How Radial Oscillations Can Help to Probe the Onset of the Deconfinement Phase Transition and Special Points in Hybrid Stars

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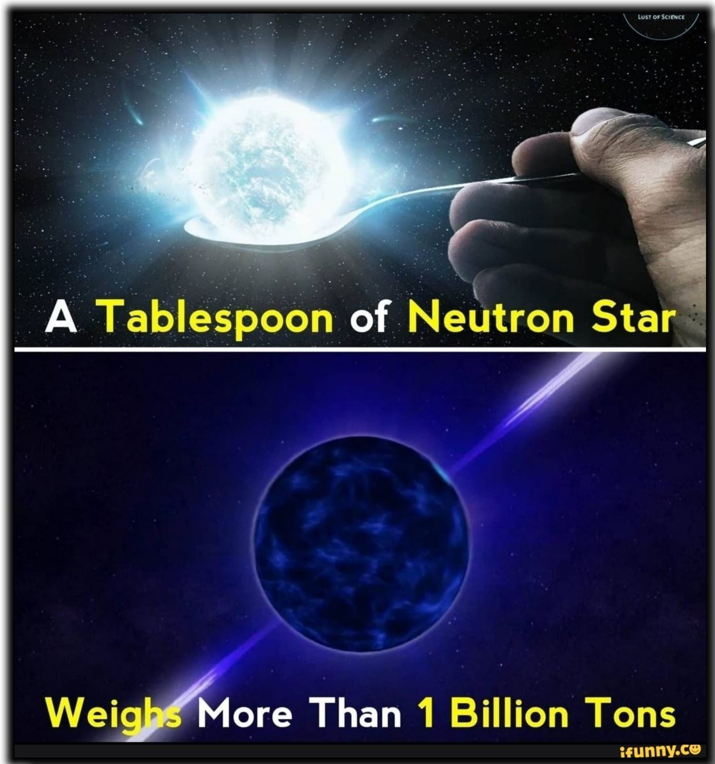
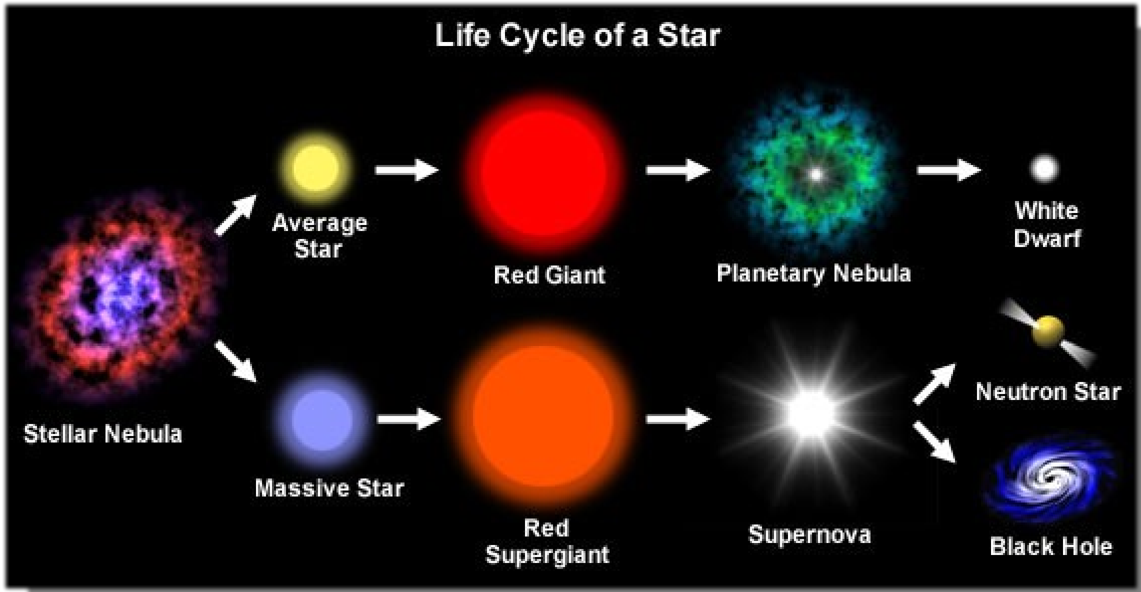
Outline

