^{1/2}.$$

How to cure it?

- Modify the mass term: $$\mu^2 \rightarrow \mu(T)^2 = \mu^2\left(1 - \frac{T^2}{T_0^2}\right)$$

- Use a quark-meson model (see later)

- Introduce the Polyakov loop

$$l(x) = N_c^{-1}\,\mathrm{Tr}\left[\mathcal{P}\exp\left(\imath g_{\mathrm{QCD}}\int_0^{1/T} A_0(\tau, x)d\tau\right)\right],$$

For l =0 conf, l =1 deconf.

$$\mathcal{L}_{\sigma\text{-Pol}}(N_c) = \mathcal{L}_\sigma(N_c) + \frac{\alpha N_c}{4\pi}|\partial_\mu l|^2 T^2 - \mathcal{V}(l) - \frac{h^2}{2}\Phi^2|l|^2T^2.$$

$$T_c = \frac{\mu}{\sqrt{h^2|l(T_c)|^2 + \frac{6\lambda}{N_c}}}$$

PHYSICAL REVIEW D **106**, 116016 (2022)

Editors' Suggestion

Fate of the critical endpoint at large N_c

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(Received 22 September 2022; accepted 18 November 2022; published 23 December 2022)

- Using a model a sigma-model that is as complete as possible (with (psuedo)scalar, (axial-)vector) d.o.f.)
- Linear realization of chiral symmetry
- Vacuum: D. Parganlija et al., Phys.Rev.D 87 (2013) 1, 014011 • e-Print: 1208.0585 [hep-ph]
- Extension to the medium: P. Kovacs, Phys.Rev.D 93 (2016) 11, 114014 • e-Print: 1601.05291 [hep-ph]: coupling to quarks and to the Polyakov loop.

eLSM Lagrangian, etc. Actually just a complicated vs of the Mexican hat ☺

$$\mathcal{L}_m = \mathrm{Tr}[(D_\mu M)^\dagger (D^\mu M)] - m_0 \mathrm{Tr}(M^\dagger M) - \lambda_1 [\mathrm{Tr}(M^\dagger M)]^2 - \lambda_2 [\mathrm{Tr}(M^\dagger M)^2] + c(\det M + \det M^\dagger) + \mathrm{Tr}[H(M + M^\dagger)]$$
$$- \frac{1}{4}\mathrm{Tr}[L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}] + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu L^\mu + R_\mu R^\mu)\right] + \frac{h_1}{2}\mathrm{Tr}(\phi^\dagger \phi)\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu]$$
$$+ h_2 \mathrm{Tr}[(MR_\mu)^\dagger (MR^\mu) + (L_\mu M)^\dagger (L^\mu M)] + 2h_3 \mathrm{Tr}[R_\mu M^\dagger L^\mu M] - 2g_2 \mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\},$$

$$\mathcal{L}_Y = \bar{\psi}(i\gamma_\mu \partial^\mu - g_F(S + i\gamma_5 P))\psi.$$

$$M = S + iP = \sum_a (S_a + iP_a) T_a,$$
$$L^\mu = V^\mu + A^\mu = \sum_a (V_a^\mu + A_a^\mu) T_a,$$
$$R^\mu = V^\mu - A^\mu = \sum_a (V_a^\mu - A_a^\mu) T_a,$$

$$D^\mu = \partial^\mu M - ig_1(L_\mu M - MR_\mu) - ieA^\mu[T_3, M],$$
$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu[T_3, L^\mu]\},$$
$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu[T_3, R^\mu]\},$$

- Polyakov loop potential.

$$\Omega(T, \mu_q) = U_{Cl} + \Omega_{\bar{q}q}(T, \mu_q) + U_{\mathrm{Pol}}(T, \mu_q) \quad (2)$$

$$\Omega_{\bar{q}q}^{\mathrm{v}} = -2N_c \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_f(p),$$
$$\Omega_{\bar{q}q}^{\mathrm{T}}(T, \mu_q) = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_c \left[\ln\left(1 + L^\dagger e^{-\beta(E_f(p) - \mu_q)}\right) + \ln\left(1 + L e^{-\beta(E_f(p) + \mu_q)}\right)\right]$$

$$\Omega(T, \mu_q) = U(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(T, \mu_q) + U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle)$$

Parameters and large-Nc scaling

TABLE I. Parameter sets. Left column is taken from [11] (set A) and right column is taken from [38] (set B).

Parameter	Set A	Set B
ϕ_N [GeV]	0.1411	0.1290
ϕ_S [GeV]	0.1416	0.1406
m_0^2 [GeV^2]	2.3925_{E-4}	-1.2370_{E-2}
m_1^2 [GeV^2]	6.3298_{E-8}	0.5600
λ_1	−1.6738	−1.0096
λ_2	23.5078	25.7328
c_1 [GeV]	1.3086	1.4700
δ_S [GeV^2]	0.1133	0.2305
g_1	5.6156	5.3295
g_2	3.0467	−1.0579
h_1	37.4617	5.8467
h_2	4.2281	−12.3456
h_3	2.9839	3.5755
g_F	4.5708	4.9571
M_0 [GeV]	0.3511	0.3935

TABLE II. N_c dependence of the parameters.

m_0^2, m_1^2, δ_S	N_c^0
g_1, g_2, g_f	$1/\sqrt{N_c}$
$\lambda_2,\ h_2,\ h_3$	N_c^{-1}
$\lambda_1,\ h_1$	N_c^{-2}
c_1	$N_c^{-3/2}$
$h_{N/S}$	$\sqrt{N_c}$
g_F	$1/\sqrt{N_c}$

Chiral condensate vs T

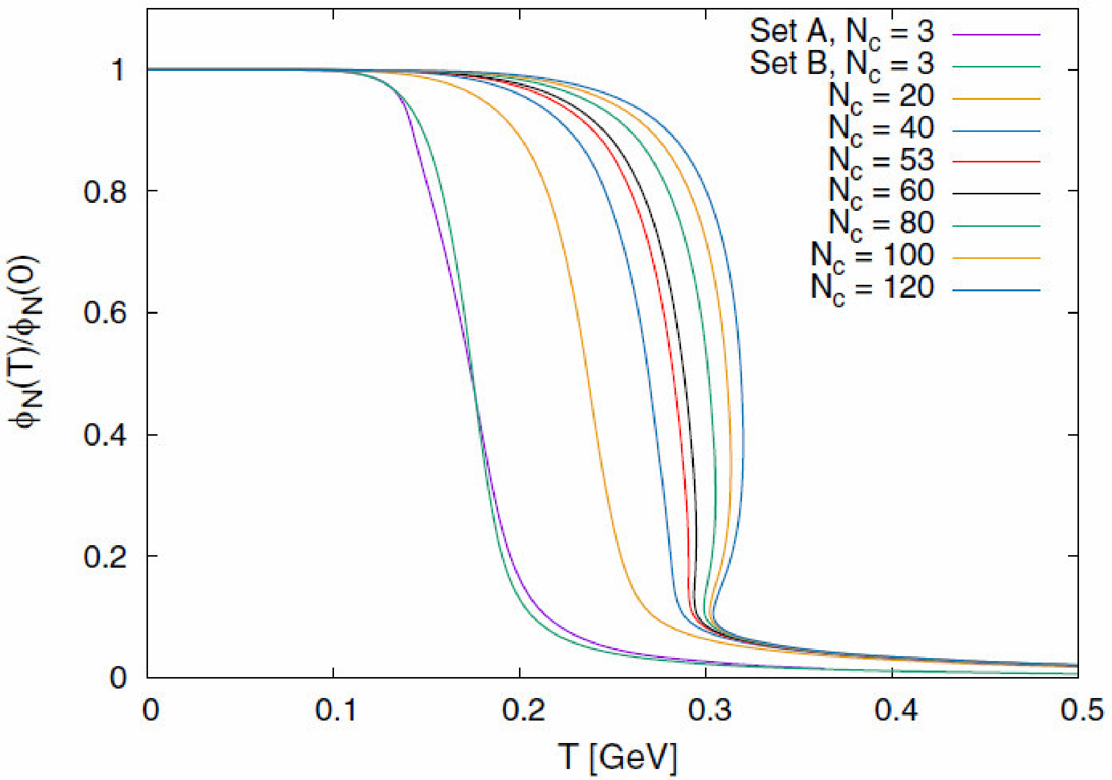


FIG. 4. The temperature dependence of the normalized chiral condensate ϕ_N.