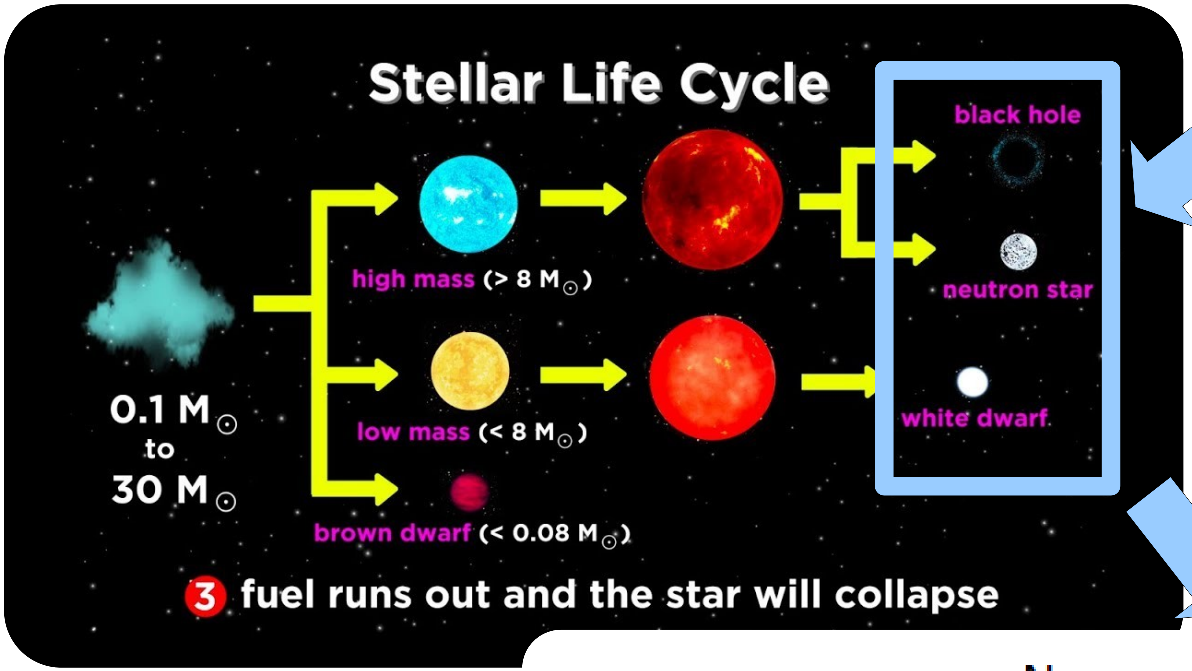
- Compact Stars
- Building up an NS
- Special points
- Radial Oscillations
- Conclusions

Compact Stars

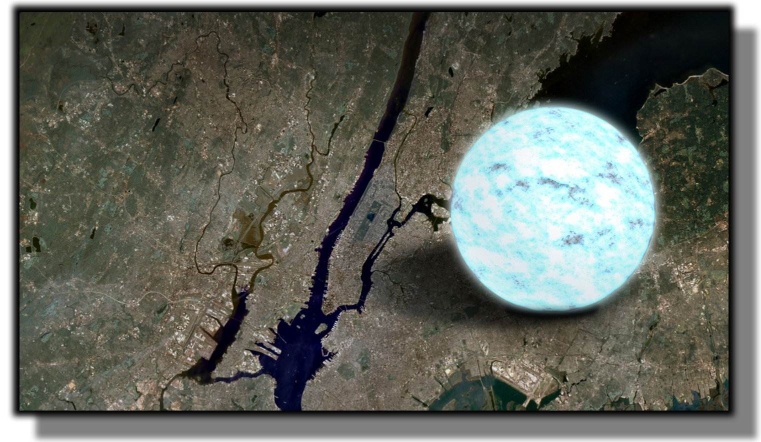




Neutron Stars



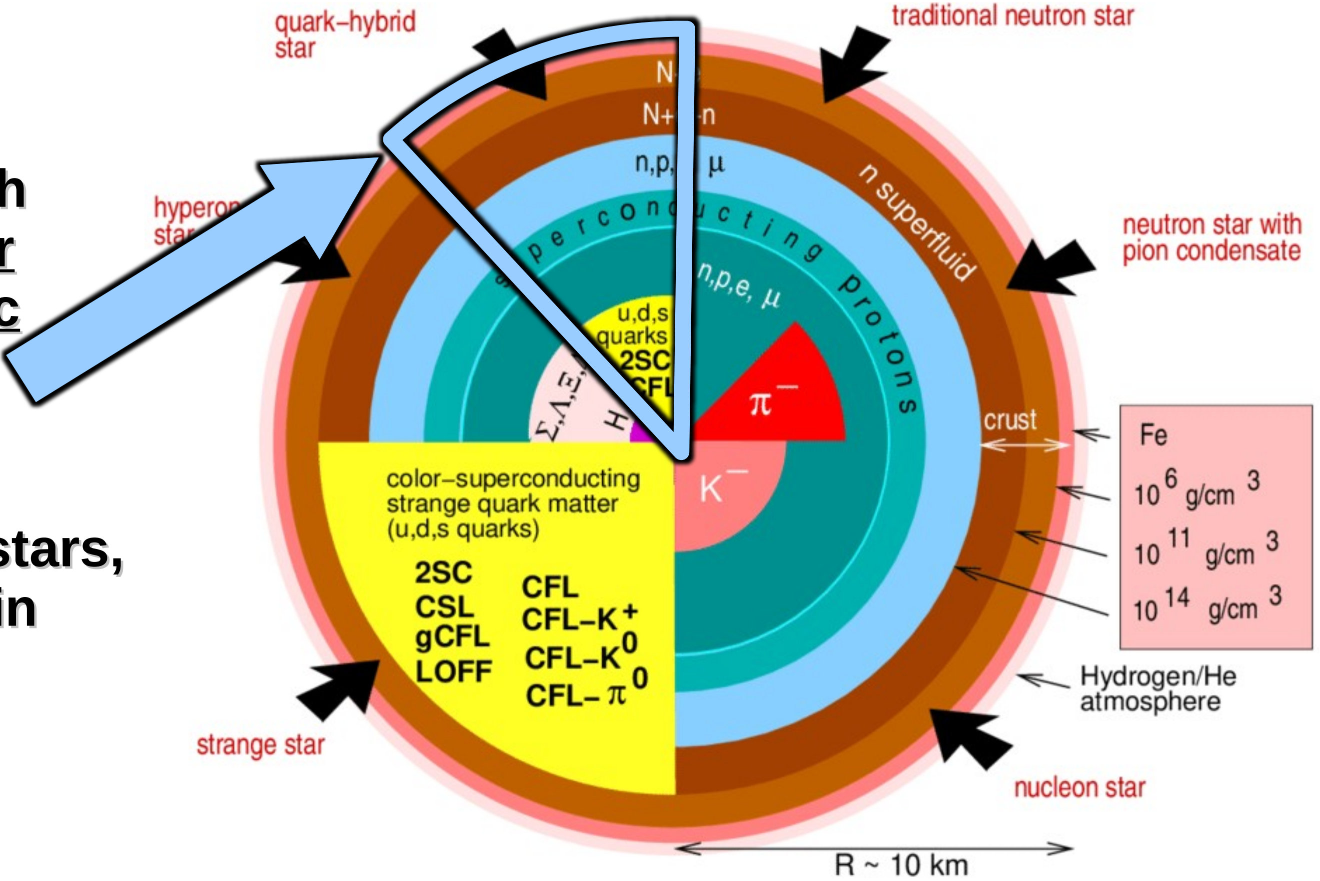
Last stage of a star



	Neutron star	White dwarf	Sun
$M_{max}(M_\odot)$	2	1.44	1
R (km)	11-12	10^4	$7 \cdot 10^5$
n_c (g/cm^3)	$10^{14} - 10^{15}$	10^7	10^2
rotation speed (s)	$10^{-3} - 1$	100	$2 \cdot 10^6$
B (G)	$10^8 - 10^{16}$	100	1
T (K)	$10^6 - 10^{11}$	10^3	10^5

Variety in the composition

- different models for the composition
- focussing on a type with <u>deconfined quark matter</u> inside core and <u>baryonic matter in outer shells</u>
- other compositions as strange stars, hyperon stars, etc. are not considered in our study



Weber+ (2007)

Construction the Equation of State (EoS)

Idea:

- **combine the EoS for <u>hadronic</u> <u>matter</u> and our model for <u>quark matter</u> via Maxwell construction**

DD2npY-T model Shahrbaf, Blaschke+ (2022)

(hadronic) neutrons and protons + hyperonic degrees of freedom

Quark matter: Ivanytskyi,Blaschke (2022)

confining relativistic density functional approach (RDF)

Underlying Lagrangian:

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_V + \mathcal{L}_D - \mathcal{U}$$

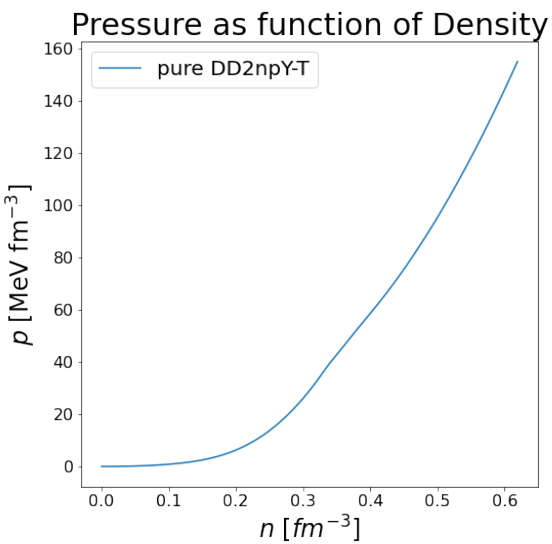
Meaning what?

Including **<u>vector</u> <u>repulsion</u>** (η_V) and **<u>diquark</u> <u>pairing</u>** (η_D) controlled by dimensionless couplings

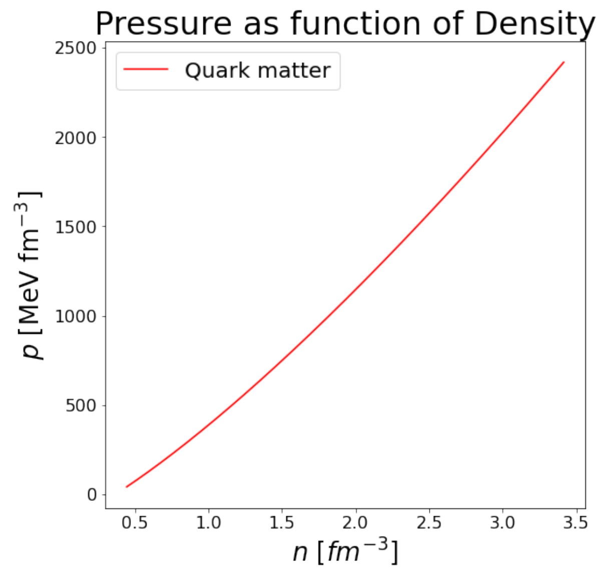
→ Confinement + Conformal limit + Color Superconductivity