Chiral condensate vs chemical potential

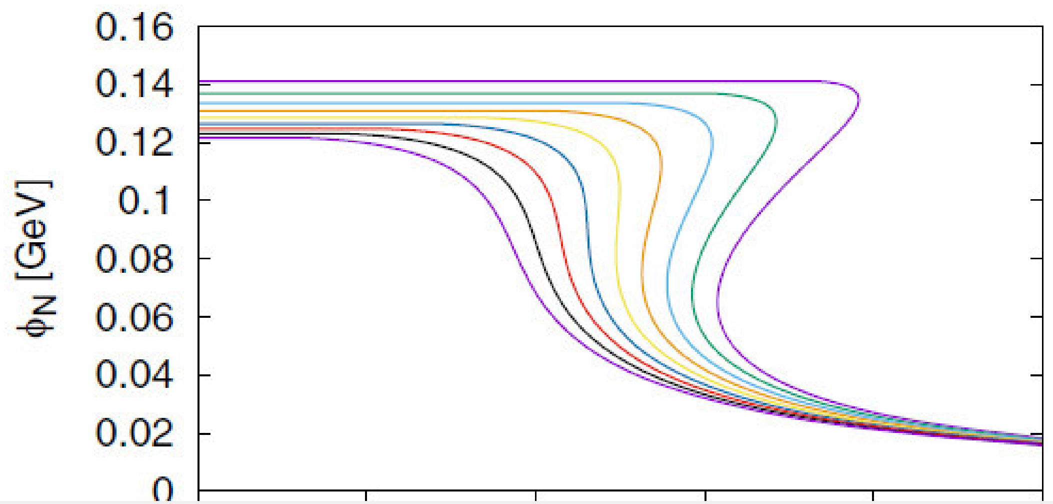
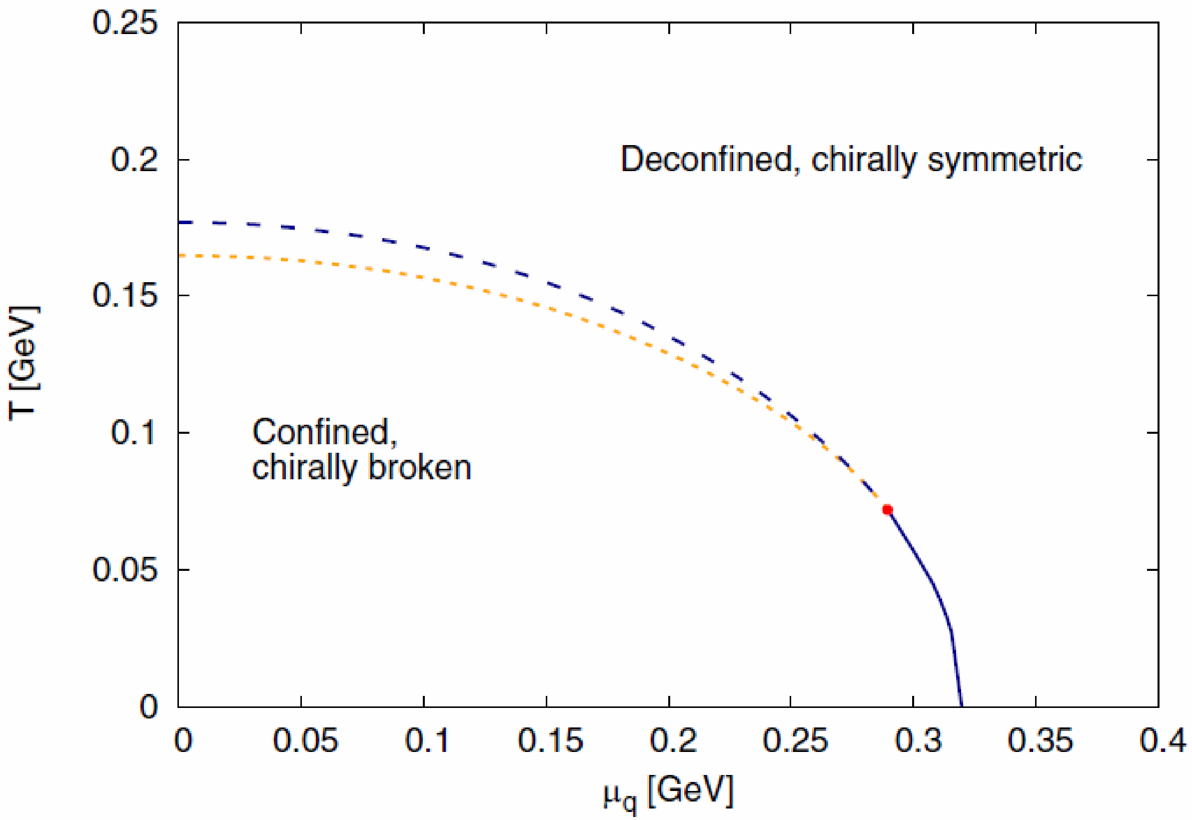
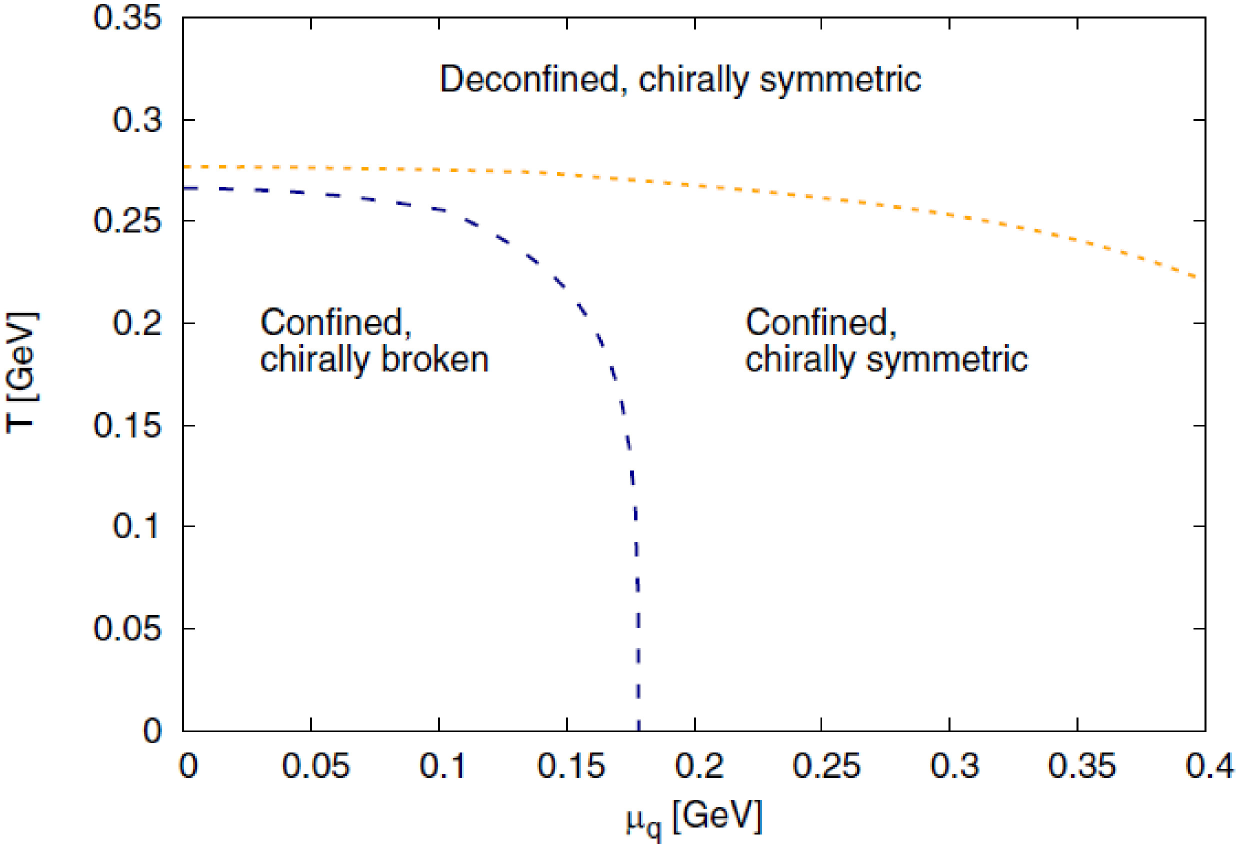


FIG. 3. The μ_q quark chemical potential dependence of the ϕ_N condensate at different N_c values. $N_c = 3.00$ corresponds to the rightmost curve, while $N_c = 3.45$ corresponds to the leftmost curve. The top figure is obtained with set A, while the bottom figure with set B of Table I.

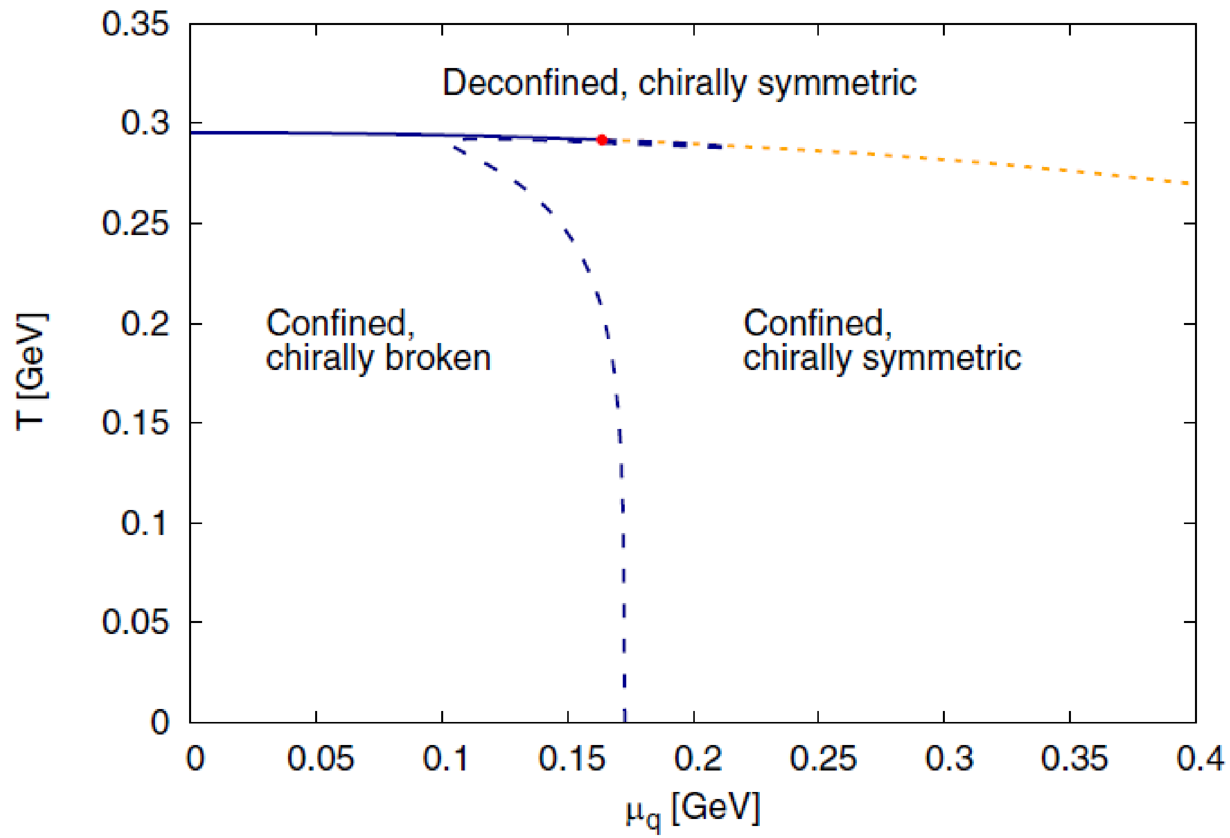
Phase diagram: Nc =3

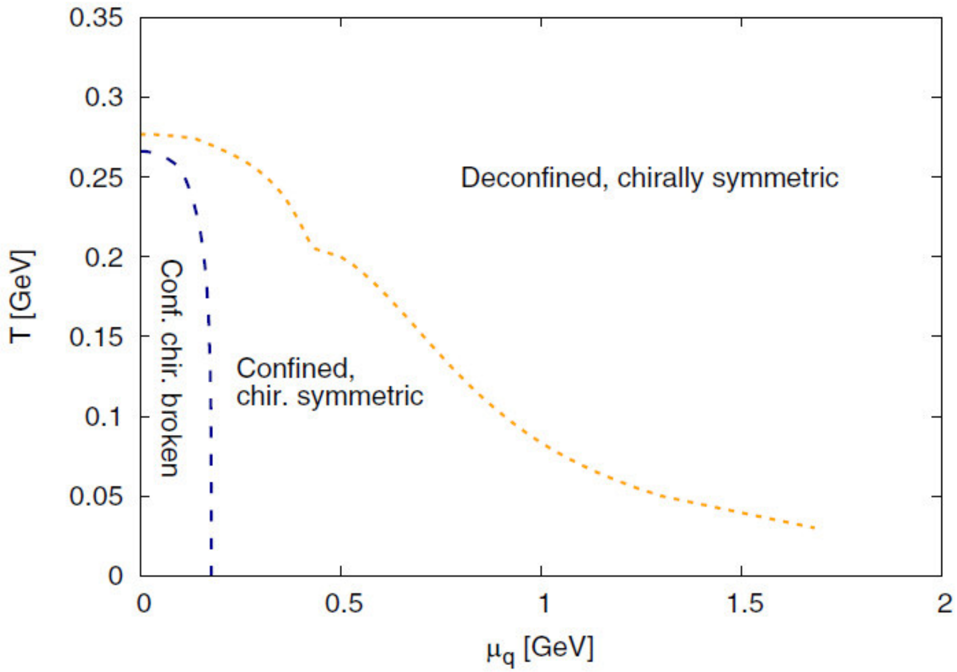


Phase diagram: Nc = 33
(only cross-over, no CP)

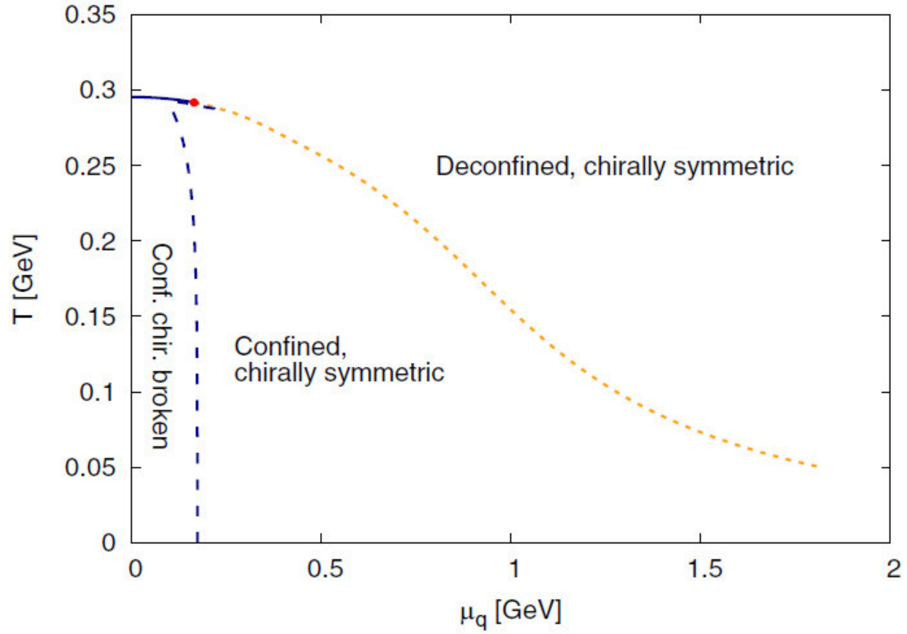


Phase diagram: Nc = 63




0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
T [GeV]
μq [GeV]
Deconfined, chirally symmetric
Conf. chir. broken
Confined,
chir. symmetric

Nc = 33


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
T [GeV]
μq [GeV]
Deconfined, chirally symmetric
Conf. chir. broken
Confined,
chirally symmetric