Hybrid EoS

Low densities – Outer regions of NS

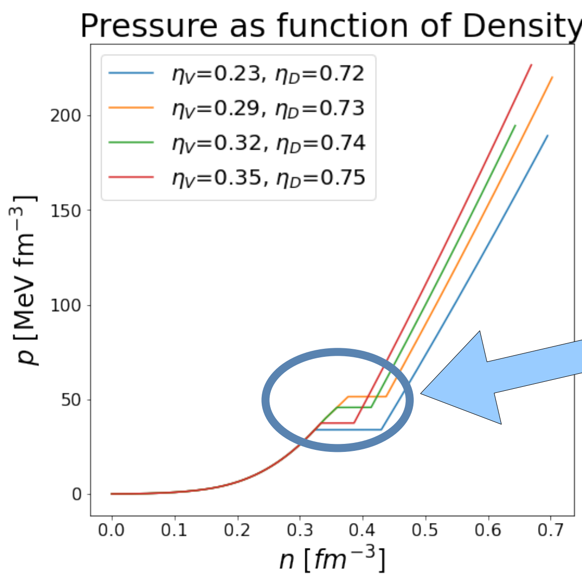


High densities – Inner regions of NS



Maxwell construction

first order phase transition



Typical plateaus of 1st order phase transitions

Construction:

$$p_{quark}(\mu_c) = p_{hadron}(\mu_c) \Rightarrow \mu_c$$

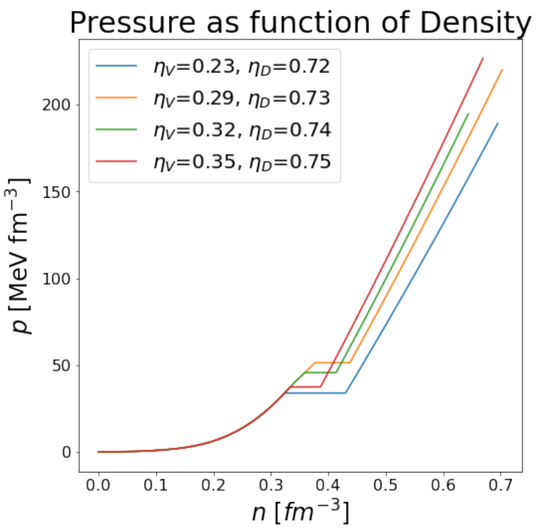
via chemical potential

Mass-Radius Curves

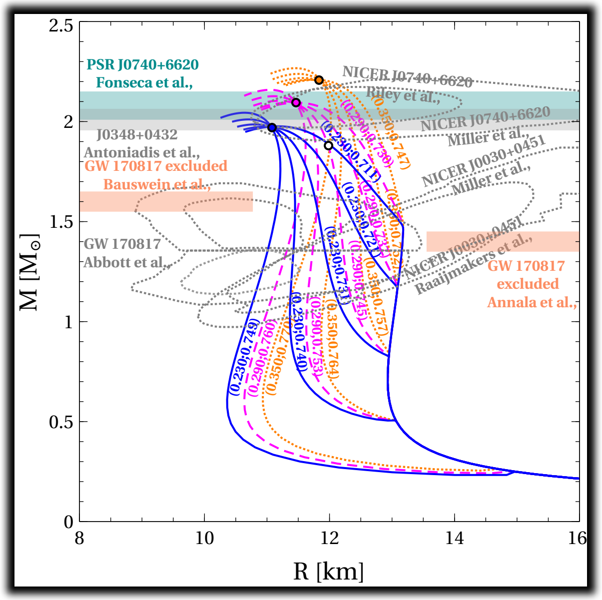
Tolman-Oppenheimer-Volkoff (TOV)

$$\frac{dp}{dr} = -(\epsilon + p)\frac{m + 4\pi p r^3}{r^2 - 2rm}, \quad \text{and} \quad \frac{dm}{dr} = 4\pi r^2 \epsilon,$$