Nc = 63

Schematic phase diagram at large Nc

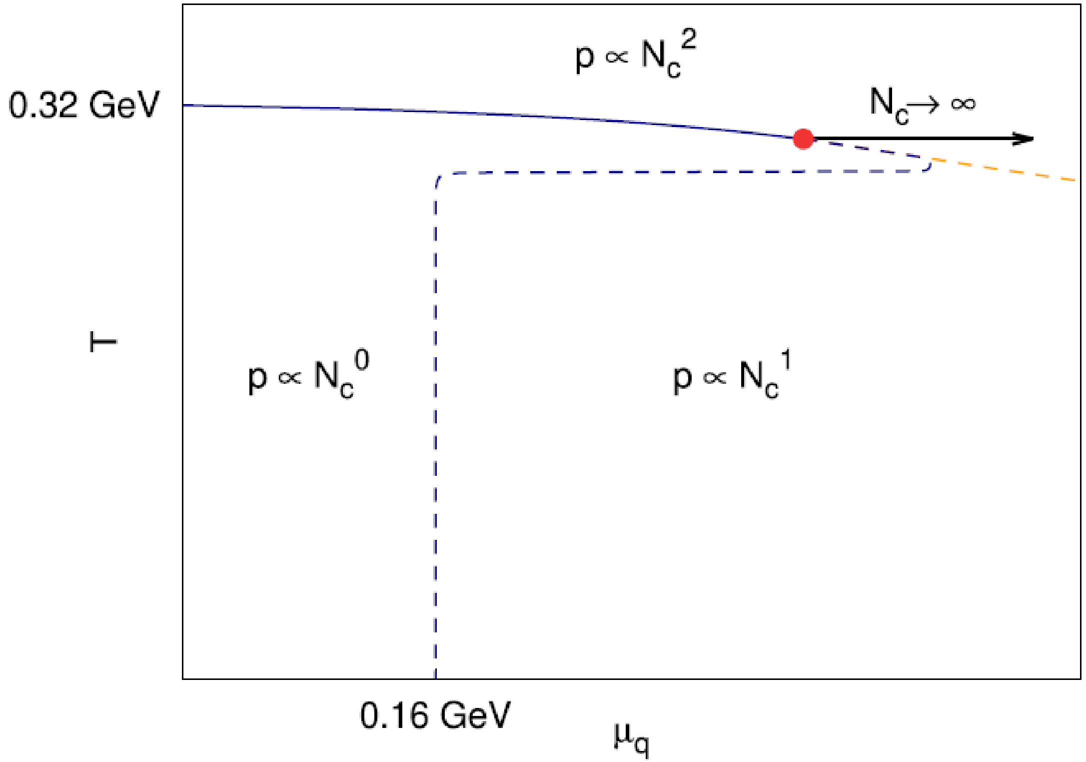
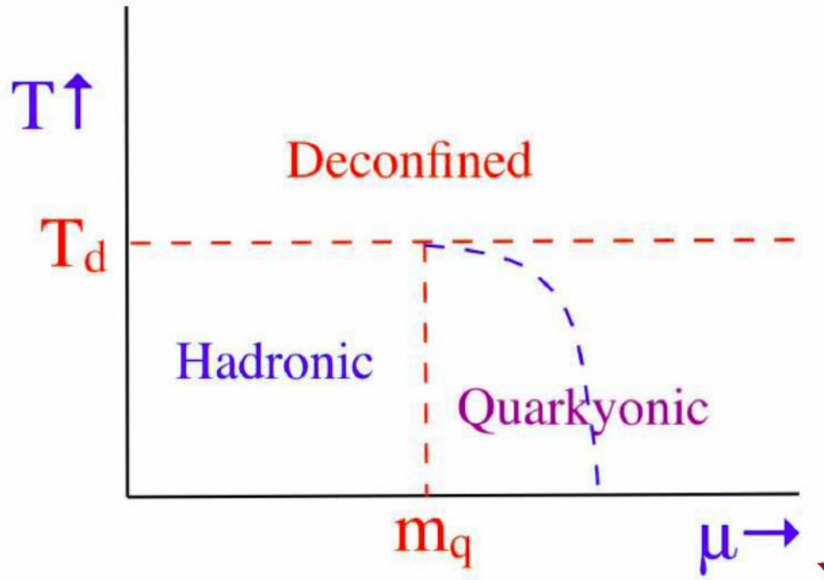


FIG. 13. The schematic phase diagram for large N_c and the N_c scaling of the pressure in the different phases.

Then, for the QCD diagram: 3 is not a large number!!!!

…agrees well with quarkyonic…



- Confined, quarkyonic phase may appears for large density

McLerran, Pisarski: **Nucl. Phys. A 796, 83-100 (2007)**

McLerran, Redlich, Sasaki: **Nucl. Phys. A 824, 86-100 (2009)**

Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics A 859 (2011) 49–62
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

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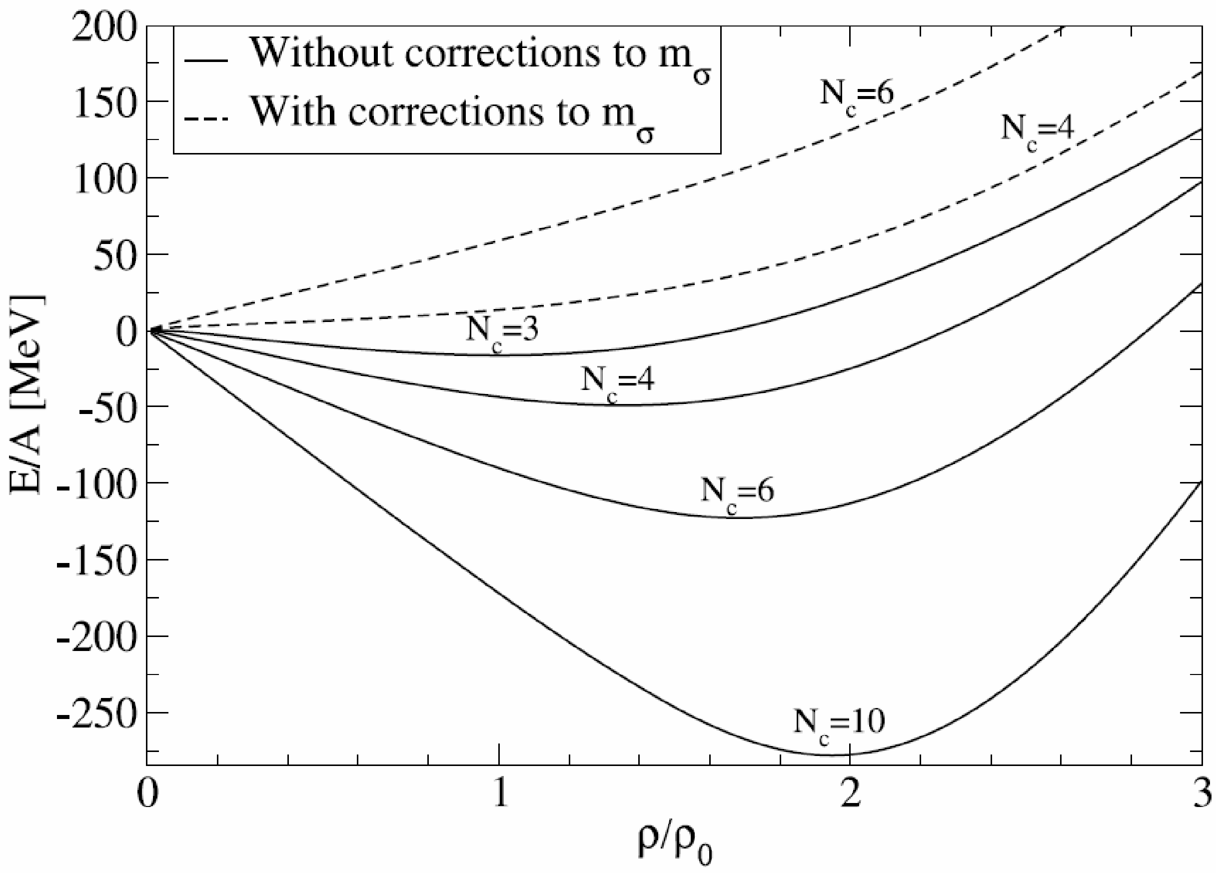
Does nuclear matter bind at large N_c?

Luca Bonanno *, Francesco Giacosa

The Lagrangian of the Walecka model reads [9]:

$$\mathcal{L} = \bar{\psi}\left[\gamma^{\mu}(i\partial_{\mu} - g_{\omega}\omega_{\mu}) - (m_N - g_{\sigma}\sigma)\right]\psi + \frac{1}{2}\partial^{\mu}\sigma\,\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^2\sigma^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$+ \frac{1}{2}m_{\omega}^2\omega^{\mu}\omega_{\mu} - V_{\sigma}(\sigma),$$

$$m_{\sigma} \longrightarrow m_{\sigma};$$
$$m_{\omega} \longrightarrow m_{\omega}, \qquad m_N \longrightarrow m_N\frac{N_c}{3};$$
$$g_{\sigma} \longrightarrow g_{\sigma}\sqrt{\frac{N_c}{3}}, \qquad g_{\omega} \longrightarrow g_{\omega}\sqrt{\frac{N_c}{3}}.$$



$$m_\sigma^2(N_c) = m_\sigma^2 + b_\sigma^2\left(\frac{1}{3} - \frac{1}{N_c}\right).$$

Minimal variation of the scaling…
quark model places this state higher.
Enough to unbind nuclear matter

What if the lightest scalar is a tetraquark?

$$\mathcal{L} = \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega \omega_\mu) - (m_N - g_\chi \chi)]\psi + \frac{1}{2}\partial^\mu \chi \partial_\mu \chi - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu,$$

$$\chi = [\bar{R}, \bar{B}][R, B] + [\bar{G}, \bar{B}][G, B] + [\bar{R}, \bar{G}][R, G]$$

for Nc = 3

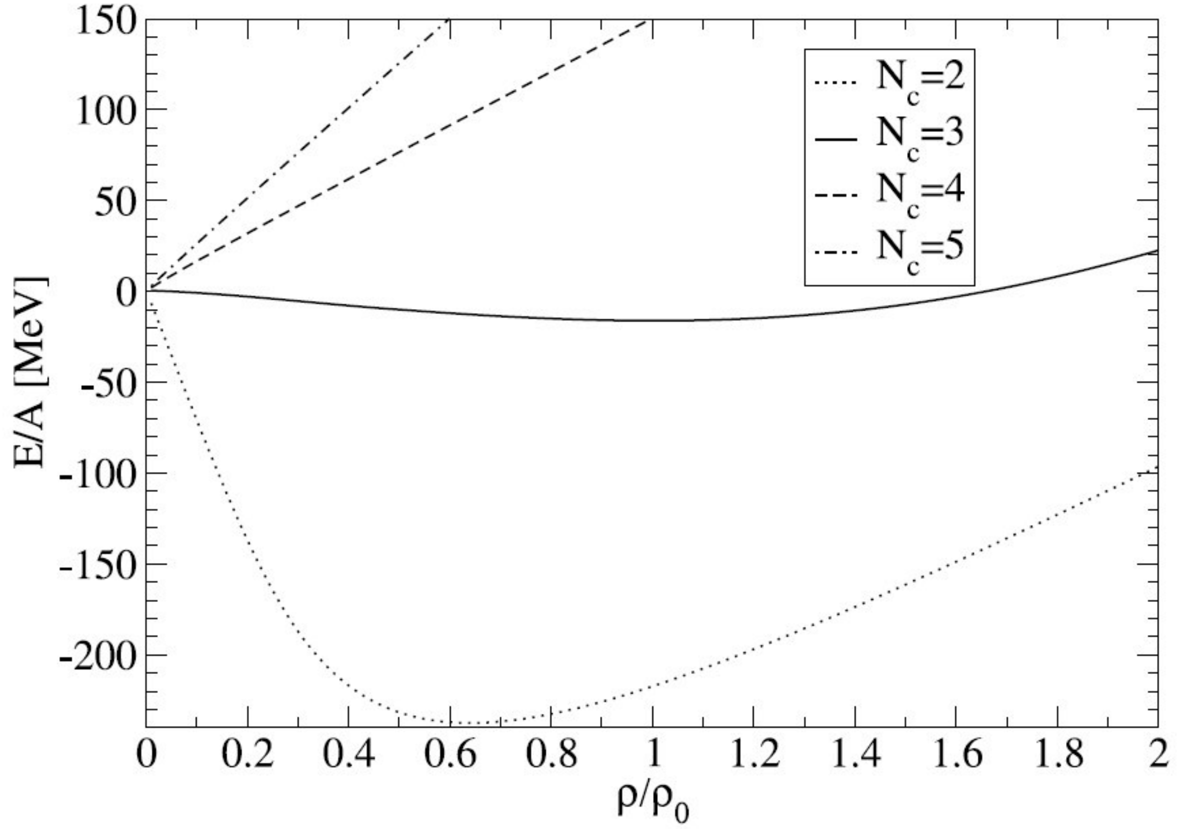
$$d_{a_1} = \varepsilon_{a_1 a_2 a_3 \cdots a_{N_c}} q^{a_2} q^{a_3} \cdots q^{a_{N_c}} \quad \text{with} \quad a_2, \ldots, a_{N_c} = 1, \ldots, N_c.$$

for Nc> 3

$$\chi = \sum_{a_1=1}^{N_c} d_{a_1}^\dagger d_{a_1}$$

Extended 'tetraquark' version!
(indeed, a well-defined one)

$$m_\chi \to m_\chi \frac{2N_c - 2}{4}, \qquad g_\chi \to g_\chi.$$



Summary: for nuclear matter, 3 is not a large number!!!!

Other scenarios

- Two scalar fields: tetraquark+quarkonium, no nuclear matter.
- f0(500) as pion-pion molecular states, dissolves at large Nc, no nuclear matter.
- One-pion-exchange: what does eventually happen at very large Nc? (not taken into account here because beyond MFE)

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Neutron stars in the large-N_c limit

Francesco Giacosa [a,b], Giuseppe Pagliara [c,*]

$$p_q = b_1 N_c \mu_q^4 - N_c^2 B$$

$$b_1 = \frac{N_f}{12\pi^2}.$$

Quark matter athigh density: free gas plus bag

$$p_b = a_1 \mu_b^\alpha - K$$

Baryonic matter at high density (starting from 2ρ0): parameter α unknown

$$a_1(N_c) \propto \left(\frac{g_V^2}{m_V^2}\right)^{\frac{\alpha-4}{2}} \propto N_c^{\frac{\alpha-4}{2}}$$

$$v_b = \sqrt{\frac{dp_b}{d\varepsilon_b}} = \frac{1}{\sqrt{\alpha - 1}}.$$

$$\alpha \geq 2$$

$$K = \tilde{K} N_c^{(3\alpha-4)/2}$$

The stiffest equation of state corresponds, in agreement with causality, to $\alpha = 2$.

$$p_b = \tilde{a}_1 N_c \mu_q^2 - \tilde{K} N_c$$

The speed of sound is 1 in this case

Stiffest equation and transition

The stiffest equation of state corresponds, in agreement with causality, to $\alpha = 2$.