EoS → **Mass** and **Radius** of NS sequence

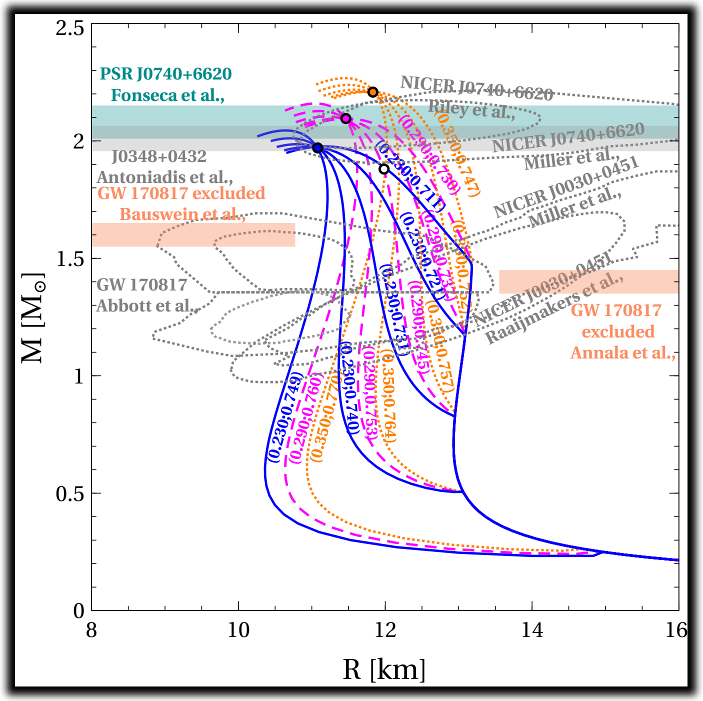


→



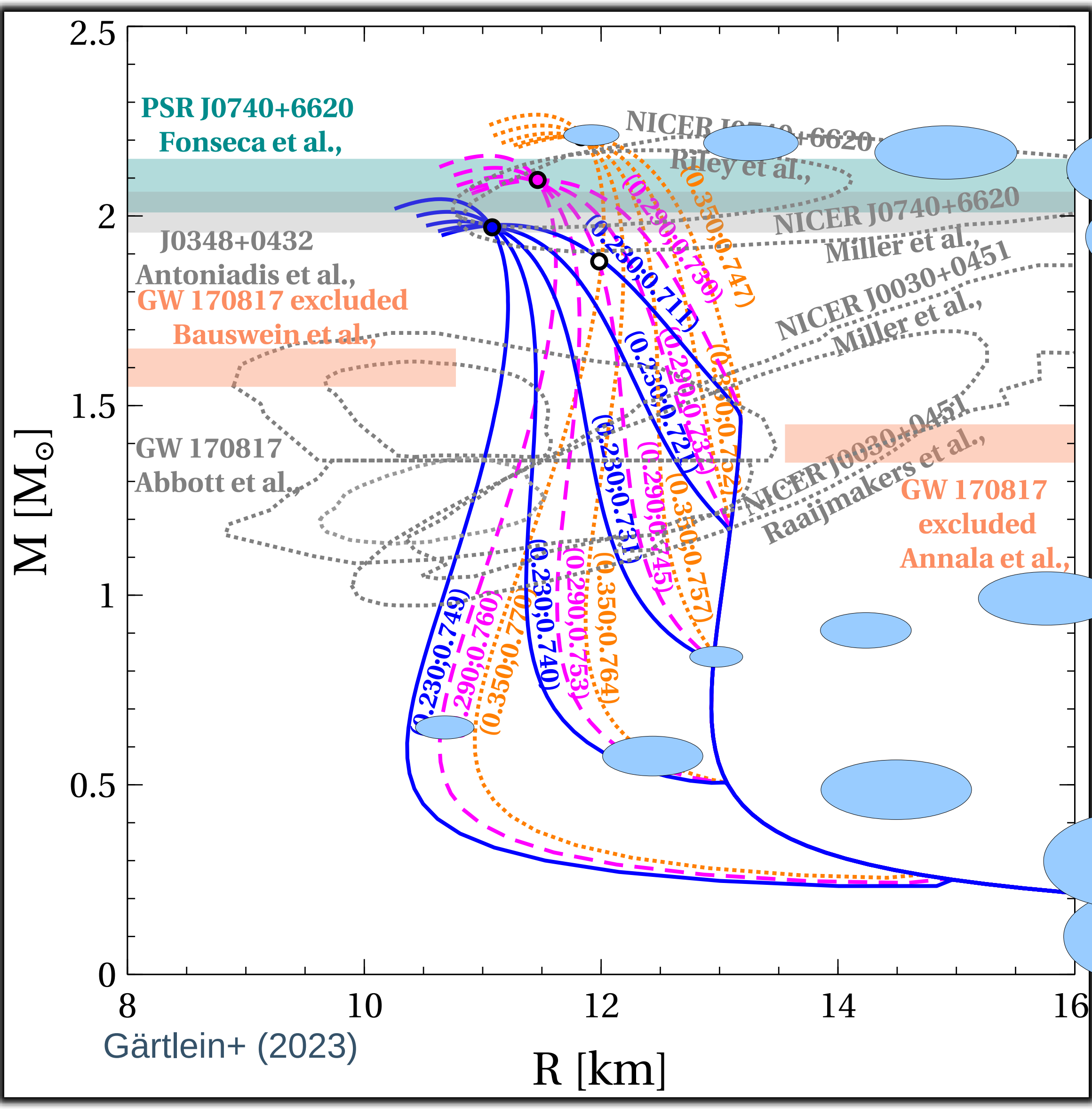
Special Points and Onset Mass

- M-R curves show many interesting characteristics
- Plotting curves characterized by the couplings of the quark phase
- Special points and onset mass (deconfinement phase transition)



Gärtlein+ (2023)