$$p_b = \tilde{a}_1 N_c \mu_q^2 - \tilde{K} N_c$$

The speed of sound is 1 in this case

$$p_b = \tilde{a}_1 N_c^{\frac{3\alpha-4}{2}} \mu_q^\alpha - \tilde{K} N_c^{\frac{3\alpha-4}{2}} = b_1 N_c \mu_q^4 - N_c^2 B = p_q$$

$$\mu_q^{\rm crit} = \left(\frac{B N_c}{b_1}\right)^{1/4} [1 + \ldots] \text{ for } 2 \leq \alpha \leq \frac{16}{7}$$

Results

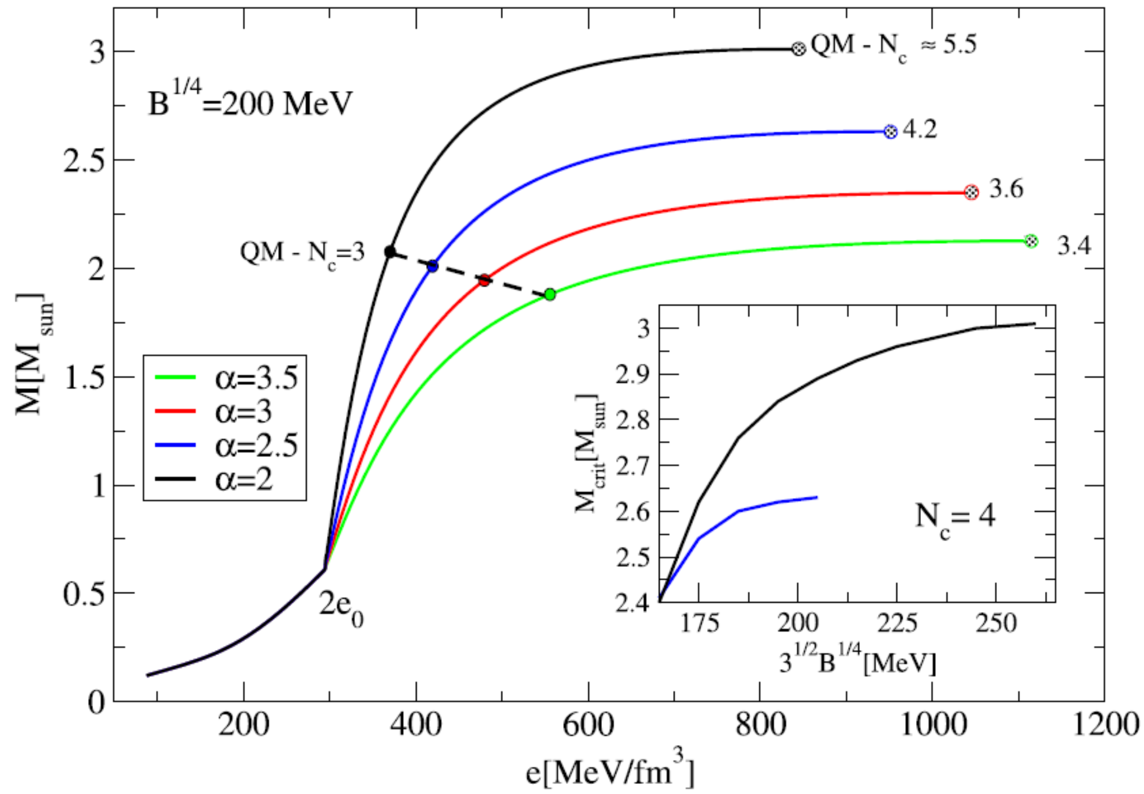


Fig. 1. Masses of neutron stars as functions of their central energy densities for different values of α. The filled points correspond to the critical mass for the transition to quark matter in the case $N_c = 3$. The filled dashed points correspond to the critical values of N_c for which the phase transition occurs in correspondence of the maximum mass configuration.

Neutron stars, main outcome

From the $N_c = 3$ analysis, one further reduces the range (20), namely α must be *smaller* than about $\alpha_{\max} \simeq 2.5$ (the blue line) in order to explain the existence of $2M_\odot$ stars:

$$2 \leq \alpha \lesssim 2.5 \text{ for } N_c = 3.$$

Moreover one should not observe stars with masses larger than about $2.1M_\odot$

Also for neutron stars: Nc = 3 is not large!

Conclusions and outlook

Large-Nc

• useful tool for QCD (as well as for a variety of models/theories)

• Phenomenology in the vacuum can be better understood (i.e. OZI), certains terms appear as dominant, other are suppressed… 3 is a large number.

• Applications at nonzero temperature and density, in most cases 3 is not a large number.

Thanks!

Large-Nc: basics/1

$SU_c(3)$

$|q\rangle = \begin{pmatrix} R \\ G \\ B \end{pmatrix}$; $|q\rangle \mapsto U|q\rangle$, $U \in SU_c(3)$

$q_a \quad a = 1, 2, 3$

$|MESON\rangle = \sqrt{\frac{1}{3}}\,(\bar{R}R + \bar{G}G + \bar{B}B)$

invariant under $SU_c(3) \equiv$ 'white'

$|BARYON\rangle = N \cdot \varepsilon_{abc}\, q^a q^b q^c$

$= N\,(RGB + BRG + GBR - GRB - BGR - RBG)$

invariant under $SU_c(3)$

$|BARYON\rangle \mapsto |BARYON\rangle$

$SU_c(N_c)$

$|q\rangle = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_{N_c} \end{pmatrix}$; $|q\rangle \mapsto U|q\rangle$, $U \in SU(N_c)$

$q_a \quad a = 1, 2, \ldots, N_c$

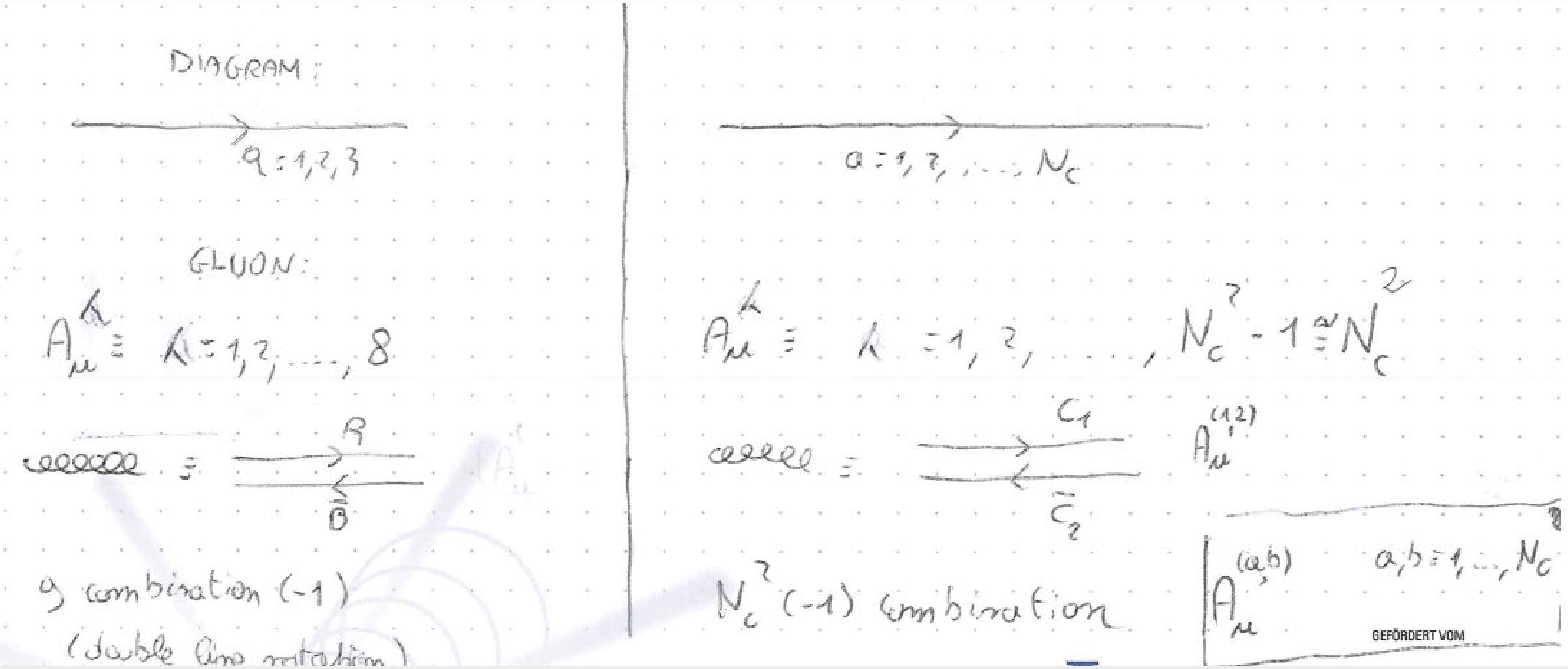
$|MESON\rangle = \sqrt{\frac{1}{N_c}}\,(\bar{C}_1 C_1 + \ldots + \bar{C}_{N_c} C_{N_c})$

$|BARYON\rangle = N\, \varepsilon_{a_1 \ldots a_{N_c}}\, q^{a_1} q^{a_2} \ldots q^{a_{N_c}}$

$a_1, \ldots, a_{N_c} = 1, 2, \ldots, N_c$

Invariant under $SU_c(N_c)$

Large-Nc: basics/2



Glueball production and decays: gluon-rich processes

Glueballs should be find in gluon-rich processes
(such as J/ψ decays, proton-antiproton fusion, …)

Glueball should have suppressed decay into flavor breaking channels (eg eta-etaprime)

Moreover, glueballs should have a suppressed (but nonzero!) decay into photons.

In the $N_c \to \infty$ limit eg.:

- Stable, noninteracting mesons and glue-balls (infinite number with fixed qn.) in the hadronic phase with $m \propto N_c^0$ masses.
- Baryon masses diverges as $m_B \propto N_c^1$.
- Hadronic phase built from noninteracting mesons and glueballs, energy density scales as $\propto N_c^0$
- Phase boundary to quark-gluon plasma at a temperature $\propto N_c^0$
- Energy density of quark-gluon phase N_c^2.
  $\Rightarrow$ First or second order phase transition expected.
- Quark loops are suppressed: the thermodynamics expected to became similar to Yang-Mills.
- Confined, quarkyonic phase may appears for large density

McLerran, Pisarski: **Nucl. Phys. A 796, 83-100 (2007)**

McLerran, Redlich, Sasaki: **Nucl. Phys. A 824, 86-100 (2009)**

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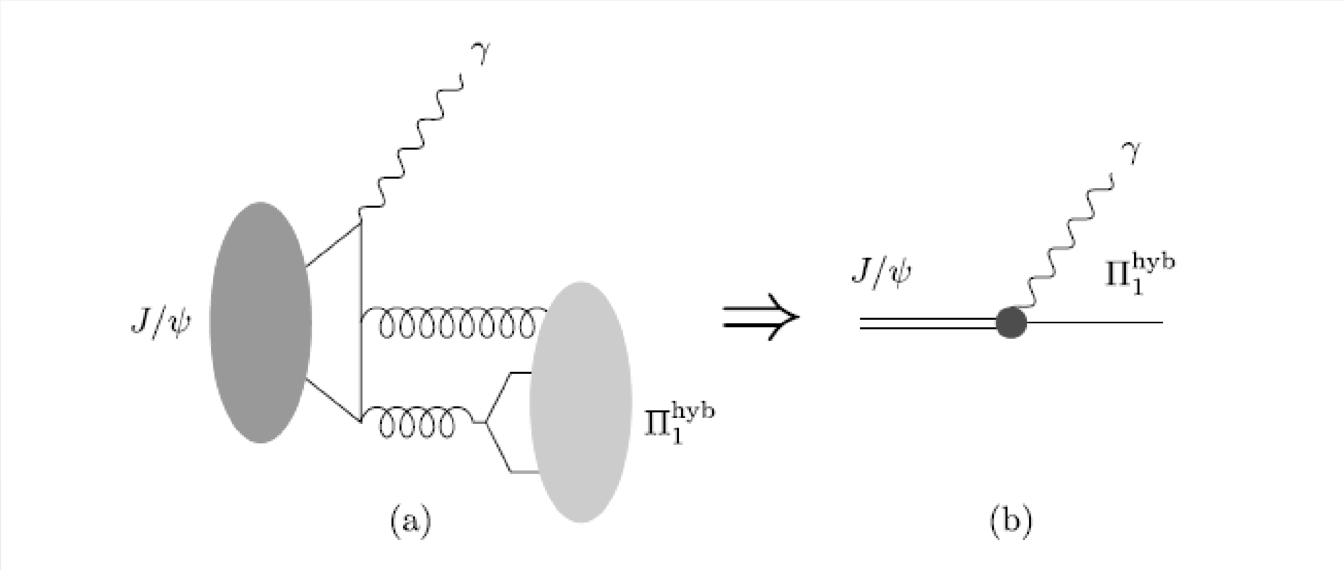
Radiative production and decays of the exotic $\eta_1'(1855)$ and its siblings

Vanamali Shastry [a,*], Francesco Giacosa [a,b]



Glueball spectrum from lattice QCD