Characteristics

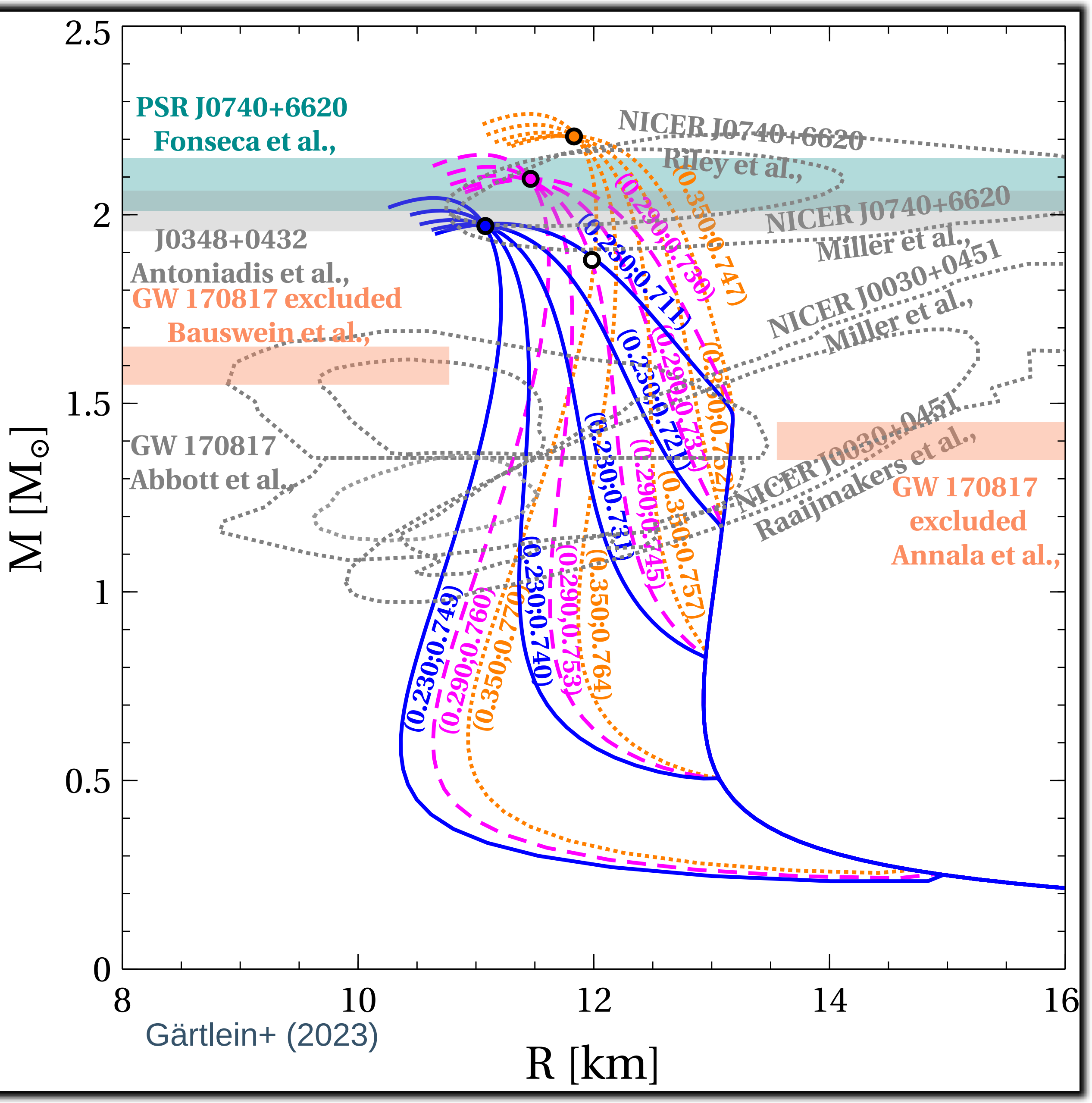


Special "points": area of intersection for certain sets of couplings

Points of Phase transition (onset mass) ≙ Deconfinement phase transition

Diquark coupling controls onset of deconfinement

Vector repulsion controls stiffness of EoS

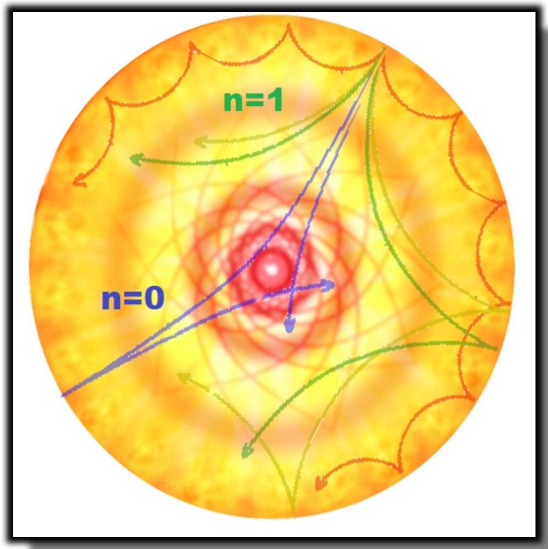


Relation:

$$M_{Max} = M_{SP} + \delta |M^{*}_{Onset} - M_{Onset}|^{\kappa}$$

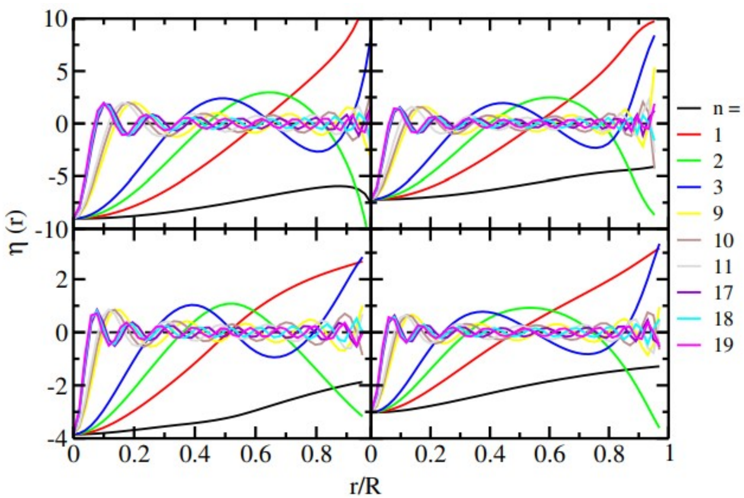
- Special characteristics of the curves are not completely independent
- M^{*}_{Onset} is fix parameter (fitting)
- $\kappa \approx 2$
- $\delta \propto \eta_V$ and very small

Radial Oscillations



- Similar to stars (well studied for the Sun) radial and nonradial oscillations are considerable
- Changes of radius and pressure
  (harmonic oscillations)
- Radial: n=0 → zero or fundamental mode (f-mode)
- n > 1: p-modes

Pressure deviation



Rather+ (2023)

How do they work?

Equations:

$$\frac{d\xi}{dr} = -\left(\frac{3}{r} + \frac{1}{\epsilon + p}\frac{dp}{dr}\right)\xi - \frac{\eta}{r\gamma},$$

$\xi \equiv \frac{\Delta r}{r}$ radial perturbations

$$\frac{d\eta}{dr} = \omega^2\left[\frac{\epsilon + p}{p} r e^{(\lambda-\nu)}\right]\xi - \left[\frac{4}{p}\frac{dp}{dr} + 8\pi(\epsilon + p) r e^{\lambda} - \frac{r}{p(\epsilon + p)}\left(\frac{dp}{dr}\right)^2\right]\xi - \left[\frac{\epsilon}{p(\epsilon + p)}\frac{dp}{dr} + 4\pi\zeta r e^{\lambda}\right]\eta,$$

$\eta \equiv \frac{\Delta p}{p}$ pressure perturbations

- **radial pressure oscillations (p-waves) ⇒ no GW emission**
- **could couple to nonradial oscillations ⇒ emit GW**
- **used to probe composition of compact stars**

What to do with them?

Same special point (crossing)

Fixed value

Unstable branch, as frequency becomes imaginary

SP	M_{SP} [$M_\odot$]	R_{SP} [km]	η_V	η_D	M_{onset} [$M_\odot$]	[M_{max}] [$M_\odot$]	f [kHz]
blue	1.973	11.06	0.230	0.749	0.251	2.044	0.840
				0.740	0.506	2.011	0.821
				0.731	0.826	1.986	0.770
				0.721	1.169	1.974	0.318
				0.711	1.483	1.976	–
magenta	2.092	11.46	0.290	0.760	0.251	2.159	0.760
				0.753	0.506	2.130	0.755
				0.745	0.826	2.104	0.750
				0.737	1.169	2.094	0.410
				0.730	1.483	2.095	–
orange	2.207	11.85	0.35	0.770	0.251	2.267	0.760
				0.764	0.506	2.241	0.742
				0.757	0.826	2.218	0.740
				0.752	1.169	2.210	0.491
				0.747	1.483	2.209	–
white	1.885	12.00	0.35	0.770	0.251	2.267	0.89
			0.29	0.745	0.826	2.104	0.76
			0.23	0.711	1.483	1.976	0.70

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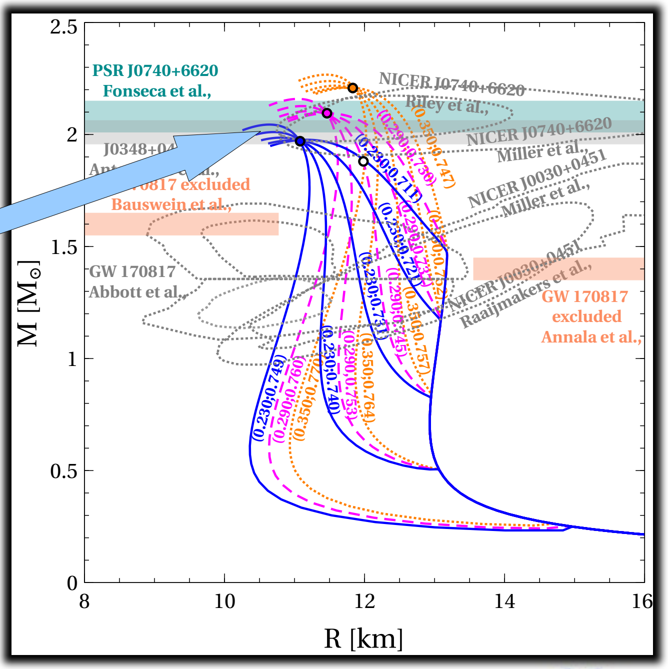
Fundamental frequencies



frequency lowers for increasing M_{Onset}

until, $f = 0$

per definition: last stable f-mode for $M = M_{Max}$

M^*_{Onset}

see relation: $M_{Max} = M_{SP}$

$M_{Onset} = M^*_{Onset}$

$$M_{Max} = M_{SP} + \delta |M^*_{Onset} - M_{Onset}|^\kappa$$

Conclusions

1. Phenomenological EoS → in agreement with astrophysical constraints

2. Variation of η_D while keeping η_V fixed → family of hybrid quark-hadron EoS

also other possibilities

→ corresponding mass-radius curves intersect in <u>**SP**</u>

3. Finding special points within microscopic approach

4. Study of f-modes (belonging to SPs)

↳ **for same gravitational mass, radius and vector coupling**

↳ **frequency decreases with decreasing diquark coupling**

5. Earlier deconfinement phase transition → higher frequency

6. Observing low f-modes → closer to maximum mass

↳ **with relation: constraining NS mass at which deconfinement transition occurs**

SP	M_{SP} $[M_\odot]$	R_{SP} [km]	η_V	η_D	M_{onset} $[M_\odot]$	$[M_{max}]$ $[M_\odot]$	f [kHz]
blue	1.973	11.06	0.230	0.749	0.251	2.044	0.840
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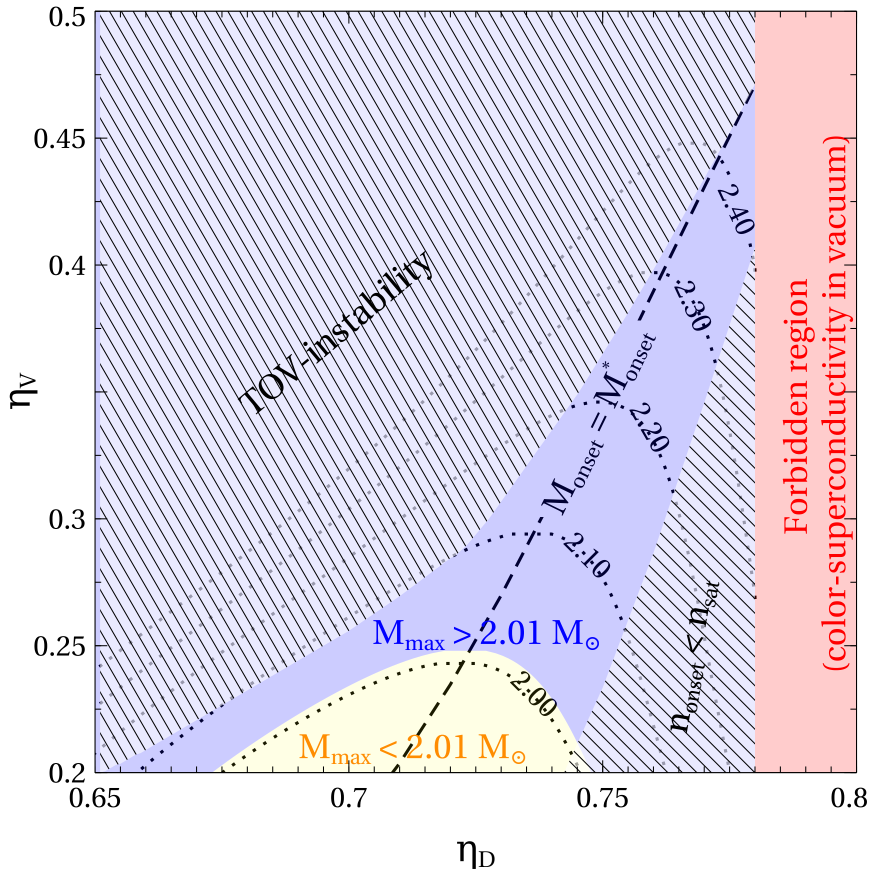
Outlook and further findings

- **Constraining the possible values of couplings**
- **range suggests an early deconfinement of quark matter and deconfinement onset masses below 1 solar mass**
- **consistent with the existence of heavy NSs with up to 2.4 solar masses**



Gärtlein+ (2023)

Thank you for your attention

Any Questions?

Back up Slides

Lagrangian of the quark matter theory:
including Up and Down quarks

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - m)q + \mathcal{L}_V + \mathcal{L}_D - \mathcal{U},$$

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2 + \Theta_V,$$

$$\mathcal{L}_D = G_D(\overline{q}i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q) - \Theta_D.$$

$$\mathcal{U} = D_0\left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\gamma_5\vec{\tau}q)^2\right]^{\frac{1}{3}}$$

Definition of couplings:

$$G_V = \frac{G_{V0}}{1 + \frac{8}{9M_g^2}\left(\frac{\pi^2 \langle q^+ q \rangle}{2}\right)^{2/3}},$$

$$G_D = \frac{G_{D0}}{1 + \frac{8}{9M_g^2}\left(\frac{\pi^2 |\langle \overline{q}^c i \tau_2 \gamma_5 \lambda_2 q \rangle|}{2}\right)^{2/3}},$$

$$\eta_V \equiv \frac{G_{V0}}{G_{S0}} \quad \text{and} \quad \eta_D \equiv \frac{G_{D0}}{G_{S0}}$$

Vacuum value of diquark coupling

Vacuum value of scalar coupling

dimensionless

Fixing parameters via mass and decay constant of the pion as well as of the scalar meson

m [MeV]	Λ [MeV]	α	$D_0\Lambda^{-2}$	M_g [MeV]
4.2	573	1.43	1.39	600

Speed of sound profiles